

# Tipos abstractos de datos básicos

Algoritmos y Estructuras de Datos II, DC, UBA.

## Índice

|   |    |
|---|----|
| 1. TAD <b>BOOL</b>                                  | 2  |
| 2. TAD <b>NAT</b>                                   | 2  |
| 3. TAD <b>TUPLA</b> ( $\alpha_1, \dots, \alpha_n$ ) | 3  |
| 4. TAD <b>SECUENCIA</b> ( $\alpha$ )                | 4  |
| 5. TAD <b>CONJUNTO</b> ( $\alpha$ )                 | 5  |
| 6. TAD <b>MULTICONJUNTO</b> ( $\alpha$ )            | 6  |
| 7. TAD <b>ARREGLO DIMENSIONABLE</b> ( $\alpha$ )    | 7  |
| 8. TAD <b>PILA</b> ( $\alpha$ )                     | 8  |
| 9. TAD <b>COLA</b> ( $\alpha$ )                     | 9  |
| 10. TAD <b>ÁRBOL BINARIO</b> ( $\alpha$ )           | 9  |
| 11. TAD <b>DICCIONARIO</b> (CLAVE, SIGNIFICADO)     | 10 |
| 12. TAD <b>COLA DE PRIORIDAD</b> ( $\alpha$ )       | 11 |

## 1. TAD Bool

### TAD Bool

**géneros**      bool

**exporta**      bool, generadores,  $\neg$ ,  $\vee$ ,  $\wedge$ ,  $\Rightarrow$ ,  $\vee_L$ ,  $\wedge_L$ ,  $\Rightarrow_L$

**igualdad observacional**

$$((\text{true} =_{\text{obs}} \text{true}) \wedge (\text{false} =_{\text{obs}} \text{false}) \wedge \neg(\text{true} =_{\text{obs}} \text{false}) \wedge \neg(\text{false} =_{\text{obs}} \text{true}))$$

**generadores**

true      :                       $\longrightarrow$  bool

false     :                       $\longrightarrow$  bool

**otras operaciones**

$\neg \bullet$       : bool                       $\longrightarrow$  bool

$\bullet \vee \bullet$    : bool  $\times$  bool               $\longrightarrow$  bool

$\bullet \wedge \bullet$    : bool  $\times$  bool               $\longrightarrow$  bool

$\bullet \Rightarrow \bullet$  : bool  $\times$  bool               $\longrightarrow$  bool

$\bullet \vee_L \bullet$  : bool  $\times$  bool               $\longrightarrow$  bool

$\bullet \wedge_L \bullet$  : bool  $\times$  bool               $\longrightarrow$  bool

$\bullet \Rightarrow_L \bullet$  : bool  $\times$  bool               $\longrightarrow$  bool

**axiomas**       $\forall x, y: \text{bool}$

$\neg \text{true}$        $\equiv$  false

$\neg \text{false}$       $\equiv$  true

$\text{true} \vee x$       $\equiv$  true

$\text{false} \vee x$      $\equiv x$

$\text{true} \wedge x$       $\equiv x$

$\text{false} \wedge x$      $\equiv$  false

$x \Rightarrow y$         $\equiv \neg x \vee y$

$x \wedge_L y$         $\equiv$  **if**  $x$  **then**  $y$  **else** false **fi**

$x \vee_L y$         $\equiv$  **if**  $x$  **then** true **else**  $y$  **fi**

$x \Rightarrow_L y$        $\equiv \neg x \vee_L y$

**Fin TAD**

## 2. TAD Nat

### TAD Nat

**géneros**      nat

**exporta**      nat, generadores, observadores,  $+$ ,  $-$ ,  $\times$ ,  $<$ ,  $\leq$ , mín, máx

**usa**            Bool

**igualdad observacional**

$$(\forall n, m : \text{nat}) \left( n =_{\text{obs}} m \iff \left( (n = 0? =_{\text{obs}} m = 0?) \wedge_L (\neg(n = 0?) \Rightarrow_L (\text{pred}(n) =_{\text{obs}} \text{pred}(m))) \right) \right)$$

**observadores básicos**

$\bullet = 0?$  : nat  $\longrightarrow$  bool

pred : nat  $n \longrightarrow$  nat

$\{\neg(n = 0?)\}$

**generadores**

0 :  $\longrightarrow$  nat

suc : nat  $\longrightarrow$  nat

**otras operaciones**

$\bullet + \bullet$  : nat  $\times$  nat  $\longrightarrow$  nat

$\bullet - \bullet$  : nat  $n \times$  nat  $m \longrightarrow$  nat

$\{m \leq n\}$

$\bullet \times \bullet$  : nat  $\times$  nat  $\longrightarrow$  nat

$\bullet < \bullet$  : nat  $\times$  nat  $\longrightarrow$  bool

$\bullet \leq \bullet$  : nat  $\times$  nat  $\longrightarrow$  bool

mín : nat  $\times$  nat  $\longrightarrow$  nat

máx : nat  $\times$  nat  $\longrightarrow$  nat

**axiomas**  $\forall n, m: \text{nat}$ 

$0 = 0?$   $\equiv$  true

$\text{suc}(n) = 0?$   $\equiv$  false

$\text{pred}(\text{suc}(n)) \equiv n$

$n + m \equiv$  **if**  $m = 0?$  **then**  $n$  **else**  $\text{suc}(n + \text{pred}(m))$  **fi**

