Game Theory Lecture Notes v1.0

Contents

1 Intro to Game Theory **2**

2 L20: Getting Started with Games **2**

2.1 2×2 Games . . . 2
2.2 Rationality in Games . . . 3
2.2.1 Unilateral Deviations . . . 3
2.2.2 Best Responses . . . 4
2.2.3 Dominant Strategies . . . 4
2.3 Nash Equilibria . . . 5
2.4 Pareto Optimal Outcomes . . . 5
2.5 Sequential Move Games . . . 6

3 L21: Looking "Inside" Games **7**

3.1 Pure and Mixed Strategies . . . 7
3.1.1 Motivating Mixed Strategies: The Umbrella Game . . . 8
3.2 Best Response Curves . . . 9
3.3 Phase Spaces: Looking "Inside" Games . . . 10

4 L22: Repeated Prisoner's Dilemma **11**

5 L23: Evolutionary Dynamics **13**

6 L24: Decomposing Games **13**

1 Intro to Game Theory

In our class—microeconomic theory—we've discussed various forms of **decision-making**, such as utility-maximizing consumers and profit-maximizing firms. Generally, in these cases, an agent is optimizing against a *fixed* constraint.

In **game theory**, we consider *interdependent optimization*. One agent's decision can influence the decision of another agent; so, how can—and how should—an agent optimize? In our lectures, we explored these questions through a few avenues:

- What is achievable from *moving alone* versus *moving together*.
- Mixed strategies and playing under beliefs and / or with probabilities.
- The *shadow of the future* and repeated interaction.

In these approaches, two agents played among themselves in a "closed environment." But agents are often enmeshed in a larger group: a society or a population. For this reason, we also studied games played by populations.

In short, game theory is a powerful tool to investigate social behavior and phenomena. Given the ubiquity of social interaction in our lives and in the world we participate in, game theory has been used to investigate a range of topics beyond economics, including political science, international relations, biology, philosophy, among others.

2 L20: Getting Started with Games

2.1 2×2 Games

The simplest interaction is a 2-agent, 2-strategy game—a 2×2 game. Two agents, each with two strategies, can achieve one of four possible outcomes. The consequences of these outcomes can be represented using **payoffs**.[1] All of this information can be neatly organized in a **game matrix**.

		Agent 2: C	Agent 2: D
Agent 1	C	$3, 3$	$1, 4$
	D	$4, 1$	$2, 2$

Table 1: A prisoner's dilemma

In the 2×2 prisoner's dilemma in Table 1, the two agents are Agent 1 and Agent 2, and they both have the same two strategies: cooperate (C) and defect (D). We sometimes

[1] Payoffs can be utility, money, evolutionary fitness, and so on.

refer to Agent 1 as "row player" and Agent 2 as "column player." Each **strategy profile** describes the actions of each agent and corresponds to an outcome of the game (*i.e.*, a cell in the game matrix). Within each cell of the matrix, row player's payoff is on the left, and column player's payoff is on the right. For example, in the game in Table 1, the strategy profile (C, D) results in a payoff of 1 for row player and a payoff of 4 for column player.

2.2 Rationality in Games

How can we draw insight from a game matrix? How do agents make decisions? The standard decision-making assumption is **rationality**: a rational agent maximizes her utility.[2]

Still, since games capture *interaction*, how does an agent maximize their utility when the outcome doesn't solely depend on their action? This is not an easy question to answer.

Several tools consistent with rationality can be employed to help us get answers. For instance, we've discussed **Nash equilibria** and **dominant strategies**. There is also the notion of **Pareto optimality**, which sometimes disagrees with rationality. Pareto optimality is not so much a *prediction tool* as it is a *measure* that lets us say one outcome is "better" for *all agents*. We will get back to Pareto optimality; first, let's talk about Nash equilibria and dominant strategies.

Nash equilibria and dominant strategies depend on *individual action*, which is captured with the notions of **unilateral deviations** and **best responses**.

2.2.1 Unilateral Deviations

Given a cell in the game matrix, an agent has a **unilateral incentive to deviate** if, by choosing an alternate strategy, the agent in question receives a higher payoff. It only makes sense to talk about unilateral deviations from a strategy profile—it's a deviation from how the agent is supposedly already playing. For example, consider the prisoner's dilemma in Table 2.

