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Alperen & Saumya
A+S
Research Services, Inc.

Programming Assignment #1

Report

We worked alone on this assignment and followed all other guidelines:

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EN.600.445 Computer Integrated Surgery - Fall 2010

Prof. Russell Taylor

Grade Factor		
Program (40)		
Design and overall program structure	20	
Reusability and modularity	10	
Clarity of documentation and programming	10	
Results (20)		
Correctness and completeness	20	
Report (40)		
Description of formulation and algorithmic approach	15	
Overview of program	10	
Discussion of validation approach	5	
Discussion of results	10	
TOTAL	100	

<u>Mathematical Approach Taken</u>:

The main mathematical methods required for solving a point cloud to point cloud registration, as described in various literature and on the in-class lectures, is using a form of least squares analysis. Least-squares allows us to solve for the unknown variables in the frame transformation using a handful of experimental observations.

For calculating the frame transformation between two coordinate systems, that of the electro-magnetic tracker and the calibration body, we decided to use quaternion data structures and algorithms. The quaternion "least-squares" method has been shown to be robust and quick (according to Liu and Shi) for many point cloud scenarios. Furthermore, using the powerful native matrix mathematics in MATLAB, quaternion calculations were easily performed.

Our function returns a frame transformation (quaternion) as a 4x4 matrix, which consists of the following elements:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ p_x & & & \\ p_y & & [R] & \\ p_z & & & \end{bmatrix}$$

where $[R]$ is the rotation matrix and $[p_j]$ is the translation matrix.

For the pivot calibration, we used an approach detailed in Lorsakul et al, which uses least squares regression to find the position of the calibration tool tip. This method involves a rearrangement of the standard equation, $P_{dimple} = F_G * t_G$, into its rotation and translation parts (R_G, p_G).

The equation becomes: $R_G * t_G - P_{dimple} = -p_G$, which is now in a form capable of being transformed into a least squares problem:

$$\left[R_G \,\vdots\, -I \right] \begin{bmatrix} t_G \\ P_{dimple} \end{bmatrix} = \left[\vdots \atop -p_G(k) \atop \vdots \right]$$

We can rewrite this as $\widetilde{A}\,\widetilde{b} = \widetilde{p}$. Thus $\widetilde{b} = (\widetilde{A}^T\widetilde{A})^{-1}\widetilde{A}^T\,\widetilde{p}$ where $(\widetilde{A}^T\widetilde{A})^{-1}\widetilde{A}^T$ is the pseudoinverse.