$n - m \equiv$  **if**  $m = 0?$  **then**  $n$  **else**  $\text{pred}(n) - \text{pred}(m)$  **fi**

$n \times m \equiv$  **if**  $m = 0?$  **then** 0 **else**  $n \times \text{pred}(m) + n$  **fi**

$n < m \equiv \neg(m = 0?) \wedge_L (n = 0? \vee_L \text{pred}(n) < \text{pred}(m))$

$n \leq m \equiv n < m \vee n = m$

$\text{mín}(n, m) \equiv$  **if**  $m < n$  **then**  $m$  **else**  $n$  **fi**

$\text{máx}(n, m) \equiv$  **if**  $m < n$  **then**  $n$  **else**  $m$  **fi**

**Fin TAD**

### 3. TAD TUPLA( $\alpha_1, \dots, \alpha_n$ )

**TAD TUPLA**( $\alpha_1, \dots, \alpha_n$ )

**igualdad observacional**

$$(\forall t, t' : \text{tupla}(\alpha_1, \dots, \alpha_n)) (t =_{\text{obs}} t' \iff (\Pi_1(t) =_{\text{obs}} \Pi_1(t') \wedge \dots \wedge \Pi_n(t) =_{\text{obs}} \Pi_n(t')))$$

**parámetros formales**

**géneros**  $\alpha_1, \dots, \alpha_n$

**géneros**  $\text{tupla}(\alpha_1, \dots, \alpha_n)$

**exporta**  $\text{tupla}, \text{generadores}, \text{observadores}$

**observadores básicos**

$$\begin{aligned}\Pi_1 & : \text{tupla}(\alpha_1, \dots, \alpha_n) \longrightarrow \alpha_1 \\ & \vdots \\ \Pi_n & : \text{tupla}(\alpha_1, \dots, \alpha_n) \longrightarrow \alpha_n\end{aligned}$$

**generadores**

$$\langle \bullet, \dots, \bullet \rangle : \alpha_1 \times \dots \times \alpha_n \longrightarrow \text{tupla}(\alpha_1, \dots, \alpha_n)$$

**axiomas**  $\forall a_1: \alpha_1 \dots \forall a_n: \alpha_n$

$$\Pi_1(\langle a_1, \dots, a_n \rangle) \equiv a_1$$

$$\vdots \equiv \vdots$$

$$\Pi_n(\langle a_1, \dots, a_n \rangle) \equiv a_n$$

**Fin TAD**

## 4. TAD SECUENCIA( $\alpha$ )

**TAD SECUENCIA( $\alpha$ )**

**igualdad observacional**

$$(\forall s, s' : \text{secu}(\alpha)) \left( s =_{\text{obs}} s' \iff \left( \begin{array}{l} \text{vacía?}(s) =_{\text{obs}} \text{vacía?}(s') \wedge_{\text{L}} \\ (\neg \text{vacía?}(s) \Rightarrow_{\text{L}} (\text{prim}(s) =_{\text{obs}} \text{prim}(s') \wedge \text{fin}(s) =_{\text{obs}} \text{fin}(s'))) \end{array} \right) \right)$$

**parámetros formales**

**géneros**  $\alpha$

**géneros**  $\text{secu}(\alpha)$

**exporta**  $\text{secu}(\alpha)$ , generadores, observadores, &, o, ult, com, long, está?

**usa** **BOOL**, **NAT**

**observadores básicos**

$$\text{vacía?} : \text{secu}(\alpha) \longrightarrow \text{bool}$$

$$\text{prim} : \text{secu}(\alpha) \ s \longrightarrow \alpha \quad \{ \neg \text{vacía?}(s) \}$$

$$\text{fin} : \text{secu}(\alpha) \ s \longrightarrow \text{secu}(\alpha) \quad \{ \neg \text{vacía?}(s) \}$$

**generadores**

$$\langle \rangle : \longrightarrow \text{secu}(\alpha)$$

$$\bullet \bullet \bullet : \alpha \times \text{secu}(\alpha) \longrightarrow \text{secu}(\alpha)$$

