Experiment 7: Laplace Transforms

This experiment is intended to make the student to use MATLAB for experiments, relating to the Laplace Transform of continuous time signals. It is expected that the student will write a "readable" code in a file and execute.

In this experiment, we will find the Laplace Transform of given signals and Inverse Laplace Transform and verify the property that convolution of two time domain signals is equivalent to the multiplication of their Laplace Transforms

NOTE:

In order to perform Laplace Transform in Matlab "**symbolic**" mathematics need to be used, so students are expected to explore that functionality of matlab using **HELP**

Run # 01 : Forward Laplace Transform

(i) Write Matlab program to find Laplace Transform of the following standard causal signals

NOTE : verify the result with your hand calculations

a) t

b) e^{-at}

c) cos ωt

d) e^{-at} sin ωt

e) t^2 -3t

(ii) Explore the use of matlab buit-in function "simplify" and apply this function to simplify the above expressions

ANS(i)

```
clc;
clear all;
close all;
syms a w t;

x1=t;
x1s=laplace(x1);
display(x1s);

x2=exp(-a.*t);
x2s=laplace(x2);
display(x2s);

x3=cos(w.*t);
x3s=laplace(x3);
display(x3s);

x4=exp(-a.*t).*cos(w.*t);
x4s=laplace(x4);
display(x4s);

x5=t^2-3.*t;
x5s=laplace(x5);
display(x5s);
```

OUTPUT

```
x1s =1/s^2
x2s =1/(a + s)
x3s =s/(s^2 + w^2)
x4s =(a + s)/((a + s)^2 + w^2)
x5s =2/s^3 - 3/s^2
```

(ii)

```
clc;
clear all;
close all;
syms a w t;

x1=t;
x1s=laplace(x1);
simplify(x1s);
```

```
display(x1s);

x2=exp(-a.*t);
x2s=laplace(x2);
simplify(x2s);
display(x2s);

x3=cos(w.*t);
x3s=laplace(x3);
simplify(x3s);
display(x3s);

x4=exp(-a.*t).*cos(w.*t);
x4s=laplace(x4);
simplify(x4s);
display(x4s);

x5=t^2-3.*t;
x5s=laplace(x5);
simplify(x5s);
display(x5s);
```

OUTPUT

x1s =1/s^2

x2s =1/(a + s)

x3s =s/(s^2 + w^2)

x4s =(a + s)/((a + s)^2 + w^2)

x5s =2/s^3 - 3/s^2

Run # 02 : Inverse Laplace Transform

(i) Write Matlab program to find Inverse Laplace Transform of the following S-domain signals

NOTE : verify the result with your hand calculations

a) $\frac{2}{s(s+1)(s+2)^2}$

b) $\frac{1}{(s^2+s+1)(s+2)}$

c) $\frac{2}{s(s+1)(s+2)}$

(ii) After obtaining time expressions for the problem no.2, write matlab programs and plot all those signals in time scale

ANS(i)

```
clc;
clear all;
close all;
syms t s;

x1s=2/(s.*(s+1).*((s+2)^2));
x1=ilaplace(x1s);
display(x1);

x2s=1/((s^2+s+1).*(s+2));
x2=ilaplace(x2s);
display(x2);

x3s=2/(s.*(s+1).*(s+2));
x3=ilaplace(x3s);
display(x3);
```

OUTPUT

x1 =(3*exp(-2*t))/2 - 2*exp(-t) + t*exp(-2*t) + 1/2

x2 =exp(-2*t)/3 - (exp(-t/2)*(cos((3^(1/2)*t)/2) - 3^(1/2)*sin((3^(1/2)*t)/2)))/3

x3 =exp(-2*t) - 2*exp(-t) + 1

(ii)

