DATE
PAGE No.

ECE/CSC 570 : Computer Networks [HW 3]

Ans 1-

a) Signal A $\longrightarrow$ Amplitude Shift Keying Modulation
Signal B $\longrightarrow$ Phase Shift Keying Modulation
Signal C $\longrightarrow$ Frequency Shift Keying Modulation

b) Given: p is the probability of head at every toss
X = number of tosses until the first head (H) shows up

X follows Geometric Distribution

~~$P(X) = (1-p)$~~

Pmf of X :

$$P_r(X=k) = (1-p)^{k-1} p \quad \text{for } k = 1, 2, 3, \ldots \infty$$

$$\text{Expected Value, } E(X) = \sum_{k=1}^{\infty} k \times P_r(X=k)$$

$$= \sum_{k=1}^{\infty} k (1-p)^{k-1} p$$

$$= p \sum_{k=1}^{\infty} k (1-p)^{k-1}$$

$$= p\left[1 + 2(1-p) + 3(1-p)^2 + 4(1-p)^3 + \cdots\right]$$

$$= p\left[1-(1-p)\right]^{-2} \qquad \left\{ \text{From } (1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \cdots \right\}$$

Therefore, $E(X) = 1/p$

c) Main design issues of the Data Link Layer:

i) Service provided to the network layer
ii) Framing
iii) Error Control.
iv) Flow Control.

DATE
PAGE No.

d) Since there are 4 codewords, the minimum Hamming Distance is given by the minimum distance found between all 2 pair codeword combinations.

Codewords 3 and 4 have minimum hamming distance.

$C_3 = 10110$
$C_4 = 11101$
$\overline{\quad 01011}$

Therefore, $D_{min} = 3$ is the minimum hamming distance between the given set of codewords.

$$Detection_{max} = D_{min} - 1 = 3 - 1 = 2$$

$$Correction_{max} = \cancel{2D_{min}+1} = \cancel{(2\times3)+1} = \frac{(D_{min}-1)}{2} = \frac{3-1}{2} = 1$$

Hence, the maximum number of bit-errors it can detect is 2 and the maximum number of bit-errors it can correct is 1.

Ans 2- Given : $P_{tx} = 5$ watts
Channel bandwidth $= 20$ MHz
$P_{rx}(dBm) = P_{tx}(dBm) - 50 - 30 \log_{10} d\,(Km)$

Calculation :

a) Converting 5 watts into dBm $= 10 \log_{10}(5\times10^3\ mW/1\ mW)$
$= 36.99$ dBm

Now, $P_{rx}(dBM) = P_{tx}(dBm) - 50 - 30 \log_{10} d\,(km)$
$= 36.99 - 50 - 30 \log_{10}(1)$ ~~{1km = 1000m}~~
$= 36.99 - 50 - 0$ $\{\log_{10} 1 = 0\}$
~~= -103.01 dBm~~
$= -13.01$ dBm

DATE
PAGE No.

$SNR = \text{Signal Power} - \text{Noise Power}$
$= (-13.01\ dBm) - (-30\ dBm)$
$= \cancel{-73.01\ dB} = 16.99\ dB$

b) Now, converting SNR from dB into absolute value

$SNR_{dB} = 10 \log_{10} SNR$

$\therefore\ SNR = 10^{(SNR_{dB}/10)} = 10^{(16.99/10)} = 50$

$\cancel{SNR = 10^{-73.01/10} = 10^{-7.301} = 5\times10^{-8}}$

Now, $C = H\log_{2}(1+SNR)$
$= 20\times10^{6}\log_{2}(1+50)$
$= 113.45\ Mbps$

c) Time to transmit to point A $= \dfrac{2\ Mb}{113.45\ Mbps} = 0.01762$ seconds

Now, computing the channel capacity at point B,

$P_{rx}(dBM)_{point B} = 36.99 - 50 - 30\log_{10} d\ (km)$
$= 36.99 - 50 - 30\log_{10} 2$
$= 36.99 - 50 - 9.03$
$= -22.04$