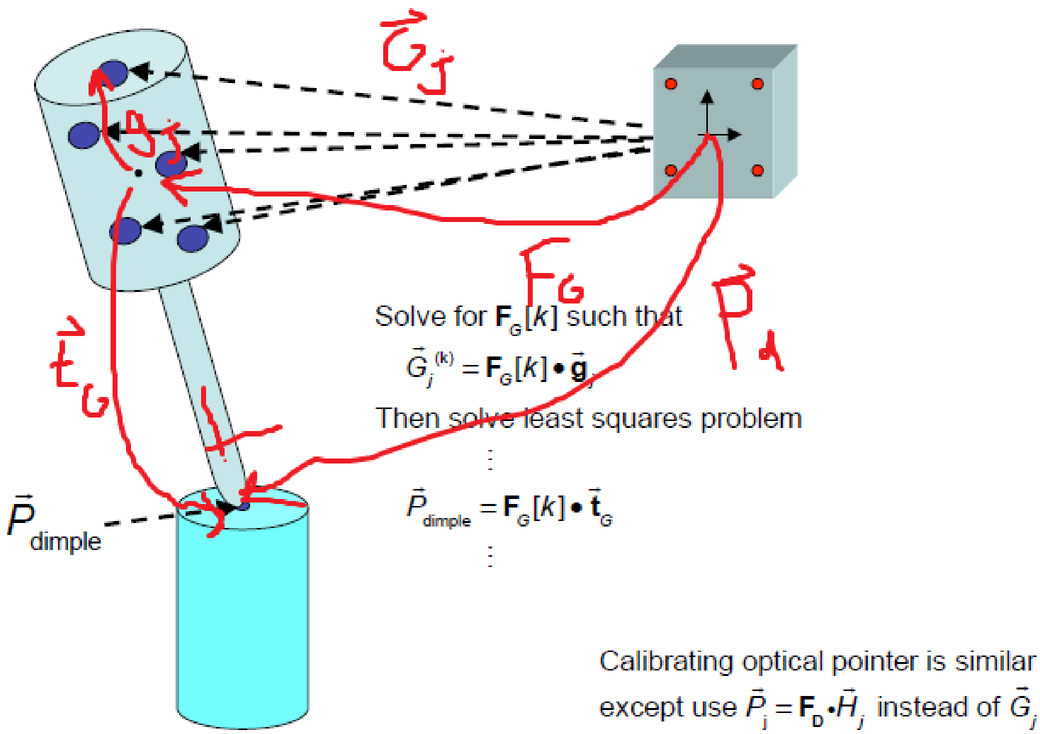


Figure 1: Figure showing transformations and vectors for the EM pivot calibration.

<u>Algorithmic Steps Followed</u>:

For calculating the frame transformation between the EM tracker and calibration body (as is done in our getTransformation.m file):

1. Calculate the centroid (mean) point of each point cloud in the given frame.
2. Calculate the 3x3 Hermitian matrix.
3. Structure this into a 4x4 matrix G, whose eigenvectors are unit quaternions.

4. Find the largest positive eigenvalue and corresponding eigenvector.
5. Convert this eigenvector into the corresponding 3x3 Rotation matrix.
6. Calculate the translation vector using the formula for a frame transformation.

For calculating the pivot calibrations using the EM tracker:

1. Use the first frame to define a local probe coordinate G_o.
2. Subtract G_o from each G to get g.
3. For each frame k, compute $F_G(k)$ such that $G = F_G(k)^* g$.
4. Solve the least squares problem to get P_{dimple}.

For calculating the pivot calibrations using the EM tracker:

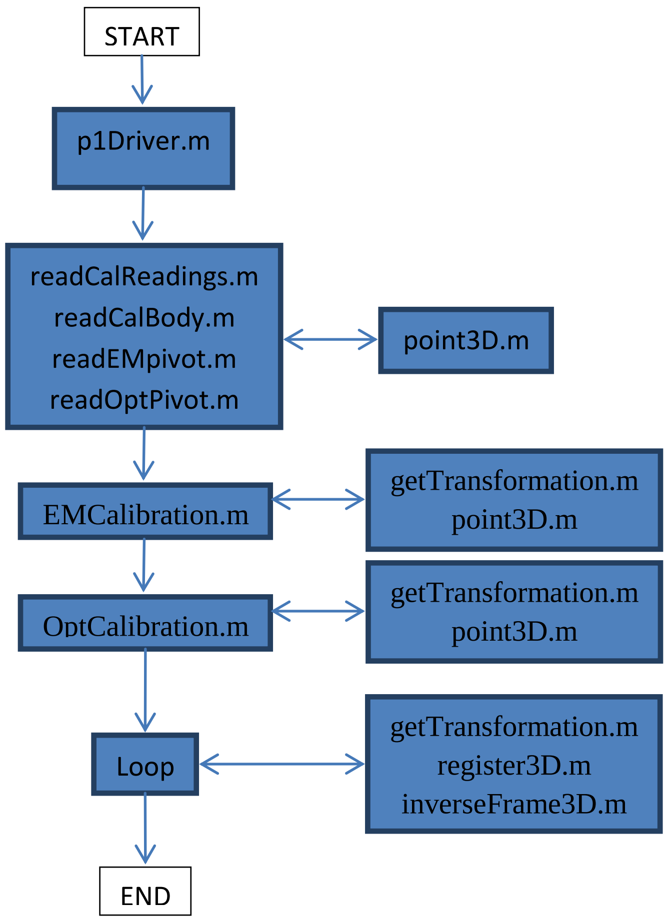
1. Find the frame transformation F_D between the optical tracker and the LEDs on the EM tracker.
2. Transform the H vector using F_D.
3. Use the first frame to define a local probe coordinate $(H_d)_o$.
4. Subtract $(H_d)_o$ from each H_D to get h.
5. For each frame k, compute $F_H(k)$ such that $H_D = F_H(k)^* h$.
6. Solve the least squares problem to get P_{dimple}.

