Started on	Saturday, February 16, 2019, 4:11 PM
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Time taken	1 min 24 secs
Grade	**20.00** out of 20.00 (**100**%)

Question 1

Complete

20.00 points out of 20.00

Problem: Bayesian Estimation of the Parameters of a Gaussian Distribution

Attach the PDF file of your solution to this problem. To ease grading, place the final answer for each question in a highlighted box.

Assumptions:

- Univariate Case: The data $X = \{x^t\}, t = 1, \ldots, n$ is the univariate data, with the i.i.d. samples.
- Gaussian (Normal) Distribution: The sample is drawn from the Gaussian (Normal) distribution, $p(x) \sim N(\mu, \sigma^2)$, with parameters μ and σ^2.
- Parameters: Unknown mean, known variance
- Priors: The conjugate prior for μ is Gaussian, $p(\mu) \sim N(\mu_0, \sigma_0^2)$

Assignment:

1. Derive the formula for the posterior distribution of μ
2. Show that the posterior distribution is the Gaussian, $p(\mu|X) \sim N(\mu_n, \sigma_n^2)$
3. Show the derivation and the final estimate for μ_n and $1/\sigma_n^2$
4. If the mean of the posterior density (which is the MAP estimate), μ_n is written as the weighted average of the prior mean, μ_0, and the sample (likelihood) mean, $\bar{X}$, then what are the formulas for the weights?
5. Are the weights in Question #4 directly or inversely proportional to their variances (justify)?
6. Do the weights in Questions #4 sum up to 1 (justify)?
7. Is each weight between zero and one (justify)?
8. Given your answers for Questions #4-7, what can you say about the value of μ_n w.r.t. the values of μ_0 and $\bar{X}$
9. If σ^2 is known, then for the new instance x^{new}, show that $p(x^{new}|X) \sim N(\mu_n, \sigma_n^2 + \sigma^2)$
10. Generate a plot that displays $p(x) \sim N(6, 1.5^2)$, prior $p(\mu) \sim N(4, 0.8^2)$, and posterior $p(\mu|X) \sim N(\mu_n, \sigma_n^2)$ for $n = 20$ sample points. What are the values for μ_n and σ_n^2?

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# R Code; Answer 10

n <- 20

x <- seq(0, 9.999, by=0.5)

length(x)

mean_x <- 6; var_x <- 1.5^2

mean_prior <- 4; var_prior <- 0.8^2

# Generating samples following p(x) and prior p(mu) distributions

sample_dist <- dnorm(x, mean=mean_x, sd=sqrt(var_x))
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prior_dist <- dnorm(x, mean=mean_prior, sd=sqrt(var_prior))

# Calculating the mean and variance of the posterior

# For that, calculating x_bar

x_bar <- mean(rnorm(n, mean=mean_x, sd=sqrt(var_x)))

x_bar

var_n <- (var_x*var_prior)/(var_x + n*var_prior)

w1 <- (var_x*mean_prior)/(var_x+n*var_prior)

w2 <- (n*x_bar*var_prior)/(var_x+n*var_prior)

mean_n <- w1 + w2

# Generating samples from the posterior distribution

posterior_dist <- dnorm(x, mean=mean_n, sd=sqrt(var_n))

plot(x, sample_dist, col='red', type='l', xlim=c(0,10), ylim=c(0,1), main="Probability Density Plot", xlab="X", ylab="Probability Density")

par(new=TRUE)

plot(x, prior_dist, col='green', type='l', xlim=c(0,10), ylim=c(0,1), main="Probability Density Plot", xlab="X", ylab="Probability Density")

par(new=TRUE)

plot(x, posterior_dist, col='blue', type='l', xlim=c(0,10), ylim=c(0,1), main="Probability Density Plot", xlab="X", ylab="Probability Density")

legend(x=7.3, y=0.99, legend=c('sample','prior','posterior'), pch=19, col=c('red', 'green', 'blue'))

mean_n

var_n

## Value of mean_n = 5.745827 and var_n = 0.09568106
```

sshekha4.zip

#1: $p(\mu|X)$~$p(\mu)p(X|\mu)$~$N(\mu_n, \sigma_n^2)$

#4: $w_0 = \sigma^2/(n\sigma_0^2 + \sigma^2)$

$w_1 = n\sigma_0^2/(n\sigma_0^2 + \sigma^2)$

$\mu_n = w_0\mu_0 + w_1\bar{X}$

$\sigma_n^2 = 1/\sigma_0^2 + n/\sigma^2$

#5: Inversely proportional

#6: Yes, they sum up to 1

#7: Yes, between 0 and 1

#8: It always lies between them

#10: $p(\mu|X)$~$N(5.7, 0.3^2)$

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