

CSD209 Term Paper

Analysis of ‘Search for the Wreckage of Air France Flight AF 447’ By Lawrence D. Stone, Colleen M. Keller, Thomas M. Kratzke and Johan P. Strump

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Abstract

During a flight from Rio de Janeiro to Paris on June 1, 2009, Air France Flight AF 447 vanished in stormy weather over a remote section of the Atlantic, killing all 228 passengers and crew members on board. The posterior target location distribution was created using a Bayesian procedure designed for search planning. In this review paper, we discussed the Bayesian approach that the paper “Search for the Wreckage of Air France Flight AF 447 by Lawrence D. Stone, Colleen M. Keller, Thomas M. Kratzke and Johan P. Strumpfer” used along with some additional scenarios where this proposed framework can be anticipated and applied to.

Inferences of various search techniques Used

Search Number	Method	Search Duration	Search Path	Personnel/equipment	Result
1: International search by aircraft and surface ships	the first debris and bodies were found 38 NM north of the aircraft's last known position	June 6th- June 10th, 2009	-	-	Unsuccessful
2: Search for underwater locator beacon	Search was performed by two ships employing passive acoustic sensors.	31 days from June 10, 2009	Intended flight path	Two ships employing passive acoustic sensors supplied by the U.S. Navy and operated	Unsuccessful

				by personnel from Phoenix International	
3: detect the wreckage on the ocean bottom	active acoustic search with side-looking sonar	August 2009	south of the last known position in an area not covered by the passive acoustic searches	-	Unsuccessful
4: Active acoustic search using estimated location of wreckage	Oceanographic experts estimated the currents in the area at the time of the crash. This was used along with the Results from the first search to produce a number of trajectories ending at an estimated location of the crash. Outliers were removed. A bivariate normal error was estimated for each of the remaining crash location estimates and used to produce a weighted mean with a bivariate normal error distribution.	April and May of 2010	Rectangle centered at the mean with a 95% probability of "containing" the wreck location	U.S. Navy, Phoenix International team used a towed side-scan sonar system. Woods Oceanographic Institute team used autonomous underwater vehicles with side-scan sonars and a remotely operated vehicle.	Unsuccessful in the rectangular search area, both North-West and South-West of the last known location

Bayesian Analysis

Bayesian analysis is a mathematical paradigm that uses probability statements to address research questions regarding uncertain parameters.

In Bayesian analysis, rather than one fixed value, a parameter is summarized by an entire distribution of values, as in classical frequentist analysis. The heart of Bayesian research is estimating this distribution, a posterior distribution of a parameter of interest.

A posterior distribution is made up of a prior distribution for a parameter and a probability model that provides parameter information dependent on observed data. The posterior distribution can be calculated analytically or approximated using one of the Markov chain Monte Carlo (MCMC) models, depending on the prior distribution and probability model used.

The posterior distribution is used in Bayesian inference to create different summaries for model parameters, such as posterior means, medians, percentiles, and interval projections known as credible intervals. Furthermore, all model parameter statistical analyses can be represented as probability statements dependent on the predicted posterior distribution.

The ability to add prior knowledge into the analysis, an intuitive understanding of credible intervals as fixed ranges to which a parameter is known to belong with a predetermined likelihood, and the ability to attribute an actual probability to any hypothesis of interest are all unique aspects of Bayesian analysis.

Reason for use of BAYESIAN ANALYSIS:

Complicated searches such as the one for AF 447 are onetime events. We are not able to recreate the conditions of the crash 1000 times and record the distribution of locations where the aircraft hits the water. As a result, definitions of probability distributions in terms of relative frequencies of events do not apply. Instead, we are faced with computing a probability distribution on the location of the wreckage (search object) in the presence of uncertainties and conflicting information that require the use of subjective probabilities. The probability distribution is therefore a subjective one.

Reason for selecting the 40 mile radius circle

On June 1, 2009, AF 447's last recorded location was 2.98°N latitude/30.59°W longitude, which was submitted at 02 hours 10 minutes and 34 seconds Coordinated Universal Time. The BEA concluded that the plane may not have flown more than 40 miles from its last reported location until crashing into the ocean based on the inability to receive any communications after 02 hours, 14 minutes, and 26 seconds. As a result, it was safe to assume that the debris was within a 40-mile radius based on the last reported spot, with a chance of one. Any probability distribution for the site of the wreckage that had a probability outside of this radius was truncated at the circle and renormalized.

