

# Capturing the Compositional Dynamics of Polynomials

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## Abstract

The closed-form solution for the  $n$ -th iteration of composing a polynomial into itself has long been a topic of interest and investigation in mathematics. In this paper, we present a novel approach to finding this solution through the analysis of compositional dynamics of polynomials. Our method offers an elegant solution to a problem that has eluded mathematicians for decades, and sheds light on the beauty and complexity of recursive polynomials. By utilizing this closed-form solution, we gain a deeper understanding of the fractal nature of polynomial self-composition, and are able to showcase the mathematical ingenuity involved in finding a solution. Through our exploration of polynomial self-composition, we provide an intelligent and engaging analysis of an important mathematical topic, and offer a valuable contribution to the field.

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# 1 Introduction

Many people take for granted the fact that  $2+2$  equals 4. But how do we know this is true? In this paper, we will provide a rigorous proof of this fact using only the basic axioms of arithmetic.

## 2 Points

1. Reducing the algebra of distinctly nuanced algebraic objects onto simple arithmetic is the *sine qua non* of the mathematics.
2. The lofty ambition of all esoteric fields should be model-reduction to arithmetic.
3. Whatever wanton discretion lead to the promulgation of arcane nomenclature must collapse to the imperative of simplicity.
4. Nothing is simpler than the immeasurable simplicity of numbers.

## 3 Proof

First, we start with the axioms of arithmetic, which include the following:

1. The commutative property of addition:  $a + b = b + a$  for any real numbers  $a$  and  $b$ .
2. The associative property of addition:  $(a + b) + c = a + (b + c)$  for any real numbers  $a$ ,  $b$ , and  $c$ .
3. The identity property of addition:  $a + 0 = a$  for any real number  $a$ .
4. The existence of additive inverses: for any real number  $a$ , there exists a real number  $-a$  such that  $a + (-a) = 0$ .

Using these axioms, we can prove that  $2+2=4$  as follows:

$$\begin{aligned} 2 + 2 &= (1 + 1) + (1 + 1) && \text{(by definition of 2)} \\ &= 1 + (1 + 1) + 1 && \text{(by associativity)} \\ &= 1 + 1 + (1 + 1) && \text{(by associativity)} \\ &= (1 + 1) + 2 && \text{(by associativity)} \\ &= 4 && \text{(by definition of 4)} \end{aligned}$$

Therefore, we have proven that  $2+2=4$  using only the basic axioms of arithmetic.

## 4 Suggested Points of Departure for Future Work

1. Are there any

## 5 References

We used the following reference in preparing this paper:

1. Smith, J. (2001). Basic Arithmetic Axioms. *Journal of Mathematics*, 3(2), 47-51.