		Agent 2	
		C	D
Agent 1	C	$3, 3$	$1, 4$
	D	$4, 1$	$2, 2$

Table 2: A prisoner's dilemma

Suppose the agents are cooperating (*i.e.*, the relevant cell or outcome is (C, C)). A unilateral deviation by row player would be a change in his strategy—in this case from C to D. This unilateral deviation by row player would take the agents to the outcome (D, C).

[2]Sometimes rationality goes counter to what we observe in the real world. As much as this can be a limitation of game theory, it is also an opportunity to further develop the theory. All in all, this is a deep discussion.

Since this new outcome yields a better payoff for row player, he has a unilateral incentive to deviate from (C, C) to (D, C). The key idea behind unilateral deviations is *moving alone*.

Notice that neither agent has a unilateral incentive to deviate from (D, D)—doing so would yield a lower payoff. Since they are utility maximizers, *if they are moving alone*, neither agent has an incentive to leave (D, D). Because of this, (D, D) is a **Nash equilibrium**—we'll get back to Nash equilibria soon.

2.2.2 Best Responses

A closely related idea to **unilateral deviations** is the notion of **best response**. A player's best response is the strategy she would choose holding all other agents' strategies fixed. If an agent is playing her **best response**, she does not have a **unilateral incentive** to deviate.

Suppose row player knows column player is playing C, what is the *rational* choice for row player? To play D. In this case, it is row player's best response to play D. An agent's best response is her *rational* response given a situation.

When all agents are playing their best response, the strategy profile is a **Nash equilibrium**. In fact, this is how we found Nash equilibria together in class—we underlined payoffs that correspond to the agents' best responses. Then, we looked for the strategy profile where all payoffs were underlined—*i.e.*, the strategy profile that contained all agents' best responses.[3]

2.2.3 Dominant Strategies

In a prisoner's dilemma, it is *always* each player's best response to play D—the strategy D always yields a higher payoff, no matter what the other agent is playing. **Dominant strategies** are those that are always the best response. We say that D is domin**ant** or that C is domin**ated** by D.[4]

Because C is strictly dominated by D, it is *irrational* to play C. We can cross out the strategy C for both agents, simplifying prisoner's dilemmas into a single possible outcome, as demonstrated in Table 3.

		Agent 2	
		~~C~~	D
Agent 1	~~C~~	~~$3, 3$~~	~~$1, 4$~~
	D	~~$4, 1$~~	$2, 2$

Table 3: Prisoner's dilemma; dominated strategies removed

If all agents have a dominant strategy, the resulting outcome after removing dominated

[3]We later learned that this "underlining" method does not account for *mixed* strategy Nash equilibria.

[4]There is a distinction between strictly and weakly dominant strategies. Check out the solution to Problem 3 of Assignment 10 for more details.

strategies is always a **Nash equilibrium**. However, a Nash equilibrium is not always the result of the agents having dominant strategies.

2.3 Nash Equilibria

To summarize, a **Nash equilibrium** is a strategy profile (*i.e.*, an outcome of a game):

- from which rational players do not have a unilateral incentive to deviate;
- where the agents are both already playing their best responses;
- that is composed of all agents' dominant strategies.

The first two bullet points are equivalent statements. The third bullet point is only relevant in special cases—when all agents have dominant strategies.

A simple way to put it: Nash equilibria are outcomes that *trap* agents who *move alone*—for better or worse.

2.4 Pareto Optimal Outcomes

In a game, a strategy profile is **Pareto optimal** if there is no way to improve the payoff of an agent without diminishing the payoff of the other agents. With Pareto optimality, an outcome is compared with all other outcomes—not just those achievable from unilateral deviations. Imagine you are looking at the game from the outside and are simply *commenting* on the possible outcomes. Pareto optimality is a useful way to point to the different outcomes of a game and say whether these outcomes are "special" in terms of how *all* agents are doing.

		Agent 2	
		C	D
Agent 1	C	$3, 3$	$1, 4$
	D	$4, 1$	$2, 2$

Table 4: A prisoner's dilemma

In the prisoner's dilemma in Table 4, the strategy profile (C, C) is Pareto optimal. From (C, C), it is impossible to make an agent better off without making another worse off. But notice how the strategy profiles (C, D) and (D, C) are also Pareto optimal. These Pareto outcomes seem *different*—in these outcomes, one of the agents gets the worse possible payoff in the entire game!

