Question 1. Using ‘ode-Kepler.py’, verify the Kepler’s 3rd law for five different elliptical orbits.

In [12]:

```
import numpy as np
import matplotlib.pyplot as plt
import scipy.integrate as integ
```

In [37]:

```
# A vector function. y is a two-compo vector, i.e. (x,v).
# f returns two-compo vector, i.e. (f1,f2)=(.
def F(t,z):
    x,y,vx,vy=z
    r=np.sqrt(x**2+y**2)
    fx=-x/r**3; fy=-y/r**3
    return [vx,vy,fx,fy]
```

In [40]:

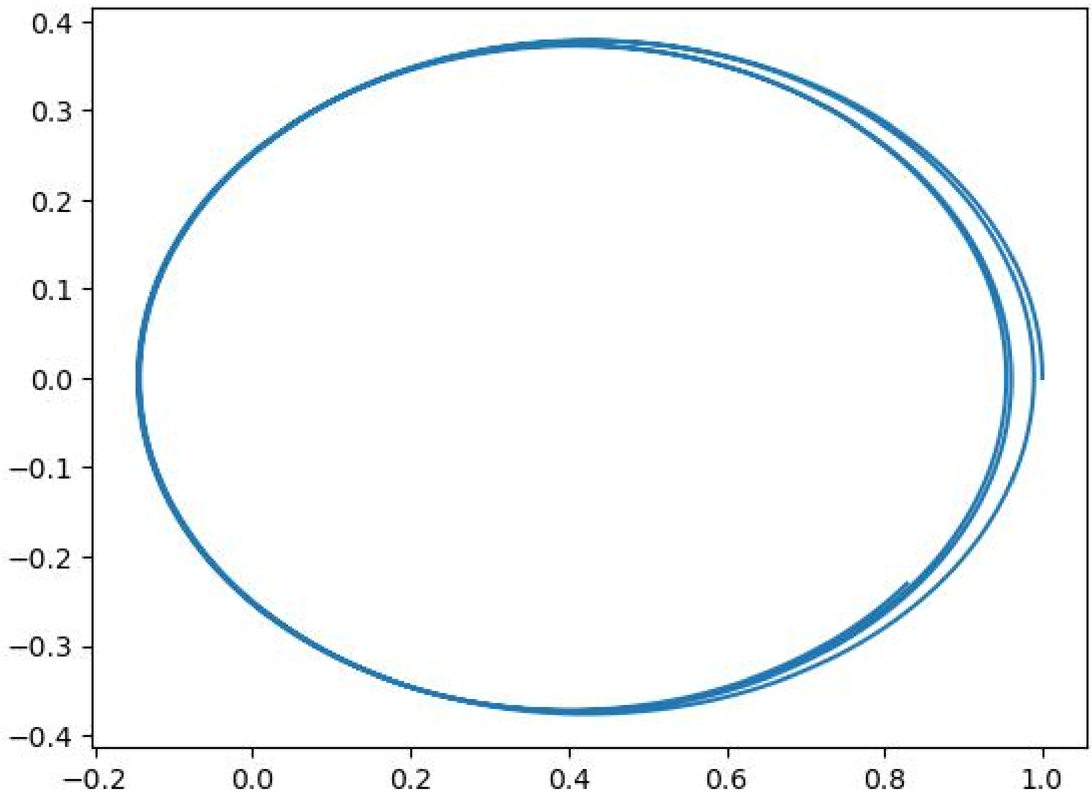
```
z0=[1.0, 0.0, 0.00, 0.5] #x0,y0,vx0,vy0
```