Steps Taken to Verify That the Program Is Working Correctly:

During our programming process, we ensured each module of our code worked before working on the next one. In this process, we built up from the basic modules--such as defining a point as a vector and quaternion--to more complex functions.

Within modules, we embedded debug lines into our code (as can be seen, for example, in the file getTransformation.m). This provided us with a means to determine what was going wrong during runtime. Furthermore, the MATLAB command window stores all variables used and allows us to visually observe the matrices to double check where errors were made.

Overview of the Structure of the Program:



Tabular Summary of the Result Obtained for Unknown Data:

On attached pages (5 pages for data set H, 5 pages for data set I).

Short discussion for the Results of Running Our Program:

We ran our program over the 9 data sets (labeled a-i) presented to us in the test_files.zip file on the CISST website. The final version of our program was able to successfully reproduce the output of the first 7 "debug" files, with very little mathematical error. A total of 9 output files can be found in the ‘output’ folder.

Our approach is non-iterative, in the sense that we do not start with a "guess" value and then try to optimize its error. Our frame transformation use some iteration (for summing and averaging over elements), but the mathematical calculations for least squares analysis are done through matrix multiplications.

Based on the high level of accuracy for our output, as compared to the test output, for the initial 7 data sets, we feel confident that our output for the two unknown data sets presented on the following pages is correct.

Short Statement of Who Did What:

Throughout this assignment, we decided to work via paired programming. We met over three days, Thursday Oct. 14, Saturday Oct. 16, and Sunday Oct. 16 and worked on the program together. We used Dropbox as a subversioning system. Though the majority of the code was written in paired programming and we worked together to debug it, the following pieces of code were done individually:

- EM Calibration algorithm: implemented by Alperen.
- Modifications to quaternion algorithm presented in class: implemented by Saumya.

Program Description - README.txt

The README.txt file, which explains the contents of our program, is on the following pages.

Reference Papers:

- "Ancient Architecture Reconstructing Based on Terrestrial 3D Laser Scanning Technology." Liu, Renyi; Shi, Guigang; Ji, Fengquan. Software Engineering and Data Mining (SEDM), 2010 2nd International Conference on. June 2010, page(s): 285-288. http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=5542910&isnumber=5542824

- "Point-Cloud-to-Point-Cloud Technique on Tool Calibration for Dental Implant Surgical Path Tracking." Lorsakul, A., Sinthanayothin, C., and Suthakorn, J. Proceedings of the SPIE Medical Imaging 2008 (SPIE2008). Feb 2008. http://www.bartlab.org/Dr.%20Jackrit's%20Papers/ney/4.Lorsakul_ProcSPIE.pdf

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===CIS Programming Assignment 1===

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Course:   Computer Integrated Surgery, EN.600.445
          Professor Russ Taylor
          Fall 2010
          http://www.cisst.org/~cista/445/445-schedule-2010.html

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This code package may be found online at http://code.sgurbani.com/cis/source/
no earlier than the submission date for PA2 (on or after November 2, 2010).

--PROGRAM STRUCTURE--

Total of 14 files located in the PROGRAMS/source/ directory.