$SNR = \text{Signal Power} - \text{Noise Power}$
$= (-22.04\ dBM) - (-30\ dBm)$

$SNR(dB) = 7.96\ dB$

$SNR = 10^{(SNR_{dB}/10)} = 10^{(7.96/10)} = 6.2517$

Now, $C = H\log_{2}(1+SNR)$
$= 20\times10^{6}\log_{2}(1+6.2517)$
$= 57.166\ Mbps$

Therefore, Time to transmit to point B $= \dfrac{2Mb}{57.166\ Mbps} = 0.035$ seconds

Therefore, addition time taken $= \cancel{(0.01762 - 0.035)\ sec}$
$= (0.035 - 0.01762)\ sec = 0.01738\ sec$

DATE

PAGE No.

Ans 3-

Given :- 2 coins are fair i.e. $P(H) = P(T) = 1/2$

1 coin is ~~un~~fair i.e. $P(H) = 2/3$ and $P(T) = 1/3$

A_i denotes the event that the i^{th} coin is biased.

a) To find: Probability of observing $\{H, T, H\}$ given first coin is biased.

Let M be the event of getting $\{H, T, H\}$

Therefore, to find : $P(M|A_1)$

$$P(M|A_1) = \frac{2}{3} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{6}$$

Here, $\frac{2}{3}$ is the probability of getting heads in 1^{st} toss.

and $\frac{1}{2}$ is the probability of getting tails in 2^{nd} toss.

and $1/2$ is the probability of getting heads in 3^{rd} toss.

b) To find : $P(A_2|T)$ where T is the event of observing $\{H, H, T\}$

Now,
$$P(A_2|T) = \frac{P(A_2 \cap T)}{P(T)}$$

$$= \frac{P(T|A_2) \cdot P(A_2)}{\sum_{i=1}^{3} P(T|A_i) \cdot P(A_i)}$$

Now, finding probabilities $P(T|A_1)$, $P(T|A_2)$ and $P(T|A_3)$

$$P(T|A_1) = P(\{H, H, T\}|A_1) = \frac{2}{3} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{6}$$

Similarly, $P(T|A_2) = P(\{H, H, T\}|A_2) = \frac{1}{2} \times \frac{2}{3} \times \frac{1}{2} = \frac{1}{6}$

$P(T|A_3) = P(\{H, H, T\}|A_3) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{3} = \frac{1}{12}$

Also, $P(A_i) = \frac{1}{3} \quad \forall i$

Now, $$P(A_2|T) = \frac{P(T|A_2) \cdot P(A_2)}{\sum_{i=1}^{3} P(T|A_i) \cdot P(A_i)}$$

$$= \frac{\left(\frac{1}{6} \times \frac{1}{3}\right)}{\left(\frac{1}{6} \times \frac{1}{3}\right) + \left(\frac{1}{6} \times \frac{1}{3}\right) + \left(\frac{1}{12} \times \frac{1}{3}\right)}$$

$$= \frac{\frac{1}{18}}{\left(\frac{1}{18} + \frac{1}{18} + \frac{1}{36}\right)}$$

$$= \frac{\frac{1}{18}}{\left(\frac{1}{9} + \frac{1}{36}\right)} = \frac{\frac{1}{18}}{\frac{4+1}{36}} = 2/5$$

$$\cancel{= \frac{\frac{1}{18}}{\frac{8+36}{36}} = \frac{2}{44} = \frac{1}{22}}$$

Ans 4- The binomial distribution tends to poisson distribution as $n \to \infty$, $p \to 0$, $\lambda = np$ stays constant.