**otras operaciones**

$$\bullet \circ \bullet : \text{secu}(\alpha) \times \alpha \longrightarrow \text{secu}(\alpha)$$

$$\bullet \& \bullet : \text{secu}(\alpha) \times \text{secu}(\alpha) \longrightarrow \text{secu}(\alpha)$$

$$\text{ult} : \text{secu}(\alpha) \ s \longrightarrow \alpha \quad \{ \neg \text{vacía?}(s) \}$$

$$\text{com} : \text{secu}(\alpha) \ s \longrightarrow \text{secu}(\alpha) \quad \{ \neg \text{vacía?}(s) \}$$

$$\text{long} : \text{secu}(\alpha) \longrightarrow \text{nat}$$

$$\text{está?} : \alpha \times \text{secu}(\alpha) \longrightarrow \text{bool}$$

**axiomas**  $\forall s, t: \text{secu}(\alpha), \forall e: \alpha$

$\text{vacía?}(<>) \equiv \text{true}$   
 $\text{vacía?}(e \bullet s) \equiv$   
 $\text{false}$   
 $\text{prim}(e \bullet s) \equiv e$   
 $\text{fin}(e \bullet s) \equiv s$   
 $s \circ e \equiv \text{if vacía?}(s) \text{ then } e \bullet <> \text{ else } \text{prim}(s) \bullet (\text{fin}(s) \circ e) \text{ fi}$   
 $s \& t \equiv \text{if vacía?}(s) \text{ then } t \text{ else } \text{prim}(s) \bullet (\text{fin}(s) \& t) \text{ fi}$   
 $\text{ult}(s) \equiv \text{if vacía?}(\text{fin}(s)) \text{ then } \text{prim}(s) \text{ else } \text{ult}(\text{fin}(s)) \text{ fi}$   
 $\text{com}(s) \equiv \text{if vacía?}(\text{fin}(s)) \text{ then } <> \text{ else } \text{prim}(s) \bullet \text{com}(\text{fin}(s)) \text{ fi}$   
 $\text{long}(s) \equiv \text{if vacía?}(s) \text{ then } 0 \text{ else } 1 + \text{long}(\text{fin}(s)) \text{ fi}$   
 $\text{está?}(e, s) \equiv \neg \text{vacía?}(s) \wedge_L (e = \text{prim}(s) \vee \text{está?}(e, \text{fin}(s)))$

**Fin TAD**

## 5. TAD CONJUNTO( $\alpha$ )

**TAD CONJUNTO( $\alpha$ )**

**igualdad observacional**

$$(\forall c, c' : \text{conj}(\alpha)) (c =_{\text{obs}} c' \iff ((\forall a : \alpha)(a \in c =_{\text{obs}} a \in c')))$$

**parámetros formales**

**géneros**  $\alpha$

**géneros**  $\text{conj}(\alpha)$

**exporta**  $\text{conj}(\alpha)$ , generadores, observadores,  $\emptyset?$ ,  $\cup$ ,  $\cap$ ,  $\#$ ,  $\bullet - \{\bullet\}$ ,  $\text{dameUno}$ ,  $\text{sinUno}$ ,  $\subseteq$ ,  $\bullet - \bullet$

**usa**  $\text{BOOL}$ ,  $\text{NAT}$

**observadores básicos**

$$\bullet \in \bullet : \alpha \times \text{conj}(\alpha) \longrightarrow \text{bool}$$

**generadores**

$$\emptyset : \longrightarrow \text{conj}(\alpha)$$

$$\text{Ag} : \alpha \times \text{conj}(\alpha) \longrightarrow \text{conj}(\alpha)$$

**otras operaciones**

$$\emptyset? : \text{conj}(\alpha) \longrightarrow \text{bool}$$

$$\# : \text{conj}(\alpha) \longrightarrow \text{nat}$$

$$\bullet - \{\bullet\} : \text{conj}(\alpha) \times \alpha \longrightarrow \text{conj}(\alpha)$$

$$\bullet \cup \bullet : \text{conj}(\alpha) \times \text{conj}(\alpha) \longrightarrow \text{conj}(\alpha)$$

$$\bullet \cap \bullet : \text{conj}(\alpha) \times \text{conj}(\alpha) \longrightarrow \text{conj}(\alpha)$$

$$\text{dameUno} : \text{conj}(\alpha) \longrightarrow \alpha$$

$$\{-\emptyset?(c)\}$$

$$\text{sinUno} : \text{conj}(\alpha) \longrightarrow \text{conj}(\alpha)$$

$$\{-\emptyset?(c)\}$$

$$\bullet \subseteq \bullet : \text{conj}(\alpha) \times \text{conj}(\alpha) \longrightarrow \text{bool}$$

$$\bullet - \bullet : \text{conj}(\alpha) \times \text{conj}(\alpha) \longrightarrow \text{conj}(\alpha)$$

