# GADTs Meet Their Match:

Pattern-Matching Warnings That Account for GADTs, Guards, and Laziness

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**Pattern Syntax**

$$
\begin{array}{rlll}
K \in & \text{Con} & & \\
x, y, a, b \in & \text{Var} & & \\
\tau, \sigma \in & \text{Type} & & \\
e \in & \text{Expr} & ::= & x : \tau \\
 & & \mid & K\ \overline{a}\ \overline{\gamma}\ \overline{e : \tau} \\
 & & \mid & \ldots \\
\gamma \in & \text{TyCt} & ::= & \tau_1 \sim \tau_2 \mid \ldots \\
g \in & \text{Grd} & ::= & \text{let } x : \tau = e; \\
 & & \mid & K\ \overline{a}\ \overline{\gamma}\ \overline{y : \tau} \leftarrow x; \\
 & & \mid & !x;
\end{array}
$$

**Oracle Syntax**

$$
\begin{array}{rlll}
\Gamma & ::= & \varnothing \mid \Gamma, x : \tau \mid \Gamma, a & \text{Context} \\
\Delta & ::= & \times \mid \checkmark \mid \Delta, \delta \mid \Delta_1 \vee \Delta_2 & \text{Delta} \\
\Delta & ::= & \times \mid \checkmark \mid \Delta, \delta \mid \Delta_1 \vee \Delta_2 & \text{Delta} \\
\delta & ::= & \gamma \mid K\ \overline{x : \tau} \leftarrow y \mid x \not\approx K \mid x \approx \bot \mid x \not\approx \bot \mid x \approx e & \text{Constraints}
\end{array}
$$

## TODO LIST

## REFERENCES


Lennart Augustsson. 1985. Compiling pattern matching. In *Proceedings of the 1985 Conference on Functional Programming and Computer Architecture*.

Edwin Brady. 2013a. Idris, a general-purpose dependently typed programming language: Design and implementation. *Journal of Functional Programming* 23 (9 2013), 552–593. Issue 05. https://doi.org/10.1017/S095679681300018X

Edwin Brady. 2013b. Programming and Reasoning with Algebraic Effects and Dependent Types. In *Proceedings of the 18th ACM SIGPLAN International Conference on Functional Programming (ICFP '13)*. ACM, New York, NY, USA, 133–144. https://doi.org/10.1145/2500365.2500581

James Cheney and Ralf Hinze. 2003. *First-class phantom types.* Technical Report. Cornell University.

Koen Claessen, Moa Johansson, Dan Rosén, and Nicholas Smallbone. 2013. Automating Inductive Proofs Using Theory Exploration.. In *CADE (Lecture Notes in Computer Science)*, Maria Paola Bonacina (Ed.), Vol. 7898. Springer, 392–406.

Thierry Coquand. 1992. Pattern matching with dependent types. In *Proceedings of the Workshop on Types for Proofs and Programs*.

Joshua Dunfield. 2007a. Refined Typechecking with Stardust. In *Proceedings of the 2007 Workshop on Programming Languages Meets Program Verification (PLPV '07)*. ACM, New York, NY, USA, 21–32. https://doi.org/10.1145/1292597.1292602

Joshua Dunfield. 2007b. *A Unified System of Type Refinements.* Ph.D. Dissertation. Carnegie Mellon University. CMU-CS-07-129.

Richard A. Eisenberg and Stephanie Weirich. 2012. Dependently Typed Programming with Singletons. In *Proceedings of the 2012 Haskell Symposium (Haskell '12)*. ACM, New York, NY, USA, 117–130. https://doi.org/10.1145/2364506.2364522

M Erwig and SL Peyton Jones. 2000. Pattern guards and transformational patterns. In *Proceedings of the 2000 Haskell Symposium*. ACM.

Jacques Garrigue and Jacques Le Normand. 2011. Adding GADTs to OCaml: the direct approach. In *Workshop on ML*.

Jean-Yves Girard, Paul Taylor, and Yves Lafont. 1989. *Proofs and Types.* Cambridge University Press, New York, NY, USA.