Prior Solution Strategy:

The prior distribution P on the location of the wreck was taken to be a mixture of:

D_1 : Uniform distribution within the 40 NM circle

D₂: Based on data from crashes that involved loss of control while a plane was at flight altitude

D₃: Analysis that drifted dead bodies found on the surface backward in time to possible crash locations

On the basis of discussions with analysts at the BEA, following subjective weights for these distributions were taken as, $p_1 = p_2 = 0.35$ and $p_3 = 0.3$

i.e. $P = p_1 * D_1 + p_2 * D_2 + p_3 * D_3$

In retrospect, it seems that interpreting D₃ as a probability function and multiplying the distribution $0.5D_1 + 0.5D_2$ by D₃ to achieve the prior seemed to be more fitting.

Distribution D₂

It is based on an analysis of data from nine commercial aircraft accidents involving loss of control at flight altitude. It showed that all impact points (adjusted to the 35,000 foot altitude at which AF 447 was cruising) were contained within a circle of radius 20 NM from the point at which the emergency began. These results were represented by a circular normal distribution centered at the last known position with standard deviation 8 NM along any axis. We set D₂ equal to this distribution truncated at the 40 NM circle.

Distribution D₃

It's a case of reverse drift. The distribution was determined by reversing the motion of recovered bodies back to the moment of impact using data on tides and winds. Drift has two components: drift caused by the ocean current and drift caused by the wind. The latter is referred to as leeway. To reflect the locations of selected bodies recovered on each day, we used polygons. 16,000 positions were taken from a uniform distribution over the polygon for each polygon. Winds and tides caused each spot to drift backward in time.

A 60-minute time stage was used. For each phase and each particle, a draw was made using the wind and current distributions for the particle's time and location. Until the next time point, the negative of the vector amount of current plus wind leeway affected the particle motion. The vast uncertainties in the ocean currents resulted in a distribution for the crash site that ranged well beyond the 40 NM circle. The reverse-drift distribution generated in this manner and truncated at the 40 NM circle reveals the reverse-drift distribution produced in this fashion and truncated at the 40 NM circle. This example was assigned a lower weight than the other two due to the significant uncertainties in the currents.

- Simulating wind and currents

Wind and current estimates are provided in the form of a grid of velocity vectors (w , v) indexed by space and time where u is the speed in the east-west direction and v is the speed in the north-south direction. We interpret these as mean values of the actual wind and current velocities and add a stochastic component.

To obtain (u , v) for a wind or current at a time that corresponds to a grid time but for a spatial point that is not equal to one of the spatial grid points, we take the three closest spatial grid points and use a weighted average of the values at those points, where the weights are proportional to the inverses of the distances from the desired point to the chosen grid points.

Most often we will need (w, v) for times that are not equal to one of the time grid values. To get (m, v) in this case, we use the values as calculated above for the two closest times in the data and then linearly interpolate between these values. For every time step and every particle, the simulation perturbs the speeds u and v obtained from the data by adding a random draw from a normal distribution with a standard deviation of 0.22 kts for current speeds and 2.0 kts for wind speeds. These draws are independent for u and v and from particle to particle, but for a given particle and speed the draws are correlated in time. Specifically, if Δt is the increment in time, measured in minutes, between two time steps, then the correlation is given by

$$p(\Delta t) = e^{(-\alpha \Delta t)} \text{ where } \alpha \text{ is chosen so that } e^{-\alpha 60} = 0.5$$

-Simulating drift

Effects of current:

The particle's velocity due to the current is equal to the velocity of the current.

Effects of wind:

The sum of the force of the wind acting on the open surfaces of the object and the drag of the water acting on the submerged surfaces of the object induces wind drift (leeway). The wind does not propel an object at the same speed as the wind, and it does not drive an object in the same direction as the wind.

Leeway has two components: downwind and crosswind. Downwind refers to the direction in which the wind is blowing. The crosswind portion is perpendicular to the downwind component, and the crosswind leeway is unpredictable in course. To account for this, when constructing a particle path, the simulation flips between the two crosswind paths at exponentially scattered intervals.

The following formula is used to calculate the magnitudes of the downwind and crosswind components:

For a specific particle at a specific moment, we computed wind velocity from gridded wind data in the same way as we computed ocean currents, computed mean downwind and crosswind leeway, and applied a random variable to the mean values by adding the value of a draw from a regular distribution with mean 0 and standard deviation equal to the standard error computed for the regression used to approximate the mean downwind and crosswind leeway.