Pareto optimality is not a perfect measure of collective well-being. Because of this, in game theory, we are often concerned with Pareto optimal outcomes that *maximize social welfare*. We didn't get into this, so don't worry about it—just note that, unlike (C, D) and (D, C), the Pareto optimal (C, C) is collectively desirable.

The key feature of a prisoner's dilemma is the *tension* behind the collectively desirable Pareto outcome (C, C) and the Nash equilibrium (D, D). To achieve (C, C), the agents would have to *move together*. But even then, (C, C) is fragile against unilateral deviations and incentives to *move alone*.

2.5 Sequential Move Games

So far we have discussed games where the agents make their decisions **simultaneously**, but we can also model interactive decision-making where the agents move **sequentially**—they move in turns.

Consider the coordination game in Table 5 that we've discussed in class.

		Friend	
		R	J
You	R	$2, 3$	$0, 0$
	J	$0, 0$	$3, 2$

Table 5: Coordination game

This game has two pure strategy Nash, the strategy profiles (R, R) and (J, J). As a simultaneous game, it is hard to tell which equilibrium the agents might play.[5] But what if the agents don't move simultaneously? **If you move first, how would that change the coordination game?**

Let's put the coordination game from Table 5 into **extensive form** with *you* moving first (see Figure 1).

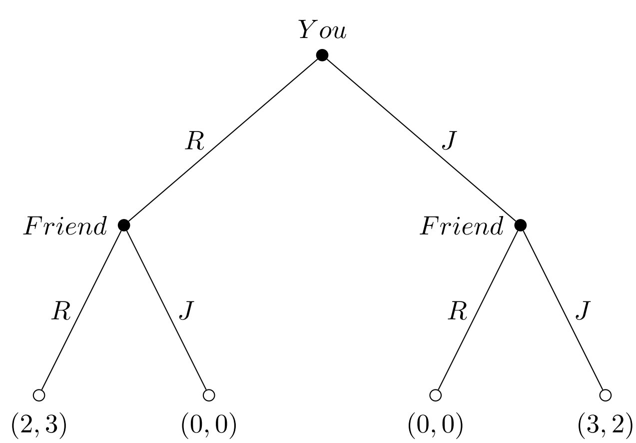

Figure 1: Sequential Move Coordination Game

Extensive form games aptly capture sequential moves, and are drawn as a tree. The first node is the first agent to play—in this example, *you*—and the branches represent the first

[5]In the lecture 21 (April 11th) recording and slides, we discuss how mixed strategies, best response curves, and the phase spaces can enhance our understanding of the coordination game.

mover's strategies. At the end of these branches, there is a node for the second agent to move—in this example, your friend—where the branches extending from this node represent the second mover's strategies. At the end of the branches are pairs off payoffs. The first mover's payoffs are on the left, and the second mover's payoffs are on the right.

We can reason backwards in sequential move games using **backward induction**. The first agent to play can *look ahead* and work backwards to make a decision that optimizes her payoff. You know that if you pick *Rock*, your friend will also pick *Rock*—you know your friend wants a payoff of 3 rather than 0. On the other hand, if you pick $Jazz$, you know your friend will also pick $Jazz$—she wants a payoff of 2, not 0. This boils down to you choosing between *Rock*, with a payoff of 2 for yourself, and $Jazz$, with a payoff of 3 for yourself.

The branches of the extensive form game can be colored to highlight these decisions (see Figure 2). We see that (J, J) becomes the equilibrium of this sequential move coordination game.[6]