-point3D.m               - struct defining a 3-dim point
-rotate3D.m              - applies a frame rotation to a point in space
-frame3D.m               - defines a frame transformation (rotate + translate)
-inverseFrame3D.m        - calculates the inverse transformation of a given one
-transform3D.m           - performs a frame transformation between 2 points
-register3D.m            - performs a series of frame transformations
-getTransformation.m     - calculates the transformation between 2 point clouds
-EMCalibration.m         - determine the position of dimple relative to EM tracker
-OptCalibration.m        - determine the position of dimple rel. to Opt tracker
-readCalBody.m           - reads in the CalBody.txt files
-readCalReadings.m       - reads in the CalReadings.txt files
-readEMpivot.m           - read in the empivot.txt files
-readOptPivot.m          - read in the optpivot.txt files
-p1Driver.m              - A MATLAB script file to run the program


--INPUT / OUTPUT--

Input files are located in the PROGRAMS/input/ directory.

Input files should be of the form "pa1-debug-X-TYPE.txt" or "pa1-unknown-X-TYPE.txt",
where X is the letter of the data set and TYPE is the type of input data (calreadings, calbody,
etc).
The form "pa1-debug-" will be used if a corresponding "pa1-debug-X-output1.txt" is found,
otherwise
the "pa1-unknown-" form will be used.

