

Let $P(i,j)$ be the probability having a link between node i and j .

\therefore Probability that node i, j, l form a triangle $P(i,j)P(j,l)P(l,i)$

- expected number of triangles in which node l with degree k_l = sum of probability that node l participates in triangles with arbitrary chosen nodes i and j in the network

$$N_{k_l}(\Delta) = \sum_{i=1}^N \sum_{j=1}^N d_j P(i,j) P(i,l) P(j,l)$$

Let assume $t_j = j$ the time when node j arrived.

probability that node j links to node i when $\underbrace{t=j}_{\text{arrival of } j}$ $P(i,j) = m \pi(k_i(j)) = m \frac{k_i(j)}{\sum_{j=1}^j k_l(j)} = m \frac{k_i(j)}{2mj}$

$$k_i(t) = m \left(\frac{t}{t_i} \right)^{\frac{1}{2}} = m \left(\frac{j}{i} \right)^{\frac{1}{2}} = m(ij)^{-\frac{1}{2}}$$

$$\therefore P(i,j) = \frac{1}{2j} m(ij)^{-\frac{1}{2}}$$

$$N_{k_l}(\Delta) = \sum_{i=1}^N d_i \sum_{j=1}^N d_j P(i,j) P(i,l) P(j,l) = \frac{m^2}{8} \sum_{i=1}^N d_i \sum_{j=1}^N d_j (ij)^{-\frac{1}{2}} (il)^{-\frac{1}{2}} (jl)^{-\frac{1}{2}}$$

$$= \frac{m^3}{8} \sum_{i=1}^N \frac{d_i}{i} \sum_{j=1}^N \frac{d_j}{j} = \frac{m^3}{8l} (\ln N)^2$$

$$C_l = \frac{2N_{k_l}(\Delta)}{k_l(k_l-1)} = \frac{m^3 k_l (\ln N)^2}{k_l(N) (k_l(N)-1)}$$

$$k_l(N) = m \left(\frac{N}{l} \right)^{\frac{1}{2}} \quad \text{from } k_i(t) = m \left(\frac{t}{t_i} \right)^{\frac{1}{2}}$$

$$k_l(N) (k_l(N)-1) \approx k_l^2(N) = m^2 \frac{N}{l}$$

$$\therefore C_l = \frac{\frac{m^3}{8l} (\ln N)^2}{m^2 \frac{N}{l}} = \frac{m}{4} \frac{(\ln N)^2}{N} \rightarrow \text{independent on } l.$$

$$\therefore \langle C \rangle = \frac{m}{4} \frac{(\ln N)^2}{N}$$