$$
\begin{aligned}
a_p a_q^\dagger a_r^\dagger &= n[a_p a_q^\dagger a_r^\dagger] + n[\overbrace{a_p a_q^\dagger} a_r^\dagger] + n[\overbrace{a_p a_q^\dagger a_r^\dagger}] + \cancel{n[a_p \overbrace{a_q^\dagger a_r^\dagger}]} \\
&= a_q^\dagger a_r^\dagger a_p + \overbrace{a_p a_q^\dagger} n[a_r^\dagger] + \overbrace{a_p a_r^\dagger} n[a_q^\dagger] \\
&= a_q^\dagger a_r^\dagger a_p + \delta_{pq} a_r^\dagger - \delta_{pr} a_q^\dagger
\end{aligned}
$$

Would you like to do it with hand or using computer?

$$
a_p a_q a_r^\dagger a_s a_t^\dagger a_u^\dagger = ???
$$