Georgios Karachalias, Tom Schrijvers, Dimitrios Vytiniotis, and Simon Peyton Jones. 2015. *GADTs meet their match (extended version).* Technical Report. KU Leuven. http://people.cs.kuleuven.be/~george.karachalias/papers/gadtpm_ext.pdf

Neelakantan R. Krishnaswami. 2009. Focusing on Pattern Matching. In *Proceedings of the 36th Annual ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages (POPL '09)*. ACM, New York, NY, USA, 366–378. https://doi.org/10.1145/1480881.1480927

**Clause tree**

$$
\begin{array}{rcl}
\mathcal{T}[r] & ::= & \texttt{Rhs} \\
& | & \texttt{Many}\ \overline{r} \\
\\
t_G \in \text{Gdt} & ::= & \mathcal{T}[t_G] \\
& | & \texttt{Guard}\ g\ t_G \\
\\
t_A \in \text{Ant} & ::= & \mathcal{T}[t_A] \\
& | & \texttt{MayDiverge}\ t_A \\
& | & \texttt{Inaccessible}\ t_A
\end{array}
$$

**Checking guard trees**

$$\boxed{\mathcal{U}(\Delta, \text{Gdt}) = \Delta}$$

$$
\begin{array}{lcl}
\mathcal{U}(\Delta, \texttt{Rhs}) & = & \varnothing \\
\mathcal{U}(\Delta, \texttt{Many}\ \overline{t}) & = & \mathcal{R}(\Delta, \overline{t})_{|\{\overline{t}\}|} \\
\mathcal{U}(\Delta, \texttt{Guard}\ (!x;)\ t) & = & \mathcal{U}(\Delta \oplus (x \not\approx \bot), t) \\
\mathcal{U}(\Delta, \texttt{Guard}\ (\text{let}\ x = e;)\ t) & = & \mathcal{U}(\Delta \oplus (x \approx e), t) \\
\mathcal{U}(\Delta, \texttt{Guard}\ (K\ \overline{a}\ \overline{\gamma}\ \overline{y : \tau} \leftarrow x;)\ t) & = & (\Delta \oplus (x \not\approx K) \oplus (x \not\approx \bot)) \cup \mathcal{U}(\Delta \oplus (K\ \overline{y : \tau} \leftarrow x) \oplus \overline{\gamma}, gs)
\end{array}
$$

$$\boxed{\mathcal{R}(\Delta, \text{Gdt}) = \Delta_{\mathbb{N}}}$$

$$
\begin{array}{lcl}
\mathcal{R}(\Delta, \overline{t})_0 & = & \Delta \\
\mathcal{R}(\Delta, \overline{t})_{n+1} & = & \mathcal{U}(\mathcal{R}(\Delta, \overline{t})_n, t_n)
\end{array}
$$

$$\boxed{\mathcal{A}(\Delta, \text{Gdt}) = \text{Ant}}$$

$$
\begin{array}{lcl}
\mathcal{A}(\Delta, \texttt{Rhs}) & = & \begin{cases} \texttt{Inaccessible Rhs}, & \mathcal{V}(\Gamma, \Delta) = \varnothing \\ \texttt{Rhs}, & \text{otherwise} \end{cases} \\
\mathcal{A}(\Delta, \texttt{Many}\ \overline{t}) & = & \texttt{Many}\ (\mathcal{A}(\mathcal{R}(\Delta, \overline{t})_0, t_1), \ldots, \mathcal{A}(\mathcal{R}(\Delta, \overline{t})_{n-1}, t_n)) \\
\mathcal{A}(\Delta, \texttt{Guard}\ (!x;)\ t) & = & \mathcal{U}(\Delta \oplus (x \not\approx \bot), t) \\
\mathcal{A}(\Delta, \texttt{Guard}\ (\text{let}\ x = e;)\ t) & = & \mathcal{U}(\Delta \oplus (x \approx e), t) \\
\mathcal{A}(\Delta, \texttt{Guard}\ (K\ \overline{a}\ \overline{\gamma}\ \overline{y : \tau} \leftarrow x;)\ t) & = & (\Delta \oplus (x \not\approx K) \oplus (x \not\approx \bot)) \cup \mathcal{U}(\Delta \oplus (K\ \overline{y : \tau} \leftarrow x) \oplus \overline{\gamma}, gs)
\end{array}
$$


