

## Algorithms – HW6

1.

- a) Yes, Heuristic function:  $h(x)$  is admissible. A heuristic function is admissible if it never overestimates the actual (minimum) cost of reaching its goal. Here,  $h(x)$  is defined as a Euclidean Distance function. In 3D space, Euclidean distance (is the straight-line distance) between any two points is the minimum distance of reaching from 1 point to another and hence this heuristic function never overestimates the actual cost of reaching the goal.
- b) The maximum number of edges that any node can have is 6.  
This include 4 edges along the 4 directions in a plane and 2 edges along the top and bottom direction.

2.

Reference:

- [Coin Change – Breadth First Search](#)
- [Coin Change – Best First Search](#)

- a) The heuristic makes an estimate that a solution can be found from the current node by adding 1 more coin. For the final state this assumption is wrong since being in the final state, we do not need any more coins to reach the goal (the final state itself) and hence the heuristic function [ $h(x) = 1$  coin] is not admissible.
- b) A graph structure to solve the coin change problem using A\* search algorithm:  
A state  $S_x$  is defined as the minimum number of coins that we would need to take from the set of coins provided to get the total needed (goal state – assume it N). Now, if we take each state ( $S_x$ ) as a node in the graph such that each node is connected to ( $S_{x-c[0]}, S_{x-c[1]}, S_{x-c[2]}, \dots, S_{x-c[N-1]}$ ), then we can reach from node  $S_x$  to any of its immediate neighbors with a cost of 1 (this is because we will have to add 1 coin to reach any of its neighbors, given the above graph). Thus, our  $g(n) = 1$  represents the path distance from any node to its neighbor and is a constant function. Also, if we use a heuristic function,  $h(n)$  for each node which tells the estimated cost to reach from a node  $n$  to the goal node (ie. the state which generates the required amount), then A\* search can be applied using an evaluation function [fitness function –  $f(n)$ ] at each node [ $f(n) = g(n) + h(n)$ ] to decide the next node to be considered amongst the set of nodes to be expanded next. With this additional information provided by the Heuristic function, we can make informed decisions which are also optimal to reach our goal state.

3.

- a)  $h(x)$  = cost of direct flight from  $x$  to the goal city is not admissible because even with a direct flight, it may be cheaper to fly to the destination using 1 or more hops. Since, this is not considered by the heuristic function, it overestimates the actual (minimum) cost of reaching the goal. Hence, the heuristic function given is not admissible.
- b) No,  $h(x)$  is not consistent. This is because the cost to reach from city  $x$  to city  $y$  via city  $z$  may be less than the cost to reach directly from city  $x$  to city  $y$ . Since the heuristic measures the direct cost between two nodes, the value of fitness function  $f(n)$  will go down when city  $y$  is visited again via city  $z$  compared to its previous known value when city  $y$  was directly visited from city  $x$ .

4.

Resource: [Solving 8-Puzzle using A\\* Algorithm](#)