Output: creates one file named "pa1-X-output1.txt" in the PROGRAMS/output/ directory for every
input
data set, where X is the letter of the data set.
```

--EXECUTING PROGRAM--

The script file p1Driver.m located in PROGRAMS/source/ is to be run from the MATLAB command window.

--REPORT--

The REPORT/ directory contains our programming journal and report.
The REPORT/related_papers/ directory contains PDFs of journal articles we cite and use for our program.

DATA - Set H

	27	8	pa1-h-output1.txt
EM Probe	199.55	197.11	205.24
Opt. Probe	398.26	404.89	208.47
Frame 1	190.01	224.73	210.34
	190.55	223.9	335.34
	191.08	223.07	460.33
	189.82	349.73	211.17
	190.36	348.9	336.17
	190.89	348.07	461.17
	189.64	474.73	212
	190.17	473.9	337
	190.71	473.07	462
	315.01	224.93	209.81
	315.55	224.1	334.8
	316.08	223.26	459.8
	314.82	349.92	210.64
	315.36	349.09	335.64
	315.89	348.26	460.63
	314.63	474.92	211.47
	315.17	474.09	336.47
	315.71	473.26	461.46
	440.01	225.12	209.27
	440.54	224.29	334.27
	441.08	223.46	459.27
	439.82	350.11	210.11
	440.36	349.28	335.1
	440.89	348.45	460.1
	439.63	475.11	210.94
	440.17	474.28	335.93
	440.7	473.45	460.93
Frame 2	221.19	191.8	448.21
	219.43	190.57	573.19
	217.66	189.35	698.17
	222.96	316.78	449.46
	221.2	315.56	574.44
	219.43	314.33	699.42
	224.73	441.76	450.71
	222.97	440.54	575.69
	221.2	439.31	700.67
	346.17	190.01	449.96
	344.4	188.79	574.94
	342.64	187.56	699.92
	347.94	314.99	451.21
	346.17	313.77	576.19
	344.41	312.54	701.17
	349.71	439.97	452.46

	347.94	438.75	577.44
	346.18	437.52	702.42
	471.14	188.22	451.7
	469.38	187	576.68
	467.61	185.77	701.67
	472.91	313.21	452.95
	471.15	311.98	577.93
	469.38	310.76	702.92
	474.68	438.19	454.2
	472.92	436.96	579.18
	471.15	435.74	704.17
Frame 3	202.34	448.1	207.97
	204.09	447.11	332.95
	205.84	446.12	457.94
	201.11	573.09	208.98
	202.86	572.1	333.96
	204.61	571.11	458.95
	199.88	698.08	209.99
	201.63	697.09	334.97
	203.38	696.1	459.96
	327.32	449.35	206.23
	329.07	448.35	331.21
	330.82	447.36	456.2
	326.09	574.34	207.24
	327.84	573.34	332.22
	329.59	572.35	457.21
	324.86	699.33	208.25
	326.61	698.33	333.23
	328.36	697.34	458.22
	452.3	450.59	204.49
	454.05	449.6	329.47
	455.8	448.61	454.45
	451.07	575.58	205.5
	452.82	574.59	330.48
	454.57	573.6	455.46
	449.84	700.57	206.51
	451.59	699.58	331.49
	453.34	698.59	456.47
Frame 4	212.32	465.55	456.91
	211.74	466.78	581.9
	211.16	468.01	706.89
	211.02	590.54	455.67
	210.44	591.77	580.66
	209.86	593	705.66
	209.72	715.53	454.43
	209.14	716.76	579.43
	208.56	717.99	704.42

	337.31	466.86	457.47
	336.73	468.09	582.46
	336.16	469.32	707.46
	336.01	591.85	456.23
	335.43	593.08	581.23
	334.86	594.31	706.22
	334.71	716.83	455
	334.13	718.06	579.99
	333.55	719.3	704.98
	462.3	468.17	458.03
	461.72	469.4	583.03
	461.15	470.63	708.02
	461	593.15	456.8
	460.42	594.38	581.79
	459.85	595.61	706.78
	459.7	718.14	455.56
	459.12	719.37	580.55
	458.55	720.6	705.55
Frame 5	435.31	203.84	204.17
	435.6	206.36	329.14
	435.88	208.88	454.11
	434.88	328.81	201.64
	435.16	331.34	326.62
	435.45	333.86	451.59
	434.44	453.79	199.12
	434.72	456.31	324.1
	435.01	458.83	449.07
	560.31	204.27	203.87
	560.6	206.79	328.85
	560.88	209.32	453.82
	559.88	329.25	201.35
	560.16	331.77	326.32
	560.45	334.29	451.3
	559.44	454.22	198.83
	559.72	456.74	323.8
	560.01	459.26	448.78
	685.31	204.7	203.58
	685.6	207.23	328.55
	685.88	209.75	453.53
