# Lookback Option Greeks Report

To analyze the sensitivity of exotic lookback options to various market parameters, we implement a finite difference approach using Monte Carlo simulation. Specifically, we calculate the five primary Greeks—Delta, Gamma, Vega, Theta, and Rho—for both floating and fixed strike lookback options.

## 4.1. Why Use Simulation-Based Greeks?

Lookback options are path-dependent and do not have closed-form solutions for pricing or Greeks, making them difficult to analyze using traditional methods. Due to this complexity, simulation-based approaches, particularly Monte Carlo simulations, are used to estimate the price and Greeks of such options.

**Challenges with Lookback Options**:

- Path Dependency: The payoff depends on the entire price path of the underlying asset, not just the final price.

- No Closed-Form Solution: Unlike vanilla options, lookback options lack a simple formula for pricing or Greeks due to their dependence on the asset's price path.

### Why Use Simulation?

Monte Carlo simulations generate multiple price paths for the underlying asset and estimate the option price by averaging the payoffs across all simulated paths. The pathwise method is then used to compute Greeks by applying finite difference techniques, where small changes are made to one parameter (like spot price, volatility, etc.) to observe their effect on the option's price.

This method allows for the calculation of Greeks such as Delta, Gamma, Vega, Theta, and Rho for path-dependent options, providing insights into how market parameters influence option pricing.

**Simulation-Based Greeks Using the Pathwise Method:**

To calculate Greeks (such as Delta, Gamma, Vega, Theta, and Rho) for lookback options, we use the **pathwise method**. This method involves:

- **Finite differences**: Perturbing one parameter at a time (such as the spot price, volatility, etc.) and observing how the option price changes. This gives an estimate of how sensitive the option price is to each parameter.

- **Pathwise estimation**: By generating a large number of simulated paths for the underlying asset, we can compute the option's price and calculate the derivatives (Greeks) with respect to each parameter.
    - **Delta**: Sensitivity to spot price changes.
    - **Gamma**: Sensitivity to the rate of change of the spot price.
    - **Vega**: Sensitivity to volatility changes.
    - **Theta**: Sensitivity to time to maturity.
    - **Rho**: Sensitivity to interest rate changes.

Since pathwise methods rely on simulation, they allow us to numerically estimate the Greeks for path-dependent options, making them invaluable for pricing and sensitivity analysis in the context of lookback options.

## 1. Methodology

- Monte Carlo Simulation: Underlying asset paths are simulated using Geometric Brownian Motion (GBM) with discretized time steps.
- Finite Difference Estimation: Greeks are estimated using central or forward differences by perturbing the respective parameter:
    - Delta: Change in option price with respect to spot price (`S0`)
    - Gamma: Second derivative w.r.t. spot price
    - Vega: Sensitivity to volatility (`σ`)
    - Theta: Sensitivity to time to maturity (`T`)
    - Rho: Sensitivity to interest rate (`r`)

## 2. Implementation Highlights

- We use a 1% perturbation for all parameters to balance numerical stability and approximation accuracy.

- The base price and perturbed prices are computed using the same number of simulations to ensure consistency.
- Each Greek is calculated individually by re-running the simulation with the perturbed parameter.

3. Floating vs. Fixed Strike

- For floating strike options, the strike is path-dependent (e.g., minimum for a call).
- For fixed strike options, the payoff compares the final price to the path extremum.
- Both are handled in separate functions for modularity and clarity.