**axiomas**  $\forall c, d : \text{conj}(\alpha), \forall a, b : \alpha$

|                                |          |   |
|--------------------------------|----------|---|
| $a \in \emptyset$              | $\equiv$ | false   |
| $a \in \text{Ag}(b, c)$        | $\equiv$ | $(a = b) \vee (a \in c)$  |
| $\emptyset?(\emptyset)$        | $\equiv$ | true  |
| $\emptyset?( \text{Ag}(b, c))$ | $\equiv$ | false   |
| $\#(\emptyset)$                | $\equiv$ | 0   |
| $\#(\text{Ag}(a, c))$          | $\equiv$ | $1 + \#(c - \{a\})$   |
| $c - \{a\}$                    | $\equiv$ | $c - \text{Ag}(a, \emptyset)$   |
| $\emptyset \cup c$             | $\equiv$ | $c$   |
| $\text{Ag}(a, c) \cup d$       | $\equiv$ | $\text{Ag}(a, c \cup d)$  |
| $\emptyset \cap c$             | $\equiv$ | $\emptyset$   |
| $\text{Ag}(a, c) \cap d$       | $\equiv$ | <b>if</b> $a \in d$ <b>then</b> $\text{Ag}(a, c \cap d)$ <b>else</b> $c \cap d$ <b>fi</b> |
| $\text{dameUno}(c) \in c$      | $\equiv$ | true  |
| $\text{sinUno}(c)$             | $\equiv$ | $c - \{\text{dameUno}(c)\}$   |
| $c \subseteq d$                | $\equiv$ | $c \cap d = c$  |
| $\emptyset - c$                | $\equiv$ | $\emptyset$   |
| $\text{Ag}(a, c) - d$          | $\equiv$ | <b>if</b> $a \in d$ <b>then</b> $c - d$ <b>else</b> $\text{Ag}(a, c - d)$ <b>fi</b>       |

**Fin TAD**

## 6. TAD MULTICONJUNTO( $\alpha$ )

**TAD MULTICONJUNTO( $\alpha$ )**

**igualdad observacional**

$$(\forall c, c' : \text{multiconj}(\alpha)) \ (c =_{\text{obs}} c' \iff ((\forall a : \alpha)(\#(a, c) =_{\text{obs}} \#(a, c'))))$$

**parámetros formales**

**géneros**  $\alpha$

**géneros**  $\text{multiconj}(\alpha)$

**exporta**  $\text{multiconj}(\alpha)$ , generadores, observadores,  $\in$ ,  $\emptyset?$ ,  $\#$ ,  $\cup$ ,  $\cap$ ,  $\in$ ,  $\bullet - \{\bullet\}$ ,  $\text{dameUno}$ ,  $\text{sinUno}$

**usa** **BOOL**, **NAT**

**observadores básicos**

$$\# : \alpha \times \text{multiconj}(\alpha) \longrightarrow \text{nat}$$

**generadores**

$$\emptyset : \longrightarrow \text{multiconj}(\alpha)$$

$$\text{Ag} : \alpha \times \text{multiconj}(\alpha) \longrightarrow \text{multiconj}(\alpha)$$

**otras operaciones**

$$\bullet \in \bullet : \alpha \times \text{multiconj}(\alpha) \longrightarrow \text{bool}$$

$$\emptyset? : \text{multiconj}(\alpha) \longrightarrow \text{bool}$$

$$\# : \text{multiconj}(\alpha) \longrightarrow \text{nat}$$

$$\bullet - \{\bullet\} : \text{multiconj}(\alpha) \times \alpha \longrightarrow \text{multiconj}(\alpha)$$

$$\bullet \cup \bullet : \text{multiconj}(\alpha) \times \text{multiconj}(\alpha) \longrightarrow \text{multiconj}(\alpha)$$

$$\begin{aligned}
\bullet \cap \bullet & : \text{multiconj}(\alpha) \times \text{multiconj}(\alpha) \longrightarrow \text{multiconj}(\alpha) \\
\text{dameUno} & : \text{multiconj}(\alpha) \ c \longrightarrow \alpha \quad \{-\emptyset?(c)\} \\
\text{sinUno} & : \text{multiconj}(\alpha) \ c \longrightarrow \text{multiconj}(\alpha) \quad \{-\emptyset?(c)\}
\end{aligned}$$

**axiomas**  $\forall c, d: \text{multiconj}(\alpha), \forall a, b: \alpha$

$$\begin{aligned}
\#(a, \emptyset) & \equiv 0 \\
\#(a, \text{Ag}(b, c)) & \equiv \text{if } a = b \text{ then } 1 \text{ else } 0 \text{ fi} + \#(a, c) \\
a \in c & \equiv \#(a, c) > 0 \\
\emptyset?(\emptyset) & \equiv \text{true} \\
\emptyset?(\text{Ag}(a, c)) & \equiv \text{false} \\
\#(\emptyset) & \equiv 0 \\
\#(\text{Ag}(a, c)) & \equiv 1 + \#(c) \\
\emptyset - \{a\} & \equiv \emptyset \\
\text{Ag}(a, c) - \{b\} & \equiv \text{if } a = b \text{ then } c \text{ else } \text{Ag}(a, c - \{b\}) \text{ fi} \\
\emptyset \cup c & \equiv c \\
\text{Ag}(a, c) \cup d & \equiv \text{Ag}(a, c \cup d) \\
\emptyset \cap c & \equiv \emptyset \\
\text{Ag}(a, c) \cap d & \equiv \text{if } a \in d \text{ then } \text{Ag}(a, c \cap (d - \{a\})) \text{ else } c \cap d \text{ fi} \\
\text{dameUno}(c) \in c & \equiv \text{true} \\
\text{sinUno}(c) & \equiv c - \{\text{dameUno}(c)\}
\end{aligned}$$

