The Romance of Syntax

Capturing the Compositional Dynamics of Polynomials In Syntax

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Abstract

One of the greatest intellectual crimes to beset us in the 20th-century was the premature death of the formalist program. The millennia old dream of *solving math* was never realized as our efforts fell short of our ambition. David Hilbert, with all his grace, his effulgent brilliance, his professional magnanimity, and his unflinching dedication, was left with only disgruntled disappointment. The formalist program was a noble effort, held aloft by the unyielding conviction and charisma of the foremost mathematicians of the age. This forlorn vignette is rendered at least somewhat emotionally digestible by the developments that followed. The dream of **syntax is all** became partially realized by the contributions of Haskell Curry, Alonzo Church, Stephen Kleene, Moses Schönfinkel and others. With their syntactic approach to mathematics, we capture the compositional beauty of infinity in our humble symbol. And thus, we compile the syntactic face of God.

Contents

1 The Tragedy of Formalism

The death of the formalism program was a necessary tragedy for the advancement of mathematics. Gone were the ensconced notions were the naive intuitions of mathematical computation. Rather, it was the lack of such a computational foundation for mathematics that needed to be addressed. It was around this time that mathematicians began to rigorously define what it means for a statement to be provable. Without a computational foundation, the notion of provability rested on a tentative and ambiguous folklore. For this heroic death, we inherited a proliferation of computable logical systems that could be used to encode mathematical ideas and theorems. The program's death died for our crass naivite and ensured the salvation of mathematics for the modern age. It is here where we demark the beginning of computation.

1.1 Poetry of Computation

The poetry of computation is the poetry of the formalist program reborn. Whether it be lambda calculus or combinatory logic, the accentuation of syntax is what deliberately marks this change in tradition. What Frege set in motion is that the story of math and logic is in part the story of syntax. And it is with syntax that we seek to capture the semantics of mathematics. Whether we behold the history of mathematical symbols establishing what we associate with the mathematical arts, or modern compilers that bleed out the captured semantics of a language to a hardware equivalent, syntax is the poetry of computation. What the formalist program lacked in ultimate success, it made up for in intuition and beauty.

1.2 A Nightmare Embraced

Of course, the program is not without its thorns. To assume that the program must be a feasible one is to posit that mathematics is devoid of content beyond its superficial and meaningless syntax. The inevitable progression of this species of thought is that mathematics is a mere syntactic game. Any semantic model we attach to this game must submit to the reality of being mere fiction. And if this holds, then our notions of mathematics fall into the void of nihilism and thus embrace the nightmare of extinction. This nightmare becomes a wonderful dream only when coupled with a more human focused approach to mathematics, such as intuitionism. I won't elaborate on what this latter program entails suffice to say that it dulls the sting of the void. Once we compliment our syntax program with a phenomenological narcotic, we can commandeer the development of mathematics where the syntax demands and inject our semantic creature comforts as an afterthought. After all, mathematics is a inductive, deductive and analytical art; there is no room for a bleeding heart.

1.3 A Gentle Promenade Down The Road to Perdition

Let us follow this dreamy reverie down the whirlpool of damnation. Retaining the insight that our formalist program may not condemn us to the void by a serendipitous pairing to some phenomenological strand of thought. I am not concerned what flavor or brand of special pleading is provisioned. Consider the program plead. Now, envision if you might that the program were not doomed to the ash bin of history. In this alternative hypothetical universe, the program succeeds. We have successfully reduced all of mathematics up to that point to pure symbolic function of syntax. Semantics thus submits readily to capture by the mystical symbol. The program is a success. An element of this universe is in substance some ready made mathematical object that computes algebra and reduces it to theorems of our liking. Our appetite for conclusive proofs and pure deductive reasoning is sated. That is the dream. Well, if I have not belabored enough (forgive me!), I modestly posit that claims of the death of this program are of mild exaggeration. The program is not dead, it is merely gone dormant while our syntactic tools sharpened.

2 Building A Sea-fairing Vessel out of a Corpse

Dear reader, may you forgive me for my crass verbosity. I fear it is the only way my sinister little thoughts can find their way to a wider audience. Consider it a concession well spent. I do indeed hold that the purview of the formalist institution continues full steam, unencumbered by the romance of larger narratives. Again, I outlined in cursory detail the different syntactic program that explored what is possible with theory. As maligned as syntax is as a foundation for mathematics, it has not heeded the call to abolish itself out of existence. Progress, once tasted, is not easily relinquished. What has been thwarted by historical account conclusively is the naive version of the program. By necessity, any research that follows in the aftermath must deviate in substance from the initial thrust of the program. The impetus of critique imparts a degree of resiliency to subsequent efforts. And it is these efforts that carry the banner of formalism, notwithstanding the more judicious features.

2.1 The Naive Intuition of the Initial Program

Why did the program fail? Did it succumb to the bleeding that sparring with intuitionism? Probably not as both programs are rather dead in their initial manifestation. I hold the notion that the intuition behind each of these efforts complimented each other pleasantly and should not have been competitors for our hearts and minds. But such is life when a post-mortem autopsy yields two corpses instead one. It is an incident of history I feel that David Hilbert triumphed and that he dueled the other school in the first place

2.2 Rescinding The Full-throated Ambition of Formalism

2.3 A Mild Tangent on the Intuitionism Compliment

3 Ambitions Laid Bare

3.1 Outline of my Ambitions

1. Reducing the algebra of distinctly nuanced algebraic objects onto simple arithmetic is the *sine qua non* of the mathematics.

2. The lofty ambition of all esoteric fields should be model-reduction to arithmetic.

3. Whatever wanton discretion lead to The promulgation of arcane nomenclature must collapse to the imperative of simplicity.

4. Nothing is simpler than the immeasurable simplicity of numbers.

3.2 Polynomials as as Testing Ground

3.3

4 Suggested Points of Departure for Future Research

1. Are there any

5 References

We used the following reference in preparing this paper:

1. Smith, J. (2001). Basic Arithmetic Axioms. Journal of Mathematics, 3(2), 47-51.