In [106]:

```
def orbit( z0) :
    h=float(0.001)   # time step
    nStep=int(10000)  # num. steps to run
    t=np.arange(0,h*nStep,h)  # an array of discretized time
    #sol = integ.odeint(F,z0,t) # sol is [[x0,y0,vx0,vy0],..]
    sol = integ.solve_ivp(F,[0,10],z0,method='RK45',t_eval=t) # sol is [[x0,y0,vx0,vy0],..]


#     index = np.where(sol.y[0]>=0 and sol.y[2]>=0)


    for i in range(1, len(sol.y[0])-1) : # for x coordinate
        if sol.y[0][i]>=0 and sol.y[2][i]>=0 : # when x > =0  and vx >=0
            if sol.y[1][i]*sol.y[1][i+1] <= 0 : # when y sign changes
                index = i
                t = sol.t[i] #period
                break

    plt.plot(sol.y[0],sol.y[1]); plt.show()

    print(sol.y[0][index],sol.y[1][index],sol.y[2][index],sol.y[3][index]) #coordinates for t

    return t**2/z0[0]**3 #check the Kepler's 3rd law
```

In [107]:

```
orbit([1.0, 0.0, 0.00, 0.5])
```



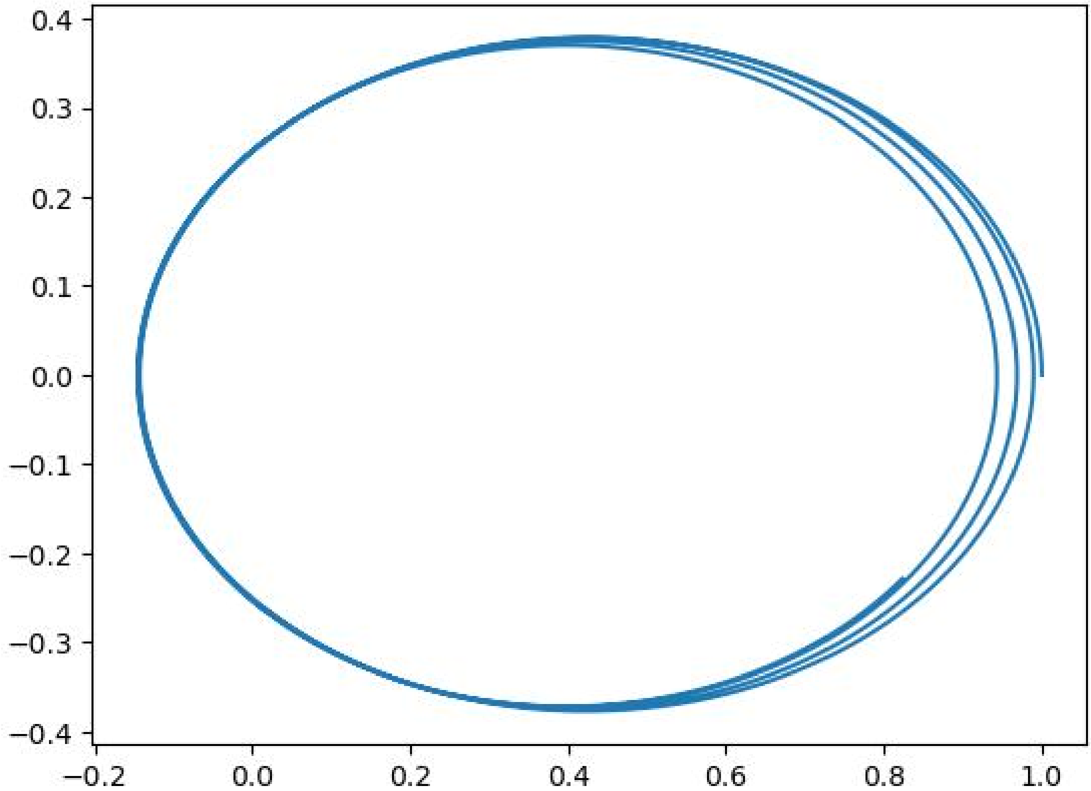

```
0.9894017096402175 -0.0001845601171006489 0.003465400169647026 0.5061712197339278
```

