Question 2. In ‘oblique.py’, split the domain in x-direction to get two regions with different refractive indices n1 and n2 (see the figure). Put the emitEMwave boundary on the left side. For n1 > n2 (e.g. n1 = 2, n2 = 1), measure the critical angle. For n1 < n2, measure the Brewster’s angle.

In [1]:

```
import numpy as np
import matplotlib.pyplot as plt
import emitEMwave as ant
import emitEMwave_update as antu
import math as mt
```

In [6]:

```
def refraction(xmax, ymax, dx, dy, dt,f,D,smax,phi,apk,n1,n2) :
    a=dt/dx;  b=dt/dy
    w=2.0*np.pi*f
    s=0

    cntr=0.6*ymax
    #upper=int(0.5*(ymax+D)/dy)
    #lower=int(0.5*(ymax-D)/dy)
    upper=int(D/dy)
    lower=int(0)

    x=np.arange(0,xmax+dx,dx)
    y=np.arange(0,ymax+dy,dy)
    Nh = int(xmax/dx/2)

    n = n2/n1 #relative refractive idnex

    X,Y=np.meshgrid(x,y)
    Ex=0*X; Ey=0*X; Ez=0*X
    Bx=0*X; By=0*X; Bz=0*X
    while s<smax:
        #Ey[:,0]= np.exp(-(y-cntr)**2/(0.2*ymax)**2)*np.sin(w*s*dt) # emission
        #Ey[lower:upper,0]= np.sin(w*s*dt) # hole
        ant.emitEMwave(s*dt,Ey[lower:upper,0],(dx,dy),'ovwrt','p',phi,0,f,0,0,6,(1.0,),2,apk)
        ant.emitEMwave(s*dt,Ez[lower:upper,0],(dx,dy),'sum','s',phi,0,f,0,0,6,(1.0,),2,apk)
        Bx[:-1,:-1] +=  -b*(Ez[1:,:-1]-Ez[:-1,:-1])
        By[1:-1,:-1] +=  a*(Ez[1:-1,1:]-Ez[1:-1,:-1])
        Bz[:-1,:-1]+= -a*(Ey[:-1,1:]-Ey[:-1,:-1]) + b*(Ex[1:,:-1]-Ex[:-1,:-1])

        Ex[1:-1,:Nh] +=  1.0*b*(Bz[1:-1,:Nh]-Bz[:-2,:Nh])
        Ex[1:-1,Nh:-1] +=  1.0*b*(Bz[1:-1,Nh:-1]-Bz[:-2,Nh:-1])/n**2

        Ey[:-1,1:Nh] += -1.0*a*(Bz[:-1,1:Nh]-Bz[:-1,:Nh-1])
        Ey[:-1,Nh:-1] += -1.00*a*(Bz[:-1,Nh:-1]-Bz[:-1,Nh-1:-2])/n**2

        Ez[1:-1,1:Nh]+=  1.0*(a*(By[1:-1,1:Nh]-By[1:-1,:Nh-1]) - b*(Bx[1:-1,1:Nh]-Bx[:-2,1:Nh])
        Ez[1:-1,Nh:-1]+=  1.00*(a*(By[1:-1,Nh:-1]-By[1:-1,Nh-1:-2]) - b*(Bx[1:-1,Nh:-1]-Bx[:-2
        s+=1

    extn=(0,xmax,0,ymax)
    plt.subplot(1,2,1);cs=plt.imshow(Ey,origin='lower',extent=extn); plt.colorbar(cs)
    plt.subplot(1,2,2);cs=plt.imshow(Ez,origin='lower',extent=extn); plt.colorbar(cs)
    plt.show()

    if n1 > n2 :
        #print(np.max(abs(Ey[:,-1])))
        #print(np.max(abs(Ey[:,Nh+int(Nh/2):])))
        m1 = np.max(abs(Ey[:,Nh+3:]))
        m2 = np.max(abs(Ey[:,Nh:]))
        return abs(m1-m2)
    else :
        print(np.sum(Ey[:,Nh]**2))
        return
```