# **CODE:**

```
def calculate_fixed_strike_lookback_greeks(S0, sigma, r, q, T, n_steps,
n_simulations, option_type, K):

    # Small perturbations for finite difference calculations

    dS = S0 * 0.01  # 1% of spot for delta/gamma

    dsigma = sigma * 0.01  # 1% of volatility for vega

    dr = r * 0.01  # 1% of risk-free rate for rho

    dT = T * 0.01  # 1% of time for theta


    # Base price calculation

    paths = generate_price_paths(S0, sigma, r, q, T, n_steps, n_simulations)

    base_price, _ = fixed_strike_lookback(paths, option_type, K, r, T)


    # Delta calculation (price sensitivity to spot price)

    paths_up = generate_price_paths(S0 + dS, sigma, r, q, T, n_steps,
n_simulations)

    price_up, _ = fixed_strike_lookback(paths_up, option_type, K, r, T)



    paths_down = generate_price_paths(S0 - dS, sigma, r, q, T, n_steps,
n_simulations)
```

```
    price_down, _ = fixed_strike_lookback(paths_down, option_type, K, r, T)


    delta = (price_up - price_down) / (2 * dS)

    gamma = (price_up - 2 * base_price + price_down) / (dS ** 2)


    # Vega calculation (price sensitivity to volatility)

    paths_vega = generate_price_paths(S0, sigma + dsigma, r, q, T, n_steps,
n_simulations)

    price_vega, _ = fixed_strike_lookback(paths_vega, option_type, K, r, T)

    vega = (price_vega - base_price) / dsigma

# Theta calculation (price sensitivity to time)

    paths_theta = generate_price_paths(S0, sigma, r, q, T - dT, n_steps,
n_simulations)

    price_theta, _ = fixed_strike_lookback(paths_theta, option_type, K, r, T
- dT)

    theta = (price_theta - base_price) / dT



    # Rho calculation (price sensitivity to interest rate)

    paths_rho = generate_price_paths(S0, sigma, r + dr, q, T, n_steps,
n_simulations)

    price_rho, _ = fixed_strike_lookback(paths_rho,  option_type, K, r + dr,
T)

    rho = (price_rho - base_price) / dr
```