Out[107]:

```
7.246864000000001
```

In [114]:

```
orbit([1.001, 0.0, 0.00, 0.5])
```



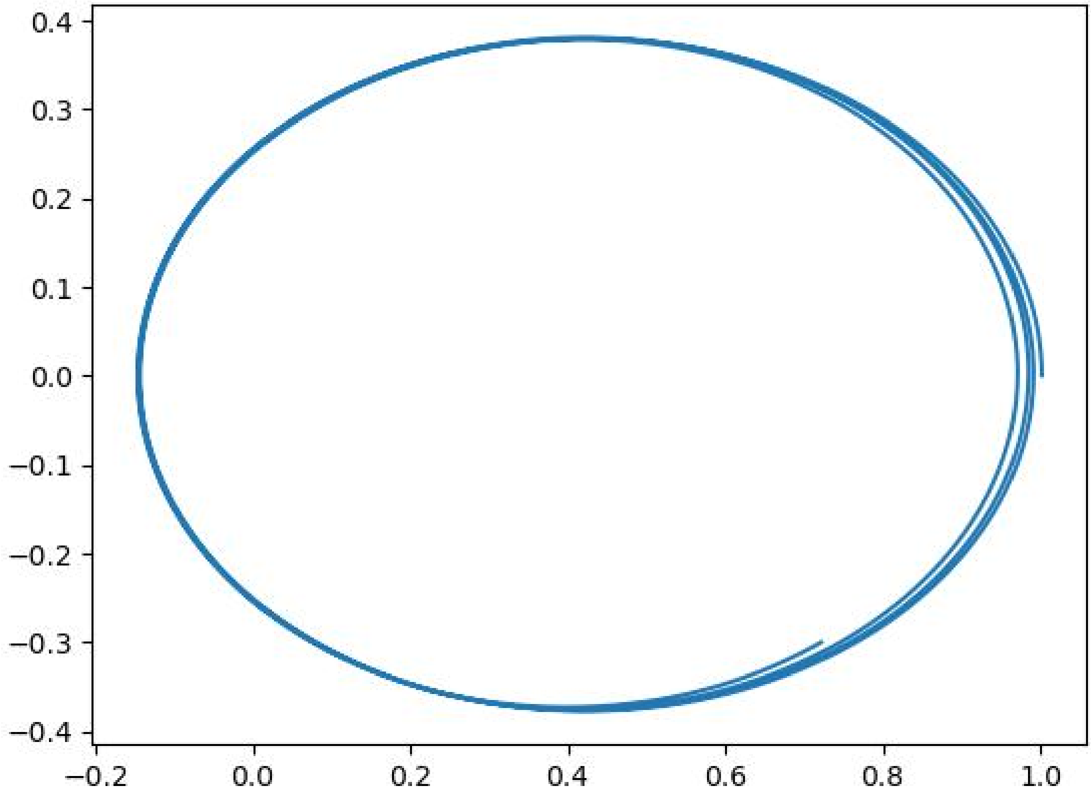

```
0.9904654030250065 -4.9034766850530875e-05 0.003207353889812664 0.506137108735396
3
```

