Question 1. Using ‘oblique.py’, generate a Gaussian beam which propagates in x-direction. Measure the beam radius and peak value of field strength at several different positions along x (including the focal point). Compare the measured radius with the envelope formula. Also check the power conservation along the propagation.

import package

In [1]:

```
import numpy as np
import matplotlib.pyplot as plt
import emitEMwave as ant
import emitEMwave_update as antu
```

Set variables

In [2]:

```
xmax=20
ymax=10
dx=0.02
dy=0.02
dt=0.01
f=2
D=10
smax=2000
```

In [3]:

```
a=dt/dx;  b=dt/dy
w=2.0*np.pi*f
```

In [4]:

```
cntr=0.5*ymax
upper=int(0.5*(ymax+D)/dy)
lower=int(0.5*(ymax-D)/dy)
```

In [5]:

```
x=np.arange(0,xmax+dx,dx)
y=np.arange(0,ymax+dy,dy)
Nh = int(xmax/dx/2)
```

In [6]:

```
X,Y=np.meshgrid(x,y)
Ex=0*X; Ey=0*X; Ez=0*X
Bx=0*X; By=0*X; Bz=0*X
```

In [7]:

```
I_sum = 0*X # commulated intensity for each step
```

Simulation of Gaussian wave

In [8]:

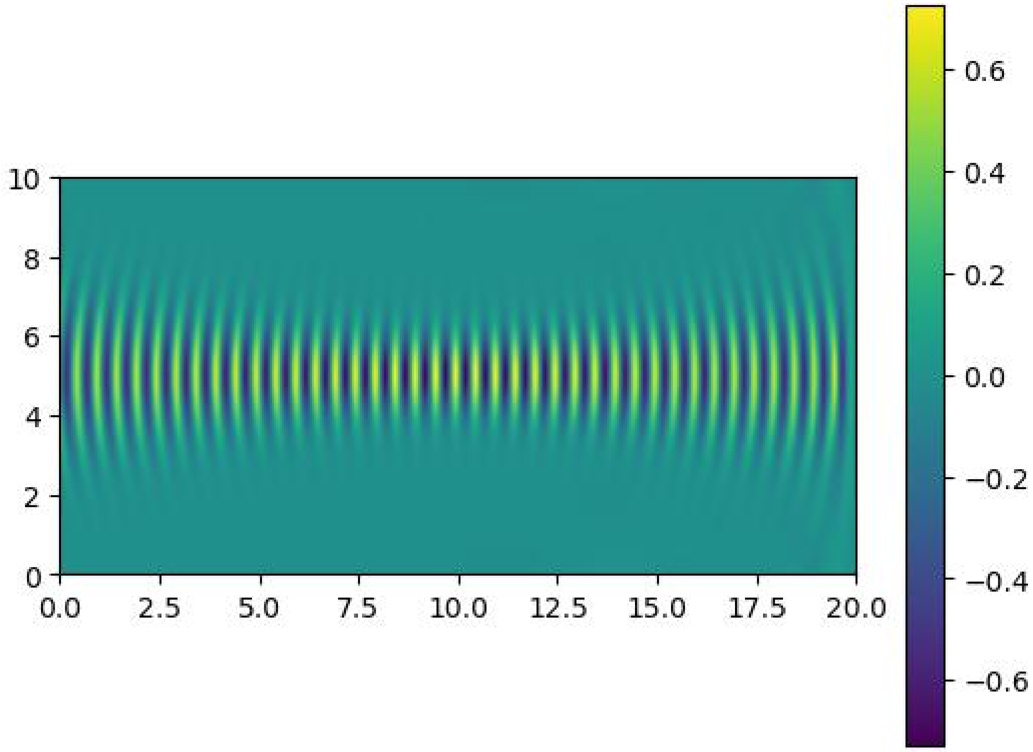
```
s=0
while s <= smax:
    #Ey[:,0]= np.exp(-(y-cntr)**2/(0.2*ymax)**2)*np.sin(w*s*dt) # emission
    #Ey[lower:upper,0]= np.sin(w*s*dt) # hole
    antu.emitEMwave(s*dt,Ey[:,0],(dx,dy),'ovwrt','p',0,0,f,0,0,10,(1.0,),1000,1)

    Bx[:-1,:-1] +=  -b*(Ez[1:,:-1]-Ez[:-1,:-1])
    By[1:-1,:-1] +=  a*(Ez[1:-1,1:]-Ez[1:-1,:-1])
    Bz[:-1,:-1]+= -a*(Ey[:-1,1:]-Ey[:-1,:-1]) + b*(Ex[1:,:-1]-Ex[:-1,:-1])
    Ex[1:-1,:Nh] +=  b*(Bz[1:-1,:Nh]-Bz[:-2,:Nh])
    Ex[1:-1,Nh:-1] +=  b*(Bz[1:-1,Nh:-1]-Bz[:-2,Nh:-1])
    Ey[:-1,1:Nh] += -a*(Bz[:-1,1:Nh]-Bz[:-1,:Nh-1])
    Ey[:-1,Nh:-1] += -a*(Bz[:-1,Nh:-1]-Bz[:-1,Nh-1:-2])
    Ez[1:-1,1:-1]+=  a*(By[1:-1,1:-1]-By[1:-1,:-2]) - b*(Bx[1:-1,1:-1]-Bx[:-2,1:-1])

    I_sum += Ey ** 2
#   Ez[1:-1,1:-1]+=  a*(By[1:-1,1:-1]-By[1:-1,:-2]) - b*(Bx[1:-1,1:-1]-Bx[:-2,1:-1])
    s+=1
```

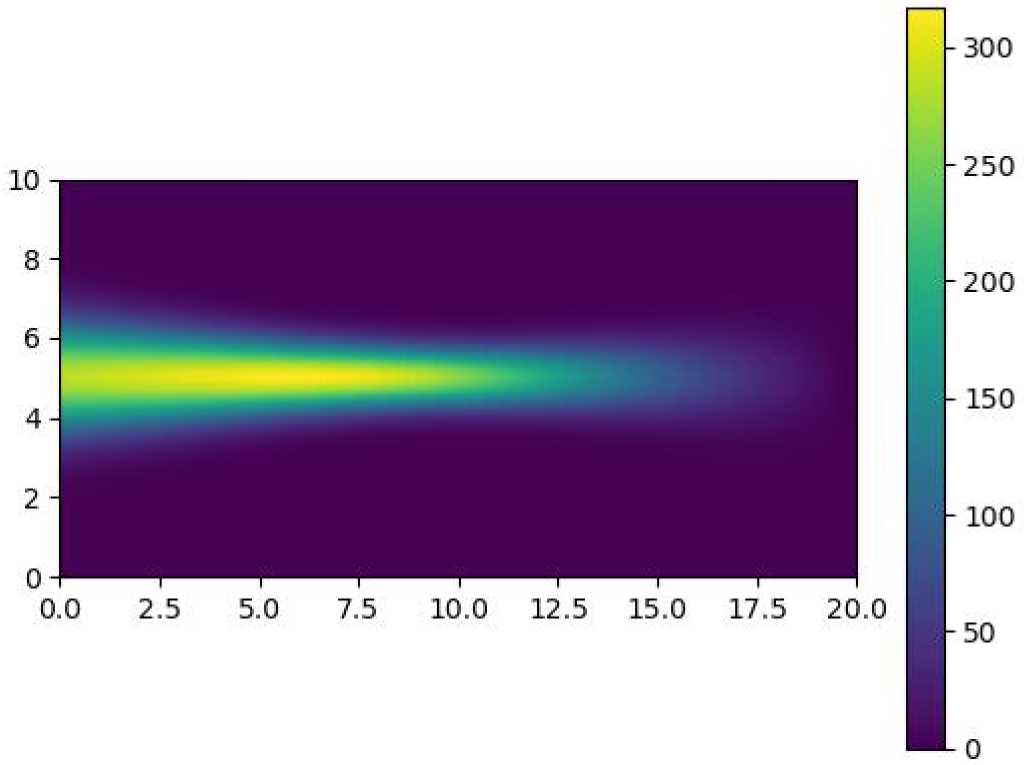
In [9]:

```
extn = (0,xmax,0,ymax)
plt.scatter(int(cntr/dy),100)
cs=plt.imshow(Ey,extent=extn); plt.colorbar(cs); plt.show()
```



In [10]:

```
extn = (0,xmax,0,ymax)
cs=plt.imshow(I_sum,extent=extn); plt.colorbar(cs); plt.show()
```



Measure radius of beam and compare it to theoretical value

Measured radius : where the intensity of wave becomes $\frac{1}{e^2}$ of center of gaussian. Since wave is basically sine-wave, we cannot take good measure when using square of E. (There's so many point which phase is not maximum or minimum, whose absolute value is amplitude. And intensity of wave is proportional to square of ampllitude.) Therefore, we will use commulated intensity for each iteration because this value can cancel the effect above.

In [11]:

```
r = 0 *x
for i in range(len(x)) :
    for j in range(int(len(y)/2)) :
        if abs(I_sum[j+int(cntr/dy),i]/np.max(I_sum[:,i])) <= np.e**(-2) :
            r[i] = j
            break

r = r*dy
```

```
C:\Users\LG\AppData\Local\Temp\ipykernel_2736\774945182.py:4: RuntimeWarning: inv
alid value encountered in double_scalars
  if abs(I_sum[j+int(cntr/dy),i]/np.max(I_sum[:,i])) <= np.e**(-2) :
```

theoretical value $r = r_0\sqrt{1 + z\lambda/\pi r_0^2}$

In [12]:

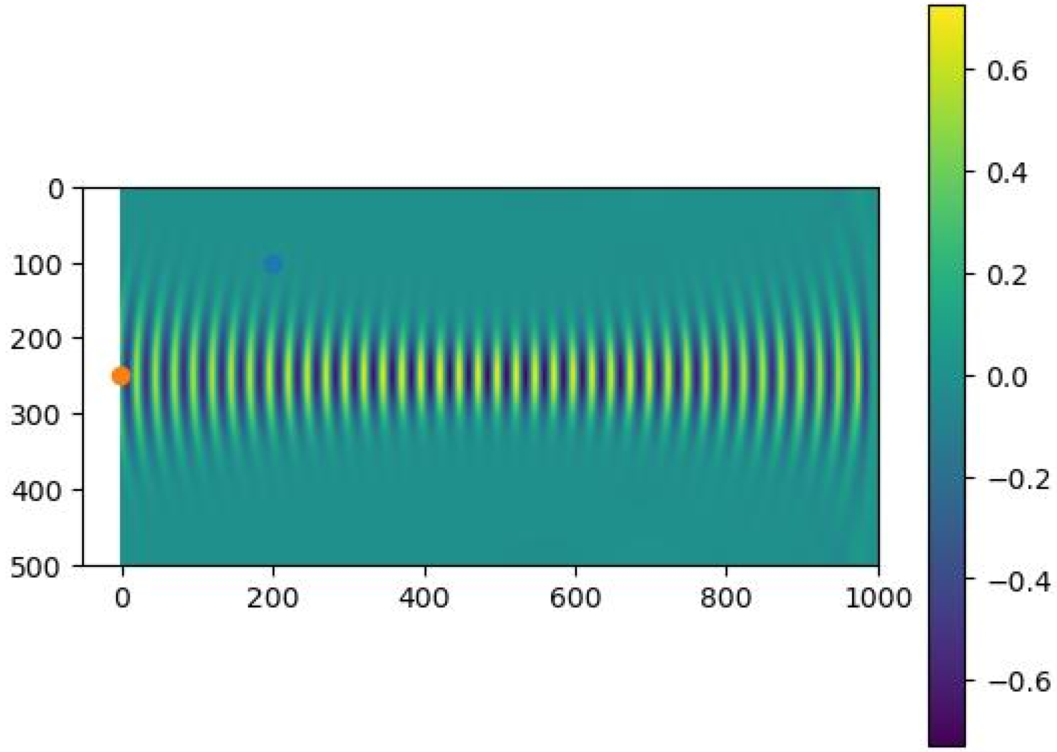
```
wavelength = 1/f
#mesh of wwave lenfth : wavelength/dx
w0 = r[int(10/dx)] # radius of focus
```

In [13]:

```
cal_r =w0*np.sqrt(1+(((x-10)/(np.pi*f*w0**2))**2))
#plt.plot(x,dx*r[0]*np.sqrt(1+(((x-10)*wl/(np.pi*r[0]**2))**2))
```

In [14]:

```
cs=plt.imshow(Ey); plt.colorbar(cs);plt.scatter(200,100)
plt.scatter(-1,int(cntr/dy)); plt.show()
```

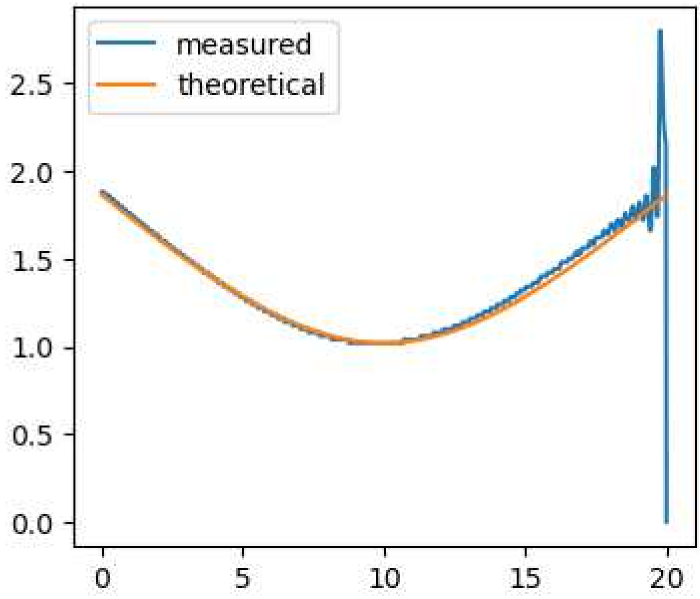


Compare theoretical and measure value

In [15]:

```
plt.figure(figsize = (4,3.5))
plt.plot(x,r,label='measured')
plt.plot(x,cal_r,label = 'theoretical')
plt.legend()

plt.show()
```