	684.87	329.68	201.06
	685.16	332.2	326.03
	685.44	334.72	451
	684.44	454.65	198.54
	684.72	457.17	323.51
	685.01	459.7	448.48
Frame 6	454.07	214.61	451.66
	450.57	212.26	576.59

	447.08	209.91	701.52
	456.06	339.57	454.06
	452.57	337.22	578.99
	449.07	334.87	703.92
	458.06	464.53	456.47
	454.56	462.18	581.4
	451.06	459.83	706.33
	579.01	212.55	455.12
	575.51	210.2	580.04
	572.01	207.85	704.97
	581	337.51	457.52
	577.5	335.16	582.45
	574.01	332.81	707.38
	582.99	462.47	459.93
	579.5	460.12	584.86
	576	457.77	709.78
	703.94	210.49	458.57
	700.44	208.14	583.5
	696.95	205.79	708.43
	705.93	335.45	460.98
	702.44	333.1	585.91
	698.94	330.75	710.84
	707.93	460.41	463.38
	704.43	458.06	588.31
	700.94	455.72	713.24
Frame 7	449.93	444.83	207.42
	451.37	441.59	332.37
	452.81	438.36	457.32
	451.8	569.77	210.63
	453.24	566.54	335.58
	454.68	563.3	460.53
	453.67	694.72	213.85
	455.11	691.48	338.8
	456.56	688.25	463.75
	574.91	442.99	205.93
	576.35	439.76	330.88
	577.79	436.53	455.83
	576.78	567.94	209.14
	578.22	564.7	334.09
	579.66	561.47	459.04
	578.65	692.88	212.36
	580.09	689.65	337.31
	581.53	686.41	462.26
	699.88	441.16	204.44
	701.33	437.93	329.39
	702.77	434.69	454.34
	701.76	566.11	207.65

	703.2	562.87	332.6
	704.64	559.64	457.55
	703.63	691.05	210.87
	705.07	687.81	335.82
	706.51	684.58	460.77
Frame 8	443.22	462.8	451.23
	442.88	463.14	576.23
	442.55	463.49	701.23
	439.5	587.74	450.88
	439.17	588.09	575.88
	438.84	588.44	700.88
	435.79	712.69	450.52
	435.45	713.03	575.52
	435.12	713.38	700.52
	568.16	466.51	451.56
	567.83	466.86	576.56
	567.49	467.21	701.56
	564.45	591.46	451.2
	564.11	591.8	576.2
	563.78	592.15	701.2
	560.73	716.4	450.84
	560.4	716.75	575.84
	560.06	717.1	700.84
	693.11	470.23	451.88
	692.77	470.57	576.88
	692.44	470.92	701.88
	689.39	595.17	451.52
	689.06	595.52	576.52
	688.72	595.87	701.52
	685.68	720.12	451.17
	685.34	720.46	576.17
	685.01	720.81	701.17

DATA - Set I

	27	8	pa1-i-output1.txt
EM Probe	203.31	194.98	203.34
Opt. Probe	406.65	392.07	209.85
Frame 1	197.91	222.48	209.01
	202.43	224.7	333.91
	206.95	226.91	458.81
	196.87	347.46	206.84
	201.39	349.67	331.73
	205.9	351.88	456.63
	195.83	472.44	204.66
	200.35	474.65	329.56
	204.86	476.86	454.46
	322.83	223.44	204.48
	327.34	225.66	329.38
	331.86	227.87	454.28
	321.79	348.42	202.3
	326.3	350.63	327.2
	330.82	352.85	452.1
	320.75	473.4	200.13
	325.26	475.61	325.03
	329.78	477.82	449.92
	447.74	224.41	199.95
	452.26	226.62	324.84
	456.77	228.83	449.74
	446.7	349.38	197.77
	451.22	351.6	322.67
	455.73	353.81	447.57
	445.66	474.36	195.59
	450.18	476.57	320.49
	454.69	478.78	445.39
Frame 2	230.83	204.22	453.65
	226.4	203.21	578.57
	221.97	202.2	703.48
	227.32	329.17	454.54
	222.9	328.16	579.45
	218.47	327.15	704.37
	223.82	454.12	455.42
	219.4	453.11	580.34
	214.97	452.1	705.26
	355.7	207.69	458.1
	351.27	206.68	583.02
	346.85	205.67	707.94
	352.2	332.64	458.99
	347.77	331.63	583.91
	343.35	330.62	708.82
	348.7	457.58	459.88

	344.27	456.57	584.79
	339.85	455.56	709.71
	480.57	211.16	462.55
	476.14	210.15	587.47
	471.72	209.13	712.39
	477.07	336.1	463.44
	472.64	335.09	588.36
	468.22	334.08	713.28
	473.57	461.05	464.33
	469.14	460.04	589.25
	464.72	459.03	714.16
Frame 3	202.94	444.31	204.39
	206.51	441.7	329.31
	210.09	439.08	454.23
	205.6	569.25	206.93
	209.18	566.64	331.85
	212.75	564.03	456.77
	208.27	694.2	209.46
	211.84	691.59	334.38
