Monte Carlo Simulation Framework For Pricing Exotic Lookback Options

Introduction

This project focuses on pricing exotic options—specifically **floating and fixed strike lookback options**—on four assets: **Gold, Oil, Bitcoin, and ICICI Bank**. These options derive their value from the optimal price path of the underlying asset, making them sensitive to market volatility.

We implement a **Monte Carlo simulation framework** using **geometric Brownian motion** to model asset price paths. Key market parameters such as **interest rates, volatility, dividend yield, time to maturity,** and **spot prices** are incorporated. To enhance accuracy and efficiency, we apply the **antithetic variates** technique for variance reduction and conduct **error estimation and convergence analysis**.

This study extends prior correlation analysis and provides insight into the pricing dynamics of exotic derivatives across diverse asset classes.

Market Parameters

This simulation evaluates exotic option pricing on four diverse assets—**Gold, Oil, Bitcoin, and ICICI Bank**—using data as of **May 1, 2025**. For each asset, the **spot price**, **annualized volatility**, and **dividend yield** are specified below.

Volatility Estimation

Volatility was calculated based on historical daily returns over the **past one year (April 1, 2024 to April 1, 2025)**. A one-year window was chosen to capture a representative level of recent market behavior while smoothing out short-term noise.

Risk-Free Rate

A **risk-free rate of 6.35%** was used, based on the **10-year Indian government bond yield** as reported on **Investing.com** on **May 1, 2025**. This reflects a long-term, market-consistent benchmark for discounting future cash flows.

Strike Price Definitions

To study how moneyness affects option prices, we consider the following strike levels:

- **At-the-money (ATM)**: Strike = Spot price
- **In-the-money (ITM)**: Strike = 90% of spot price (for calls)
- **Out-of-the-money (OTM)**: Strike = 110% of spot price (for calls)

These three categories allow us to analyze the sensitivity of fixed strike lookback options across different market scenarios and investor expectations.

Choice of Time to Maturity (1 Year):

I have taken a time to maturity of **1 year** to align with standard practice in option pricing and financial modeling.1-year horizon provides a balanced view for assessing price evolution, capturing both short-term volatility and long-term trends. It also corresponds with the annualized volatility I calculated for the period from **30 April 2024 to 30 April 2025**, ensuring consistency across model inputs.

Summary Table of Market Parameters

ASSET	SPOT PRICE	VOLATILITY	DIVIDEND YIELD	CURRENCY
GOLD	9436.00	18%	0.00%	INR
OIL	5007.38	32%	0.00%	INR
BITCOIN	8216447.00	53%	0.00%	INR
ICICI BANK	1430.10	21%	0.70%	INR

3. Monte Carlo Simulation Setup

The purpose of this section is to explain the setup and methodology behind the Monte Carlo simulations used for pricing exotic options. Monte Carlo simulations are a powerful tool for pricing financial derivatives by generating multiple possible price paths of underlying assets and computing the option's payoff under each path. The technique is especially useful for pricing complex derivatives, such as lookback options, where analytical solutions are not readily available.

Explanation of Geometric Brownian Motion (GBM)

The Geometric Brownian Motion (GBM) model is widely used in financial markets for modeling asset prices due to its simplicity and ability to capture the essential features of financial time series, such as random price fluctuations and volatility clustering. The model assumes that the asset price follows a continuous path, which is influenced by both deterministic factors (like drift) and stochastic factors (like randomness).

The GBM equation is given by:

Where:

$$S(t) = S(0) \cdot \exp\left(\left(r - \frac{\sigma^2}{2}\right) \cdot t + \sigma \cdot \sqrt{t} \cdot Z_t\right)$$

- S(t) is the asset price at time ttt
- S(0)is the initial spot price
- r is the risk-free rate
- σ is the volatility of the asset
- Zt is a standard normal random variable, representing the stochastic nature of price changes
- t is the time, usually measured in years

The first term $\left(r - \frac{\sigma^2}{2}\right) \cdot t$ represents the deterministic component of the price path, while the second term $\sigma \cdot \sqrt{t} \cdot Z_t$ captures the randomness of the price movement.

This model is ideal for pricing options and simulating price paths because it accounts for both the expected return and the randomness inherent in asset prices.

3.2 Simulation Design

Time Discretization:
The simulation period is divided into discrete time steps. For a one-year maturity with daily steps, we use 252 time steps, corresponding to the number of trading days in a typical financial year. This fine-grained discretization helps better approximate the continuous nature of GBM.

Number of Simulations:
To ensure statistical reliability of the simulation results, we experiment with a range of simulation counts:

- **1,000**, **5,000**, **10,000**, and **20,000** simulations
  This variation is used to perform **convergence analysis**, balancing computational efficiency and pricing accuracy.