Alain Laville. 1991. Comparison of Priority Rules in Pattern Matching and Term Rewriting. *J. Symb. Comput.* 11, 4 (May 1991), 321–347. https://doi.org/10.1016/S0747-7171(08)80109-5

Fabrice Le Fessant and Luc Maranget. 2001. Optimizing Pattern-Matching. In *Proceedings of the 2001 International Conference on Functional Programming.*

Luc Maranget. 1992. Compiling Lazy Pattern Matching. In *Proceedings of the 1992 ACM Conference on LISP and Functional Programming (LFP '92).* ACM, New York, NY, USA, 21–31. https://doi.org/10.1145/141471.141499

Luc Maranget. 2007. Warnings for pattern matching. *Journal of Functional Programming* 17 (2007), 387–421. Issue 3.

Luc Maranget. 2008. Compiling pattern matching to good decision trees. In *Proceedings of the ACM Workshop on ML.*

Luc Maranget and Projet Para. 1994. *Two Techniques for Compiling Lazy Pattern Matching.* Technical Report.

The Coq development team. 2004. *The Coq proof assistant reference manual.* LogiCal Project. http://coq.inria.fr Version 8.0.

C. McBride and J. McKinna. 2004. The view from the left. *Journal of Functional Programming* 14, 1 (2004), 69–111.

Neil Mitchell and Colin Runciman. 2008. Not All Patterns, but Enough: An Automatic Verifier for Partial but Sufficient Pattern Matching. In *Proceedings of the First ACM SIGPLAN Symposium on Haskell (Haskell '08).* ACM, New York, NY, USA, 49–60. https://doi.org/10.1145/1411286.1411293

Ulf Norell. 2007. *Towards a practical programming language based on dependent type theory.* Ph.D. Dissertation. Department of Computer Science and Engineering, Chalmers University of Technology, SE-412 96 Göteborg, Sweden.

**Test if Oracle state Delta is unsatisfiable**

$$\frac{\boxed{\nvdash_{\text{SAT}} \Gamma \vdash \Delta}}{\nvdash_{\text{SAT}} \Gamma \vdash fvs\Gamma \triangleright \Delta}$$

**Test a list of SAT roots for inhabitants**

$$\frac{\boxed{\nvdash_{\text{SAT}} \Gamma \vdash \overline{x} \triangleright \Delta}}{\nvdash_{\text{SAT}} \Gamma \vdash x_i \triangleright \Delta}$$

**Test a single SAT root for inhabitants**

$$\frac{\boxed{\nvdash_{\text{SAT}} \Gamma \vdash x \triangleright \Delta}}{\nvdash_{\text{SAT}} \Gamma \vdash \oplus\Delta x \approx \bot \quad \{\overline{K}\} \text{ COMPLETE set } \quad \overline{\forall \overline{y : \tau}.\, \nvdash_{\text{SAT}} \Gamma, \overline{y : \tau} \vdash \oplus\Delta x \approx K\ \overline{y}}}$$

**Add a single equality to $\Delta$**

$$\boxed{\nvdash_{\text{SAT}} \Gamma \vdash \oplus\Delta\delta}$$

Term stuff: Bottom, negative info, positive info + generativity, positive info + univalence

$$\frac{x \not\approx sth \in \Delta}{\nvdash_{\text{SAT}} \Gamma \vdash \oplus\Delta x \approx \bot} \qquad \frac{x \approx K\ \overline{y} \in \Delta}{\nvdash_{\text{SAT}} \Gamma \vdash \oplus\Delta x \approx \bot}$$

$$\frac{x \not\approx K \in \Delta}{\nvdash_{\text{SAT}} \Gamma \vdash \oplus\Delta x \approx K\ \overline{y}} \qquad \frac{x \approx K_i\ \overline{y} \in \Delta \quad i \neq j \quad K_i \text{ and } K_j \text{ generative}}{\nvdash_{\text{SAT}} \Gamma \vdash \oplus\Delta x \approx K_j\ \overline{z}}$$