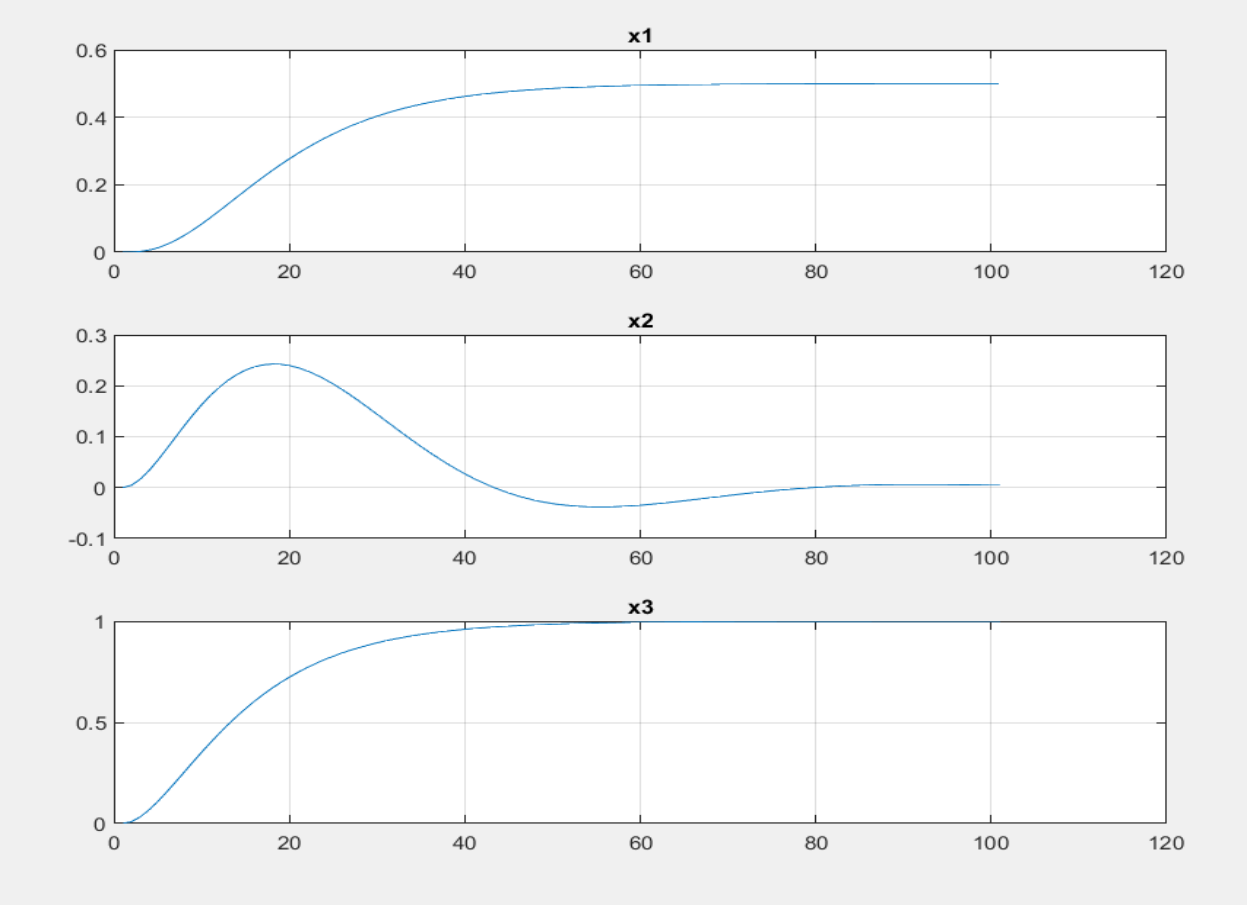
```
clc;
clear all;
close all;

t=0:0.1:10;
```

```
x1=(3.*exp(-2.*t))./2 - 2.*exp(-t) + t.*exp(-2*t) + 1./2;
subplot(3,1,1);
plot(x1);
grid on;
title('x1');

x2=exp(-2.*t)/3 - (exp(-t./2).*(cos((3^(1/2).*t)./2) -
3^(1/2).*sin((3^(1./2).*t)/2)))./3;
subplot(3,1,2);
plot(x2);
grid on;
title('x2');

x3=exp(-2.*t) - 2*exp(-t) + 1;
subplot(3,1,3);
plot(x3);
grid on;
title('x3');
```



Run # 03 : Verifying the property that convolution of two time domain signals is equivalent to the multiplication of their Laplace Transforms. → $x_1(t) * x_2(t) = X_1(S) \times X_2(S)$

Write MATLAB Program to verify the above property on the given signals

(a) $x_1(t) = t^2 - 2t \quad and \quad x_2(t) = t$

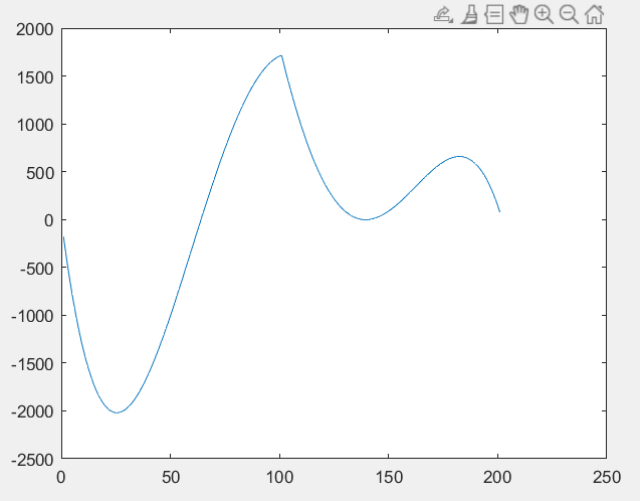
(b) $x_1(t) = u(t) - u(t-1) \quad and \quad x_2(t) = u(t-1) - u(t-2)$