Randomness Control:
A fixed random seed is used in generating the standard normal variables to ensure reproducibility. This

allows for consistent results across multiple runs, which is important for validation and comparison of techniques such as variance reduction.

3.4 Assumptions Made

The simulation framework is built on the following financial and computational assumptions:

1. **Efficient Markets:** Asset prices fully reflect all available information; no arbitrage opportunities exist.

2. **Constant Parameters:** Risk-free rate and volatility are assumed constant over the life of the option.

3. **No Transaction Costs:** There are no taxes, fees, or liquidity constraints.

4. **Lognormal Price Distribution:** GBM ensures positive prices and realistic asset evolution.

5. **No Early Exercise:** The options considered are European-style, exercised only at maturity.

6. **Homogeneous Time Steps:** Each simulation divides the total time uniformly (daily steps), assuming a continuous evolution without weekends or holidays.

7. **Independence of Simulations:** Each simulated path is independent of others, ensuring statistical validity.

3.5 Overview of Monte Carlo Process

1. **Initialization:** Set up asset-specific parameters including $S0,\sigma,r,q,T$

2. **Simulation of Price Paths:** Generate a large number of paths using GBM.

3. **Payoff Calculation:** Compute the payoff of the exotic options (floating and fixed strike lookback) for each path.

4. **Discounting:** Apply risk-free discounting to obtain present values.

5. **Statistical Estimation:** Calculate the average payoff across simulations to estimate the option price, and compute standard error for confidence intervals.

6. **Convergence Check:** Compare prices at different simulation sizes to confirm numerical stability.

This Monte Carlo simulation setup provides a robust and flexible framework for pricing complex derivatives. The GBM-based path generation, combined with realistic financial assumptions and controlled randomness, ensures that our pricing estimates are statistically sound and reflective of market behavior. The next sections discuss the implementation of exotic option payoffs and techniques used to enhance the efficiency and accuracy of these simulations.

4. Lookback Option Pricing

Lookback options are a class of exotic options that allow the holder to "look back" over the life of the option and select the optimal price—either the maximum or the minimum—for exercising the option. This feature makes them path-dependent and often more valuable than vanilla options.

We consider two types:

- **Floating Strike Lookback Options** (strike is determined at maturity)
- **Fixed Strike Lookback Options** (strike is known in advance)

All prices are estimated using Monte Carlo simulation with geometric Brownian motion (GBM) as the underlying model.

4.1 Floating Strike Lookback Option

Payoff Formulation

The strike price is not fixed in advance; instead, it is determined based on the historical minimum or maximum price observed during the option's lifetime.

- **Call Option:** $\text{Payoff} = S_T - \min(S)$
- **Put Option:** $\text{Payoff} = \max(S) - S_T$

Where:

- S_T is the terminal asset price
- $\min(S)$, $\max(S)$ are the minimum and maximum prices over the path

Pricing Approach

1. Generate multiple price paths using GBM.
2. For each path:
    - Track the minimum and maximum price over the path.

- Compute the payoff at maturity using the respective formula.

3. Discount the average payoff using the risk-free rate:

$$\text{Option Price} = e^{-rT} \cdot \mathbb{E}[\text{Payoff}]$$

4. Estimate the standard error to quantify uncertainty.

ASSET	Payoff(Floating Call)	Payoff(Floating Put)
Gold	1508.4255 ± 9.5175	1055.9901 ± 6.0705
Oil	1240.8586 ± 9.3200	1149.4944 ± 5.8187
Bitcoin	3018079.3425 ± 28234.4533	3488300.97 ± 16099.88
ICICI Bank	249.8137 ± 1.6605	198.2627 ± 1.0913

Payoff Formulation

- **Call:**Payoff=max(max(St)−K,0)
- **Put:** Payoff=max(K−min(St),0)   Where K is the strike price (ATM, OTM, or ITM).

Pricing Approach

- Monte Carlo simulation of price paths.
- For each strike level (ATM, OTM, ITM), lookback payoffs are calculated using the max/min of the path..