$$\frac{x \approx K\ \overline{\tau}\ \overline{y} \in \Delta \quad \nvdash_{\text{SAT}} \Gamma \vdash \oplus\Delta\tau_i \sim \sigma_i}{\nvdash_{\text{SAT}} \Gamma \vdash \oplus\Delta x \approx K\ \overline{\sigma}\ \overline{z}} \qquad \frac{x \approx K\ \overline{\tau}\ \overline{y} \in \Delta \quad \nvdash_{\text{SAT}} \Gamma \vdash \oplus\Delta y_i \approx z_i}{\nvdash_{\text{SAT}} \Gamma \vdash \oplus\Delta x \approx K\ \overline{\sigma}\ \overline{z}}$$

Type stuff: Hand over to unspecified type oracle

$$\frac{\tau_1 \text{ and } \tau_2 \text{ incompatible to Givens in } \Delta \text{ according to type oracle}}{\nvdash_{\text{SAT}} \Gamma \vdash \oplus\Delta\tau_1 \sim \tau_2}$$

Mixed: Instantiate K and see if that leads to a contradiction TODO: Proper instantiation

$$\frac{\overline{\nvdash_{\text{SAT}} \Gamma \vdash y \triangleright \Delta \cup y \not\approx \bot}}{\nvdash_{\text{SAT}} \Gamma \vdash \oplus\Delta x \approx K\ \overline{y}}$$

Ulf Norell. 2008. Dependently typed programming in Agda. In *In Lecture Notes from the Summer School in Advanced Functional Programming.*

Simon Peyton Jones, Dimitrios Vytiniotis, Stephanie Weirich, and Geoffrey Washburn. 2006. Simple Unification-based Type Inference for GADTs. In *Proceedings of the Eleventh ACM SIGPLAN International Conference on Functional Programming (ICFP '06)*. ACM, New York, NY, USA, 50–61. https://doi.org/10.1145/1159803.1159811

Norman Ramsey, João Dias, and Simon Peyton Jones. 2010. Hoopl: A Modular, Reusable Library for Dataflow Analysis and Transformation. In *Proceedings of the Third ACM Haskell Symposium on Haskell (Haskell '10)*. ACM, New York, NY, USA, 121–134. https://doi.org/10.1145/1863523.1863539

Patrick M. Rondon, Ming Kawaguci, and Ranjit Jhala. 2008. Liquid Types. In *Proceedings of the 2008 ACM SIGPLAN Conference on Programming Language Design and Implementation (PLDI '08)*. ACM, New York, NY, USA, 159–169. https://doi.org/10.1145/1375581.1375602

Tom Schrijvers, Simon Peyton Jones, Manuel Chakravarty, and Martin Sulzmann. 2008. Type Checking with Open Type Functions. In *Proceedings of the 13th ACM SIGPLAN International Conference on Functional Programming (ICFP '08)*. ACM, New York, NY, USA, 51–62. https://doi.org/10.1145/1411204.1411215

Tom Schrijvers, Simon Peyton Jones, Martin Sulzmann, and Dimitrios Vytiniotis. 2009. Complete and Decidable Type Inference for GADTs. In *Proceedings of the 14th ACM SIGPLAN International Conference on Functional Programming (ICFP '09)*. ACM, New York, NY, USA, 341–352. https://doi.org/10.1145/1596550.1596599

R. C. Sekar, R. Ramesh, and I. V. Ramakrishnan. 1995. Adaptive Pattern Matching. *SIAM J. Comput.* 24, 6 (Dec. 1995), 1207–1234. https://doi.org/10.1137/S0097539793246252

Peter Sestoft. 1996. ML pattern match compilation and partial evaluation. In *Partial Evaluation*, Olivier Danvy, Robert Glück, and Peter Thiemann (Eds.). Lecture Notes in Computer Science, Vol. 1110. Springer Berlin Heidelberg, 446–464. https://doi.org/10.1007/3-540-61580-6_22

Tim Sheard. 2004. Languages of the Future. In *In OOPSLA '04: Companion to the 19th annual ACM SIGPLAN conference on Object-oriented programming systems, languages, and applications*. ACM Press, 116–119.

