HOMEWORK-3 (G06_HW3)

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Q1)

a. P(D|B, A) = P(D,B)

= P(D|B) * P(B)

= 0.2*0.75

= 0.15

b. P(C) = P(C|A) * P(A) + P(C|~A) * P(~A)

= (0.3*0.5) + (0.6*0.5)

= 0.15 + 0.30

= 0.45

c. P(F) = P(F|C) * P(C) + P(F|~C) * P(~C)

= (0.4*0.45) + (0.5*0.55)

= 0.18 + 0.275

= 0.455

d. P(B, ~C, D, E, F) = P(B) * P(~C) * P(D|B) * P(F|~C) * P(E|~C,D)

= 0.75 * 0.55 * 0.2 * 0.5 * 0.2

= 0.00825

Q2) a.

i) SVM tries to maximize the margin between two classes. When we see the plot, it can be observed that the decision boundary passes through the point (3,4) which is the midpoint between (2,3) and (4,5). The weight vector will be of the form (w1,w2). There are 2 more conditions that has to be satisfied.

$2w_1 + 3w_2 + b = 1$
$4w_1 + 5w_2 + b = -1$
But we have $w_1 = w_2 = w$ which reduces our above equations to the following:

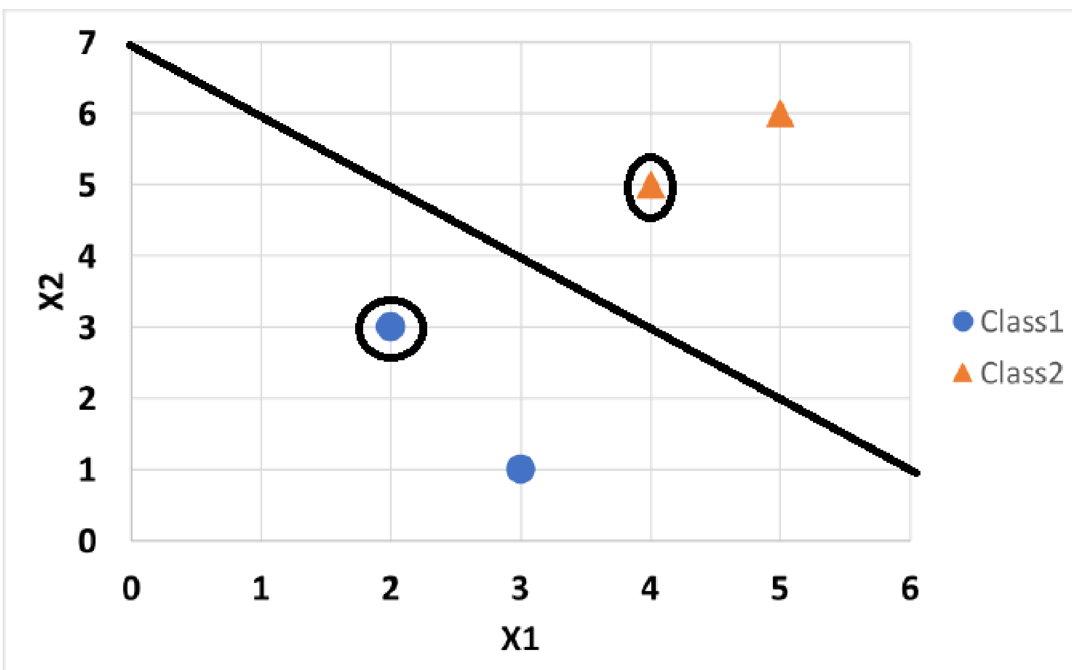
$5w+b = 1$
$9w+b = -1$

Solving the above two equations we get w = -1/2 and b = 7/2

Therefore the weight vector is [-1/2, -1/2].

The decision boundary equation is $w^T x + b = 0$.
Therefore the decision boundary equation is $x_1+x_2 = 7$.

ii) The points circled are the support vectors and the line drawn is the decision boundary.



Q2 b)

i) We have the two dimensional vectors $X=[x_1, x_2]$ and $Z = [z_1, z_2]$.
The Kernel function we have is $K(X,Z) = (1 + 2.X.Z)^2$. So upon expansion of our Kernel function, we get the value of the function to be:

$$K(X,Z) = 1 + 4X_1^2Z_1^2 + 8X_1X_2Z_1Z_2 + 4X_2^2Z_2^2 + 4X_1Z_1 + 4X_2Z_2$$

Above kernel can be written as inner product of ø(X) and ø(Z).

So upon resolving the value into a product of two vectors, we get
$ø(X) = [1, 2X_1^2, 2\sqrt{2}\, X_1X_2, 2X_2^2, 2X_1, 2X_2]$

ii) The four transformed points are as follows:

$ø(X^1) = [1,0,0,2,0,2]$
$ø(X^2) = [1,0,0,2,0,-2]$
$ø(X^3) = [1,2,0,0,2,0]$
$ø(X^4) = [1,2,0,0,-2,0]$

iii) To determine the linear decision boundary in the transformed higher dimensional space, we make use of the following 3 conditions as the basis.

$\Sigma\alpha_i y_i = 0$ -------------------- (a)

$w_0 = y_k - w.x_k$ --------------------------------(b)

$w = \Sigma\alpha_i y_i x_i$ -----------------------(c)

Using (a) from above equations we get,

$-\alpha_1 - \alpha_2 + \alpha_3 + \alpha_4 = 0 \Rightarrow \alpha_1 + \alpha_2 = \alpha_3 + \alpha_4$ -----------------(1)

Using (b) we get the following equations,

$w_0 = -1 - (w_1 + 2w_4 + 2w_6)$ -----------------(2)

$w_0 = -1 - (w_1 + 2w_4 - 2w_6)$ ------------------(3)

$w_0 = 1 – (w_1 + 2w_2 + 2w_5)$ -------------------(4)

$w_0 = 1 – (w_1 + 2w_2 - 2w_5)$ ------------------(5)

From the above set of equations we can see some trivial solutions for some weights. We get $w_5 = 0$ and $w_6 = 0$.

Using (c), we get

$[w_1 , w_2 , w_3, w_4, w_5, w_6] = [-\alpha_1 -\alpha_2 +\alpha_3 + \alpha_4 , 2\alpha_3 + 2\alpha_4 , 0 , -2\alpha_1 -2\alpha_2 , 2\alpha_3 - 2\alpha_4 , 2\alpha_1 -2\alpha_2]$ ------------(6)

Solving above equations we get $w_1 = 0$ and $w_3 = 0$. We also get $\alpha_1 =\alpha_2 =\alpha_3 = \alpha_4 = \alpha$

Solving equations (3) and (5), we get $w_2 + w_4 = 0$ ----------(7)

From equation (6) we get $w_2 – w_4 = 1$ ----------(8)

Solving (7) and(8) we get $w_2 = ½$ and $w_4 = -1/2$.

$\alpha = 1/8$ and $w_0 = 0$.

Thus the maximum margin linear decision boundary is $w\,ø(X) + w_0 = 0$

which we get as $X_1^2 – X_2^2 = 0$.

Q3) i)

Points given are (2,5), (0,-2) and (3,-3),

Solving the linear regression using least square minimization, we get:

$SSE = \Sigma [y_i - (w_1 x^2-w_0)]^2$.

$SSE = (5-w_0-4w_1)^2 + (-2-w_0)^2 +(-3-w_0-9w_1)^2$

After partial derivative with respect to w_1 and equating to 0, we get :

$0 = \Sigma\, 2 * [y_i - (w_1 x^2-w_0)] * (-x^2)$.

Upon substituting the points we get the equation as:

$(-40 + 32w_1 + 8w_0) + (54 +162\ w_1+18w_0) = 0$

$14 + 194\ w_1 + 26\ w_0 = 0.$ ---------------------- (1)

After partial derivative with respect to w_0 , we get:

$0 = \Sigma\ 2 * [y_i - (w_1 x^2-w_0)] * (-1).$

Upon substituting the points we get the equation as:

$-2(5-4w_1-w_0) -2(-2-w_0) – 2(-3-9w_1-w_0) = 0$

$26\ w_1 + 6\ w_0 = 0.$ ---------------------- (2)

From (2) we can substitute $w_1 = -6/26\ w_0$ in (1).

$14 + 194 *(-6/26) * w_0 + 26\ w_0 = 0.$

Solving for w_0 and w_1, we get the value as 0.7518 and -0.1725 respctively.

Final regression equation is $y = 0.7518 + (-0.1725)\ x^2$.

ii) RMSE

X	Y	Y’	$(Y-Y')^2$
2	5	0.0618	24.3858
0	-2	0.7518	7.5724
3	-3	-0.8007	4.8369

$RMSE = sqrt(\Sigma\ [Y_i - Y'_i\ ]^2 /n)$

$RMSE = sqrt(12.2650)$

$= 3.50214$