

CSC505 Jennings Homework 1, Spring 2020 There are 16 questions. Please answer all of them.

1. Name: Shashank Shekhar (Student Id: 200262327)

2. Consider the following pseudo-code:

```
Algorithm ApproxArea
Inputs:   $f$  (a function),
          $x_0, x_1$  (floating point numbers),
          $n$  (a positive integer)
Output: an approximation of the area under  $f(x)$  between  $x_0$  and  $x_1$ 
1  $sum \leftarrow 0$ 
2  $delta \leftarrow (x_1 - x_0) / n$ 
3  $x \leftarrow x_0$ 
4 for  $i = 1$  to  $n-1$ 
5    $x \leftarrow x + delta$ 
6    $sum \leftarrow sum + f(x)$ 
7 return  $(delta/2) * (f(x_0) + (2 * sum) + f(x_1))$ 
```

We are interested in the asymptotic complexity of **ApproxArea** as n grows large. In particular, we want to know $T(n)$ = the number of times we evaluate the function f , for a given value of n .

Give a formula for $T(n)$ and support it using an argument that refers to specific lines (by number) in the pseudo-code above.

Note that in pseudo-code, the convention is that the bounds on *for* loops are inclusive.

Ans: $T(n) = n+1$

Supporting Argument: Function f is called $(n-1)$ times in the for loop in line 4 and two more times in line 7. Therefore, total = $(n-1) + 2 = (n+1)$ times

3. Consider the following pseudo-code:

```
Algorithm FelixHausdorff
Inputs:   S, a non-empty set of points (x, y),
          T, a non-empty set of points (x, y)
Output: a vector  $v=(v_x, v_y)$  that minimizes the maximum value of
distance(s+v, t) for any points s in S and t in T
1 min_dist  $\leftarrow$  infinity
2 min_vec  $\leftarrow$  null
3 max_dist  $\leftarrow$  0
4 max_vec  $\leftarrow$  null
5 for each s in S
6     for each t in T
7         v  $\leftarrow$  subtract(s, t)           // v+s = t
8         max_dist  $\leftarrow$  0
9         for each s in S
10            d  $\leftarrow$  distance(v+s, t)
11            if d > max_dist then
12                max_dist  $\leftarrow$  d
13                max_vec  $\leftarrow$  v
14            if max_dist < min_dist then
15                min_vec = v
16                min_dist = max_dist
17 return min_vec
```

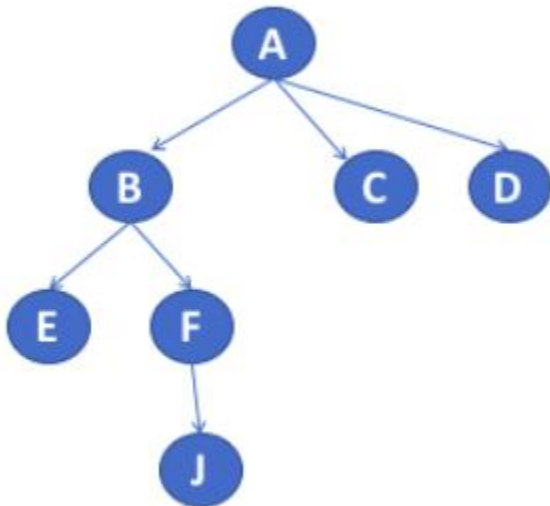
We are interested in the worst-case asymptotic complexity of **FelixHausdorff** as the sizes of S and T grow large. In particular, we want to know $T(n, m)$ = the number of times we evaluate the function distance().

Assume S has n elements, and T has m elements, and give a formula for $T(n, m)$. Support it by referencing (by line number) the pseudo-code above.

Ans: $T(n, m) = n^2m$

Support Argument: Function distance() is called inside 3 for loops. 1st for loop iterates n times (line 5), 2nd for loop iterates m times (line 6) and the 3rd for loop iterates n times (line 9). Since, these for loops are nested, $T(n) = n*m*n = n^2m$

Use the following tree for the next 3 questions:



4. Write the order in which the nodes of the tree above will be visited in a *pre-order traversal*.

Ans: Preorder Traversal: A->B->E->F->J->C->D

5. Write the order in which the nodes of the tree above will be visited in a *post-order traversal*.

Ans: Postorder Traversal: E->J->F->B->C->D->A

6. Write the order in which the nodes of the tree above will be visited in a *level order traversal*, also known as a *breadth-first traversal*.

Ans: Level order (Breadth First Search) Traversal: A->B->C->D->E->F->J

7. Which of these worst-case running times is better (grows more slowly) than the others?

- ☐ n^2
- ☐ $n^2 \log n$
- ☒ $\log n$
- ☐ $n \log n$

8. Which of these worst-case running times is worse (grows more quickly) than the others?

- ☐ n^2
- ☒ 2^n
- ☐ $n^2 \log n$
- ☐ n^3

9. Consider a singly-linked list with a head pointer. There are n items in the list. To find a given value (element) in the list requires how much time, in the worst case?

- ☐ $O(\log n)$
- ☐ $O(n \log n)$
- ☒ $O(n)$
- ☐ $O(1)$

10. Consider a singly-linked list with a head pointer. There are n items in the list. To insert an item at the front of the list requires how much time, in the worst case?

- ☐ $O(\log n)$
- ☐ $O(n \log n)$
- ☐ $O(n)$
- ☒ $O(1)$

11. Consider a doubly-linked list with head and tail pointers. There are n items in the list. To insert an item at the end of the list requires how much time, in the worst case?

- ☐ $O(\log n)$
- ☐ $O(n \log n)$
- ☐ $O(n)$
- ☒ $O(1)$

12. Consider an array-based list with the head of the list at index 0 (zero). Inserting a new element at the head of the list requires how much time, in the worst case? (Assume the array has enough capacity for the new element.)

- $O(\log n)$
- ✓ $O(n)$
- $O(n \log n)$
- $O(1)$

13. Which single answer best describes binary search in a linear data structure?

- Needs only $O(n)$ comparisons to search n elements
- ✓ Needs only $O(\log n)$ comparisons to search n elements
- The input array of elements can start off in any order
- Works for already-sorted arrays and linked lists

14. How much space is needed to store an arbitrary positive integer?

- 32 bits
- ✓ Varies with machine architecture, e.g. 16-, 32-, or 64-bits
- $\log(n)$ bits needed to store the integer n
- n bits needed to store the integer n

15. A recursive function...

- ✓ Calls itself
- Calls itself in an infinite loop
- Calls another function which calls another function, and so on
- Maintains its own stack

16. If $T(n)$ counts the number of times a function performs a specific operation X , then which of these $T(n)$ describes a recursive function that uses X ?

- $T(n) = 4$
- $T(n) = 2n + 4$
- $T(n) = n + 1$
- ✓ $T(n) = 2T(n/2) + 1$