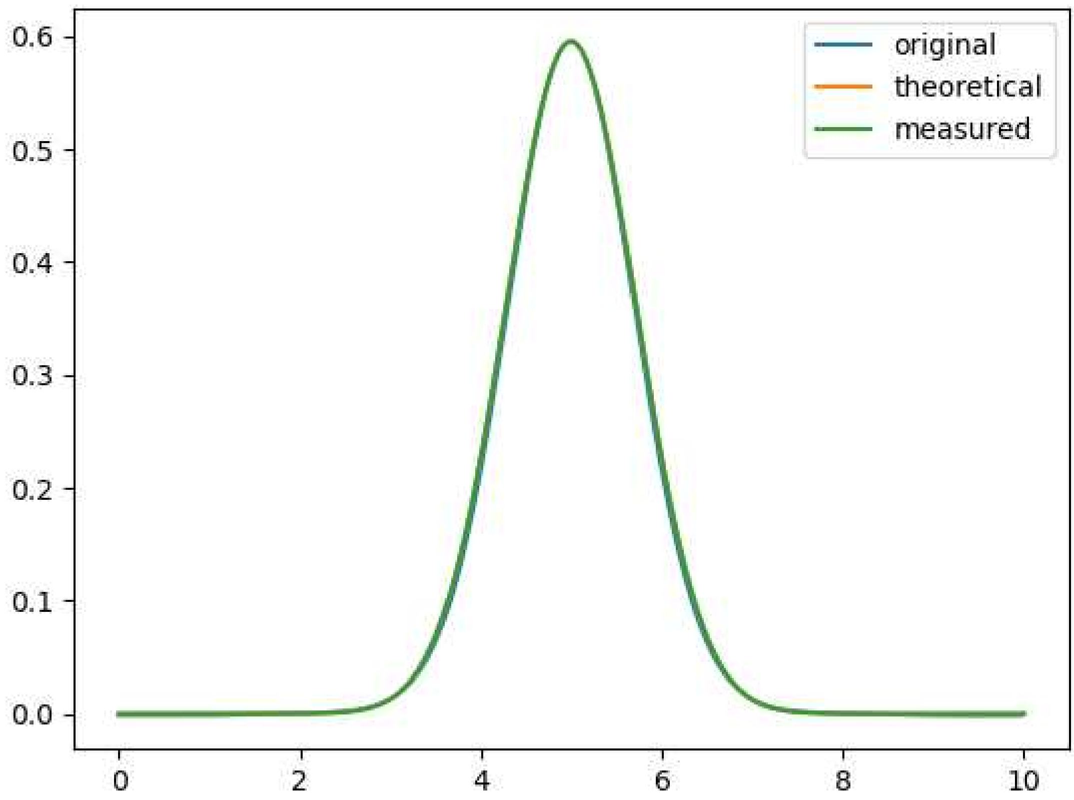


Compare for fixed x

In [16]:

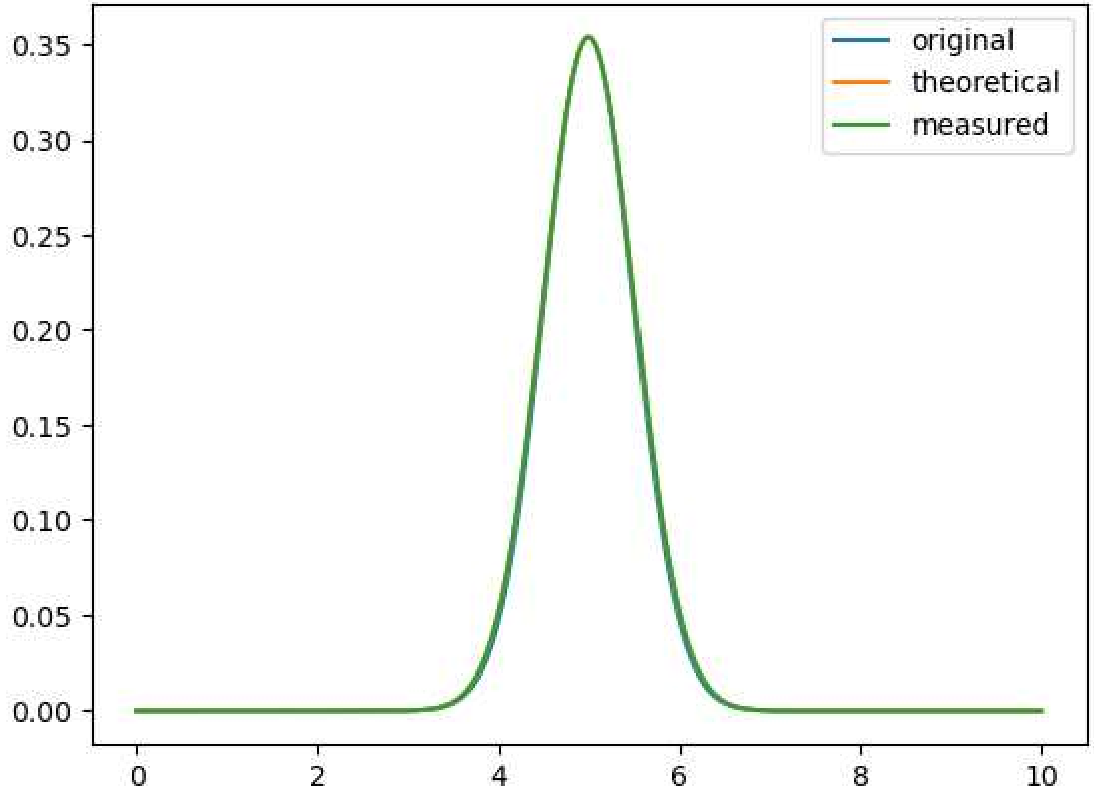
```
xc = 10
n = int(xc/dx)
print(n)
plt.plot(y,Ey[:,n],label = 'original')
plt.plot(y,max(Ey[:,n])*np.exp(-(y-cntr)**2/cal_r[n]**2),label='theoretical')
plt.plot(y,max(Ey[:,n])*np.exp(-(y-cntr)**2/r[n]**2),label = 'measured')
plt.legend()
plt.show()
```