ANS(a)

```
TIME DOMAIN

clc;
clear all;
close all;

t=-5:0.1:5;
x1=t.*t-2.*t;
x2=t;
x3=conv(x1,x2);
plot(x3);
```



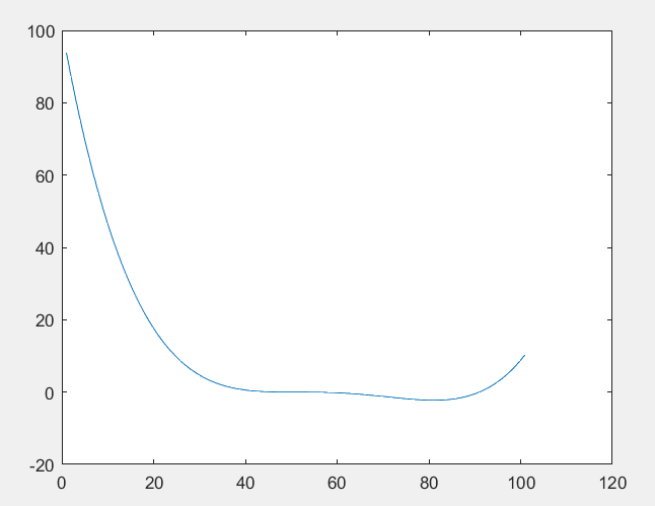
LAPLACE DOMAIN

```
clc;
clear all;
close all;
syms t

x1=t.*t-2.*t;
x1s=laplace(x1);
x2=t;
x2s=laplace(x2);
x3s=x1s.*x2s;
x3=ilaplace(x3s);
```

running the laplace file in time domain file

```
x3_lap=(t.^4)./12-(t.^3)./3;
plot(x3_lap);
```



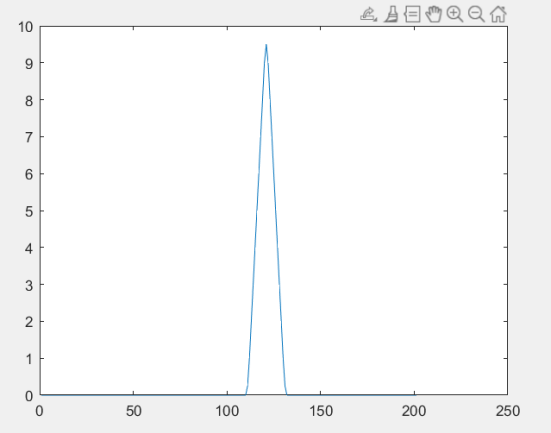
(b)

TIME DOMAIN

```
clc;
clear all;
close all;

t=-5:0.1:5;
```

```
x1=heaviside(t)-heaviside(t-1);
x2=heaviside(t-1)-heaviside(t-2);
x3=conv(x1,x2);
plot(x3);
```



LAPLACE DOMAIN

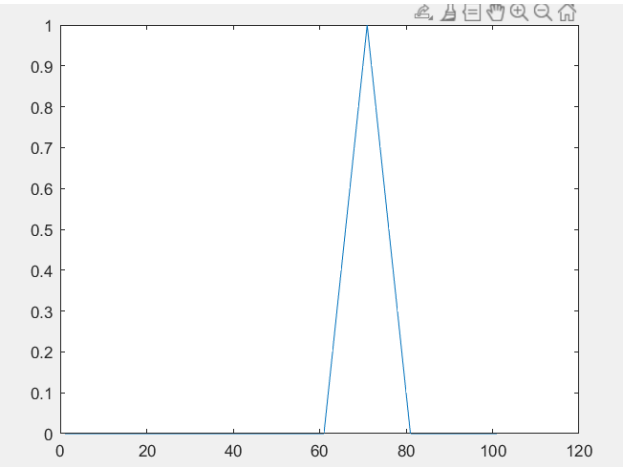
```
clc;
clear all;
close all;
syms t s;

x1=heaviside(t)-heaviside(t-1);
x2=heaviside(t-1)-heaviside(t-2);
x1s=laplace(x1);
x2s=laplace(x2);
x3s=x1s.*x2s;
x4=ilaplace(x3s);
```

running the laplace file in time domain file

```
x3_lap=heaviside(t - 1)*(t - 1) - 2*heaviside(t - 2)*(t -2)
+ heaviside(t - 3)*(t - 3);

plot(x3_lap);
```



Additional Problems

1) Using Matlab find Laplace Transform of the following Causal Signals

a) $t^n e^{-at}$

b) $t^{n-1} e^{-at}/(n-1)$!

c) $3 \sin 2t + 3 \cos 2t$

d) $1+0.4e^{-2t} \sin 3t$