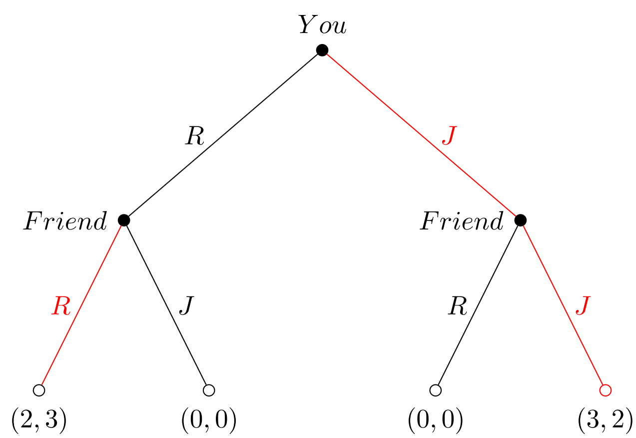

Figure 2: Sequential Move Coordination Game

3 L21: Looking "Inside" Games

3.1 Pure and Mixed Strategies

The strategies in a game are referred to as **pure strategies**. For example, a prisoner's dilemma has two pure strategies: *cooperate* and *defect*. The Nash equilibrium of a prisoner's dilemma is (D, D); it is a **pure strategy Nash equilibrium** since the strategy profile is composed of pure strategies.

Some games don't have pure Nash equilibria. For instance, consider the matching pennies game in Table 6.

[6] The idea of equilibria in sequential move games can be investigated in much more detail and depth by considering **subgame perfect equilibria**. Although this is certainly interesting, you will not be tested on it.

		Agent 2	
		H	T
Agent 1	H	$1, -1$	$-1, 1$
	T	$-1, 1$	$1, -1$

Table 6: Matching Pennies

Even though matching pennies does not have a pure Nash, we are able to look *between* strategies. A **mixed strategy** assumes the agents are able to *mix* their strategies using probabilities or proportions. To handle mixed strategies, we consider **expected payoffs**. The economist John Nash showed that **every game has a Nash equilibrium**—if there is no pure Nash equilibrium, there must be a mixed Nash equilibrium.[7]

3.1.1 Motivating Mixed Strategies: The Umbrella Game

Consider a simple game between you and "the weather." The weather can be *rainy* or *sunny*, and you can choose between *bringing an umbrella* (strategy U) and *not bringing an umbrella* (strategy $\neg U$).[8] You don't know what the weather will be, but you can check the forecast for the corresponding probabilities. It makes intuitive sense that, after some probabilistic threshold of rain, you should bring an umbrella—at a certain probability, it becomes your best response to bring an umbrella. This game is shown in Table 7, where the weather's payoffs are omitted.[9]

		The Weather	
		Rain	*Sun*
You	U	0	-1
	$\neg U$	-1	1

Table 7: The umbrella game

Suppose there is a probability q that it will rain. Your expected payoff of bringing an umbrella is $E_{you}(U) = 0 \cdot q + (-1) \cdot (1 - q) = q - 1$. On the other hand, your expected payoff of *not* bringing an umbrella is $E_{you}(\neg U) = -1 \cdot q + 1 \cdot (1 - q) = 1 - 2q$.

To find the probability q of rain that makes "bring an umbrella" your best response, we compare expected payoffs: for which values of q does bringing an umbrella give you a higher

[7] This is true for **finite games**—games with a finite number of players and a finite number of strategies. It is not guaranteed that an *infinite* game will have Nash equilibria. Don't worry about this. Just pointing it out.

[8] The sign $\neg$ means "not."

[9] In some ways, this is not a game because we are assuming the weather does not make choices *strategically* and there is no true *interaction* between you and the weather. We are assuming the weather makes choices *probabilistically*—it has no payoff so it has no concern for maximizing its payoff; it is just sometimes rainy and other times sunny. Still, this is a good example of how an agent best responds to probabilistic outcomes.

expected payoff? In other words, when is $E_{you}(U) > E_{you}(\neg U)$?

This inequality becomes $q - 1 > 1 - 2q$, which simplifies to $q > \frac{2}{3}$. In other words, you bring your umbrella whenever the chance of rain is high enough (above 67%); you bring it because it's your best response to bring it—you bring it because given your potential payoffs and those odds of rain, you're better off risking carrying an umbrella around on a sunny day than getting caught in the rain without an umbrella.

Alternatively, when $q < \frac{2}{3}$, your best response is to not bring an umbrella. And when the probability of rain is exactly equal to 67% (*i.e.*, $q = \frac{2}{3}$), you are indifferent.

The "umbrella game" helps us understand mixed strategies and best responses to mixed strategies in a relatable and intuitive way.

To see how to calculate mixed Nash, check out lecture 21 (April 11) and the assignments.