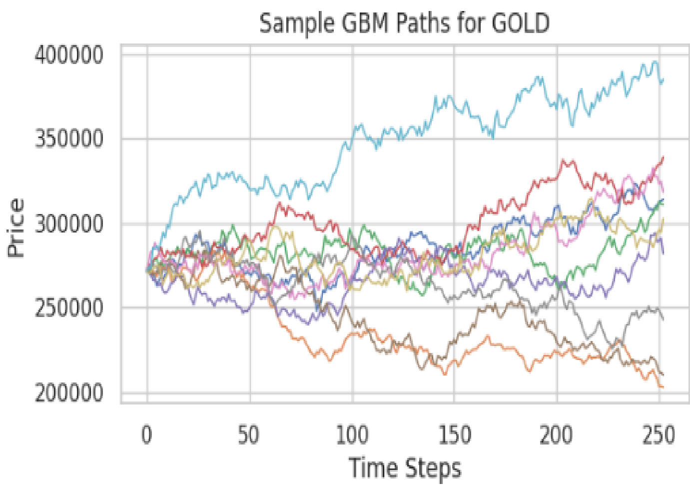
ASSET	CALL PRICE(ATM)	CALL PRICE(ITM)	CALL PRICE(OTM)
GOLD	1627.4460 ± 13.8209	2512.9902 ± 13.8209	905.0589 ± 12.2024
OIL	1446.2020 ± 14.0011	1916.1317 ± 14.0011	1043.5757 ± 13.3397
BITCOIN	3949199.11 ± 44614.74	4720291.39±44614.74	3269964.21± 43783.49
ICICI BANK	274.5584 ± 2.4312	408.7696 ± 2.4312	164.2373 ± 2.1963

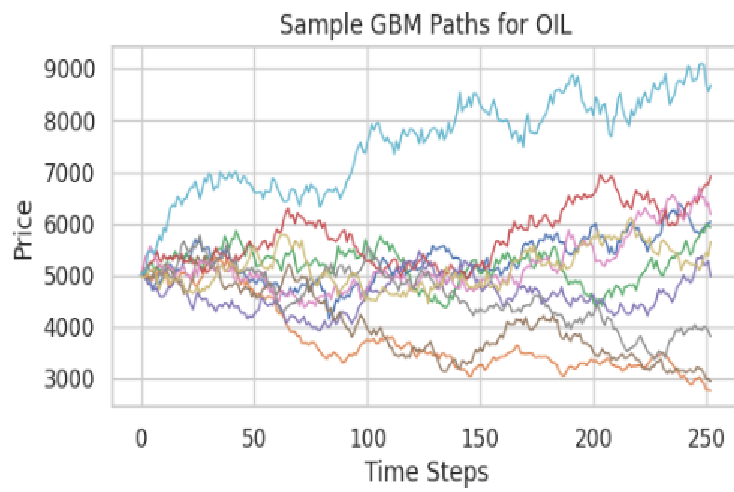
Note: Using ATM, ITM, and OTM helps understand how the option’s value and risk vary with moneyness. Higher intrinsic value in ITM cases leads to more expensive options, while OTM cases reflect lower probabilities of exercise, resulting in lower premiums.

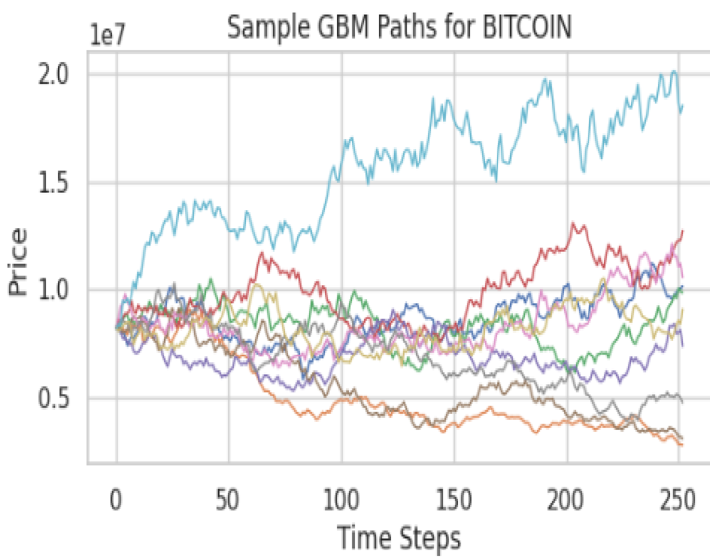
5. Visualizations

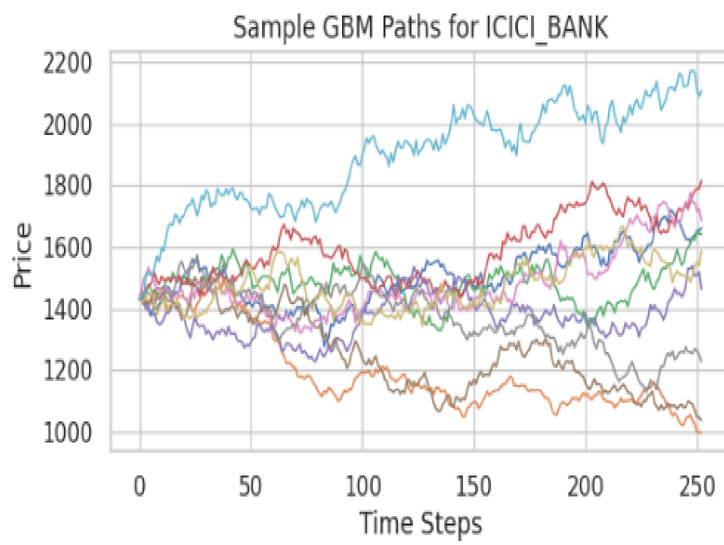
This section presents key visual insights derived from the Monte Carlo simulations. The goal is to provide an intuitive understanding of price behavior, payoff distributions, sensitivity to parameters, and cross-asset comparisons.

