ECE/CSC570: Computer Networks

Spring 2020

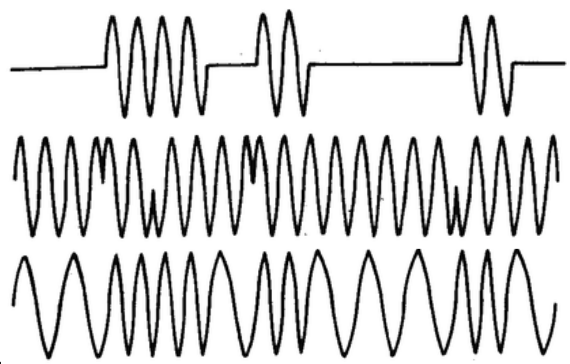
Prof. Shih-Chun Lin

Homework 3

Due: 11:59 pm, Monday, March 9th, 2020

1. Conceptual Question (20 points)

a. Given that the baseband signal is 00110100010, identify the modulation schemes of the following modulated signals. (6 points)



Signal A.

Signal B.

Signal C.

b. Suppose you are tossing a biased coin, which shows head (H) with probability p at every toss. Let random variable (RV) X denote the number of tosses until the first H shows up. What distribution does X follow? Give the PMF of RV X, and derive its expected value $\mathbf{E}\{X\}$ as a function of variable p. (5 points)

c. What are the main design issues of the Data Link layer in the OSI reference model? (4 points)

d. What is the Hamming distance of a code scheme with the following four code words?

[00000], [01011], [10110], [11101]

How many bit-errors can it detect, and how many can it correct? (5 points)

2. Bandwidth and Data Rates (20 points)

An eNodeB (base station) is transmitting to a UE (mobile phone) at power P_{tx} = 5 Watts, on a 20 MHz downlink channel. Suppose the received power at the UE can be obtained as

$$P_{rx}\text{ (dBm)} = P_{tx}\text{ (dBm)} - 50 - 30\log_{10} d\text{ (Km)},$$

where d (Km) is the distance between the eNodeB and the UE. Suppose the measured noise power within the channel bandwidth is always -30 dBm at the UE, and the UE moves from point A (distance to the eNodeB d_A = 1 Km) to point B (d_B = 2 Km).

a. What is the SNR of the UE at point A (in dB)? (5 points)
b. What is the capacity of the downlink channel when UE is at point A (in Mbps)? (8 points)
c. After UE moved to point B, how much more time does it take to transmit a file of 2 Mb over this downlink channel (at the maximum data rate), compared to that at point A? (7 points)

3. Probability Basics - Conditional Probability (15 points)

You have three coins. Two of them are 'fair' (P{H} = P{T}=1/2), while one of them is biased in that, for the biased coin, P{H} = 2/3 and P{T} = 1/3. All three coins look alike, so that you don't know a priori which coin is biased. Let A_i denote the event that the i-th coin is biased.

a. Suppose the first coin is biased. You flip three coins one by one for one time. What is the probability that you observe {H, T, H}? (5 points)
b. Now you don't know which coin is biased. You flip the three coins one by one, and observe {H, H, T}. Based on this observation, what is the probability that the second coin is biased? (10 points)
   Hint: You are expected to derive probability P{A_2| {H, H, T}}. You may find the Bayes' Rule P{C|B} = $\frac{P(B|C)\,P(C)}{P(B)}$ useful. In addition, probability P{B} = $\sum_i$P{A_i B}, where $\{A_i\}_i$ denotes all the possible outcomes of an event, i.e. $\sum_i$P{A_i}=1.

4. Probability Basics - Poisson & Binomial Distribution (10 points)

Let X be a Binomial random variable with parameter $0<p<1$, and n=0, 1, 2, ... In other words,

$$P\{X = k\} = \frac{n!}{k!(n-k)!}p^k(1-p)^{n-k}.$$

If $n \to \infty$ and $p \to 0$, such that $np = \lambda$, show that the distribution of X converges to that of a Poisson random variable with mean λ. In this problem, simply assume that $np = \lambda$ all the time, i.e., $p = \lambda/n$. Specifically, show that

$$P\{X = k\} \to \frac{\lambda^k\, e^{-\lambda}}{k!} \text{as } n \to \infty \text{ (while } p = \lambda/n \to 0).$$

Hint: $(1 - \lambda/n)^n \to e^{-\lambda}$ as $n \to \infty$.

5. Probability Basics - Poisson Arrival Process (15 points)

Process $N(t)$ is called a Poisson Process with rate λ, if $N(t)$ satisfies the following conditions:
(i) $N(0)=0$.
(ii) $N(t) \sim$ Poisson(λt), i.e., $P\{N(t) = n\} = \frac{(\lambda t)^n\, e^{-(\lambda t)}}{n!}$.
(iii) Independent Increment: for any s>0, t>0, and $m \geq n \geq 0$, we have

$$P\{N(t+s)=m \mid N(s) = n\} = P\{N(t) = m-n\}.$$

Consider a Poisson process $N(t)$ that denotes the number of packets arriving at a router, up till time t=0, 1, 2, …

a. Find the average number of packet arrivals **E**{$N(t)$} during time interval (0, t]. (5 points)
b. What is the probability that the first packet arrives after t=10? (5 points)
   Hint: Event {The first packet arrives after t=10} is equivalent to event {No (Zero) packet arrives at the router from time t=0 to time t=10.} in probability.
c. What is the probability that there has been 4 packets arrivals up till time t=3, and there are 10 packets arrivals up till time t=7, i.e., P{$N(3)$=4, $N(7)$ = 10}? (5 points)

6. Data Link - Bit Error and Packet Error(10 points)

User A is transmitting a file of size F Kb to user B via an erroneous link. Suppose in the transmission, each bit of the file is flipped with probability p, and the error on any bit is independent of errors on any other bits.

a. What is the probability that B receives an error-free file after one transmission? (5 points)

b. Now consider a simple error-correction mechanism between A and B: A repeats each bit 3 times at transmission, and B performs a majority vote for each bit (such that when one bit is flipped 1 out of 3 times, this bit error can be identified and corrected). What is the probability that B has an error-free file after its error-correction? (5 points)

7. Data Link - Hamming Code (10 points)

A 9-bit message with binary value 100101011 is to be encoded using an even-parity Hamming code. What is the binary value after encoding? Show all your steps to receive credits (draw the table as in the lecture notes).