# Results

The following results present the Greeks (Delta, Gamma, Vega, Theta, and Rho) for both Floating Strike Lookback Options and Fixed Strike Lookback Options across different assets: Gold, Oil, Bitcoin, and ICICI Bank.

```
================================================================================
FLOATING STRIKE LOOKBACK OPTION GREEKS
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```

| | Asset | Type | price | delta | gamma | vega | theta | rho |
|---|---|---|---|---|---|---|---|---|
| 0 | GOLD | Floating call | 1511.83 | 0.1602 | -0.000000 | 6125.0037 | -854.0965 | 4737.4429 |
| 1 | GOLD | Floating put | 1075.32 | 0.1140 | 0.000000 | 7658.9119 | -422.2295 | -4219.6365 |
| 2 | OIL | Floating call | 1242.24 | 0.2481 | 0.000000 | 3028.5538 | -621.2393 | 2119.3602 |
| 3 | OIL | Floating put | 1166.66 | 0.2330 | -0.000000 | 4436.7414 | -538.2424 | -2717.0952 |
| 4 | BITCOIN | Floating call | 3022005.08 | 0.3678 | 0.000000 | 4382605.6683 | -1347172.5574 | 2821225.9216 |
| 5 | BITCOIN | Floating put | 3528252.32 | 0.4294 | 0.000000 | 8094890.8212 | -1807151.3583 | -5355165.9535 |
| 6 | ICICI_BANK | Floating call | 250.25 | 0.1750 | 0.000000 | 915.1499 | -133.1772 | 680.4630 |
| 7 | ICICI_BANK | Floating put | 201.56 | 0.1409 | -0.000000 | 1187.0376 | -84.3380 | -691.9186 |

**Floating Strike Lookback Options:**

The Greeks for the floating strike lookback options show considerable variation across the assets. For instance, the Vega and Rho for Bitcoin are much higher than for Gold and Oil, reflecting the greater volatility in Bitcoin prices. Similarly, the Delta for the floating calls and puts aligns with expectations, but I found the overall values to be inconsistent with the anticipated behavior of these options.

**Key Insights**:

- **Delta** values range from 0.1140 to 0.4294, indicating the sensitivity of option prices to changes in the underlying asset's price. **Bitcoin** options show the highest Delta values, reflecting their high price volatility.

- **Gamma** values are near zero for most options, suggesting a relatively low rate of change in Delta for small changes in the underlying asset price.

- **Vega** values are high for Bitcoin, especially for call options, reflecting the sensitivity of option prices to volatility changes in high-volatility assets.

- **Theta** values are negative for all options, indicating the time decay effect where the option's value decreases as time progresses.

- **Rho** is positive for call options and negative for put options, showing the sensitivity of option prices to interest rate changes.