5.1 Sample Price Paths









🔷 *Observation:* Assets like Bitcoin exhibit wider dispersion due to higher volatility, whereas ICICI Bank and Gold show more constrained paths.

5.2: Payoff Distribution

To avoid redundancy and focus the analysis, payoff distributions are illustrated using ICICI Bank as a representative asset. This choice reflects realistic market conditions due to its moderate volatility and presence of a dividend yield, which allows analysis under all relevant factors.

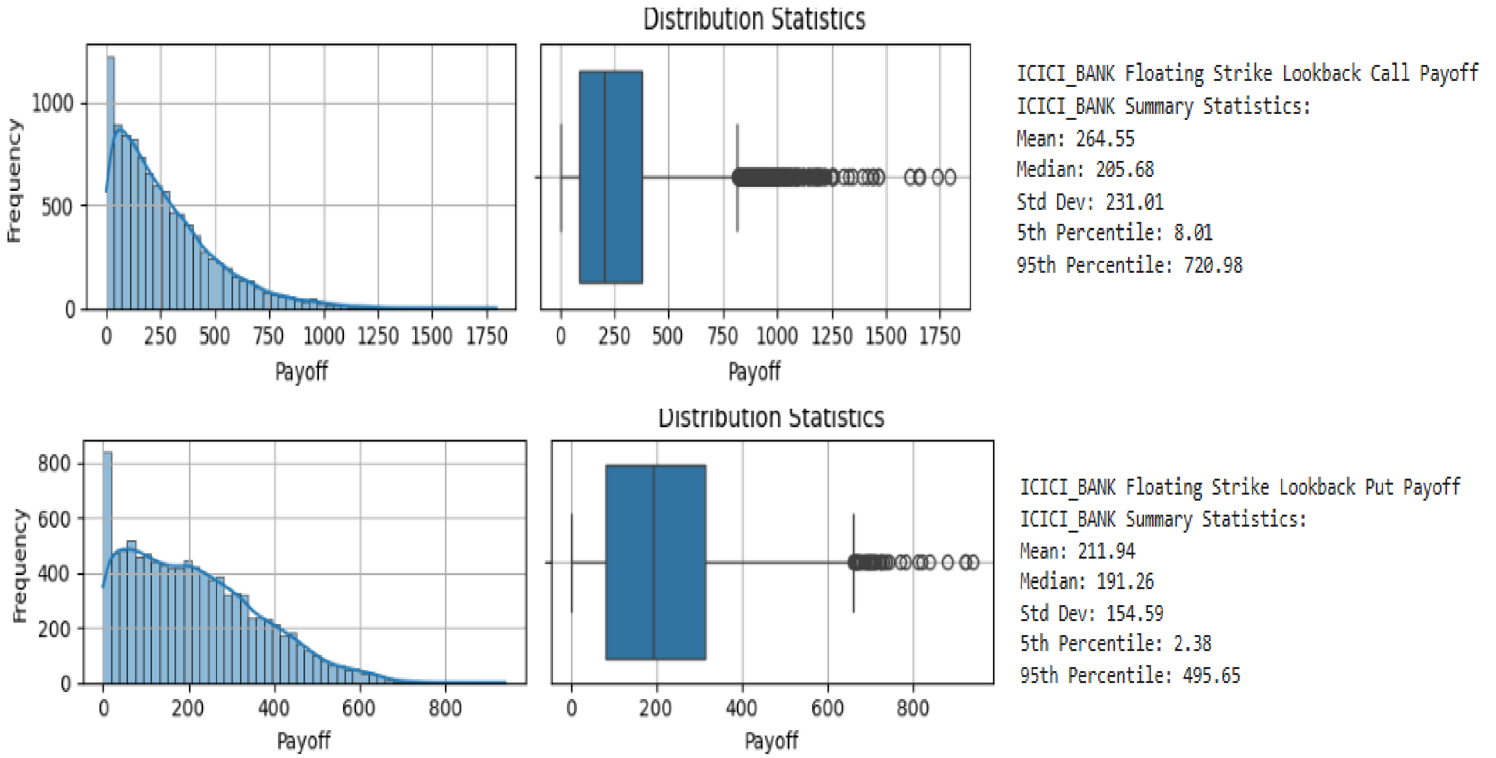
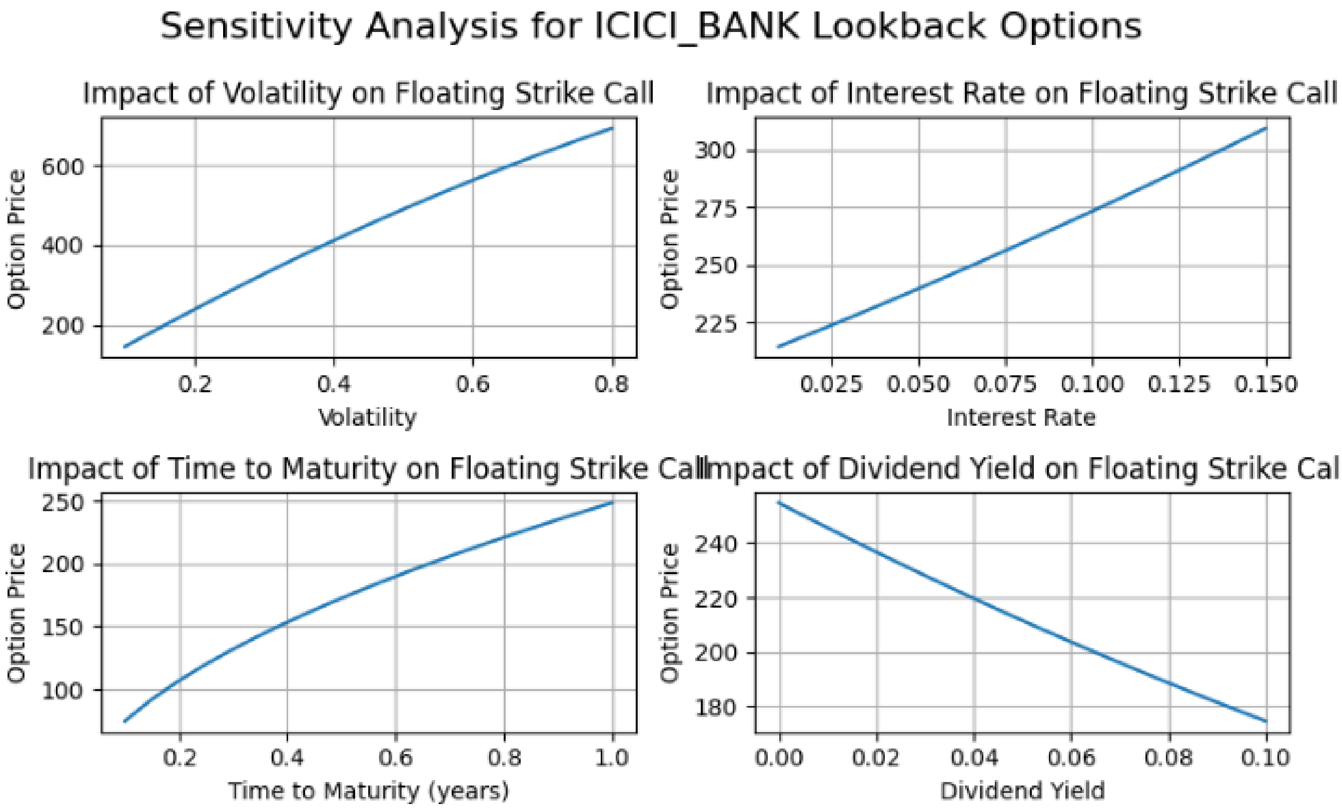


