GADTs Meet Their Match:

Pattern-Matching Warnings That Account for GADTs, Guards, and Laziness

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Pattern Syntax

$$
\begin{array}{rlll}
K \in & \text{Con} & & \\
x, y, a, b \in & \text{Var} & & \\
\tau, \sigma \in & \text{Type} & & \\
e \in & \text{Expr} & ::= & x : \tau \\
 & & \mid & K\ \overline{a}\ \overline{\gamma}\ \overline{e : \tau} \\
 & & \mid & \ldots \\
\gamma \in & \text{TyCt} & ::= & \tau_1 \sim \tau_2 \mid \ldots \\
g \in & \text{Grd} & ::= & \text{let } x : \tau = e \\
 & & \mid & K\ \overline{a}\ \overline{\gamma}\ \overline{y : \tau} \leftarrow x \\
 & & \mid & !x
\end{array}
$$

Oracle Syntax

$$
\begin{array}{rlll}
\Gamma & ::= & \varnothing \mid \Gamma, x : \tau \mid \Gamma, a & \text{Context} \\
\Delta & ::= & \times \mid \checkmark \mid \Delta, \delta \mid \Delta_1 \vee \Delta_2 & \text{Delta} \\
\Delta & ::= & \times \mid \checkmark \mid \Delta, \delta \mid \Delta_1 \vee \Delta_2 & \text{Delta} \\
\delta & ::= & \gamma \mid K\ \overline{x : \tau} \leftarrow y \mid x \not\approx K \mid x \approx \bot \mid x \not\approx \bot \mid x \approx e & \text{Constraints}
\end{array}
$$

TODO LIST

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Clause tree

$$
\begin{array}{rcl}
\mathcal{T}[r] & ::= & \texttt{Rhs} \\
& | & \texttt{Many}\ \overline{r} \\
\\
t_G \in \text{Gdt} & ::= & \mathcal{T}[t_G] \\
& | & \texttt{Guard}\ g\ t_G \\
\\
t_A \in \text{Ant} & ::= & \mathcal{T}[t_A] \\
& | & \texttt{MayDiverge}\ t_A \\
& | & \texttt{Inaccessible}\ t_A
\end{array}
$$

Checking guard trees

$$\boxed{\mathcal{U}(\Delta, \text{Gdt}) = \Delta}$$

$$
\begin{array}{lcl}
\mathcal{U}(\Delta, \texttt{Rhs}) & = & \varnothing \\
\mathcal{U}(\Delta, \texttt{Many}\ \overline{t}) & = & \mathcal{U}(...\mathcal{U}(\mathcal{U}(\Delta, t_1), t_2)..., t_n) \\
\mathcal{U}(\Delta, \texttt{Guard}\ (!x)\ t) & = & \mathcal{U}(\Delta \oplus (x \not\approx \bot), t) \\
\mathcal{U}(\Delta, \texttt{Guard}\ (\text{let}\ x = e)\ t) & = & \mathcal{U}(\Delta \oplus (x \approx e), t) \\
\mathcal{U}(\Delta, \texttt{Guard}\ (K\ \overline{a}\ \overline{\gamma}\ \overline{y : \tau} \leftarrow x)\ t) & = & (\Delta \oplus (x \not\approx K) \oplus (x \not\approx \bot)) \cup \mathcal{U}(\Delta \oplus (K\ \overline{y : \tau} \leftarrow x) \oplus \overline{\gamma}, gs)
\end{array}
$$

$$\boxed{\mathcal{A}(\Delta, \text{Gdt}) = \text{Ant}}$$

$$
\begin{array}{lcl}
\mathcal{A}(\Delta, \texttt{Rhs}) & = & \begin{cases} \texttt{Inaccessible Rhs}, & \mathcal{V}(\Gamma, \Delta) = \varnothing \\ \texttt{Rhs}, & \text{otherwise} \end{cases} \\
\mathcal{A}(\Delta, \texttt{Many}\ \overline{t}) & = & \texttt{Many}\ (\mathcal{A}(\Delta'_0, t_1), \ \ldots\ , \mathcal{A}(\Delta'_{n-1}, t_n)) \text{ where } \begin{cases} \Delta'_0 & = \Delta \\ \Delta'_{n+1} & = \mathcal{U}(\Delta'_n, t_{n+1}) \end{cases} \\
\mathcal{A}(\Delta, \texttt{Guard}\ (!x)\ t) & = & \mathcal{D}(\Delta, \mathcal{A}(\Delta \oplus (x \not\approx \bot), t)) \\
\mathcal{A}(\Delta, \texttt{Guard}\ (\text{let}\ x = e)\ t) & = & \mathcal{A}(\Delta \oplus (x \approx e), t) \\
\mathcal{A}(\Delta, \texttt{Guard}\ (K\ \overline{a}\ \overline{\gamma}\ \overline{y : \tau} \leftarrow x)\ t) & = & \mathcal{D}(\Delta, \mathcal{A}(\Delta \oplus (K\ \overline{y : \tau} \leftarrow x) \oplus \overline{\gamma}, t))
\end{array}
$$