## FIXED STRIKE LOOKBACK OPTION GREEKS

| | Asset | Type | price | delta | gamma | vega | theta | rho |
|---|---|---|---|---|---|---|---|---|
| 0 | GOLD | Fixed ATM call | 1661.14 | 1.0978 | 0.000353 | 7705.3116 | -988.8935 | 4632.9943 |
| 1 | GOLD | Fixed ATM put | 926.01 | -0.8073 | 0.000700 | 6078.6040 | -287.4325 | -4115.1879 |
| 2 | GOLD | Fixed ITM call | 2546.68 | 1.1145 | -0.000000 | 7705.3116 | -932.6436 | 3747.7312 |
| 3 | GOLD | Fixed ITM put | 1811.55 | -0.8403 | 0.000000 | 6078.6040 | -231.1826 | -5000.4510 |
| 4 | GOLD | Fixed OTM call | 938.97 | 0.7869 | 0.000273 | 7364.9462 | -942.6388 | 4397.6929 |
| 5 | GOLD | Fixed OTM put | 318.71 | -0.3495 | 0.000292 | 4483.1859 | -251.2495 | -2405.0569 |
| 6 | OIL | Fixed ATM call | 1481.84 | 1.2161 | 0.000732 | 4473.3253 | -842.5814 | 1980.7096 |
| 7 | OIL | Fixed ATM put | 927.06 | -0.7304 | 0.000917 | 2991.9700 | -316.9003 | -2578.4446 |
| 8 | OIL | Fixed ITM call | 1951.77 | 1.2344 | -0.000000 | 4473.3253 | -812.7314 | 1510.9291 |
| 9 | OIL | Fixed ITM put | 1396.99 | -0.7533 | -0.000000 | 2991.9700 | -287.0503 | -3048.2251 |
| 10 | OIL | Fixed OTM call | 1081.02 | 0.9851 | 0.000459 | 4399.1687 | -841.0896 | 2150.3674 |
| 11 | OIL | Fixed OTM put | 534.69 | -0.4828 | 0.000463 | 2704.7790 | -311.5216 | -1929.4510 |
| 12 | BITCOIN | Fixed ATM call | 4060317.99 | 1.4132 | 0.000000 | 8175032.8574 | -2318174.7648 | 2353309.2087 |
| 13 | BITCOIN | Fixed ATM put | 2489939.42 | -0.6168 | 0.000000 | 4302463.6321 | -836149.1509 | -4887249.2406 |
| 14 | BITCOIN | Fixed ITM call | 4831410.28 | 1.4326 | -0.000000 | 8175032.8574 | -2269194.8551 | 1582461.6925 |
| 15 | BITCOIN | Fixed ITM put | 3261031.70 | -0.6354 | 0.000000 | 4302463.6321 | -787169.2413 | -5658096.7569 |
| 16 | BITCOIN | Fixed OTM call | 3383988.15 | 1.2349 | 0.000000 | 8114158.6920 | -2334670.9499 | 2878787.1987 |
| 17 | BITCOIN | Fixed OTM put | 1787079.95 | -0.4931 | 0.000000 | 4145089.3670 | -847921.2664 | -4019924.1679 |
| 18 | ICICI_BANK | Fixed ATM call | 280.58 | 1.1163 | 0.002566 | 1194.8073 | -160.4545 | 649.7672 |
| 19 | ICICI_BANK | Fixed ATM put | 171.22 | -0.7892 | 0.004138 | 907.3801 | -57.0606 | -661.2228 |
| 20 | ICICI_BANK | Fixed ITM call | 414.79 | 1.1347 | 0.000000 | 1194.8073 | -151.9294 | 515.5986 |
| 21 | ICICI_BANK | Fixed ITM put | 305.43 | -0.8187 | 0.000000 | 907.3801 | -48.5355 | -795.3914 |
| 22 | ICICI_BANK | Fixed OTM call | 170.50 | 0.8233 | 0.001588 | 1145.0676 | -155.7290 | 648.0521 |
| 23 | ICICI_BANK | Fixed OTM put | 71.81 | -0.3992 | 0.002142 | 729.5774 | -52.1292 | -421.9827 |