Figure: Boxplot showing the distribution of payoffs for the Floating Strike Lookback Call Option on ICICI Bank. The plot highlights the spread, median, and presence of extreme positive outliers due to favorable price movements over the option’s life.

5.3 Sensitivity Analysis

ICICI Bank was selected for sensitivity analysis due to its realistic dividend yield, moderate volatility, and its relevance as a widely traded equity. The floating strike lookback call option was chosen to highlight the path-dependent nature of exotic options, particularly in how volatility, maturity, and dividend yield influence pricing. For other assets, code has provided.



Observations:
1.**Option Price vs Volatility**-Lookback options show strong positive sensitivity to volatility, especially floating-strike calls.
2.**Option Price vs Time to Maturity**-Longer maturity generally leads to higher option values due to more opportunity for extreme price paths.
3.**Option Price vs Dividend Yield-**Only applicable to ICICI Bank. The plot shows how increasing dividend yield lowers call prices and increases put prices (as expected under the GBM model with continuous dividend yield).
4.**Option Price vs Interest Rate**-As interest rates rise, the floating call option price increases due to higher expected asset prices, with the effect tapering at higher rates and slightly offset by the dividend yield.

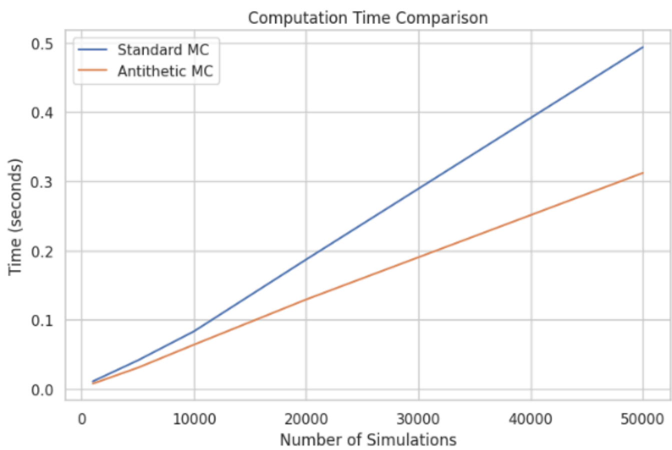
6. Variance Reduction Techniques

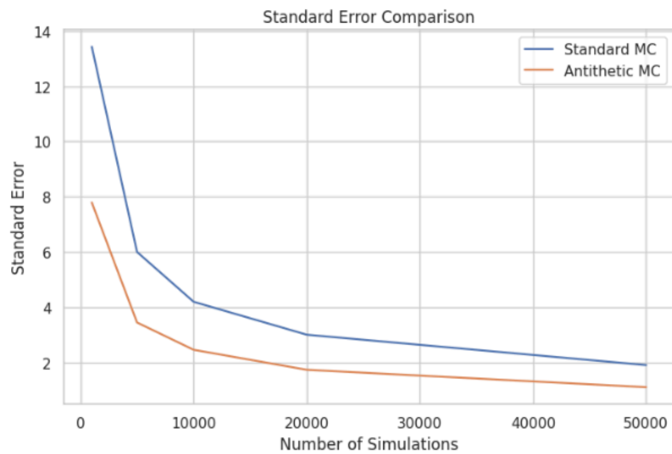
6.1 Antithetic Variates

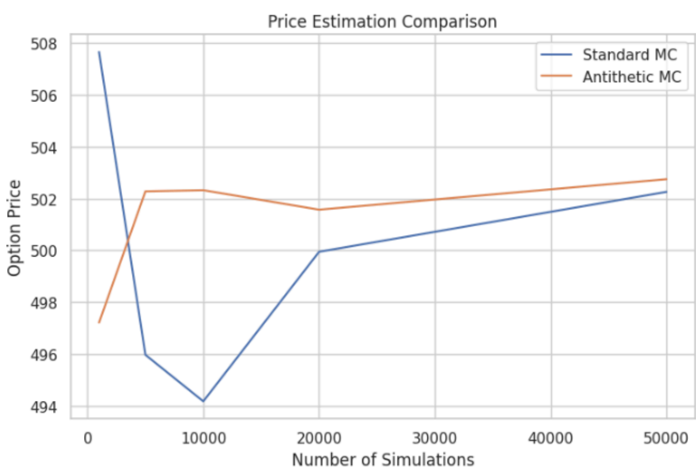
🔷 Method Explanation

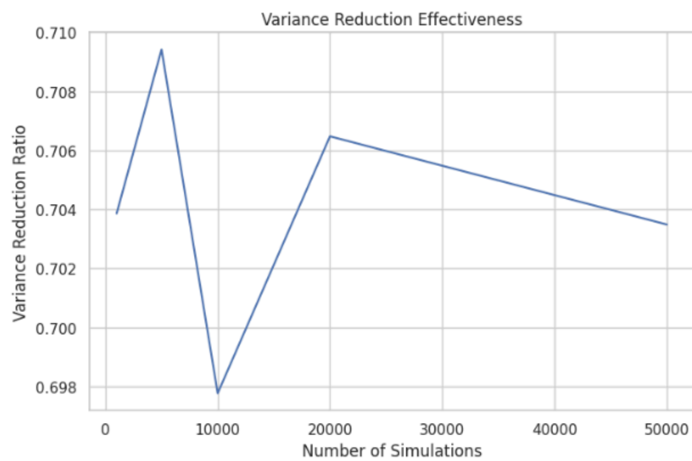
> Antithetic variates reduce variance by using negatively correlated simulations. For each random sample `Z`, we also use `-Z`. The two paths tend to "cancel out" random deviations, stabilizing the average payoff and reducing the estimator’s variance.

