Eksempel: Quadratic Programming.

$$c^T x + \frac{1}{2} x^T Q x \quad \text{hvor } Q \in \mathbb{R}^{n \times n} \text{ pos. semi-def.}$$

Med constraints:

(*) $$\min_x \left\{ c^T x + \frac{1}{2} x^T Q x \mid Ax \geq b \text{ og } x \geq 0 \right\}.$$

Da har vi at Wolfe dual:...

$$(*) \Rightarrow \min_x \left\{ c^T x + \frac{1}{2} x^T Q x \mid b - Ax \leq 0 \text{ og } -x \leq 0 \right\}$$

$$\Longleftrightarrow$$

$$\max_{x, y \geq 0} \; c^T x + \frac{1}{2} x^T Q x + y_1^T (b - Ax) - y_2^T x$$

således $$\nabla_x \left(c^T x + \frac{1}{2} x^T Q x \right) + y_1^T \nabla_x (b - Ax) - y_2^T \nabla_x x = 0$$

dvs. $$c + Qx - A^T y_1 - y_2 = 0$$

$\left[\frac{\partial}{\partial x} b^T A x = A^T b \right]$; $\left[\frac{\partial}{\partial x} a^T x \right] a]$.

Dvs $$c = -Qx + A^T y_1 + y_2$$

og indsættes i objektet:

$$(-Qx + A^T y_1 + y_2)^T x + \frac{1}{2} x^T Q x + y_1^T (b - Ax) - y_2^T x$$

$$= -x^T Q x + y_1^T A x + y_2^T x + \frac{1}{2} x^T Q x + y_1^T b - y_1^T A x - y_2^T x$$

$$= -\frac{1}{2} x^T Q x + y_1^T b$$

Hvorfor vi her at Wolfe dualet er

$$\max_{x, y \geq 0} \left\{ -\frac{1}{2} x^T Q x + y_1^T b \mid c + Qx - A^T y_1 - y_2 = 0 \right\}$$

Idet $y_2 \geq 0$ her vi ydevere

$$\max_{y_1 \geq 0, x} \left\{ -\frac{1}{2} x^T Q x + y_1^T b \mid A^T y_1 - Q x \leq c \right\}$$

$$c - y_2 c \geq c - y_2 = A^T y_1 - Q x$$