Posterior Solution Strategy:

We approximated the continuous spatial distribution for the location of the wreckage by a discrete distribution represented by a set of N point masses or particles (x_n, w_n) for $n = 1, 2, 3, \dots, N$, where w_n is the probability mass attached to particle n . The probabilities sum to 1. In the case of a stationary search object, x_n is a latitude-longitude point. In the case of a moving object, x_n is a continuous space and time path over the time interval of interest.

We use $N=75000$ for both prior and posterior distributions.

Computing the posterior distribution

The continuous spatial distribution for the location of the wreckage by a discrete distribution represented by a set of N point masses or particles (x_n, w_n) for $n=1,2,3, \dots, N$, where w_n is the probability mass attached to particle n .

We take $N=75000$ and for the n th particle we set x_n equal to the location of the n th draw and $w_n = 1/N$ for $n = 1, \dots, N$. If an unsuccessful search takes place, we compute the probability $p_d(n)$ that the search would have detected the search object if it were located at x_n . The posterior probabilities w_n' on the particles are computed using Bayes' rule as follows:

$$\tilde{w}_n = \frac{(1 - p_d(n))w_n}{\sum_{n'=1}^N (1 - p_d(n'))w_{n'}} \\ \text{for } n = 1, \dots, N.$$

Reference for the Image: 'Search for the Wreckage of Air France Flight AF 4 by Lawrence D. Stone, Colleen M. Keller, Thomas M. Kratzke and Johan P. Strump'

P1: Accounting for search 1

Assumptions:

- As the plane collided with the surface of the ocean, it plummeted 14000 feet to the ocean depths.
- The detectability of the galley is equivalent to that of a four-man raft
- Independent detection opportunity on each leg and multiplied the failure probability for each leg and sortie to obtain an overall probability of failure to detect for each particle

For each particle, a path for each particle was constructed from the position of the particle projected up to the surface of the ocean. The particle was drifted forward in time for six days using the wind and current drift model, using the drift vector. As with the reverse drift scenario, the leeway component of drift was based on that of a body in the water.

After a lot of search using manpower and boats, function of failure probability $q(n)$ was calculated for the air and ship searches for each particle for the six days of unsuccessful search. After 6 days, an effective estimate was made that the search has been ineffective with probability 0.7 and effective with probability 0.3. As a result we set $1 - p_d\{n\} = 0.7 + 0.3q(n)$ and computed the posterior probabilities $w' = w_n$ on the path of the n th particle by (4.1), which is also the posterior probability on the wreck location being equal to x_n given failure of the six days of surface search.

P2: Accounting for search 2

We determined that the sensors had a chance of at least 0.9 of detecting the beacons within a lateral range of 1730 m based on a measurement concerning the beacons' source level and propagation loss through the sea. Detection predictions based on manufacturer requirements and operator estimates have shown to be optimistic in previous searches.

As a result, we restrict estimates of sensor detection probabilities to a maximum of 0.9. The search paths were planned so that every point in the search area would have a maximum lateral range of 1730 meters to the nearest path.

$$PD = (1-(0.1)2)(0.8)2+(0.9)(2(0.8)(0.2))=0.92$$

If the beacons were installed sufficiently close to each other, then their survival becomes an independent event, thus dropping our PD to $(0.9*0.8) = 0.72$. It was decided to use a weighted average of 0.25 for independent and 0.75 for dependent probabilities that gives us $P_d = 0.23$.

Thus, $P_d' = 0.77$.

P3: Accounting for search 3

We assumed a 0.90 probability of detection in the searched region. This represents a conservative, subjective estimate of the probability of this sensor detecting a field of debris.

$1-pd(n)=0.1$, if x_n is located in the search rectangle

$1-pd(n)=1$, otherwise

P4: Accounting for search 4

As with the previous active sonar search, we estimated that within these regions the search sensors achieved detection probability 0.9. We felt this was a conservative subjective estimate based on the careful execution of the search, the quality of sonar records and the numerous small articles detected during this search.

P5: Posterior after 1-4

Even though this posterior allows for the possibility that the beacons did not work, due to doubts about the beacons an alternate posterior was created which assumes the beacons did not function. Testing by the BEA showed that when the beacon was connected to a fully charged battery, it did not produce a signal. This indicates that the beacons were damaged in the crash and did not function.

Retrospective change:

A better way to handle the doubts that we had about the beacons would have been to compute a joint distribution on beacon failure and wreck location. The marginal distribution on wreck location would then be the appropriate posterior on which to base further search.

It would have been more appropriate to view D3 as a likelihood function and multiply the distribution $0.5D1 + 0.5D2$ by D3 to obtain the prior.