VISUALIZATION(GOLD ASSET)









RESULTS

Performance Comparison Summary:

n_simulations	price_std	price_anti	price_diff	std_err_std	std_err_anti
1000	42910.711434	42027.465952	883.245481	1210.589653	658.785780
5000	41922.366085	42455.833935	533.467850	540.613954	291.420607
10000	41770.592424	42459.653419	689.060995	378.158068	207.895099
20000	42258.667640	42396.071387	137.403747	270.717588	146.668791
50000	42454.197202	42495.621375	41.424173	171.640589	93.463443

Average Variance Reduction: 70.4%
Average Time Ratio (Antithetic/Standard): 0.70

Interpretation and Insights:

1. **Improved Accuracy**:The use of antithetic variates significantly reduced the standard error of the price estimates across all simulation sizes. On average, a 70.4% variance reduction was observed, meaning the

estimates became much more stable and reliable without needing more samples.Because antithetic variates use negatively correlated random paths, which tend to cancel out each other's random noise. This leads to smoother estimates and lower variance.

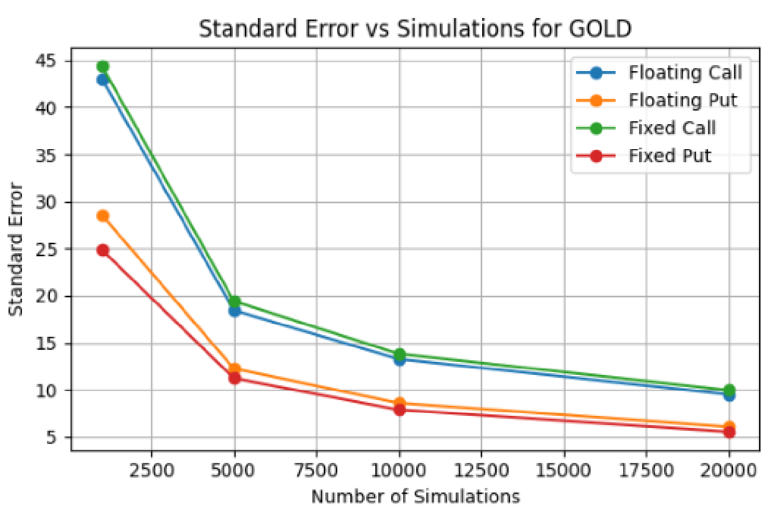
2. **Pricing Consistency**: The difference in average option price between the standard and antithetic methods (price_diff) decreased as the number of simulations increased. This indicates convergence and accuracy improvement with larger simulations, and validates that the antithetic method does not introduce bias.Because as sample size increases, the law of large numbers kicks in — both methods converge toward the true expected value. Hence, the difference between standard and antithetic pricing shrinks.

3. **Faster Computation**:With antithetic variates, the time to simulate the option prices was reduced by about 30%, with an average time ratio of 0.70. This shows that antithetic variates not only enhance accuracy but also provide computational savings.Antithetic variates use the same base random numbers, so they require **fewer function calls**. Also, the smoother convergence allows us to hit accuracy targets with fewer runs.

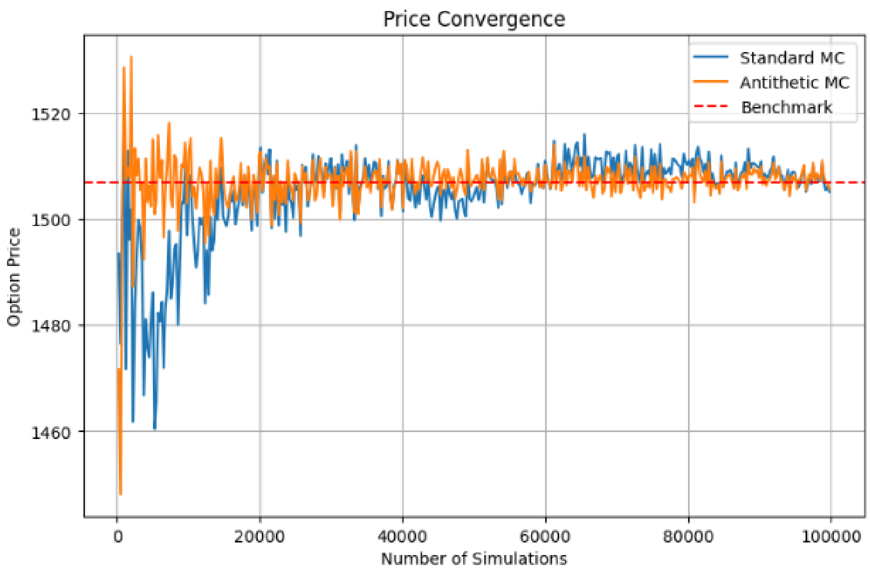
ERROR ESTIMATION AND CONVERGENCE ANALYSIS

The underlying asset (Gold) was modeled using Geometric Brownian Motion (GBM), which reflects realistic market behavior.