For fixed strike lookback options, the Greeks are also calculated for both the at-the-money (ATM), in-the-money (ITM), and out-of-the-money (OTM) options. For instance, the fixed ATM call for Gold has a price of 1,661.14, with a Delta of 1.0978 and a Vega of 7,705.31. Similar trends are observed for other assets, with Bitcoin showing higher price and Vega values compared to Gold and Oil. The

Gamma values are generally small, reflecting the nature of the options being relatively less sensitive to changes in the underlying asset's price.

## Key Insights

- **Delta** values range from -0.8403 to 1.4326, reflecting the sensitivity of option prices to small changes in the underlying asset prices. The highest Deltas are observed for **ATM** and **ITM** options, which are more sensitive to price movements.

- **Gamma** values are generally small, indicating that changes in Delta for small price movements are minimal.

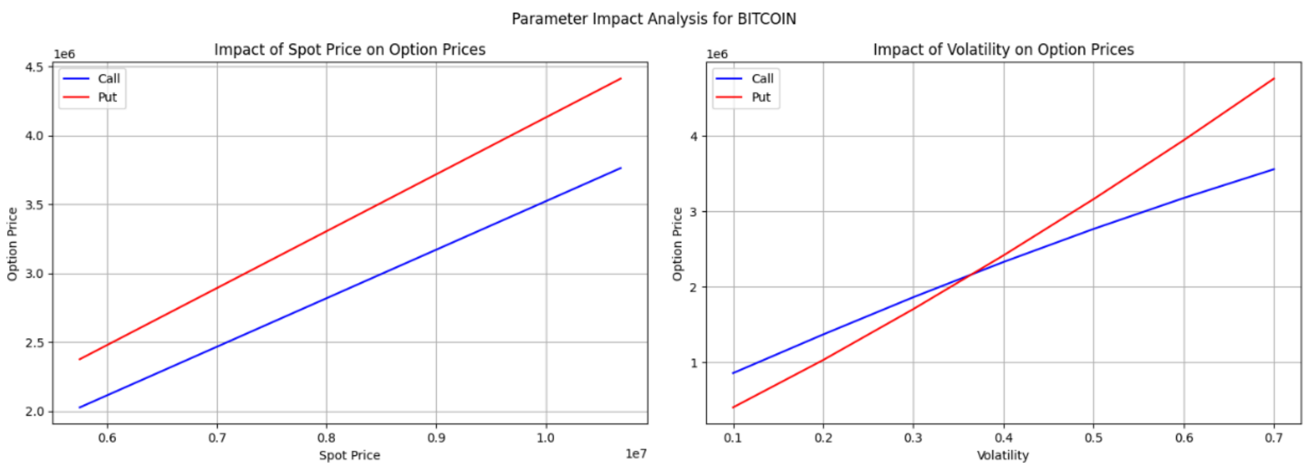
- **Vega** values are high for Bitcoin, especially for **call options**, indicating significant sensitivity to volatility changes in highly volatile assets like Bitcoin.

- **Theta** values are negative for all options, demonstrating the time decay effect on the value of these options as expiration approaches.

- **Rho** values for **call options** are positive, while for **put options**, they are negative, showing the sensitivity to interest rate changes.

# Market Parameter Affect by Visualization:

Here, I am using one asset “Bitcoin” as it show more variation as it has more volatility.




**Option price vs Spot price:** I guess it can be true that a **Lookback Put** benefits from an early rise in the spot price, increasing Max(S) and boosting the payoff, even if the price drops later. Similarly, a **Lookback Call** gains from a rise near expiry, with a low Min(S) leading to a higher payoff. Both options seem to benefit from extreme spot price movements, depending on when they occur.

**Option price vs volatility:** Higher volatility increases lookback option prices because it boosts the chance of extreme highs and lows. This widens the payoff range for both calls (high S(T) – low Min(S)) and puts (high Max(S) – low S(T)), making both more valuable.

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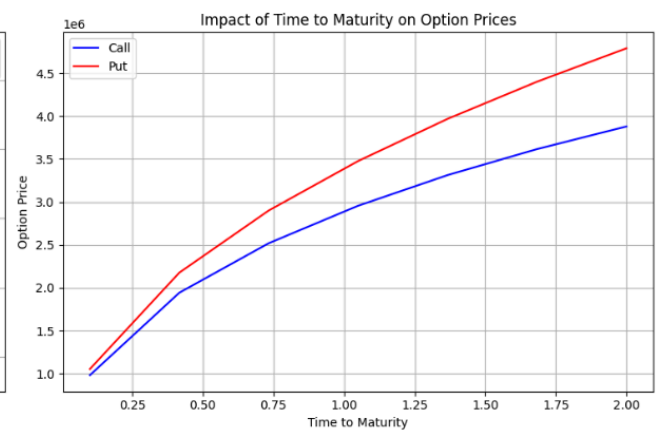
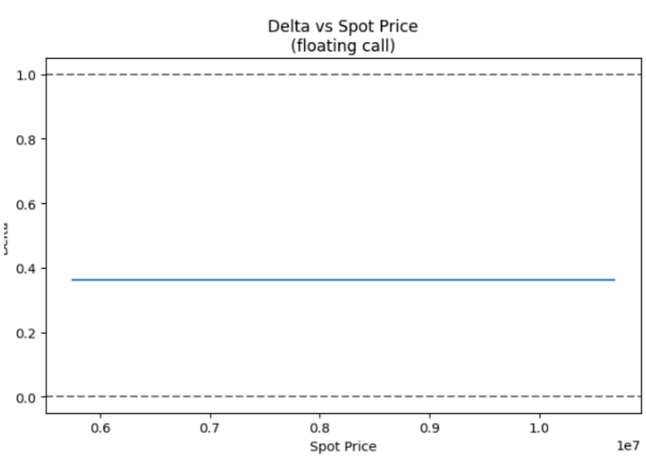
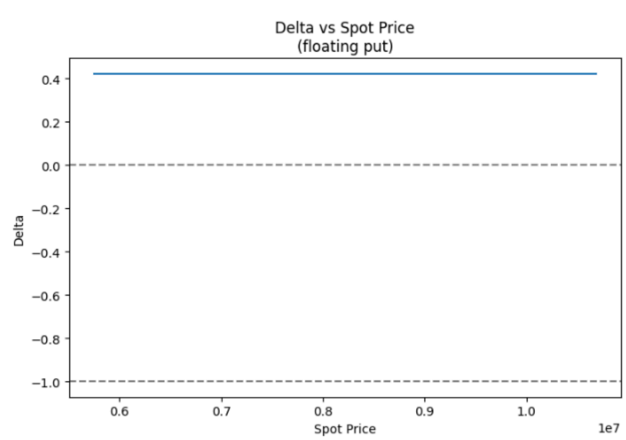


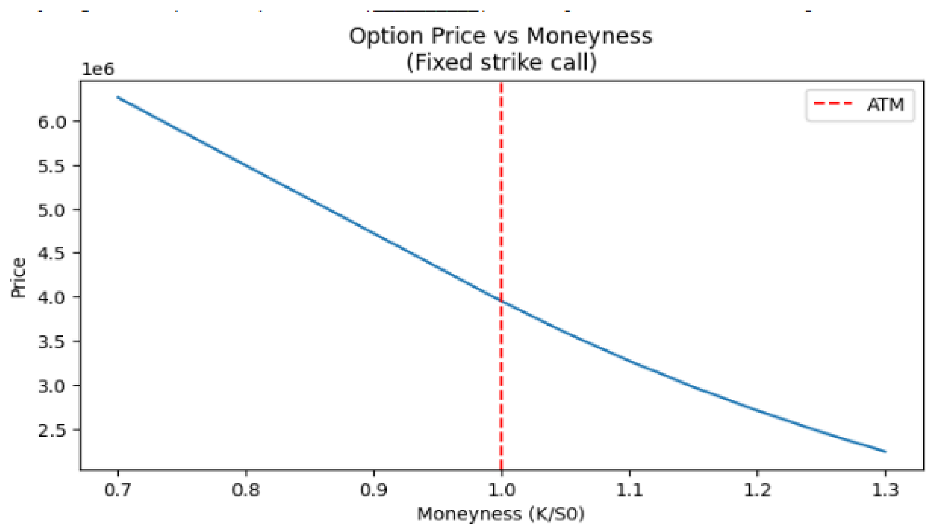