To show:
$$\binom{n}{x} p^x (1-p)^{n-x} \to \frac{\lambda^x e^{-\lambda}}{x!} \quad \text{as } n \to \infty \text{ \& } \lambda = np \text{ is const}$$

Now, $\binom{n}{x} p^x (1-p)^{n-x}$ for binomial distribution

$$= \binom{n}{x} \left(\frac{\lambda}{n}\right)^x \left(1 - \frac{\lambda}{n}\right)^{n-x}$$

$$= \frac{n!}{x!(n-x)!} \left(\frac{\lambda}{n}\right)^x \left(1 - \frac{\lambda}{n}\right)^{n-x}$$

$$= \frac{\lambda^x}{x!} \frac{n!}{(n-x)!} \frac{1}{n^x} \left(1 - \frac{\lambda}{n}\right)^{n-x} \quad (1)$$

DATE
PAGE No.

Now, taking expression $\frac{n!}{(n-x)!} \times \frac{1}{n^x}$

$$\frac{n!}{(n-x)!\,n^x} = \frac{n(n-1)(n-2)\cdots(n-x+1)\cancel{(n-x)!}}{\cancel{(n-x)!}\,n^x}$$

$$= \frac{n(n-1)(n-2)\cdots(n-x+1)}{n^x}$$

$$= \frac{n}{n}\cdot\left(\frac{n-1}{n}\right)\cdot\left(\frac{n-2}{n}\right)\cdot\cdots\cdots\frac{(n-x+1)}{n}$$

$$\therefore \frac{n!}{(n-x)!\,n^x} = 1\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdot\left(1-\frac{3}{n}\right)\cdot\cdots\left(1-\frac{x-1}{n}\right) \quad \text{—— (2)}$$

From (1), we have

$$\binom{n}{x}p^x(1-p)^{n-x} = \frac{\lambda^x}{x!}\,\frac{n!}{(n-x)!\,n^x}\left(1-\frac{\lambda}{n}\right)^{n-x}$$

Substituting $\frac{n!}{(n-x)!\,n^x}$ from eq^n (2) in above eq^n.

$$\binom{n}{x}p^x(1-p)^{n-x} = \frac{\lambda^x}{x!}\left[\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\cdots\left(1-\frac{x-1}{n}\right)\right]\left(1-\frac{\lambda}{n}\right)^{n-x}$$

Now, taking $\lim_{n\to\infty}$ on both sides

$$\lim_{n\to\infty}\binom{n}{x}p^x(1-p)^{n-x} = \lim_{n\to\infty}\frac{\lambda^x}{x!}\left[\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{x-1}{n}\right)\right]\left(1-\frac{\lambda}{n}\right)^{n-x}$$

$$= \frac{\lambda^x}{x!}\lim_{n\to\infty}\left[\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\cdots\left(1-\frac{x-1}{n}\right)\right]\left(1-\frac{\lambda}{n}\right)^{n-x}$$

$$= \frac{\lambda^x}{x!}\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^{n-x} \qquad \left\{\lim_{n\to\infty}\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{x-1}{n}\right)=1\right\}$$

DATE
PAGE No.

$$= \frac{\lambda^x}{x!} \lim_{n\to\infty} \left(1-\frac{\lambda}{n}\right)^n \left(1-\frac{\lambda}{n}\right)^{-x}$$

$$= \frac{\lambda^x}{x!} \lim_{n\to\infty} \left(1-\frac{\lambda}{n}\right)^n \qquad \left\{ \lim_{n\to\infty} \left(1-\frac{\lambda}{n}\right)^{-x} = 1 \right\}$$

$$= \frac{\lambda^x e^{-\lambda}}{x!} \qquad \left\{ \text{Since, } \lim_{n\to\infty} \left(1-\frac{\lambda}{n}\right)^n = e^{-\lambda} \right\}$$

Hence, $\lim_{n\to\infty} \binom{n}{x} p^x (1-p)^{n-x} = \frac{\lambda^x e^{-\lambda}}{x!}$

Hence, proved.

Ans 5- Given a Poisson process $N(t)$ with rate λ

$$N(t) \sim \text{Poisson}(\lambda t) \text{ ie. } P\{N(t)=n\} = \frac{(\lambda t)^n e^{-(\lambda t)}}{n!}$$

a) To find: Average number of packet arrivals $E\{N(t)\}$ during time interval $[0,t]$.