3.2 Best Response Curves

We can summarize your best responses in the umbrella game:

- When the probability of rain is high enough ($q > \frac{2}{3}$), it is your best response to bring an umbrella.
- When the probability of rain is low enough ($q < \frac{2}{3}$), it is your best response to *not* bring an umbrella.
- In between these regions, at a single point ($q = \frac{2}{3}$), you are indifferent about bringing an umbrella.

In the umbrella game, we talked about the weather playing probabilistically. Suppose you are also playing probabilistically.[10] You bring an umbrella with probability p and you don't bring an umbrella with probability $(1 - p)$. When $p = 0$, the probability that you **don't** bring an umbrella is 1, so you don't bring it (*i.e.*, you play $\neg U$). When $p = 1$, the probability that you **do** bring an umbrella is 1, so you do bring it (*i.e.*, you play U). In other words,

- $p = 0$ corresponds to you playing $\neg U$
- $p = 1$ corresponds to you playing U

We can reinterpret your best responses in the umbrella game as follows:

- When $q > \frac{2}{3}$, it is your best response to play $p = 1$.
- When $q < \frac{2}{3}$, it is your best response to play $p = 0$.

[10] This may not make as much intuitive sense as the weather playing probabilistically. But it's necessary to draw your best response curve. If you get stuck on this, let me know.

- When $q = \frac{2}{3}$, you are indifferent to playing any $0 \leq p \leq 1$.

The information above can be plotted in the mixed strategy space (see Figure 3).

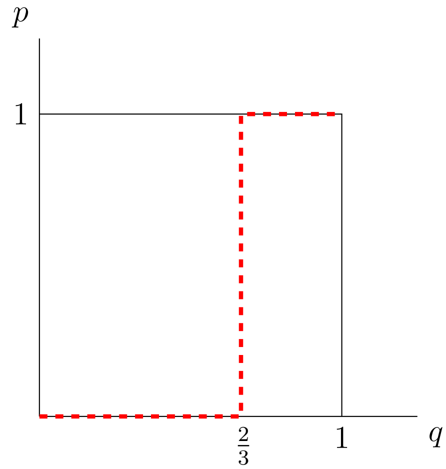

Figure 3: Your best response in the weather game

In this example, it does not make sense to plot the weather's best response. In fact, we wrote the game matrix in Table 7 without payoffs for the weather. The purpose of this example is to give you a stronger intuition on how one agent's best response (in this example, *your* best response) changes as the probability of the other agent's actions (in this example, the weather) changes.

Please study the lecture 21 (April 11th) slides and video, and the solutions to assignment 11 problem 2 to see best response curves for *both* agents.

3.3 Phase Spaces: Looking "Inside" Games

In class, we drew "phase spaces" for some 2×2 games. When drawing these together, we *pushed the pattern of the "arrows" on the edges to the inside of the square*. There can be *more than one way to do this*, but the main idea is not to let the pattern change too drastically once "inside." If you are asked to draw these pictures on the final exam, as long as the general pattern on the outside edges is somehow preserved on the inside, like we did together in class—you will get full points.

The arrows we drew on the "outside" represent the agents' pure strategy best responses. If the agents are *moving alone*, these arrows—their best responses—show the *direction* and *flow* of the game in pure strategies. By pushing this *flow of best responses* to the "inside," we are extending the flow to **mixed strategies**. When we did this for a coordination game, we saw that to continue this flow on the "inside," we had to introduce a *special point* that allows the inside arrows to all flow in a way that *always respects the outside flow*. This "special point" is a mixed Nash equilibrium.

Each point in the phase space of a 2×2 game is described by the probabilities p and q that the agents are using to mix their strategies. In other words, in the phase spaces, **the corners are always pure strategies** and **all other points involve at least one of the agents mixing.**

If both agents were continuously adjusting their probabilities by *just a tiny amount*—in a way that follows their best responses—they would follow the *arrows* we drew in the phase spaces. For this reason, these phase diagrams paint *dynamic pictures* of games. Given any way the agents are playing—whether with pure or mixed strategies—what would be the direction of movement if the agents are following their best responses? The phase spaces we drew provide an answer to this question.

Please study the lecture 21 (April 11th) slides and video, and the solutions to assignment 11 problem 1 for practice on 2×2 phase spaces.