**Fin TAD**

## 7. TAD ARREGLO DIMENSIONABLE( $\alpha$ )

**TAD ARREGLO DIMENSIONABLE( $\alpha$ )**

**igualdad observacional**

$$(\forall a, a' : \text{ad}(\alpha)) \left( a =_{\text{obs}} a' \iff \left( \text{tam}(a) =_{\text{obs}} \text{tam}(a') \wedge \left( (\forall n : \text{nat}) (\text{definido?}(a, n) =_{\text{obs}} \text{definido?}(a', n) \wedge (\text{definido?}(a, n) \Rightarrow a[n] =_{\text{obs}} a'[n])) \right) \right) \right)$$

**parámetros formales**

**géneros**  $\alpha$

**géneros**  $\text{ad}(\alpha)$

**exporta**  $\text{ad}(\alpha)$ , generadores, observadores

**usa**  $\text{BOOL}, \text{NAT}$

**observadores básicos**

$$\begin{aligned}
\text{tam} & : \text{ad}(\alpha) \longrightarrow \text{nat} \\
\text{definido?} & : \text{ad}(\alpha) \times \text{nat} \longrightarrow \text{bool} \\
\bullet [\bullet] & : \text{ad}(\alpha) \ a \times \text{nat} \ n \longrightarrow \alpha \quad \{\text{definido?}(a, n)\}
\end{aligned}$$

**generadores**

$$\text{crearArreglo} : \text{nat} \longrightarrow \text{ad}(\alpha)$$

$\bullet [ \bullet ] \leftarrow \bullet : \text{ad}(\alpha) \ a \times \text{nat} \ n \times \alpha \longrightarrow \text{ad}(\alpha) \quad \{n < \text{tam}(a)\}$

**axiomas**  $\forall a: \text{ad}(\alpha), \forall e: \alpha, \forall n, m: \text{nat}$

$\text{tam}(\text{crearArreglo}(n)) \equiv n$

$\text{tam}(a [ n ] \leftarrow e) \equiv \text{tam}(a)$

$\text{definido}(\text{crearArreglo}(n), m) \equiv \text{false}$

$\text{definido}(a [ n ] \leftarrow e, m) \equiv n = m \vee \text{definido?}(a, m)$

$(a [ n ] \leftarrow e) [ m ] \equiv \text{if } n = m \text{ then } e \text{ else } a [ m ] \text{ fi}$

**Fin TAD**

## 8. TAD PILA( $\alpha$ )

**TAD PILA( $\alpha$ )**

**igualdad observacional**

$(\forall p, p' : \text{pila}(\alpha)) \left( p =_{\text{obs}} p' \iff \left( \text{vacía?}(p) =_{\text{obs}} \text{vacía?}(p') \wedge_{\text{L}} (\neg \text{vacía?}(p) \Rightarrow_{\text{L}} \right. \right. \\ \left. \left. (\text{tope}(p) =_{\text{obs}} \text{tope}(p') \wedge \text{desapilar}(p) =_{\text{obs}} \text{desapilar}(p')) \right) \right)$

**parámetros formales**

**géneros**  $\alpha$

**géneros**  $\text{pila}(\alpha)$

**exporta**  $\text{pila}(\alpha), \text{generadores}, \text{observadores}, \text{tamaño}$

**usa**  $\text{BOOL}, \text{NAT}$

**observadores básicos**

$\text{vacía?} : \text{pila}(\alpha) \longrightarrow \text{bool}$

$\text{tope} : \text{pila}(\alpha) \ p \longrightarrow \alpha$

$\{\neg \text{vacía?}(p)\}$

$\text{desapilar} : \text{pila}(\alpha) \ p \longrightarrow \text{pila}(\alpha)$

$\{\neg \text{vacía?}(p)\}$

**generadores**

$\text{vacía} : \longrightarrow \text{pila}(\alpha)$

$\text{apilar} : \alpha \times \text{pila}(\alpha) \longrightarrow \text{pila}(\alpha)$

**otras operaciones**

$\text{tamaño} : \text{pila}(\alpha) \longrightarrow \text{nat}$

**axiomas**  $\forall p: \text{pila}(\alpha), \forall e: \alpha$

$\text{vacía?}(\text{vacía}) \equiv \text{true}$

$\text{vacía?}(\text{apilar}(e, p)) \equiv \text{false}$

$\text{tope}(\text{apilar}(e, p)) \equiv e$

$\text{desapilar}(\text{apilar}(e, p)) \equiv p$

$\text{tamaño}(p) \equiv \text{if } \text{vacía?}(p) \text{ then } 0 \text{ else } 1 + \text{tamaño}(\text{desapilar}(p)) \text{ fi}$