$$\boxed{\mathcal{D}(\Delta, \text{Ant}) = \text{Ant}}$$

$$
\mathcal{D}(\Delta, t) = \begin{cases} t, & \mathcal{V}(\Gamma, \Delta \oplus (x \approx \bot)) = \varnothing \\ \texttt{MayDiverge}\ t & \text{otherwise} \end{cases}
$$

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Test if Oracle state Delta is unsatisfiable

$$\frac{\boxed{\nvdash_{\text{SAT}} \Gamma \vdash \Delta}}{\nvdash_{\text{SAT}} \Gamma \vdash fvs\Gamma \triangleright \Delta}$$

Test a list of SAT roots for inhabitants

$$\frac{\boxed{\nvdash_{\text{SAT}} \Gamma \vdash \overline{x} \triangleright \Delta} \quad \nvdash_{\text{SAT}} \Gamma \vdash x_i \triangleright \Delta}{\nvdash_{\text{SAT}} \Gamma \vdash \overline{x} \triangleright \Delta}$$

Test a single SAT root for inhabitants

$$\frac{\boxed{\nvdash_{\text{SAT}} \Gamma \vdash x \triangleright \Delta} \quad \nvdash_{\text{SAT}} \Gamma \vdash \oplus\Delta x \approx \bot \quad \{\overline{K}\} \text{ COMPLETE set} \quad \overline{\forall \overline{y : \tau}.\ \nvdash_{\text{SAT}} \Gamma, \overline{y : \tau} \vdash \oplus\Delta x \approx K\ \overline{y}}}{\nvdash_{\text{SAT}} \Gamma \vdash x \triangleright \Delta}$$

Add a single equality to Δ

$$\boxed{\nvdash_{\text{SAT}} \Gamma \vdash \oplus\Delta\delta}$$

Term stuff: Bottom, negative info, positive info + generativity, positive info + univalence

$$\frac{x \not\approx sth \in \Delta}{\nvdash_{\text{SAT}} \Gamma \vdash \oplus\Delta x \approx \bot} \qquad \frac{x \approx K\ \overline{y} \in \Delta}{\nvdash_{\text{SAT}} \Gamma \vdash \oplus\Delta x \approx \bot}$$

$$\frac{x \not\approx K \in \Delta}{\nvdash_{\text{SAT}} \Gamma \vdash \oplus\Delta x \approx K\ \overline{y}} \qquad \frac{x \approx K_i\ \overline{y} \in \Delta \quad i \neq j \quad K_i \text{ and } K_j \text{ generative}}{\nvdash_{\text{SAT}} \Gamma \vdash \oplus\Delta x \approx K_j\ \overline{z}}$$

$$\frac{x \approx K\ \overline{\tau}\ \overline{y} \in \Delta \quad \nvdash_{\text{SAT}} \Gamma \vdash \oplus\Delta\tau_i \sim \sigma_i}{\nvdash_{\text{SAT}} \Gamma \vdash \oplus\Delta x \approx K\ \overline{\sigma}\ \overline{z}} \qquad \frac{x \approx K\ \overline{\tau}\ \overline{y} \in \Delta \quad \nvdash_{\text{SAT}} \Gamma \vdash \oplus\Delta y_i \approx z_i}{\nvdash_{\text{SAT}} \Gamma \vdash \oplus\Delta x \approx K\ \overline{\sigma}\ \overline{z}}$$

Type stuff: Hand over to unspecified type oracle

$$\frac{\tau_1 \text{ and } \tau_2 \text{ incompatible to Givens in } \Delta \text{ according to type oracle}}{\nvdash_{\text{SAT}} \Gamma \vdash \oplus\Delta\tau_1 \sim \tau_2}$$

Mixed: Instantiate K and see if that leads to a contradiction TODO: Proper instantiation

$$\frac{\overline{\nvdash_{\text{SAT}} \Gamma \vdash y \triangleright \Delta \cup y \not\approx \bot}}{\nvdash_{\text{SAT}} \Gamma \vdash \oplus\Delta x \approx K\ \overline{y}}$$

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