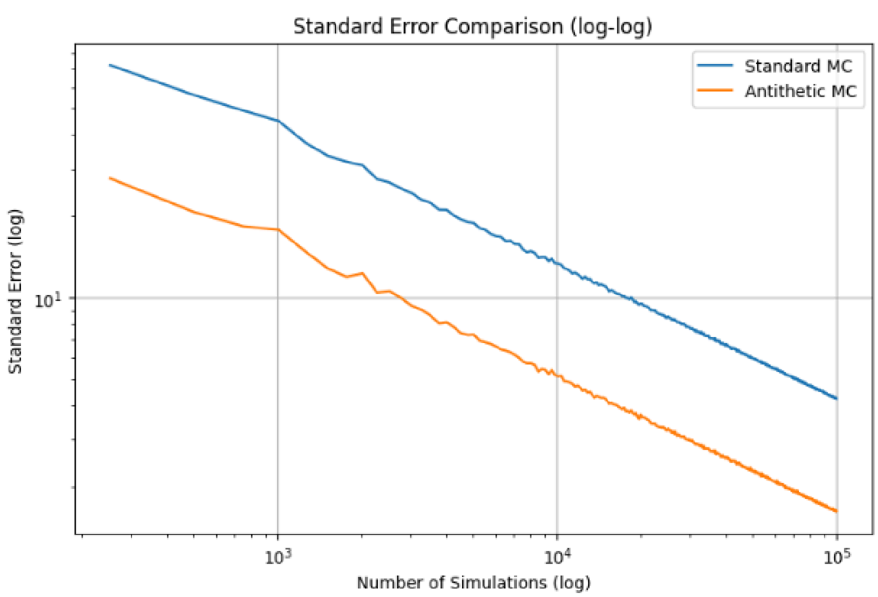
1.To ensure accuracy, I performed error estimation and convergence analysis. As expected, the standard error decreases with $1/\sqrt{N}$, where N is the number of simulations. This matches the theoretical property of the Monte Carlo method, where higher sample sizes lead to more accurate results.



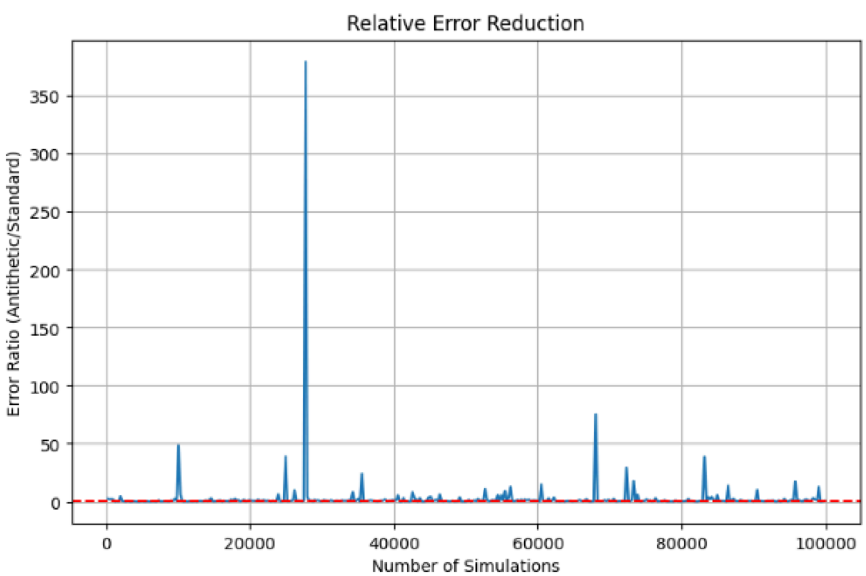
2.I also studied the convergence of option prices with increasing number of simulations. Both the standard Monte Carlo and the Antithetic Variates method (used for variance reduction) showed clear convergence in option prices as simulation size increased. This confirms that the pricing becomes more stable and reliable with larger simulations.



3.To further validate this, I plotted the standard error against the number of simulations for both standard and antithetic methods. The graph clearly shows that the standard error decreases steadily for both methods as simulation size grows.



4.Lastly, I analyzed the error ratio (antithetic error divided by standard error). Although the ratio shows some peaks and fluctuations, it remains within a reasonable range. This suggests that the antithetic method provides consistent variance reduction, though not perfectly smooth across all simulations.



Summary and Conclusion

In this project, I used Monte Carlo simulation with Geometric Brownian Motion (GBM) to price floating and fixed strike lookback options on Gold. I calculated option payoffs, analyzed market sensitivities (Greeks), and observed how they react to changes in spot price, volatility, interest rate, and time.

I also applied variance reduction using Antithetic Variates, and performed error and convergence analysis, confirming that errors decrease with more simulations. The results showed stable option pricing and aligned well with theoretical expectations, proving the effectiveness of Monte Carlo methods for path-dependent options