Out[114]:

```
7.252031143224866
```

In [115]:

```
orbit([1.002, 0.0, 0.00, 0.5])
```



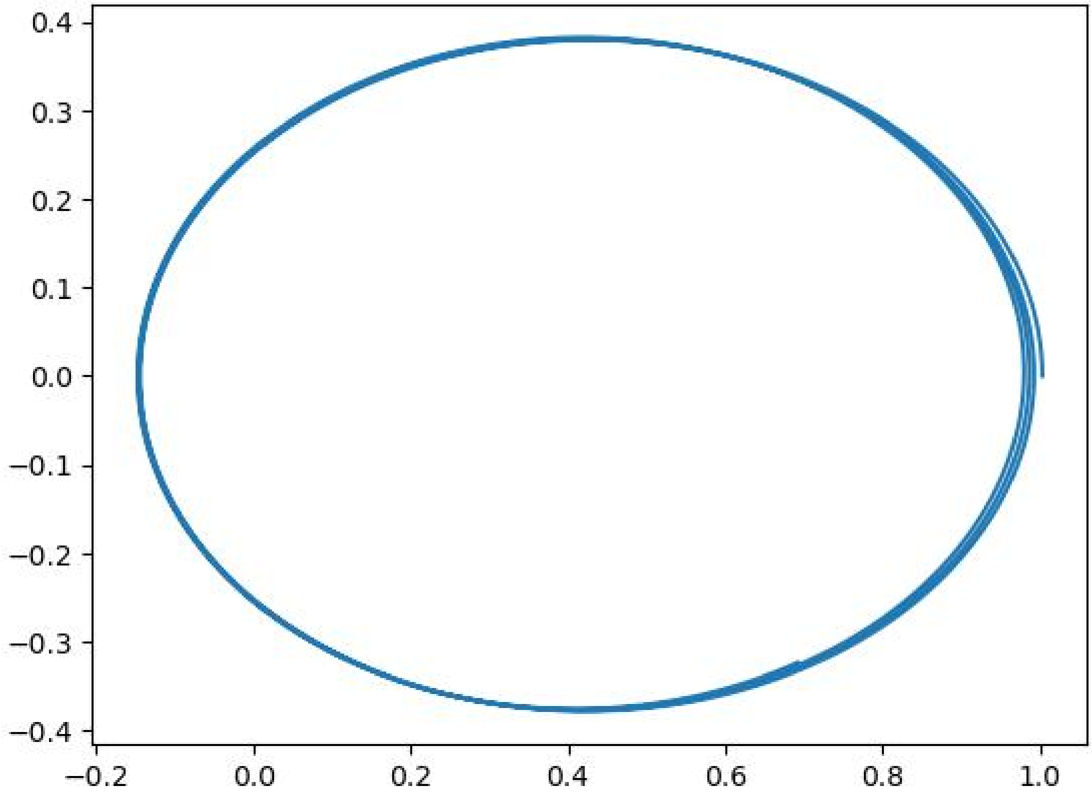

```
0.9915214594410792 -0.00041979693939070417 0.003965687751289748 0.506104496038000
3
```

Out[115]:

```
7.251803101737927
```

In [116]:

```
orbit([1.003, 0.0, 0.00, 0.5])
```



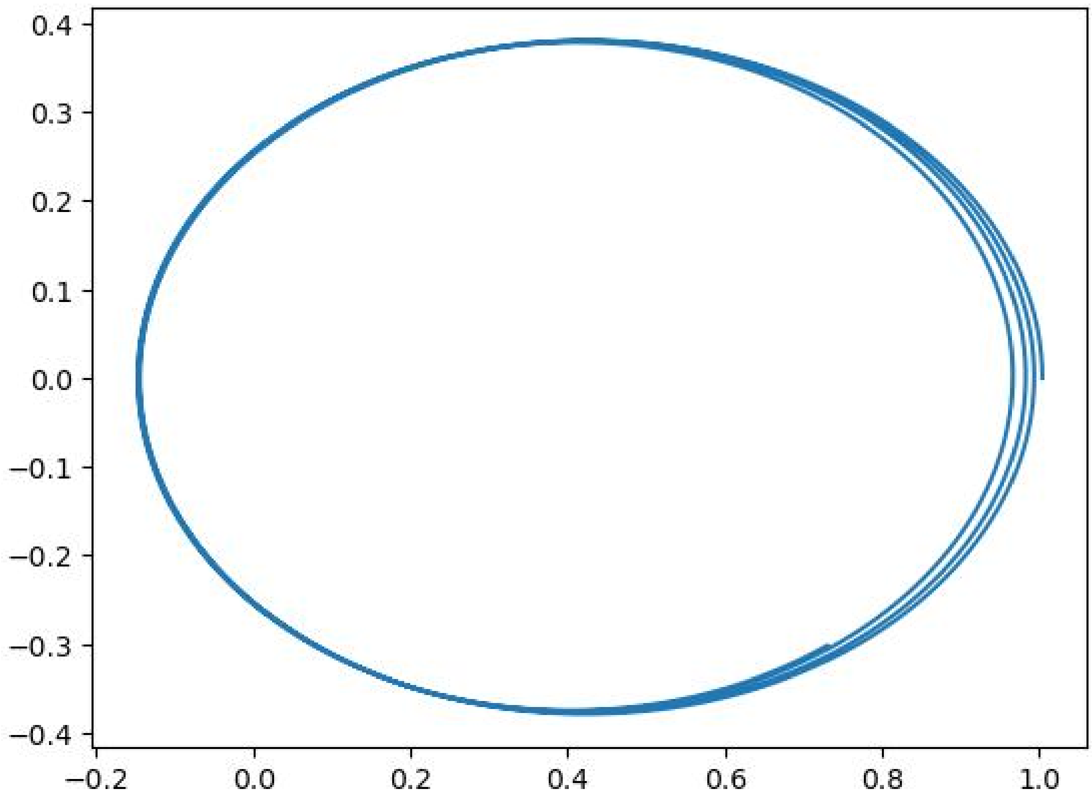

```
0.9925781443071747 -0.0002837114862019874 0.003703980978298215 0.506073931588241
8
```