W Sonnex, S Drossopoulou, and S Eisenbach. 2012. Zeno: An Automated Prover for Properties of Recursive Data Structures. Springer-Verlag Berlin, 407–421. https://doi.org/10.1007/978-3-642-28756-5_28

Martin Sulzmann, Manuel M. T. Chakravarty, Simon Peyton Jones, and Kevin Donnelly. 2007. System F with Type Equality Coercions. In *Proceedings of the 2007 ACM SIGPLAN International Workshop on Types in Languages Design and Implementation (TLDI '07)*. ACM, New York, NY, USA, 53–66. https://doi.org/10.1145/1190315.1190324

Peter Thiemann. 1993. Avoiding Repeated Tests in Pattern Matching. In *3rd International Workshop on Static Analysis*, Gilberto Filé (Ed.). Padova, Italia, 141–152.

Niki Vazou, Eric L. Seidel, Ranjit Jhala, Dimitrios Vytiniotis, and Simon Peyton-Jones. 2014. Refinement Types for Haskell. In *Proceedings of the 19th ACM SIGPLAN International Conference on Functional Programming (ICFP '14)*. ACM, New York, NY, USA, 269–282. https://doi.org/10.1145/2628136.2628161

Dimitrios Vytiniotis, Simon Peyton jones, Tom Schrijvers, and Martin Sulzmann. 2011. Outsidein(x) Modular Type Inference with Local Assumptions. *J. Funct. Program.* 21, 4-5 (Sept. 2011), 333–412. https://doi.org/10.1017/S0956796811000098

Philip Wadler. 1987a. Efficient compilation of pattern matching. In *The implementation of functional programming languages*, SL Peyton Jones (Ed.). Prentice Hall, 78–103.

P. Wadler. 1987b. Views: A Way for Pattern Matching to Cohabit with Data Abstraction. In *Proceedings of the 14th ACM SIGACT-SIGPLAN Symposium on Principles of Programming Languages (POPL '87)*. ACM, New York, NY, USA, 307–313. https://doi.org/10.1145/41625.41653

Hongwei Xi. 1998a. Dead Code Elimination Through Dependent Types. In *Proceedings of the First International Workshop on Practical Aspects of Declarative Languages (PADL '99)*. Springer-Verlag, London, UK, 228–242.

Hongwei Xi. 1998b. *Dependent Types in Practical Programming*. Ph.D. Dissertation. Carnegie Mellon University.

Hongwei Xi. 2003. Dependently typed pattern matching. *Journal of Universal Computer Science* 9 (2003), 851–872.

Hongwei Xi, Chiyan Chen, and Gang Chen. 2003. Guarded Recursive Datatype Constructors. In *Proceedings of the 30th ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages (POPL '03)*. ACM, New York, NY, USA, 224–235. https://doi.org/10.1145/604131.604150

Dana N. Xu. 2006. Extended Static Checking for Haskell. In *Proceedings of the 2006 ACM SIGPLAN Workshop on Haskell (Haskell '06)*. ACM, New York, NY, USA, 48–59. https://doi.org/10.1145/1159842.1159849

Dana N. Xu, Simon Peyton Jones, and Koen Claessen. 2009. Static Contract Checking for Haskell. In *Proceedings of the 36th Annual ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages (POPL '09)*. ACM, New York, NY, USA, 41–52. https://doi.org/10.1145/1480881.1480889

Brent A. Yorgey, Stephanie Weirich, Julien Cretin, Simon Peyton Jones, Dimitrios Vytiniotis, and José Pedro Magalhães. 2012. Giving Haskell a Promotion. In *Proceedings of the 8th ACM SIGPLAN Workshop on Types in Language Design and Implementation (TLDI '12)*. ACM, New York, NY, USA, 53–66. https://doi.org/10.1145/2103786.2103795