Applications of Bayesian Analysis

Bayes theorem was integral to the above discussed search because the BEA team conducting the search was working with very limited information. Data pertaining to plane crashes was unavailable and very limited in scope. This meant BEA researchers had to develop a model based on subjective assumptions, and a model based on assumptions is more likely to give unsuccessful results than one based on historical occurrences of an event. Bayes theorem is what allows the series of unsuccessful searches to remain useful in finding the wreckage of the plane. Bayes theorem provides a framework with which new information can be incorporated into an existing model, thereby improving the old model.

This ability to take into account new information is very useful, and as a result Bayes theorem is widely applied in a variety of disciplines. Industries where Bayes theorem is used include, but

are not limited to Health care, industrial production, the financial and tech industry, etc. To illustrate this point, we will briefly discuss a few applications of Bayes theorem in different disciplines.

Example of Bayesian Analysis in medical sciences

One characteristic unique to clinical psychology is the absence of Gold standard tests. Gold standard tests are tests which possess 100% sensitivity, that is results in no false negatives, and 100% specificity, that is results in no false positives. This makes it so that the results of a test have to be interpreted in a manner that takes into account this uncertainty, avoiding which could result in improper treatment being prescribed to an individual. Bayes Theorem is employed to interpret the results of these kinds of tests in a meaningful fashion.

For example, assume that we are using a psychological test for diagnosing the presence of depression in people. Psychological tests typically result in a score, and come with a scale with which to interpret the score. This scale usually involves a cut off value; a score above the cut off value is seen as a positive indicator of depression and a score below the cut off is seen as a negative indicator of depression.

Let the events A, B are defined in the following manner,

A: individual is depressed

B: depression test is positive, that is score from test exceeds cut off

From this,

A/B is the event where a person is depressed, given that they exceeded the cutoff.

Therefore using Bayes theorem, we get

$$P(A/B) = [P(B/A).P(A)] / [P(B)] ,$$

where B/A is the event where a person tests positive for depression, given that they are depressed.

The National Institute of Mental Health conducted a survey of 17.3 million people to determine the prevalence of depression in adults based in the United States of America. According to this survey, 7.1% of adults in the US have depression.(Source: <https://www.nimh.nih.gov/health/statistics/major-depression.shtml>)

Therefore, we have $P(A) = 0.071$

$P(B)$ is the sum of percentages false positives(FP) and true positives(TP). Assuming the percentage of FPs and TPs is 5% and 80% respectively, we have,

$$P(B) = 0.05(1 - 0.071) + 0.80(0.071) = 0.10$$

The conditional probability that a person's score is above the cutoff, given they are depressed is 0.80, and hence

$$P(B/A) = 0.80.$$

Using the values of $P(A)$, $P(B)$ and $P(B/A)$, we get

$$P(A/B) = [P(B/A).P(A)] / [P(B)] = [0.80 * 0.071] / [0.10] = 0.54$$

This means that only 54% of all positive tests are actually depressed. Despite the test having a relatively high TP rate (80%) and low FP rate (5%), only 54% of all positives are actually representative of cases of depression.

Water Quality Index Using Bayesian Analysis

Given the high numbers of deaths and the debilitating nature of diseases caused by the use of unclean water, we must have an understanding of the factors that control the dispersion of waterborne pathogens and their respective indicators.

The metric affecting water suitability for a particular use case (like drinking, agriculture, cleaning) is whether the amounts of pollutants surpass the water quality standard numeric thresholds. The correlation between indicator concentration and the amount of pollutant varies widely depending majorly on the intrinsic properties of the pathogens. The pathogens are usually

Taking the case of shellfish harvesting waters, the water properties can be estimated by the amounts of FIB or the faecal indicator bacteria (FIB, for example, faecal coliforms and *Escherichia coli*). FIB describes the range of bacteria that inhabit the gastrointestinal tract of homoeothermic animals. These microorganisms form a conservative index of faecal pollution and of the potential existence of hurtful waterborne pathogens, which, while affecting human and ecological wellbeing, are additionally considerably more difficult and expensive to calculate. Hence, to detect these waterborne pathogens at a limited cost, FIB is used as a proxy for pathogenic bacteria.

There are studies on other aspects of pollution also like sediment deposit which seem to use similar techniques, but we plan on focusing on FIB as of now.

The parameters which affect the pathogens:

The ecosystem plays a significant role. Factors like temperature, humidity affect the concentration of the pathogens. Natural aquatic bacterial communities have strong latitudinal gradients in diversity.

