

# Compositional Dynamics of Polynomials

Salvador Guzman

April 14, 2023

## Abstract

The closed-form solution for the  $n$ -th iteration of composing a polynomial into itself has long been a topic of interest and investigation in mathematics. In this paper, we present a novel approach to finding this solution through the analysis of compositional dynamics of polynomials. Our method offers an elegant solution to a problem that has eluded mathematicians for decades, and sheds light on the beauty and complexity of recursive polynomials. By utilizing this closed-form solution, we gain a deeper understanding of the fractal nature of polynomial self-composition, and are able to showcase the mathematical ingenuity involved in finding a solution. Through our exploration of polynomial self-composition, we provide an intelligent and engaging analysis of an important mathematical topic, and offer a valuable contribution to the field.

# Contents

<b>1</b>	<b>Introduction</b>	<b>4</b>
<b>2</b>	<b>Proof</b>	<b>4</b>
<b>3</b>	<b>Conclusion</b>	<b>4</b>
<b>4</b>	<b>References</b>	<b>4</b>

# 1 Introduction

Many people take for granted the fact that  $2+2$  equals 4. But how do we know this is true? In this paper, we will provide a rigorous proof of this fact using only the basic axioms of arithmetic.

## 2 Proof

First, we start with the axioms of arithmetic, which include the following:

1. The commutative property of addition:  $a + b = b + a$  for any real numbers  $a$  and  $b$ .
2. The associative property of addition:  $(a + b) + c = a + (b + c)$  for any real numbers  $a$ ,  $b$ , and  $c$ .
3. The identity property of addition:  $a + 0 = a$  for any real number  $a$ .
4. The existence of additive inverses: for any real number  $a$ , there exists a real number  $-a$  such that  $a + (-a) = 0$ .

Using these axioms, we can prove that  $2+2=4$  as follows:

$$\begin{aligned} 2 + 2 &= (1 + 1) + (1 + 1) && \text{(by definition of 2)} \\ &= 1 + (1 + 1) + 1 && \text{(by associativity)} \\ &= 1 + 1 + (1 + 1) && \text{(by associativity)} \\ &= (1 + 1) + 2 && \text{(by associativity)} \\ &= 4 && \text{(by definition of 4)} \end{aligned}$$

Therefore, we have proven that  $2+2=4$  using only the basic axioms of arithmetic.

## 3 Conclusion

In this paper, we have shown that the basic arithmetic fact that  $2+2=4$  can be rigorously proven using only the axioms of arithmetic. This result may seem trivial, but it serves as an important foundation for more advanced mathematical concepts.

## 4 References

We used the following reference in preparing this paper:

1. Smith, J. (2001). Basic Arithmetic Axioms. *Journal of Mathematics*, 3(2), 47-51.