**Fin TAD**



## 9. TAD COLA( $\alpha$ )

**TAD COLA( $\alpha$ )**

**igualdad observacional**

$$(\forall c, c' : \text{cola}(\alpha)) \left( c =_{\text{obs}} c' \iff \left( \text{vacía?}(c) =_{\text{obs}} \text{vacía?}(c') \wedge_{\text{L}} \left( \neg \text{vacía?}(c) \Rightarrow_{\text{L}} (\text{próximo}(c) =_{\text{obs}} \text{próximo}(c') \wedge \text{desencolar}(c) =_{\text{obs}} \text{desencolar}(c')) \right) \right) \right)$$

**parámetros formales**

**géneros**       $\alpha$

**géneros**       $\text{cola}(\alpha)$

**exporta**       $\text{cola}(\alpha)$ , generadores, observadores, tamaño

**usa**             $\text{BOOL}$ ,  $\text{NAT}$

**observadores básicos**

$\text{vacía?} : \text{cola}(\alpha) \longrightarrow \text{bool}$

$\text{próximo} : \text{cola}(\alpha) \ c \longrightarrow \alpha$   $\{\neg \text{vacía?}(c)\}$

$\text{desencolar} : \text{cola}(\alpha) \ c \longrightarrow \text{cola}(\alpha)$   $\{\neg \text{vacía?}(c)\}$

**generadores**

$\text{vacía} : \longrightarrow \text{cola}(\alpha)$

$\text{encolar} : \alpha \times \text{cola}(\alpha) \longrightarrow \text{cola}(\alpha)$

**otras operaciones**

$\text{tamaño} : \text{cola}(\alpha) \longrightarrow \text{nat}$

**axiomas**     $\forall c : \text{cola}(\alpha), \forall e : \alpha$

$\text{vacía?}(\text{vacía}) \equiv \text{true}$

$\text{vacía?}(\text{encolar}(e, c)) \equiv \text{false}$

$\text{próximo}(\text{encolar}(e, c)) \equiv \text{if vacía?}(c) \text{ then } e \text{ else } \text{próximo}(c) \text{ fi}$

$\text{desencolar}(\text{encolar}(e, c)) \equiv \text{if vacía?}(c) \text{ then } \text{vacía} \text{ else } \text{encolar}(e, \text{desencolar}(c)) \text{ fi}$

$\text{tamaño}(c) \equiv \text{if vacía?}(c) \text{ then } 0 \text{ else } 1 + \text{tamaño}(\text{desencolar}(c)) \text{ fi}$

**Fin TAD**

## 10. TAD ÁRBOL BINARIO( $\alpha$ )

**TAD ÁRBOL BINARIO( $\alpha$ )**

**igualdad observacional**

$$(\forall a, a' : \text{ab}(\alpha)) \left( a =_{\text{obs}} a' \iff \left( \text{nil?}(a) =_{\text{obs}} \text{nil?}(a') \wedge_{\text{L}} (\neg \text{nil?}(a) \Rightarrow_{\text{L}} (\text{raiz}(a) =_{\text{obs}} \text{raiz}(a')) \right) \right. \\ \left. \wedge \text{izq}(a) =_{\text{obs}} \text{izq}(a') \wedge \text{der}(a) =_{\text{obs}} \text{der}(a') \right)$$

