

# GADTs Meet Their Match:

Pattern-Matching Warnings That Account for GADTs, Guards, and Laziness

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**Pattern Syntax**

$K \in$	Con
$x, y, a, b \in$	Var
$\tau, \sigma \in$	Type
$e \in$	Expr
	$::= x : \tau$
	$  K \bar{a} \bar{y} \bar{e} : \bar{\tau}$
	$  \dots$
$\gamma \in$	TyCt
	$::= \tau_1 \sim \tau_2 \mid \dots$
$g \in$	Grd
	$::= \text{let } x : \tau = e;$
	$  K \bar{a} \bar{y} \bar{y} : \bar{\tau} \leftarrow x;$
	$  !x;$

**Oracle Syntax**

$\Gamma ::= \emptyset \mid \Gamma, x : \tau \mid \Gamma, a$	Context
$\Delta ::= \times \mid \checkmark \mid \Delta, \delta \mid \Delta_1 \vee \Delta_2$	Delta
$\delta ::= \gamma \mid x_1 \approx x_2 \mid K \bar{x} : \bar{\tau} \leftarrow y \mid x \not\approx K \mid x \approx \perp \mid x \not\approx \perp \mid x \approx e$	Constraints

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**Clause tree**

$\mathcal{T}[r]$	$::=$	Rhs
		Many $\bar{r}$
$t_G \in \text{Gdt}$	$::=$	$\mathcal{T}[t_G]$
		Guard $g \ t_G$
$t_C \in \text{Ctt}$	$::=$	$\mathcal{T}[t_C]$
		$\zeta? \ \delta \ t_C$
		FallThroughIf $\delta \ t_C$
		Refine $\delta \ t_C$
$t_A \in \text{Ant}$	$::=$	$\mathcal{T}[t_A]$
		Diverges $t_A$
		Inaccessible $t_A$

**Compiling constraint trees**

$$\boxed{\text{cct Gdt} = \text{Ctt}}$$

cct Rhs	=	Rhs
cct Many $\bar{t}_G$	=	Many cct $t_G$
cct Guard (let $x = e;$ ) $t_G$	=	cctg $g$ (cct $t_G$ )
cctg (let $x = e;$ )	=	Refine ( $x \approx e$ )
cctg (! $x;$ )	=	$\zeta? (x \approx \perp) \circ \text{Refine} (x \not\approx \perp)$
cctg Guard (! $x;$ )	=	$\zeta? (x \approx \perp) \circ \text{Refine} (x \not\approx \perp)$
cct Guard ( $K \ \bar{a} \ \bar{y} : \bar{\tau} \leftarrow x;$ ) $t_G$	=	$\zeta? (x \approx \perp) (\zeta? (x \approx \perp) (\text{Refine} (x \approx K \ \bar{x} : \bar{\tau} \leftarrow y) \text{cct } t_G))$

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**Test if Oracle state Delta is unsatisfiable**

$$\frac{\boxed{\not\vdash_{\text{SAT}} \Gamma \vdash \Delta}}{\not\vdash_{\text{SAT}} \Gamma \vdash fvs\Gamma \triangleright \Delta} \quad \not\vdash_{\text{SAT}} \Gamma \vdash \Delta$$

**Test a list of SAT roots for inhabitants**

$$\frac{\boxed{\not\vdash_{\text{SAT}} \Gamma \vdash \bar{x} \triangleright \Delta} \quad \not\vdash_{\text{SAT}} \Gamma \vdash x_i \triangleright \Delta}{\not\vdash_{\text{SAT}} \Gamma \vdash \bar{x} \triangleright \Delta}$$

**Test a single SAT root for inhabitants**

$$\frac{\boxed{\not\vdash_{\text{SAT}} \Gamma \vdash x \triangleright \Delta} \quad \not\vdash_{\text{SAT}} \Gamma \vdash \oplus \Delta x \approx \perp \quad \{\bar{K}\} \text{ COMPLETE set} \quad \overline{\forall \bar{y} : \bar{\tau}. \not\vdash_{\text{SAT}} \Gamma, \bar{y} : \bar{\tau} \vdash \oplus \Delta x \approx K \bar{y}}}{\not\vdash_{\text{SAT}} \Gamma \vdash x \triangleright \Delta}$$

**Add a single equality to Delta**

$$\boxed{\not\vdash_{\text{SAT}} \Gamma \vdash \oplus \Delta \delta}$$

Term stuff: Bottom, negative info, positive info + generativity, positive info + univalence

$$\frac{x \not\approx sth \in \Delta}{\not\vdash_{\text{SAT}} \Gamma \vdash \oplus \Delta x \approx \perp} \quad \frac{x \approx K \bar{y} \in \Delta}{\not\vdash_{\text{SAT}} \Gamma \vdash \oplus \Delta x \approx \perp}$$

$$\frac{x \not\approx K \in \Delta}{\not\vdash_{\text{SAT}} \Gamma \vdash \oplus \Delta x \approx K \bar{y}} \quad \frac{x \approx K_i \bar{y} \in \Delta \quad i \neq j \quad K_i \text{ and } K_j \text{ generative}}{\not\vdash_{\text{SAT}} \Gamma \vdash \oplus \Delta x \approx K_j \bar{z}}$$

$$\frac{x \approx K \bar{\tau} \bar{y} \in \Delta \quad \not\vdash_{\text{SAT}} \Gamma \vdash \oplus \Delta \tau_i \sim \sigma_i}{\not\vdash_{\text{SAT}} \Gamma \vdash \oplus \Delta x \approx K \bar{\sigma} \bar{z}} \quad \frac{x \approx K \bar{\tau} \bar{y} \in \Delta \quad \not\vdash_{\text{SAT}} \Gamma \vdash \oplus \Delta y_i \approx z_i}{\not\vdash_{\text{SAT}} \Gamma \vdash \oplus \Delta x \approx K \bar{\sigma} \bar{z}}$$

Type stuff: Hand over to unspecified type oracle

$$\frac{\tau_1 \text{ and } \tau_2 \text{ incompatible to Givens in } \Delta \text{ according to type oracle}}{\not\vdash_{\text{SAT}} \Gamma \vdash \oplus \Delta \tau_1 \sim \tau_2}$$

Mixed: Instantiate K and see if that leads to a contradiction TODO: Proper instantiation

$$\frac{\boxed{\not\vdash_{\text{SAT}} \Gamma \vdash y \triangleright \Delta \cup y \not\approx \perp}}{\not\vdash_{\text{SAT}} \Gamma \vdash \oplus \Delta x \approx K \bar{y}}$$

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