	215.41	688.98	459.3
	327.86	441.72	200.76
	331.43	439.11	325.68
	335.01	436.49	450.61
	330.52	566.66	203.3
	334.1	564.05	328.22
	337.67	561.44	453.14
	333.19	691.61	205.83
	336.76	689	330.76
	340.33	686.39	455.68
	452.78	439.13	197.14
	456.35	436.52	322.06
	459.93	433.91	446.98
	455.44	564.07	199.67
	459.02	561.46	324.59
	462.59	558.85	449.51
	458.11	689.02	202.21
	461.68	686.41	327.13
	465.25	683.8	452.05
Frame 4	220.63	454.44	455.58
	216.57	450.93	580.46
	212.51	447.43	705.35
	224.33	579.33	459.21
	220.27	575.83	584.09
	216.21	572.32	708.98
	228.03	704.22	462.83
	223.97	700.72	587.72
	219.91	697.21	712.6

	345.5	450.62	459.53
	341.45	447.11	584.41
	337.39	443.61	709.3
	349.21	575.51	463.16
	345.15	572.01	588.04
	341.09	568.5	712.93
	352.91	700.41	466.79
	348.85	696.9	591.67
	344.79	693.39	716.55
	470.38	446.8	463.48
	466.32	443.3	588.37
	462.27	439.79	713.25
	474.09	571.7	467.11
	470.03	568.19	591.99
	465.97	564.68	716.88
	477.79	696.59	470.74
	473.73	693.08	595.62
	469.67	689.57	720.51
Frame 5	445.62	190.89	206.04
	444.21	195.62	330.94
	442.81	200.35	455.84
	446.95	315.79	201.32
	445.55	320.52	326.22
	444.14	325.26	451.12
	448.28	440.69	196.6
	446.88	445.43	321.5
	445.47	450.16	446.4
	570.6	189.61	207.49
	569.2	194.34	332.39
	567.79	199.07	457.29
	571.94	314.51	202.77
	570.53	319.24	327.67
	569.13	323.98	452.58
	573.27	439.42	198.05
	571.86	444.15	322.95
	570.46	448.88	447.86
	695.59	188.33	208.94
	694.18	193.06	333.84
	692.78	197.79	458.75
	696.92	313.23	204.22
	695.52	317.97	329.13
	694.11	322.7	454.03
	698.25	438.14	199.51
	696.85	442.87	324.41
	695.44	447.6	449.31
Frame 6	456.55	196.79	452.57
	457.49	197	577.57

	458.43	197.22	702.57
	460.8	321.72	452.33
	461.74	321.93	577.32
	462.68	322.14	702.32
	465.04	446.64	452.08
	465.98	446.86	577.08
	466.92	447.07	702.07
	581.47	192.54	451.64
	582.41	192.76	576.64
	583.35	192.97	701.63
	585.72	317.47	451.39
	586.66	317.68	576.39
	587.6	317.9	701.39
	589.96	442.4	451.15
	590.91	442.61	576.14
	591.85	442.82	701.14
	706.4	188.29	450.71
	707.34	188.51	575.7
	708.28	188.72	700.7
	710.64	313.22	450.46
	711.58	313.44	575.46
	712.52	313.65	700.45
	714.89	438.15	450.22
	715.83	438.36	575.21
	716.77	438.58	700.21
Frame 7	466.09	439.29	211.11
	463.98	436.88	336.07
	461.87	434.47	461.03
	471.39	564.15	213.61
	469.28	561.74	338.57
	467.17	559.33	463.53
	476.69	689.01	216.11
	474.58	686.6	341.06
	472.47	684.19	466.02
	590.96	433.95	213.12
	588.85	431.54	338.08
	586.74	429.13	463.04
	596.26	558.81	215.61
	594.15	556.4	340.57
	592.04	553.99	465.53
	601.56	683.67	218.11
	599.45	681.26	343.07
	597.34	678.85	468.03
	715.83	428.61	215.12
	713.72	426.2	340.08
	711.61	423.79	465.04
	721.13	553.47	217.62

	719.02	551.06	342.58
	716.91	548.65	467.54
	726.43	678.33	220.12
	724.32	675.92	345.08
	722.21	673.51	470.03
Frame 8	426.17	454.55	440.73
	426.55	456.01	565.73
	426.92	457.46	690.72
	423.56	579.52	439.29
	423.94	580.97	564.28
	424.32	582.43	689.27
	420.96	704.48	437.84
	421.33	705.94	562.83
	421.71	707.39	687.82
	551.14	457.15	440.33
	551.52	458.61	565.32
	551.89	460.06	690.31
	548.54	582.12	438.88
	548.91	583.57	563.87
	549.29	585.03	688.86
	545.93	707.08	437.43
	546.3	708.54	562.43
	546.68	709.99	687.42
	676.11	459.76	439.92
	676.49	461.21	564.91
	676.87	462.67	689.9
	673.51	584.72	438.47
	673.88	586.18	563.47
	674.26	587.63	688.46
	670.9	709.69	437.03
	671.28	711.14	562.02
	671.65	712.59	687.01