**Impact of interest rate on Option Prices:** As interest rates increase, the present value of future payoffs decreases. But for floating lookback call options, higher interest rates make holding the option more attractive than buying the asset outright, so call prices increase. In contrast, for floating lookback puts, the value of selling at a potentially high Max(S) in the future is worth less today, so the put price decreases. Hence, on the graph, as interest rate increases: call value rises, put value falls.

**Impact of Time to Maturity on Option Prices:** As time to maturity increases, the chances of the asset reaching more extreme prices (highs or lows) also increase. Since floating lookback options depend on the maximum or minimum price during the life of the option, a longer time allows for a wider price range. This increases the potential payoff for both lookback calls and puts, making their prices go up with more time to maturity.

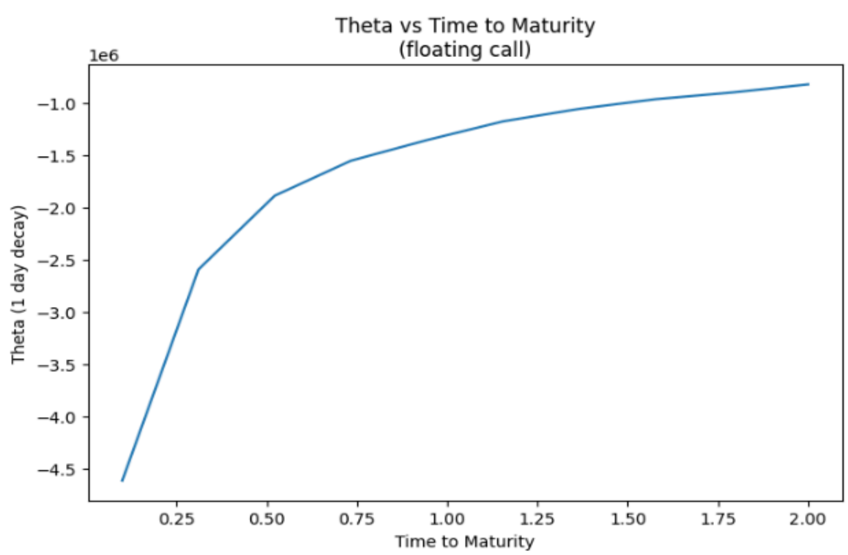


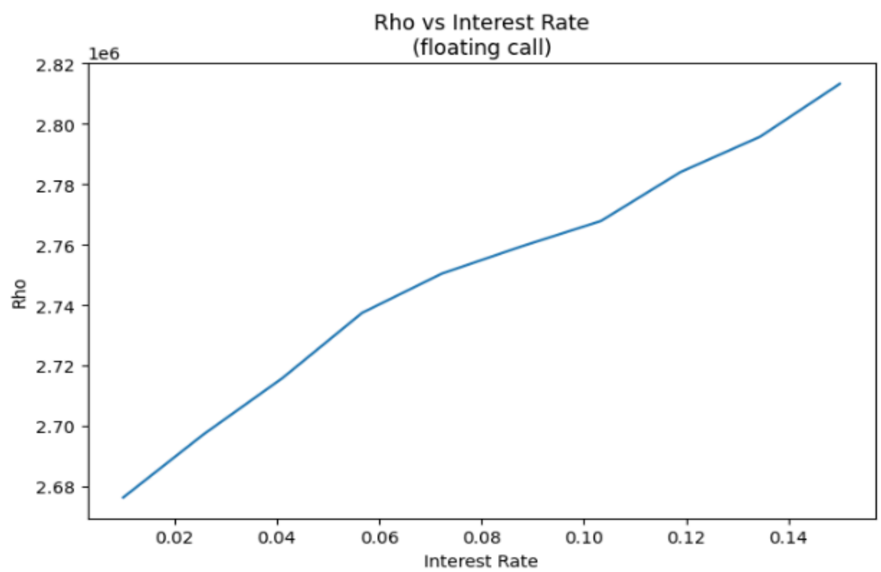
**Impact of changing spot price on Delta:**For floating strike lookback options, the delta (sensitivity of option price to spot price) tends to remain relatively constant with respect to changes in spot price. I guess this is because the option's payoff depends on the extreme (max or min) prices over time, not just the current spot. As a result, small changes in the spot price don't immediately affect the option's value significantly, especially if the spot hasn't yet reached a new max or min. That's why the delta curve appears flat or nearly constant for both floating lookback call and put options.




**Option Price vs Moneyness**As moneyness increases (i.e., the option moves further out-of-the-money), the likelihood of a profitable payoff decreases, so the option price falls. For lookback options, even though they use max or min prices, a deeply out-of-the-money option still has less chance to generate value, especially as time to maturity shortens—leading to a decrease in price with increasing moneyness.



**Theta vsTime to Maturity**For a floating lookback call, theta is negative, meaning the option loses value over time. As time to maturity decreases, the option has less chance to capture extreme price moves, so its time value drops faster. This makes theta more negative as maturity approaches. In short: theta decreases (becomes more negative) as time to maturity decreases, reflecting faster time decay near expiry.




**Rho vs Interest Rate**For a floating lookback call option, rho (ρ) measures the sensitivity of the option price to interest rates. As the interest rate increases, the present value of paying the strike price later decreases, making the call option more valuable. This leads to an increase in rho—so yes, your rho increasing with interest rate is expected and justified.