Find critical angle (n1>n2)

n1 = np.sqrt(2)

n2 = 1

In [5]:

```
#set the variables
xmax=float(6)
ymax=float(16)
dx=float(0.02)
dy=float(0.02)
dt=float(0.01)
f=float(4)
D=float(7)
smax=int(700)
dsav = 50
```

In [7]:

```
n1 = np.sqrt(2)
n2 = 1
```

$$\theta_{critical} = arcsin(\frac{n2}{n1})$$

In [8]:

```
cri_phi = mt.asin(n2/n1)
cri_phi
```

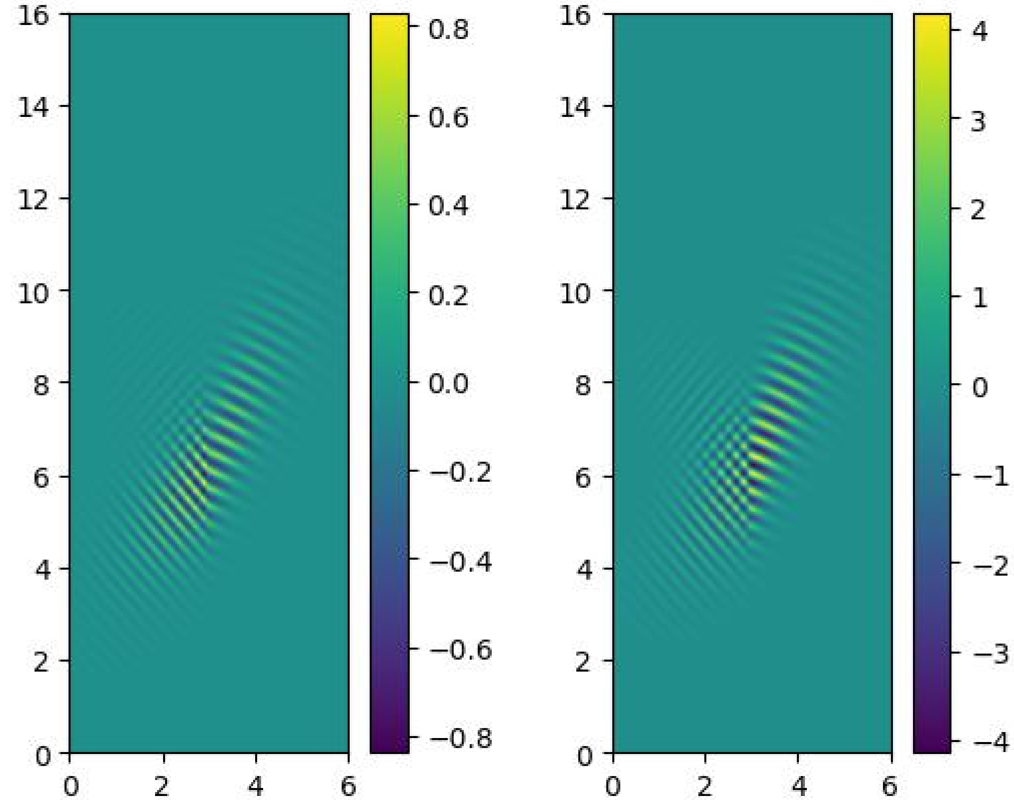
Out[8]:

```
0.7853981633974482
```

Before critical angle

In [11]:

```
refraction(xmax, ymax, dx, dy, dt,f,D,800,0.7,1,n1,n2)
```



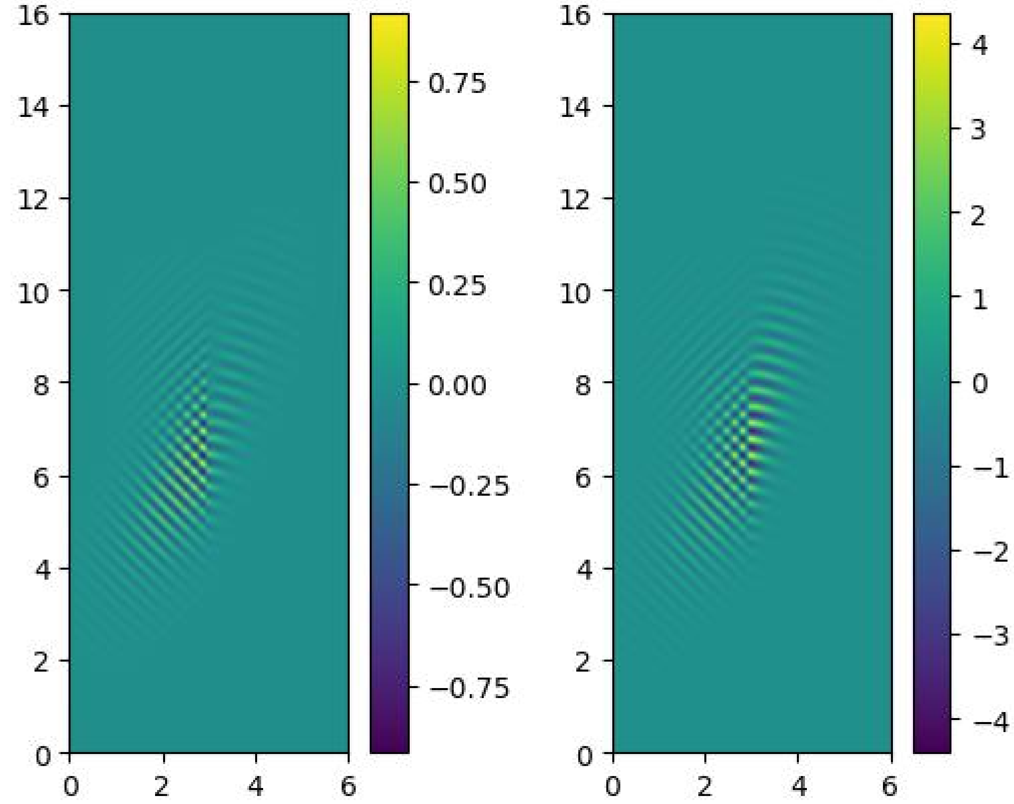
Out[11]:

```
0.0
```

At critical angle

In [10]:

```
refraction(xmax, ymax, dx, dy, dt,f,D,800,np.pi/4,1,n1,n2)
```



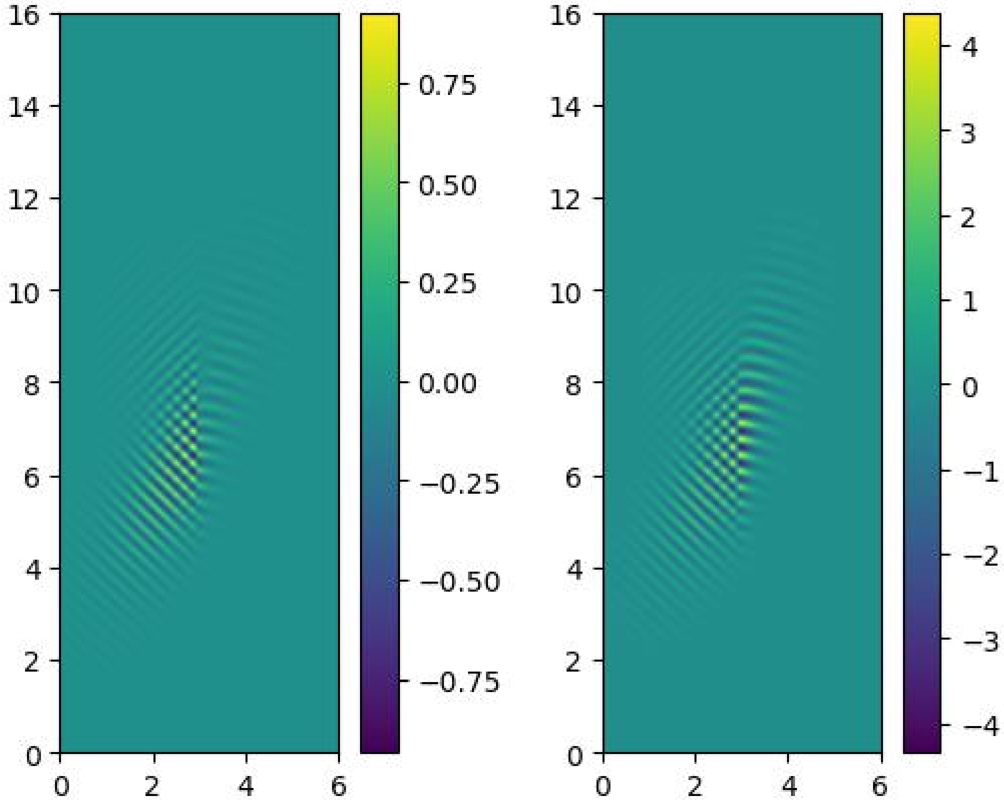
Out[10]:

0.07270534217075314

After critical angle

In [12]:

```
refraction(xmax, ymax, dx, dy, dt,f,D,800,0.8,1,n1,n2)
```

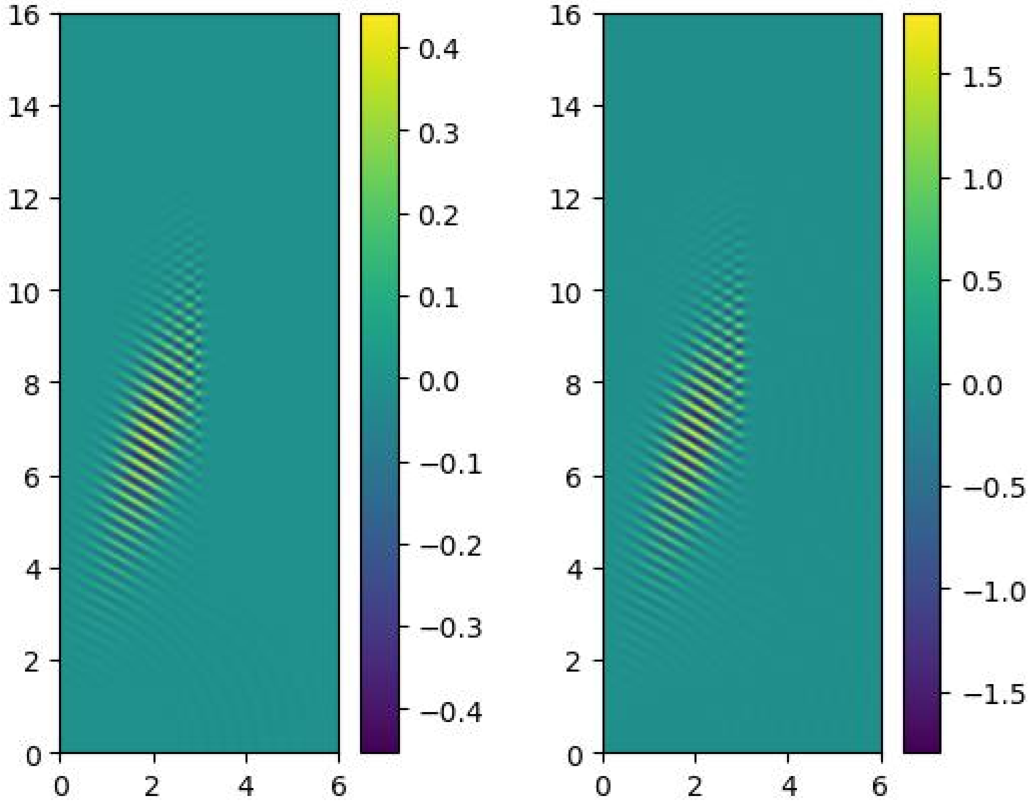


Out[12]:

0.09295891457505362

In [13]:

```
refraction(xmax, ymax, dx, dy, dt,f,D,800,np.pi/3,1,n1,n2)
```



Out[13]:

0.16291127632750432

In [53]:

```
for phi in np.arange(0,np.pi/2,0.01) :
    a = refraction(xmax, ymax, dx, dy, dt,f,D,800,phi,1,n1,n2)
    if a > 0.01 :
        break
```

```
---------------------------------------------------------------------------
----
KeyboardInterrupt                         Traceback (most recent call last)
~\AppData\Local\Temp\ipykernel_1736\268320012.py in <module>
      1 for phi in np.arange(0,np.pi/2,0.01) :
----> 2     a = refraction(xmax, ymax, dx, dy, dt,f,D,800,phi,1,n1,n2)
      3     if a > 0.01 :
      4         break

~\AppData\Local\Temp\ipykernel_1736\662373330.py in refraction(xmax, yma
x, dx, dy, dt, f, D, smax, phi, apk, n1, n2)
     24         ant.emitEMwave(s*dt,Ey[lower:upper,0],(dx,dy),'ovwrt','p',ph
i,0,f,0,0,6,(1.0,),2,apk)
     25         ant.emitEMwave(s*dt,Ez[lower:upper,0],(dx,dy),'sum','s',phi,
0,f,0,0,6,(1.0,),2,apk)
---> 26         Bx[:-1,:-1] +=  -b*(Ez[1:,:-1]-Ez[:-1,:-1])
     27         By[1:-1,:-1] +=  a*(Ez[1:-1,1:]-Ez[1:-1,:-1])
     28         Bz[:-1,:-1]+= -a*(Ey[:-1,1:]-Ey[:-1,:-1]) + b*(Ex[1:,:-1]-E
x[:-1,:-1])

KeyboardInterrupt: 
```

n1<n2 Find Brewster's angle

In [14]:

```
n1 = float(1.0)
n2 = float(2.0)
n = n2/n1
```

$$\theta_{Brewster} = arctan(\frac{n2}{n1})$$

In [15]:

```
bre_phi = mt.atan(n)
bre_phi
```

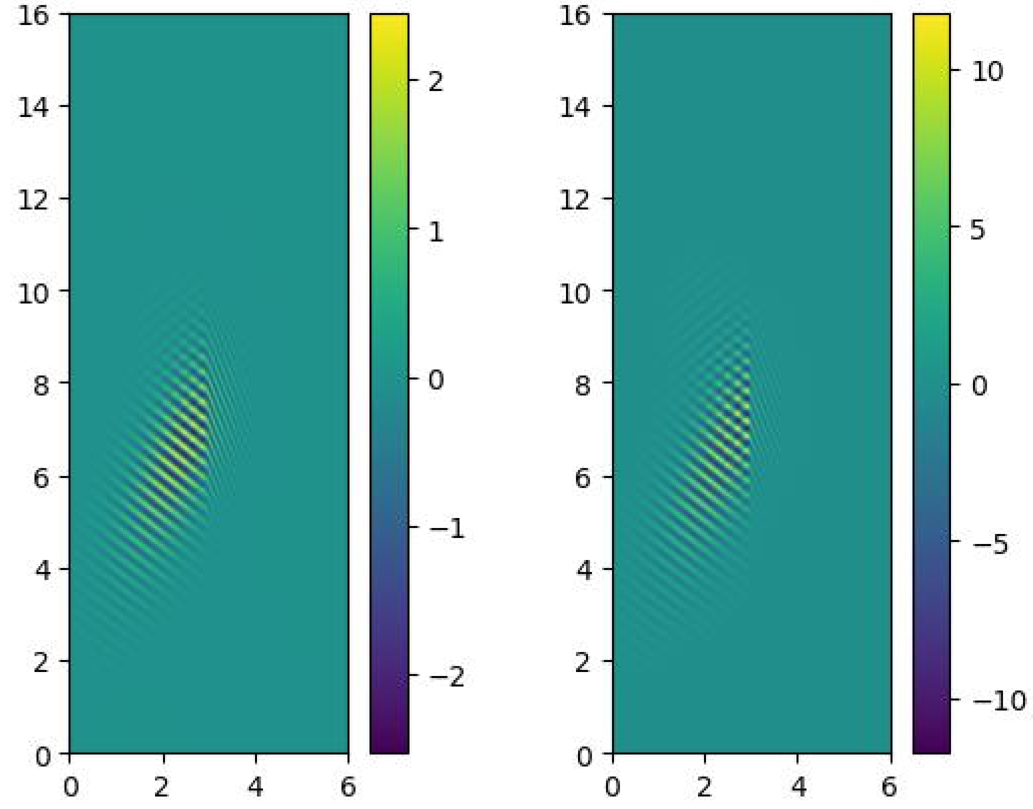
Out[15]:

```
1.1071487177940904
```

Before Brester angle

In [19]:

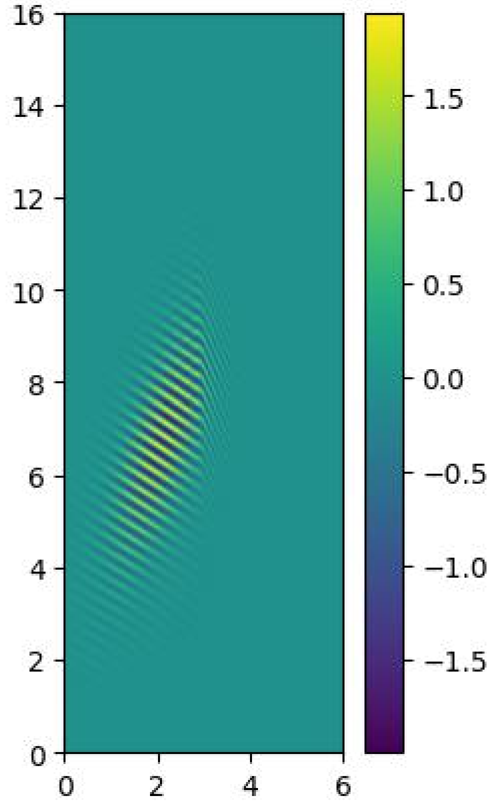
```
refraction(xmax, ymax, dx, dy, dt,f,D,800,0.9,4,n1,n2)
```