Out[116]:

```
7.256927519345824
```

In [117]:

```
orbit([1.004, 0.0, 0.00, 0.5])
```



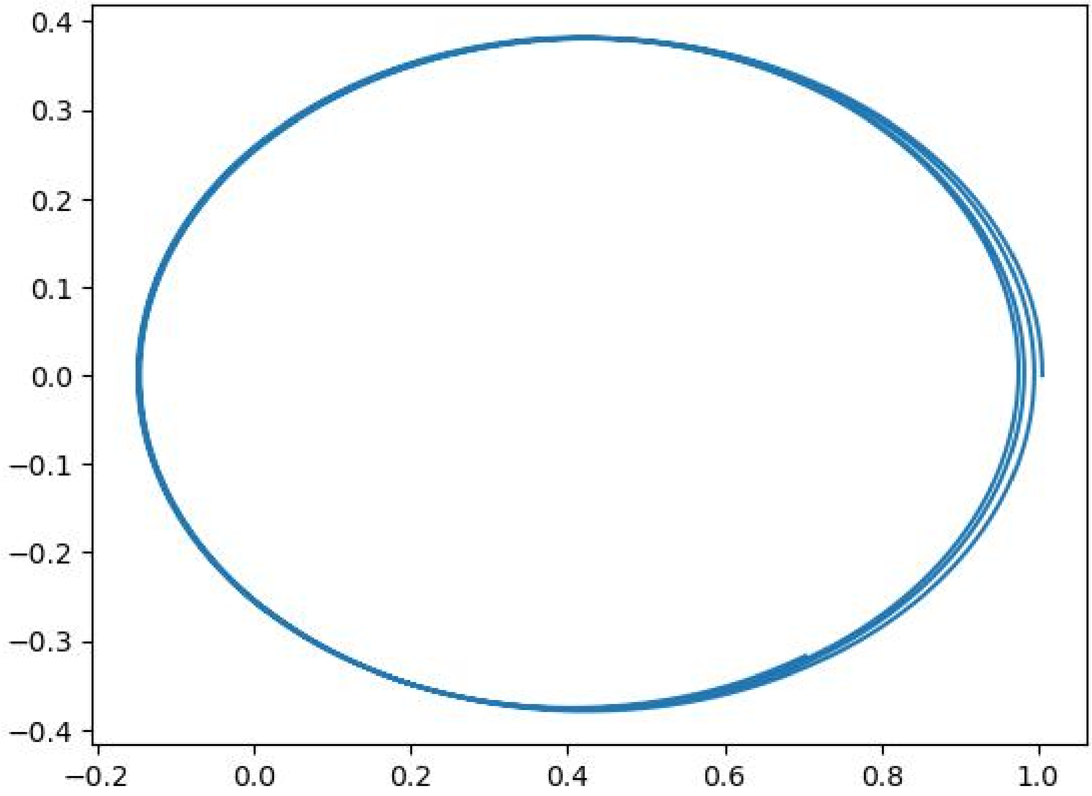
```
0.9936315242314444 -0.00014760538047389749 0.0034420824607436507 0.50604489697161
26
```

Out[117]:

```
7.262027626387514
```

In [118]:

```
orbit([1.005, 0.0, 0.00, 0.5])
```




```
0.9946817879249091 -1.1613094852438999e-05 0.003180308304861157 0.506017300202499
4
```

Out[118]:

```
7.267103506255268
```

$\frac{T^2}{r^3}$ is almost same. Therefore, Kepler's 3rd law is right