Quiz 0

1. [2 pts] What is the rank of the matrix $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 2 & 0 & 2 \end{bmatrix}$? Why?

2. [2 pts] Provide an expression for $x \in \mathbb{R}^2$ in terms of $A \in \mathbb{R}^{2\times 2}$ and $b \in \mathbb{R}^2$given that
$$Ax = 5x + b.$$

3. [4 pts] Assume $A \in \mathbb{R}^{3\times 2}$, $x \in \mathbb{R}^2$ and $y \in \mathbb{R}^3$, provide an expression (in matrix form) for the gradient of $f(x) = (y - Ax)^\top (y - Ax)$

4. [2 pts] Let H be a discrete random variable, and let E_1 and E_2 be two other discrete random variables. Which of the following sets of probabilities can be used to express $P[H \mid E_1, E_2]$?
   a. $P[E_1, E_2], P[H], P[E_1|H], P[E_2|H]$
   b. $P[E_1, E_2], P[H], P[E_1, E_2 \mid H]$
   c. $P[E_1 \mid H], P[E_2|H], P[H]$

Answer

1. It is 2, since the first two columns are clearly independent but the last one is the sum of the first two.

2. We can rewrite the expression as $(A - 5I)X = b$, where I is the identity matrix. Then, assuming that $(A - 5I)$ is invertible, we get $X = (A - 5I)^{-1}b$

3. We have $\nabla_x f(x) = -2A^\top y + 2A^\top Ax$

4. (b) $P[H \mid E_1, E_2] = \frac{P[E_1, E_2|H] \cdot P[H]}{P[E_1, E_2]}$