4 L22: Repeated Prisoner's Dilemma

Although (C, C) is the desired Pareto optimal outcome in a prisoner's dilemma, the unique Nash equilibrium is (D, D). However, we have only considered prisoner's dilemmas as **one-shot** games (*i.e.*, the agents play the game only once). In reality, social and strategic interactions often happen more than once. The fact that you may not know whether you will encounter the same person again can have a drastic effect on how you play. This idea is referred to as **the shadow of the future**.

In a repeated setting, we can consider more complicated strategies. Rather than simply playing cooperate or defect, an agent can approach the repeated prisoner's dilemma with a **responsive strategy**.[11] Two responsive strategies we discussed in class are:

- Grim Trigger
- Tit-for-Tat

The responsive strategy **Grim Trigger** is a *punishing* strategy. An agent who plays Grim Trigger starts out cooperating (*i.e.*, they play cooperate in the first round). They cooperate in all subsequent rounds unless his opponent defects. Once Grim Trigger's opponent defects, Grim Trigger plays defect in all future rounds of the game (*i.e.*, they defect forever).

Tit-for-Tat, on the other hand *punishes* and *forgives*. An agent who plays Tit-for-Tat starts out cooperating. From then on, she simply copies what her opponent did on the previous round of play. Whenever Tit-for-Tat's opponent defects on one round, Tit-for-Tat will defect on the *next* round—effectively punishing her opponent. But, if on this next round Tit-for-Tat's opponent cooperates, then Tit-for-Tat will cooperate on the following round—forgiving her opponent.

[11] Instead of an agent treating every repetition of a game as a "new game," responsive strategies allow the agent to play conditional on previous interactions with the same opponent.

		Agent 2	
		C	D
Agent 1	C	$3, 3$	$1, 4$
	D	$4, 1$	$2, 2$

Table 8: A prisoner's dilemma

In lecture 22 (April 13th), we assumed the probability of future interaction is $\frac{3}{4}$. We calculated the **expected payoffs** of someone using the responsive **Grim Trigger** strategy or the simple **Fool** strategy—a Fool always defects. This transformed the prisoner's dilemma in Table 8 into the stag hunt in Table 9.

		Agent 2	
		$Trigger$	$Fool$
Agent 1	$Trigger$	$12, 12$	$7, 10$
	$Fool$	$10, 7$	$8, 8$

Table 9: A stag hunt

In the stag hunt, there are two pure strategy Nash equilibria: $(Trigger, Trigger)$ and $(Fool, Fool)$. In this game, $(Trigger, Trigger)$ is the unique Pareto optimal outcome **and** it is also a Nash equilibrium. We could go quite a bit deeper, but the lesson here is that although cooperation is hard to sustain in a one-shot prisoner's dilemma, a repeated prisoner's dilemma can become a stag hunt—where cooperation becomes a Nash equilibrium. The repeated prisoner's dilemma makes cooperation a *rational choice*.

A simplified version of a stag hunt is shown in Table 10.

		Agent 2	
		C	D
Agent 1	C	$3, 3$	$0, 2$
	D	$2, 0$	$1, 1$

Table 10: A stag hunt

Although stag hunts support cooperation by making it a Nash equilibrium, they don't guarantee it. It is less risky to defect—a defecting agent walks out with a positive payoff no matter what their opponent does. To cooperate, the agents need each other—they both need to cooperate. If Agent 1 cooperates expecting Agent 2 to cooperate, Agent 1 risks receiving a payoff of 0 if it turns out that Agent 2 defected. For this reason, stag hunts are used frequently to talk about **trust** and **risk**.

Please study the lecture 22 recording (April 13th) and the solution to problem 4 of assignment 11 for details.

5 L23: Evolutionary Dynamics

Sometimes, two agents (or more) play the same together simultaneously, sequentially, or repeatedly; this is what we have covered so far. But agents are often enmeshed in a larger group: a society or a population.

For 3-strategy games played by a population, we learned how to visualize the *changes in the distribution of strategies played in a population* using a **simplex** (*i.e.*, a "triangle").

Each corner of the simplex represents one of the three strategies—they are where all members of the population are playing the same strategy. All other points on the simplex describe *mixed populations*. All points on the simplex (including the corners) represent **states** of the population—they give a full description of how many agents are playing which strategies.