Increasing urbanization means less area for water bodies, and due to rains and water accumulation in the area providing more opportunities for pathogens to further contaminate already contaminated water sources.

The uncertainty in the relationship between pollutant-indicator, the sampling frequency and other factors collectively can contribute to reliably forecasting environmental conditions. In this case, the Bayesian approach can be used to evaluate water quality.

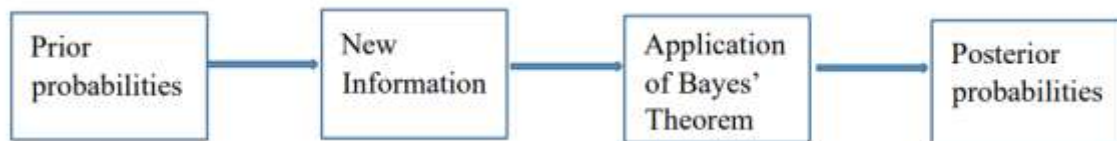
FIB concentrations are normally accepted to pursue lognormal probability distribution $LN(\mu, \sigma)$ probability distribution with a mean (μ) and a standard deviation (σ).

Why use Bayesian:

Although this simple probability model recognizes natural spatial and temporal variability in FIB dispersion patterns, it often neglects to recognize other, more not obvious sources of variability from inherent sources coming from FIB concentration estimations and how FIB amounts are determined. These can direct not only to unsureness in FIB concentration expectations but to unsureness in probability distribution parameters (μ and σ) as well.

Advantages of using Bayesian in this context:

The particular uncertainties mentioned above can be explicitly recognized by putting an earlier probability distribution on the parameters μ and σ (which may represent earlier estimates of their potential values), and building a likelihood function for μ and σ from experiential evidence (in this example, using water quality samples), and, at last, developing a joint posterior probability distribution for both. This prior probability distribution can be achieved from monitoring data used in prior assessments.



Conclusion

The paper 'Search for the Wreckage of Air France Flight AF 447' analyzed the crash of the flight AF 447 dated 1 June 2009. No person in the flight was recovered alive. The scientists applied various techniques to find the factors that lead to the air crash. The paper discusses the approach taken by these scientists in order to determine these factors. After intense discussion, the scientists decided to use Bayesian analysis to get the final outcome because they could not simulate the air crash 1000 times so they could not use probability distributions with relative frequencies. Bayesian analysis takes care of all independent as well as dependent factors therefore it was the best possible approach. They made a prior distribution consisting of three different distributions. These distributions take into account all the factors like effect of wind currents and ocean currents, the climate etc. From these distributions, they computed posterior distribution for each of them. Surprisingly, this posterior distribution precise, which begs the question of why the beacons were not discovered.

Bayes' theorem gives the probability of an event based on information that is, or may be related, to that event. The formula can also be used to see how the probability of an event occurring is affected by new information, supposing the new information will turn out to be true. As seen in the case analyzed in the paper this theorem was the most suitable, by searching about the other factors that led to the air crash we were able to derive why the air crash actually happened.

Other Approaches

Improvement of Execution

While computing the prior distribution D_2 , part of the analysis was performed by the Russian Interstate Aviation Group and the BEA. This search showed that the impact points were contained in a circle of 20 NM radius. By the modern standards of Machine Learning and Artificial Intelligence we can infer more data about these impact points using Deep Learning techniques. This data may help us to compute the result using some other probability techniques. Some techniques may also prove out to be more efficient than Bayesian analysis. As we are assuming more data is present so we can simulate experiments with more inputs, resulting in better accuracy. These experiments can also lead to additional factors that may make the research more accurate. Probably after sometime where these kind of disasters can be simulated virtually but this will not guarantee that a disaster won't happen. Just by taking care of these additional data/factors we can prevent a disaster but due to some unforeseen circumstances there could always be an issue. Bayesian analysis serves to be great for these kind of situations but with more computational power we can bring into account other probability distributions as well. Compiling the results of all these distributions we can come up with better solutions or approaches to the problems.

Alternative Framework

One possible approach can be using Genetic Algorithms. We can assign different weights to our present factors and then normalize them to get some newly generated factors accommodating the effects of all the initial factors. Using this we can also predict the factors which played major role in the given situation. This framework can also be used to accommodate more number of factors.

References:

- 'Search for the Wreckage of Air France Flight AF 4 by Lawrence D. Stone, Colleen M. Keller, Thomas M. Kratzke and Johan P. Strump'
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