**parámetros formales**

|                                    |  |                                       |                           |
|------------------------------------|--|---------------------------------------|---------------------------|
| <b>géneros</b>                     | $\alpha$   |                                       |                           |
| <b>géneros</b>                     | $\text{ab}(\alpha)$  |                                       |                           |
| <b>exporta</b>                     | $\text{ab}(\alpha)$ , generadores, observadores, tamaño, altura, tamaño, inorder, preorder, postorder  |                                       |                           |
| <b>usa</b>                         | $\text{BOOL}$ , $\text{NAT}$ , $\text{SECUENCIA}(\alpha)$  |                                       |                           |
| <b>observadores básicos</b>        |  |                                       |                           |
| $\text{nil?}$                      | $: \text{ab}(\alpha)$  | $\longrightarrow \text{bool}$         |                           |
| $\text{raiz}$                      | $: \text{ab}(\alpha) \ a$  | $\longrightarrow \alpha$              | $\{\neg \text{nil?}(a)\}$ |
| $\text{izq}$                       | $: \text{ab}(\alpha) \ a$  | $\longrightarrow \text{ab}(\alpha)$   | $\{\neg \text{nil?}(a)\}$ |
| $\text{der}$                       | $: \text{ab}(\alpha) \ a$  | $\longrightarrow \text{ab}(\alpha)$   | $\{\neg \text{nil?}(a)\}$ |
| <b>generadores</b>                 |  |                                       |                           |
| $\text{nil}$                       | $:$  | $\longrightarrow \text{ab}(\alpha)$   |                           |
| $\text{bin}$                       | $: \text{ab}(\alpha) \times \alpha \times \text{ab}(\alpha)$   | $\longrightarrow \text{ab}(\alpha)$   |                           |
| <b>otras operaciones</b>           |  |                                       |                           |
| $\text{altura}$                    | $: \text{ab}(\alpha)$  | $\longrightarrow \text{nat}$          |                           |
| $\text{tamaño}$                    | $: \text{ab}(\alpha)$  | $\longrightarrow \text{nat}$          |                           |
| $\text{inorder}$                   | $: \text{ab}(\alpha)$  | $\longrightarrow \text{secu}(\alpha)$ |                           |
| $\text{preorder}$                  | $: \text{ab}(\alpha)$  | $\longrightarrow \text{secu}(\alpha)$ |                           |
| $\text{postorder}$                 | $: \text{ab}(\alpha)$  | $\longrightarrow \text{secu}(\alpha)$ |                           |
| <b>axiomas</b>                     | $\forall a, b: \text{ab}(\alpha), \forall e: \alpha$   |                                       |                           |
| $\text{nil?}(\text{nil})$          | $\equiv \text{true}$   |                                       |                           |
| $\text{nil?}(\text{bin}(a, e, b))$ | $\equiv \text{false}$  |                                       |                           |
| $\text{raiz}(\text{bin}(a, e, b))$ | $\equiv e$   |                                       |                           |
| $\text{izq}(\text{bin}(a, e, b))$  | $\equiv a$   |                                       |                           |
| $\text{der}(\text{bin}(a, e, b))$  | $\equiv b$   |                                       |                           |
| $\text{altura}(a)$                 | $\equiv \text{if nil?}(a) \text{ then } 0 \text{ else } 1 + \text{máx}(\text{altura}(\text{izq}(a)), \text{altura}(\text{der}(a))) \text{ fi}$                     |                                       |                           |
| $\text{tamaño}(a)$                 | $\equiv \text{if nil?}(a) \text{ then } 0 \text{ else } 1 + \text{tamaño}(\text{izq}(a)) + \text{tamaño}(\text{der}(a)) \text{ fi}$                                |                                       |                           |
| $\text{inorder}(a)$                | $\equiv \text{if nil?}(a) \text{ then } <> \text{ else } \text{inorder}(\text{izq}(a)) \ \& \ (\text{raiz}(a) \bullet \text{inorder}(\text{der}(a))) \text{ fi}$   |                                       |                           |
| $\text{preorder}(a)$               | $\equiv \text{if nil?}(a) \text{ then } <> \text{ else } (\text{raiz}(a) \bullet \text{preorder}(\text{izq}(a))) \ \& \ \text{preorder}(\text{der}(a)) \text{ fi}$ |                                       |                           |
| $\text{postorder}(a)$              | $\equiv \text{if nil?}(a) \text{ then } <> \text{ else } \text{postorder}(\text{izq}(a)) \ \& \ (\text{postorder}(\text{der}(a)) \circ \text{raiz}(a)) \text{ fi}$ |                                       |                           |

Fin TAD

## 11. TAD DICCIONARIO(CLAVE, SIGNIFICADO)

TAD DICCIONARIO(CLAVE, SIGNIFICADO)

**igualdad observacional**

$$(\forall d, d' : \text{dicc}(\kappa, \sigma)) \left( d =_{\text{obs}} d' \iff \left( (\forall c : \kappa) (\text{def?}(c, d) =_{\text{obs}} \text{def?}(c, d') \wedge_{\text{L}} (\text{def?}(c, d) \Rightarrow_{\text{L}} \text{obtener}(c, d) =_{\text{obs}} \text{obtener}(c, d'))) \right) \right)$$