500



In [17]:

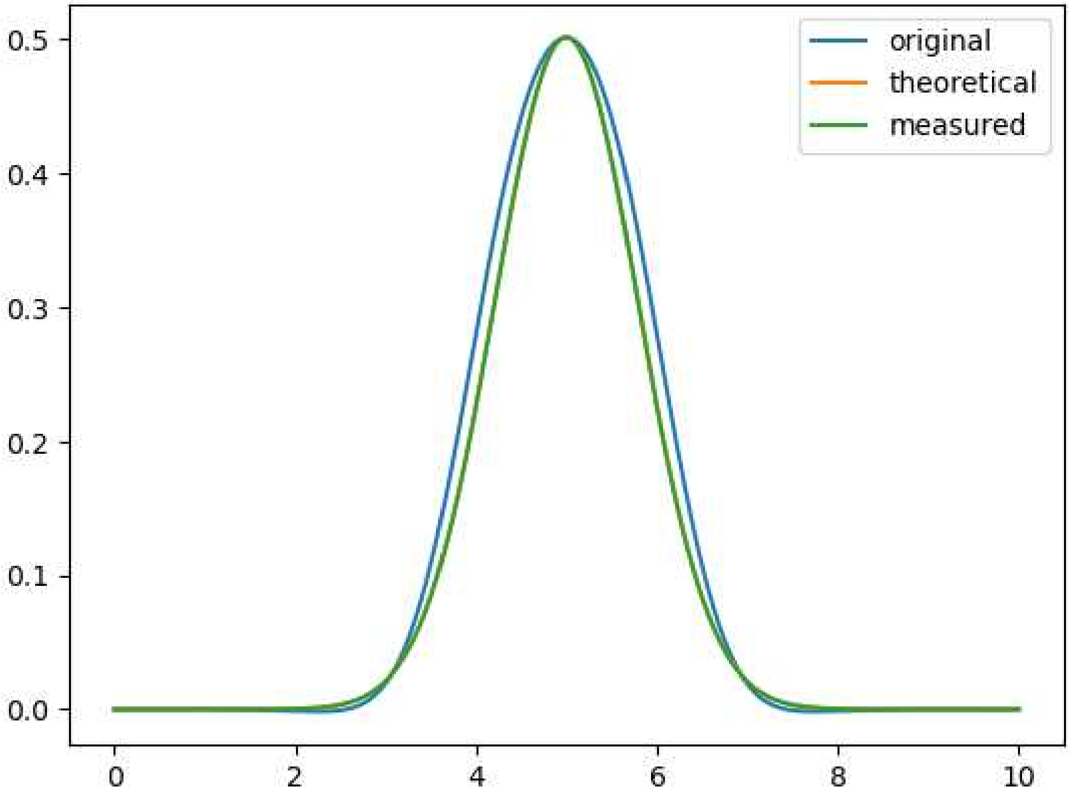
```
# intensity
plt.plot(y,Ey[:,n]**2,label = 'original')
plt.plot(y,(max(Ey[:,n])*np.exp(-(y-cntr)**2/cal_r[n]**2))**2,label='theoretical')
plt.plot(y,(max(Ey[:,n])*np.exp(-(y-cntr)**2/r[n]**2))**2,label = 'measured')
plt.legend()
plt.show()
```



In [18]:

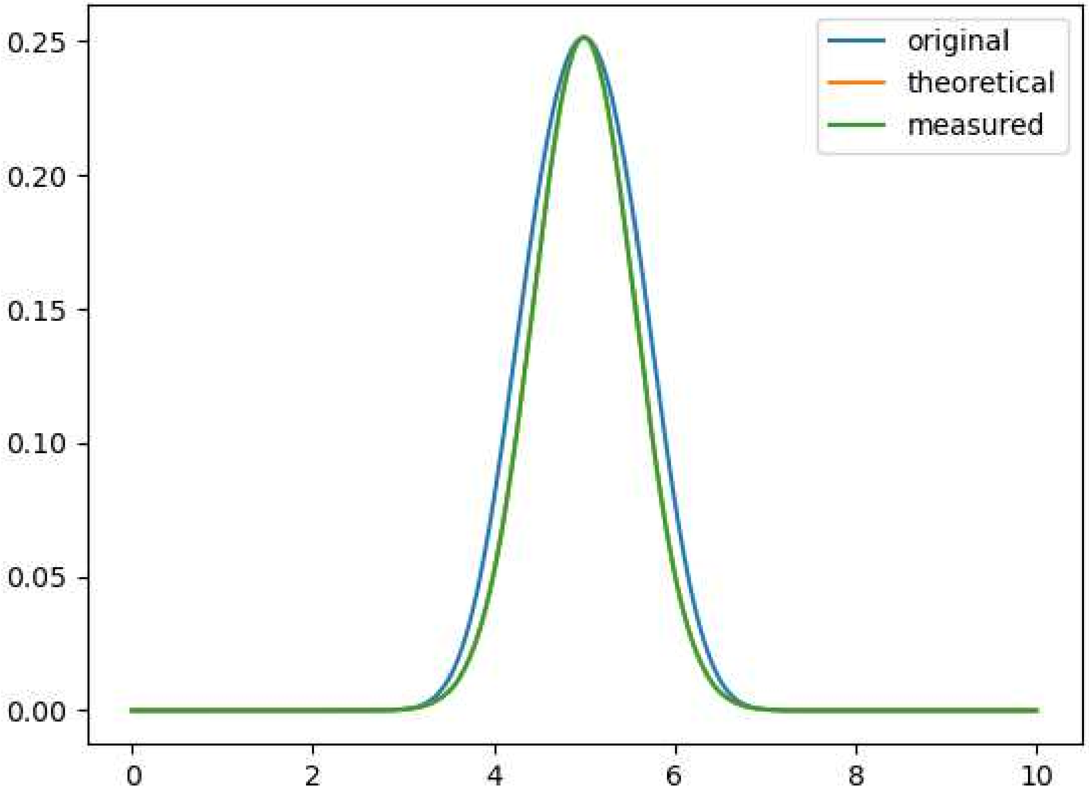
```
xc = 7
n = int(xc/dx)
print(n)
plt.plot(y,Ey[:,n],label = 'original')
plt.plot(y,max(Ey[:,n])*np.exp(-(y-cntr)**2/cal_r[n]**2),label='theoretical')
plt.plot(y,max(Ey[:,n])*np.exp(-(y-cntr)**2/r[n]**2),label = 'measured')
plt.legend()
plt.show()
```

350



In [19]:

```
#intensity
plt.plot(y,Ey[:,n]**2,label = 'original')
plt.plot(y,(max(Ey[:,n])*np.exp(-(y-cntr)**2/cal_r[n]**2))**2,label='theoretical')
plt.plot(y,(max(Ey[:,n])*np.exp(-(y-cntr)**2/r[n]**2))**2,label = 'measured')
plt.legend()
plt.show()
```



Peak value

In [20]:

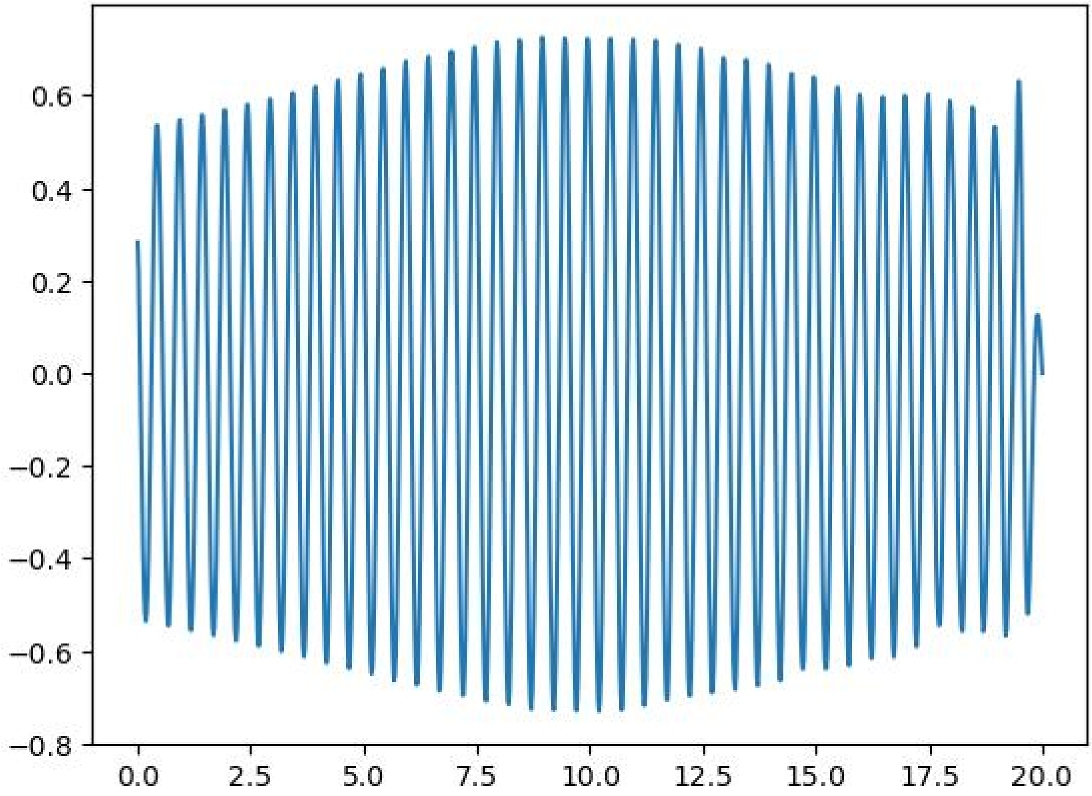
```
cntr_index = int(cntr/dy)
cntr_index
```

Out[20]:

250

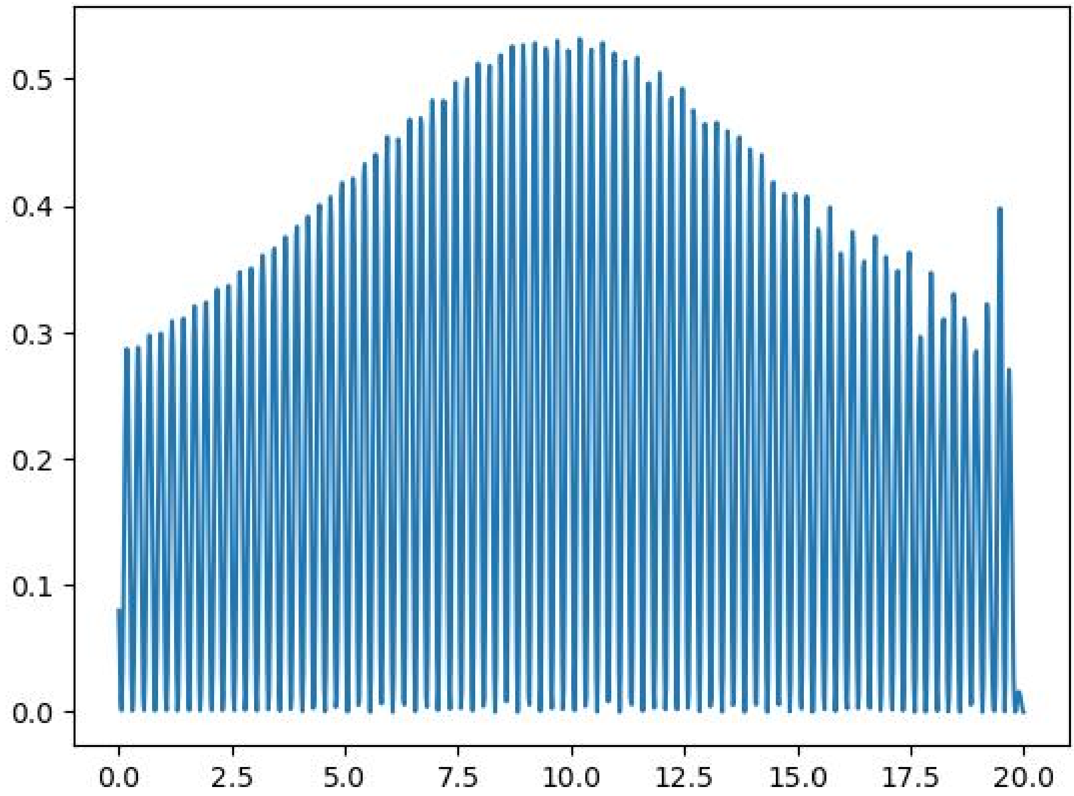
In [21]:

```
plt.plot(x,Ey[cntr_index, :]); plt.show()
```



In [22]:

```
plt.plot(x,Ey[cntr_index, :]**2); plt.show()
```



Power conservation

In general case(3D), $P = \int IdA$. But this case is 2 dimensional, so formula should be converted to $P = \int I(y)dy$ Since this is gaussian beam, $I = I_0 e^{-\frac{2y^2}{r^2}}$

$$P = \int I(y)dy = \int I_0 e^{-\frac{2y^2}{r^2}} dy = I_0 r \sqrt{\frac{\pi}{2}}$$

In [23]:

```
wavelength = 1/f
#mesh of wwave lenfth : wavelength/dx
w_index = int(wavelength/dx)
w_index
```

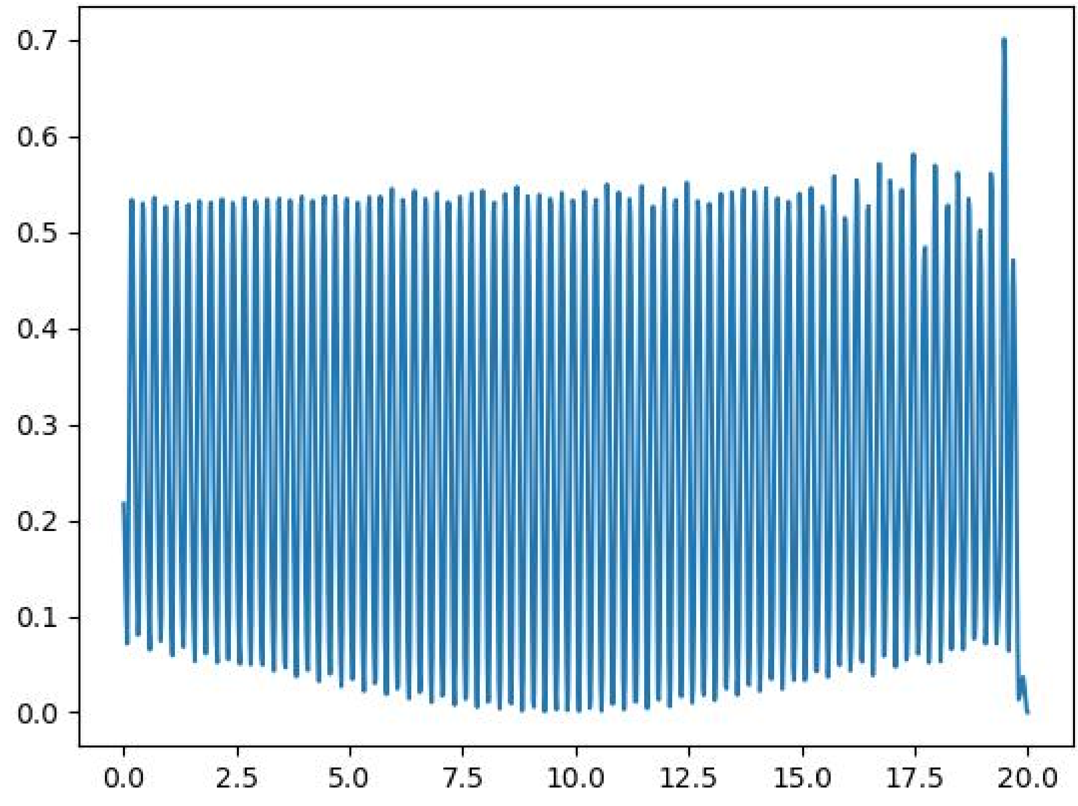
Out[23]:

25

In [24]:

```
P = r*np.max(Ey**2,axis =0)

plt.plot(x,P)
plt.show()
```



Result : For pick value,(wave has maximum-that is, amplitude) it has almost same values.

In [40]:

```
I = 0*X
for i in range(len(x)) :
    t = s - (1/a)*i # time from starting oscillating to end for x-index i
    if t < 0 :t =1
    I[: , i] = I_sum[:,i]/t

P = np.max(I,axis=0)*r
```

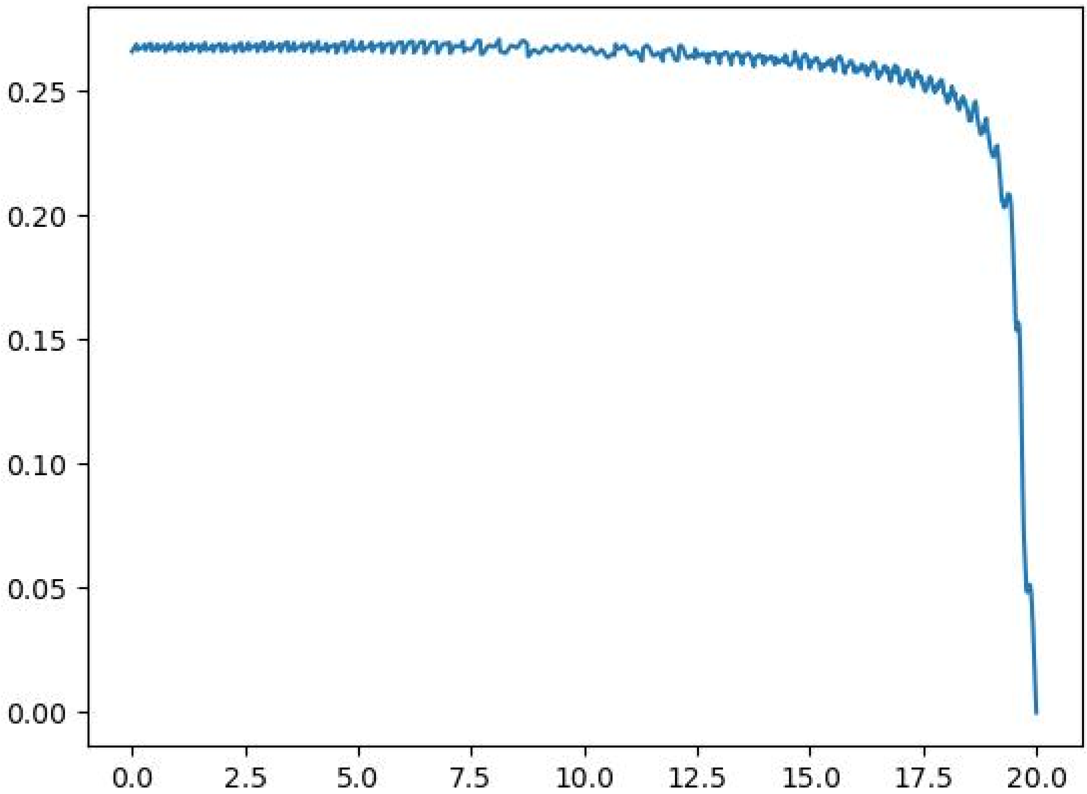
In [ ]:

```
I = 0*X
for i in range(len(x)) :
    t = s - (1/a)*i # time
    if t < 0 :t =1
    I[: , i] = I_sum[:,i]/t

P = np.max(I,axis=0)*r
```

In [42]:

```
plt.plot(x,P)
#plt.plot(x,np.sum(I,axis=0))
plt.show()
```



In [ ]:

```
```

In [43]:

```
plt.plot(x,np.sum(I,axis=0))
plt.show()
```