A **pure strategy** is said to be **evolutionarily stable** if, when all members of the population are playing this strategy, it can't be invaded by mutant strategies (or, "alternate" strategies). A **state** is said to be **evolutionarily stable** if the state—which is described as a set of proportions—can't be invaded by mutant strategies.

Every **evolutionarily stable strategy** is an **evolutionarily stable state**, but not the other way around.

Please study the lecture 23 slides and recording (April 18th) and the solutions to the practice problems.

6 L24: Decomposing Games

When we studied consumer theory, we learned about the Slutsky decompositon. This decomposition allows us to **decompose** an overall change in demand into a pure substitution effect (a change in demand coming from a change in opportunity cost) and an income effect (a change in demand coming from a change in income).[12]

Decompositions can be powerful—they allow us to see the smaller mechanisms that drive larger patterns in our objects of study. In game theory, we can **decompose games** into the **Nash component**, the **Pure Externalities component**, and **the Centers component**.

Consider the prisoner's dilemma in Table 11. Undoubtedly, (C, C) is the best group outcome. However, *moving alone*, there is temptation—for both agents—to play D. The Nash component pulls agents who move alone towards Nash equilibria—in the case of the prisoner's dilemma, it pulls them to (D, D).

[12] Just to be 100% clear, you will not be tested on the Slutsky decomposition on the final exam.

		Agent 2	
		C	D
Agent 1	C	$6,6$	$0,8$
	D	$8,0$	$2,2$

Table 11: A prisoner's dilemma

Decomposing this prisoner's dilemma yields the Nash, Pure Externalities, and Centers components shown below.

"Nash"

	C	D
C	$-1,-1$	$-1,1$
D	$1,-1$	$1,1$

$+$

"Pure Ext."

	C	D
C	$3,3$	$-3,3$
D	$3,-3$	$-3,-3$

$+$

"Centers"

	C	D
C	$4,4$	$4,4$
D	$4,4$	$4,4$

The Nash component contains the "selfish" forces in a game: it tells us what the agents would do if they were *moving alone*. It tells us *what portions of the payoffs are available by moving alone*. The Pure Externalities component contains the "collective" forces in a game: it shows what the agents could pursue *together*; it shows us what the agents have the power to *give*—and to take away from—their opponents. It shows the portion of one agent's payoff that is *fully controlled by the other agent*.[13]

With the Nash and the Pure Externalities components in mind, we can now interpret an agent unilaterally deviating from C to D in two ways: (1) to chase a higher payoff in the Nash component, or (2) to take away a payoff from the other agent—to punish the other agent.[14] But this second idea—of punishment—only works if the game is repeated. This lets us understand, in a deeper way, why Grim Trigger can work in a repeated prisoner's dilemma.

Remember, Grim Trigger starts out cooperating but if his opponent ever defects, Grim Trigger will never cooperate again. You can imagine Grim Trigger saying: *"Cooperation is good for both of us. When I cooperate, I enhance* your *payoff.*[15] *I hold some power over your payoff. If you betray me, I will never use my power to enhance your payoff again—even at a cost to myself. In other words, I will punish you."* Grim Trigger can work, but it is harsh and doesn't allow *forgiveness*. If an agent mistakenly defects against a Grim Trigger—just once—there will never be cooperation again. This is the strength of Tit-for-Tat—a strategy that both punishes and forgives.

In 2×2 games, an agent has control over their own payoffs in the Nash component, and they have control over pure externality payoffs *over the other agent*. Essentially, what is happening, is that responsive strategies in repeated games allow agents to transform the pure

[13] For most intents and purposes, the Centers component does not contribute anything meaningful to the game's payoff structure, hence we will ignore it.

[14] Carefully inspect the Pure Externalities component to see why this is true.

[15] This comes from the Pure Externalities component.

externality payoffs that they control—and that affect the other agent—into Nash payoffs that the other agent controls. Look at what Grim Trigger is saying in the previous paragraph: "I hold some power over your payoff. If you betray me, I will never use my power to enhance your payoff again." Grim Trigger is *handing off the power of the externalities he controls* to his opponent— he is transforming the externalities he has control over into "moving alone" payoffs that *his opponent has control over.*

Please check out the solutions to the practice problems for detailed steps for decomposing games.