**parámetros formales**

|   |  |  |                           |
|---|--|--|---------------------------|
| <b>géneros</b>                          | clave, significado   |  |                           |
| <b>géneros</b>                          | dicc(clave, significado)   |  |                           |
| <b>exporta</b>                          | dicc(clave, significado), generadores, observadores, borrar, claves  |  |                           |
| <b>usa</b>                              | BOOL, NAT, CONJUNTO(CLAVE)   |  |                           |
| <b>observadores básicos</b>             |  |  |                           |
| def?                                    | : clave $\times$ dicc(clave, significado)  | $\longrightarrow$ bool                     |                           |
| obtener                                 | : clave $c \times$ dicc(clave, significado) $d$  | $\longrightarrow$ significado              | $\{ \text{def?}(c, d) \}$ |
| <b>generadores</b>                      |  |  |                           |
| vacío                                   | :  | $\longrightarrow$ dicc(clave, significado) |                           |
| definir                                 | : clave $\times$ significado $\times$ dicc(clave, significado)   | $\longrightarrow$ dicc(clave, significado) |                           |
| <b>otras operaciones</b>                |  |  |                           |
| borrar                                  | : clave $c \times$ dicc(clave, significado) $d$  | $\longrightarrow$ dicc(clave, significado) | $\{ \text{def?}(c, d) \}$ |
| claves                                  | : dicc(clave, significado)   | $\longrightarrow$ conj(clave)              |                           |
| <b>axiomas</b>                          | $\forall d: \text{dicc}(\text{clave}, \text{significado}), \forall c, k: \text{clave}, \forall s: \text{significado}$  |  |                           |
| def?( $c, \text{vacío}$ )               | $\equiv \text{false}$  |  |                           |
| def?( $c, \text{definir}(k, s, d)$ )    | $\equiv c = k \vee \text{def?}(c, d)$  |  |                           |
| obtener( $c, \text{definir}(k, s, d)$ ) | $\equiv \text{if } c = k \text{ then } s \text{ else obtener}(c, d) \text{ fi}$  |  |                           |
| borrar( $c, \text{definir}(k, s, d)$ )  | $\equiv \text{if } c = k \text{ then}$<br>$\text{if def?}(c, d) \text{ then borrar}(c, d) \text{ else } d \text{ fi}$<br>$\text{else}$<br>$\text{definir}(k, s, \text{borrar}(c, d))$<br>$\text{fi}$ |  |                           |
| claves(vacío)                           | $\equiv \emptyset$   |  |                           |
| claves(definir( $c, s, d$ ))            | $\equiv \text{Ag}(c, \text{claves}(d))$  |  |                           |

Fin TAD

## 12. TAD COLA DE PRIORIDAD( $\alpha$ )

TAD COLA DE PRIORIDAD( $\alpha$ )**igualdad observacional**

$$(\forall c, c' : \text{colaPrior}(\alpha)) \left( c =_{\text{obs}} c' \iff \left( \begin{array}{l} \text{vacía?}(c) =_{\text{obs}} \text{vacía?}(c') \wedge_{\text{L}} \\ (\neg \text{vacía?}(c) \Rightarrow_{\text{L}} (\text{próximo}(c) =_{\text{obs}} \text{próximo}(c') \wedge \\ \text{desencolar}(c) =_{\text{obs}} \text{desencolar}(c'))) \end{array} \right) \right)$$

**parámetros formales****géneros**  $\alpha$

|   |  |   |
|---|--|---|
| <b>operaciones</b>  | $\bullet < \bullet : \alpha \times \alpha \longrightarrow \text{bool}$   | Relación de orden total estricto <sup>1</sup> |
| <b>géneros</b>  | $\text{colaPrior}(\alpha)$   |   |
| <b>exporta</b>  | $\text{colaPrior}(\alpha)$ , generadores, observadores   |   |
| <b>usa</b>  | BOOL   |   |
| <b>observadores básicos</b>   |  |   |
| $\text{vacía?}$   | $: \text{colaPrior}(\alpha) \longrightarrow \text{bool}$   |   |
| $\text{próximo}$  | $: \text{colaPrior}(\alpha) \ c \longrightarrow \alpha$  | $\{\neg \text{vacía?}(c)\}$                   |
| $\text{desencolar}$   | $: \text{colaPrior}(\alpha) \ c \longrightarrow \text{colaPrior}(\alpha)$  | $\{\neg \text{vacía?}(c)\}$                   |
| <b>generadores</b>  |  |   |
| $\text{vacía}$  | $: \longrightarrow \text{colaPrior}(\alpha)$   |   |
| $\text{encolar}$  | $: \alpha \times \text{colaPrior}(\alpha) \longrightarrow \text{colaPrior}(\alpha)$  |   |
| <b>axiomas</b> $\forall c: \text{colaPrior}(\alpha), \forall e: \alpha$ |  |   |
| $\text{vacía?}(\text{vacía})$   | $\equiv \text{true}$   |   |
| $\text{vacía?}(\text{encolar}(e, c))$                                   | $\equiv \text{false}$  |   |
| $\text{próximo}(\text{encolar}(e, c))$                                  | $\equiv \text{if } \text{vacía?}(c) \vee_L \text{próximo}(c) < e \text{ then } e \text{ else } \text{próximo}(c) \text{ fi}$                       |   |
| $\text{desencolar}(\text{encolar}(e, c))$                               | $\equiv \text{if } \text{vacía?}(c) \vee_L \text{próximo}(c) < e \text{ then } c \text{ else } \text{encolar}(e, \text{desencolar}(c)) \text{ fi}$ |   |

**Fin TAD**

<sup>1</sup>Una relación es un orden total estricto cuando se cumple:

**Antirreflexividad:**  $\neg a < a$  para todo  $a: \alpha$

**Antisimetría:**  $(a < b \Rightarrow \neg b < a)$  para todo  $a, b: \alpha, a \neq b$

**Transitividad:**  $((a < b \wedge b < c) \Rightarrow a < c)$  para todo  $a, b, c: \alpha$

**Totalidad:**  $(a < b \vee b < a)$  para todo  $a, b: \alpha$