Solution: $E\{N(t)\} = \sum_{n=0}^{\infty} n \cdot P(N(t)=n)$

$$= \sum_{n=0}^{\infty} \frac{n(\lambda t)^n e^{-\lambda t}}{n!}$$

Taking constants out of the summation,

$$= e^{-\lambda t} \sum_{n=0}^{\infty} \frac{n(\lambda t)^n}{n!}$$

Now, for $n=0$, $\frac{n(\lambda t)^n}{n!} = 0$,

$$= e^{-\lambda t} \sum_{n=1}^{\infty} \frac{n(\lambda t)^n}{n!}$$

DATE

PAGE No.

$$= (\lambda t) e^{-\lambda t} \sum_{n=1}^{\infty} \frac{\cancel{n} (\lambda t)^{n-1}}{\cancel{n}(n-1)!}$$

Substituting, $m = n-1$,

$$= (\lambda t) e^{-\lambda t} \sum_{m=0}^{\infty} \frac{(\lambda t)^m}{m!}$$

We have $\sum_{m=0}^{\infty} \frac{x^k}{k!} = e^x$

Therefore,

$$E\{N(t)\} = (\lambda t) e^{-\lambda t} e^{\lambda t}$$

$$E\{N(t)\} = \lambda t$$

b) To find: Probability that first packet arrives after $t = 10$ s.

Solution:

Event {1^{st} packet arrives after $t = 10$ sec} = Event {No packet arrives at router from $t=0$ to $t = 10$ sec}

Finding Probability that no packet arrives from $t=0$ to $t = 10$ s

$$P\{N(10) = 0\} = \frac{(10\lambda)^0 e^{-10\lambda}}{0!} = e^{-10\lambda}$$

c) To find: Probability that there are 4 packet arrivals up till time $t=3$ and 10 packet arrivals up till time $t=7$ ie. $P\{N(3) = 4, N(7) = 10\}$

Solution: $P\{N(3) = 4, N(7) = 10\}$

$= P\{N(3) = 4 \mid N(7) = 10\}$

$= P\{N(4) = 6\}$ Using $P\{N(t+s) = m \mid N(s) = n\} = P\{N(t) = m-n\}$

$$= \frac{(4\lambda)^6 e^{-4\lambda}}{6!}$$

Ans 6-

a) To receive a file error free in a single transmission, the entire file F Kb $= F \times 10^3 b$ needs to be transmitted without error. Since, p is the probability that a bit is flipped, $(1-p)$ is the probability that it is not flipped and thus transmitted correctly.

Hence, the probability to transfer the entire file correctly
$$= (1-p)^{F \times 10^3}$$

b) For the possibility of error correction, the only transmissions that are allowed are 111, 110, 101, 011 for transmitting bit 1 and 000, 001, 010, 100 for bit 0.

(bit 0 is the mirror case of bit 1)

Taking the case of bit 1, we have the following:

To transfer a single bit correctly, the probability is $(1-p)$

Hence, to transfer 111 correctly, probability $= (1-p)^3$

Similarly, to transfer 011/101/110 correctly, probability $= (1-p)^2 p$

Hence, the probability of transferring 111 or 011 or 101 or 110,
$$= (1-p)^3 + 3(1-p)^2 p$$

Therefore, the probability of error free transmission,
$$= \left[(1-p)^3 + 3(1-p)^2 p\right]^{F \times 10^3}$$

DATE
PAGE No.

Parity Bits

Ans 7- Below table is constructed for all bits with even parity.

1	2	3	4	5	6	7	8	9	10	11	12	13	Check bit contributed
		1		0	0	1		0	1	0	1	1	(Initial data bits)
1	–	1	–	0	–	1	–	0	–	0	–	1	1
–	1	1	–	–	0	1	–	–	1	0	–	–	2
–	–	–	1	0	0	1	–	–	–	–	1	1	4
–	–	–	–	–	–	–	1	0	1	0	1	1	8
1	1	1	1	0	0	1	1	0	1	0	1	1	⇐ Binary value after encoding

Hence, binary value after encoding = 1111001101011