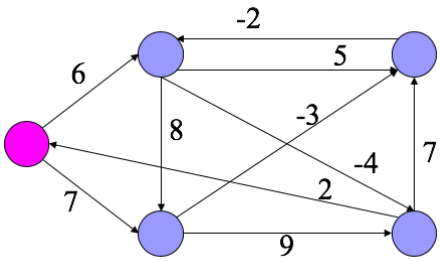
3805ICT Advanced Algorithms – Assignment (100 Marks)

This assignment must be done individually. The programming language to be used is C++ but you may use Python to generate graphs for your reports. The submission time and date is 11pm on Sunday, 17th May, 2020 and the submission method will be communicated during semester.

1. (**10 Marks**) You are traveling by a cross-country by car and there are n car rentals along the way. Before starting your journey you are given for each $1 <= i < j <= n$ the fee $f_{i,j}$ for renting a car from car rental i to car rental j. These fees are arbitrary. For example, it is possible that $f_{1,3} = 10$ and $f_{1,4} = 4$. You begin at car rental 1 and must end at car rental n using rented cars. Your goal is to minimize the rental cost. Give the most efficient algorithm you can for this problem. Be sure to prove that your algorithm yields an optimal solution and analyze the time complexity.

2. (**10 Marks**) Write a C++ program that uses the Bellman-Ford algorithm to find the shortest paths from the pink node to all other nodes:



3. (**20 Marks**) Most graph computing algorithms assume that the adjacency matrix and adjacency lists can be stored in computer memory so the following 2 operations will be fast:

   - Is vertex *v* connected to vertex *u*?
   - Produce a list of all vertices connected to *v*.

   However, the advent of very large graphs (> 50,000,000 vertices and > 100,000,00) prevents the memory storage of the adjacency matrix and standard adjacency lists for these graphs. Design and implement in C++ a data structure for storing such graphs that is able to effectively perform the 2 operations listed above. Demonstrate the efficiency of your data structure.

4. (**10 Marks**) Design and implement in C++ a data structure for storing unordered lists of integers that:
    a. Can store integers in the range 0 .. n where n is some upper bound.
    b. Duplicate integers are not allowed in the list.
    c. Is O(1) for add, delete, test for being in the list and iterating through the list.
    d. Is O(k) (where k is the number of integers in the list) for clearing the list.

5. (**15 Marks**) A very large number of random numbers are added to a list. Design and implement efficient data structures that will maintain a separate list of the k smallest numbers that are currently in the list. Space efficiency must be O(k + n). How would you handle deletions? Perform an amortised analysis of your data structure.

6. (**20 Marks**) A simple algorithm for maze generation is to start, apart from entry and exit points, with all walls present and randomly knock down walls until the entry and exit points are connected. Write a C++ program to implement this algorithm for an arbitrary sized maze – test with a 50 by 88 rectangular maze.

7. (**15 Marks**) Using C++ software obtained from the internet analyse and compare the performance of Red-Black Trees and Van Emde Boas Trees using a large number of integers. This should be done for add, find, delete and sequential access.