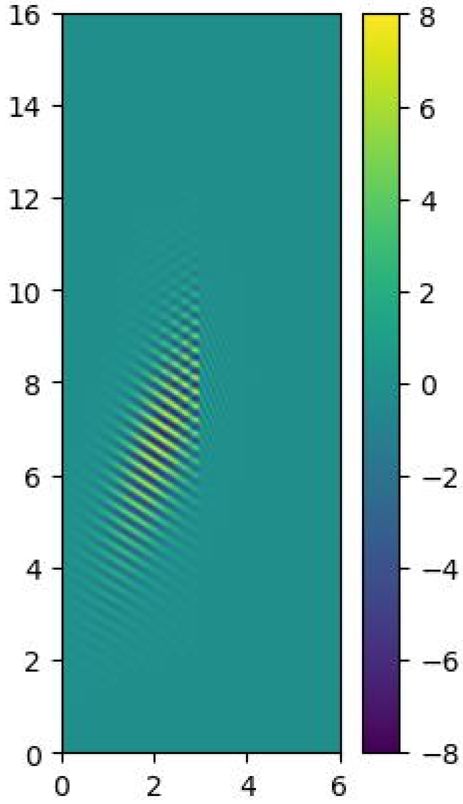


147.57763892837232

In [17]:

```
refraction(xmax, ymax, dx, dy, dt,f,D,800,1,4,n1,n2)
```



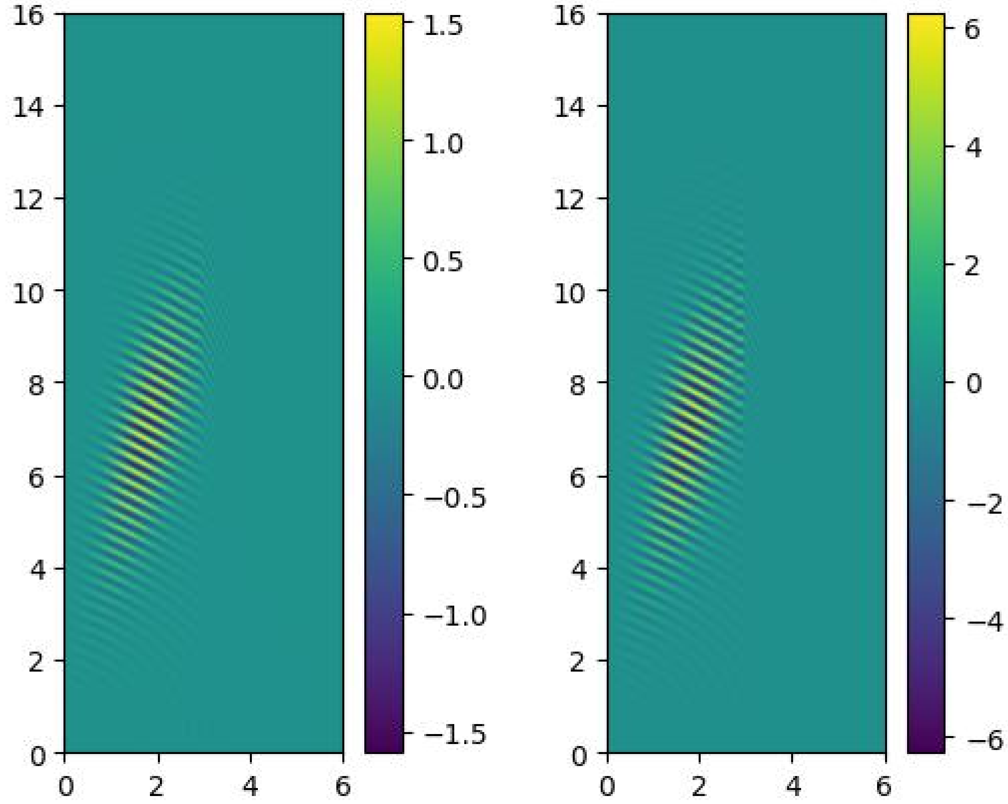


69.04020997665765

At Brester angle

In [16]:

```
refraction(xmax, ymax, dx, dy, dt,f,D,800,bre_phi,4,n1,n2)
```

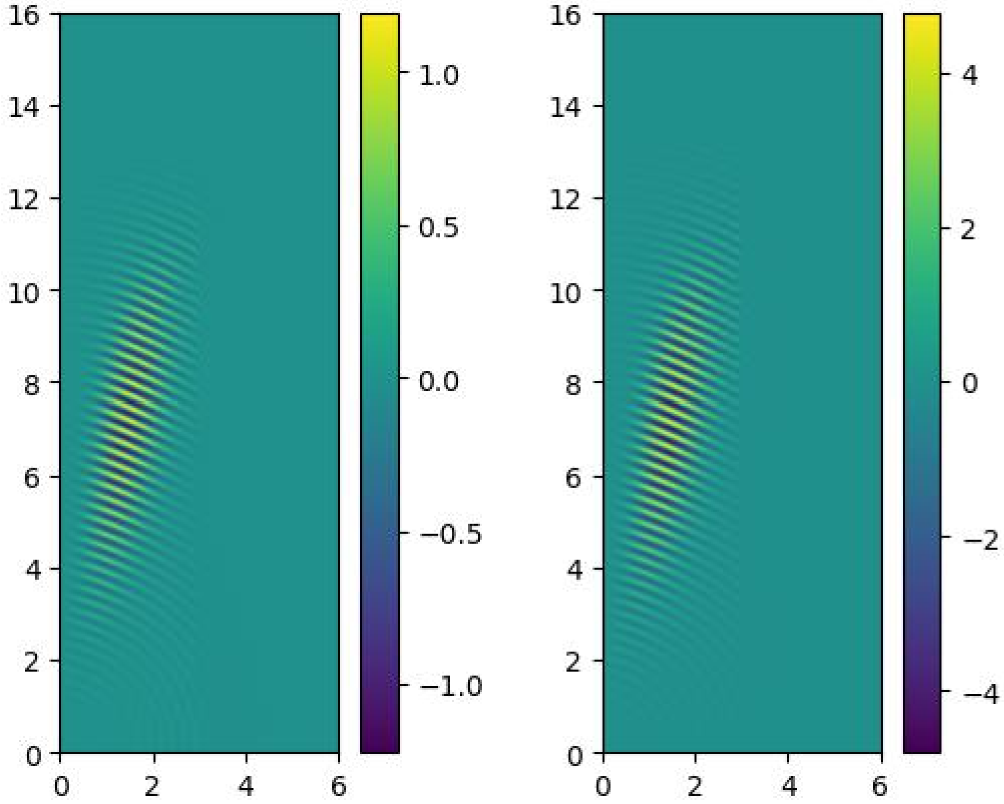


14.331901826992537

After Brester angle

In [18]:

```
refraction(xmax, ymax, dx, dy, dt,f,D,800,1.2,4,n1,n2)
```



2.1106487304024477

In [ ]: