Joyce Huang
sh2393

Introduction

In this lab, we implemented a mini SPICE program using ordinary differential equations to perform nodal analysis. The goal is to build a transient simulation of circuits into ODE. In this mini SPICE program, we only implemented a few simple circuit elements: resistor, capacitor, current source with the EKV large-signal model.

Implementation

First step is to construct generic ODE solvers. Three methods are implemented: Forward Euler method, Runge-Kutta methods for RK4 and RK34.

Forward Euler:

$$\phi = \left.\frac{dx}{dt}\right|_{t=t_i} = f(x_i, t_i)$$

Runge Kutta without time adaptation:

$$x_{i+1} = x_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h$$

$$k_1 = f(t_i, x_i)$$

$$k_2 = f\left(t_i + \frac{h}{2}, x_i + \frac{k_1 h}{2}\right)$$

$$k_3 = f\left(t_i + \frac{h}{2}, x_i + \frac{k_2 h}{2}\right)$$

$$k_4 = f(t_i + h, x_i + k_3 h)$$

Runge Kutta with time adaptation:

$$x_{i+1} = x_i + \frac{1}{9}(2k_1 + 3k_2 + 4k_3)h$$

$$k_1 = f(t_i, x_i)$$

$$k_2 = f\left(t_i + \frac{h}{2}, x_i + \frac{k_1 h}{2}\right)$$

$$k_3 = f\left(t_i + \frac{3h}{4}, x_i + \frac{3k_2 h}{4}\right)$$

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Testing and Validation

The error estimator used to validate the result is the L1 error. On each iteration, the compute error helper will calculate the ground truth and compare with the calculated results by the solver. Each solver has its own testing function helper to compute the ground truth and do the comparison.

Results and Analysis

Below

FORWARD EULER

TIME	Result	Truth	Error
0	1	2	0.00%
1	5	6.19463	19.28%
2	11.40216	14.84392	23.19%
3	25.51321	33.67717	24.24%
4	56.84931	75.33896	24.54%
5	126.55478	167.90591	24.63%

RK4

TIME	Result	Truth	Error
0	2	2	0.00%
1	6.20104	6.19463	0.10%
2	14.86248	14.84392	0.13%
3	33.72135	33.67717	0.13%
4	75.43917	75.33896	0.13%
5	168.13017	167.90591	0.13%

RK34

TIME	Step	Result	Truth	Error
0	0	2	2	0.00%
1	0.15537	6.20104	6.19463	0.10%
2	0.07336	14.84781	14.84392	0.03%
3	0.07381	33.67953	33.67717	0.01%
4	0.07395	75.3404	75.33896	0.00%
5	0.07398	167.90678	167.90591	0.00%

From the tables above we can see that forward Euler's method actually diverges, while RK34 has quadratic convergence.

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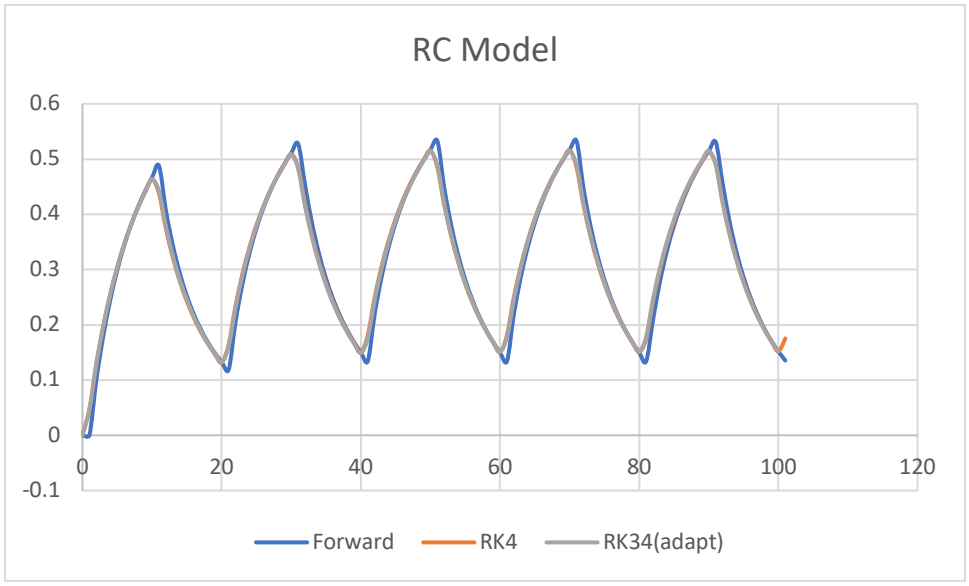
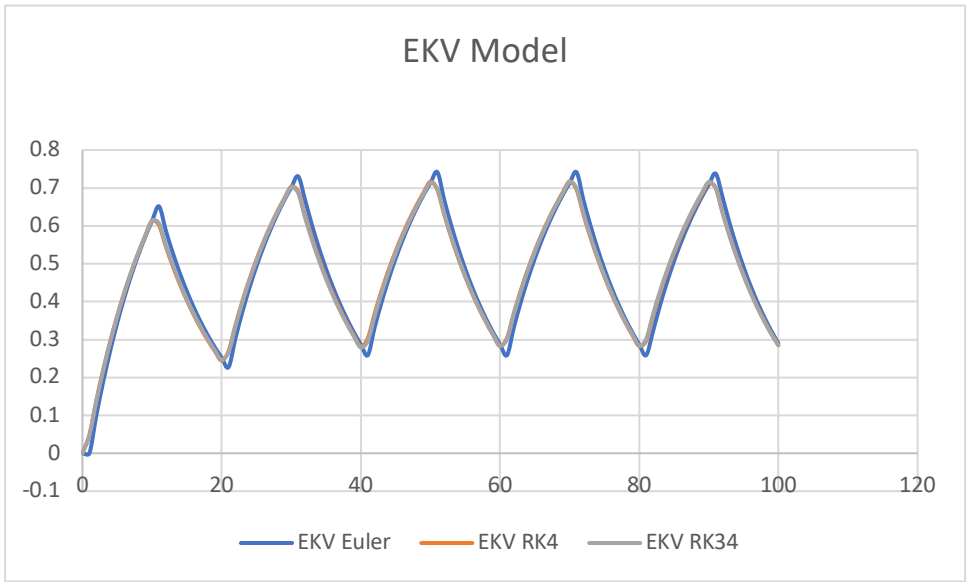
Some plots are selected to illustrate interesting results. Additional plots for all data plotting are included at the end of the report.

RC Model and EKV Model from different estimation methods





From the plots above we see that there is not much difference in the estimation from the different solvers. Among the slight difference, Runge Kutta methods produce similar results, while Euler methods has deviates more on the abrupt curves.

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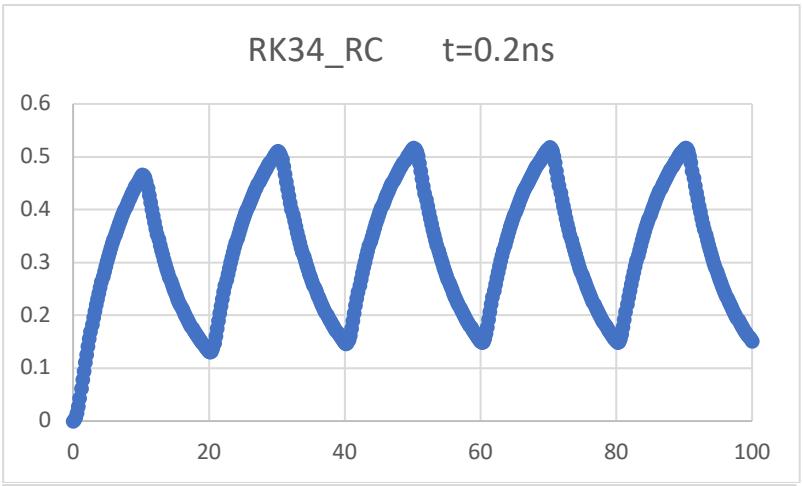
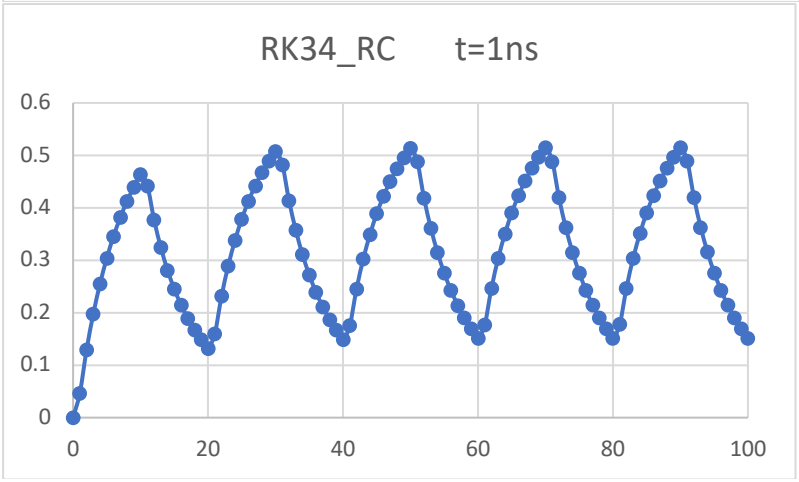
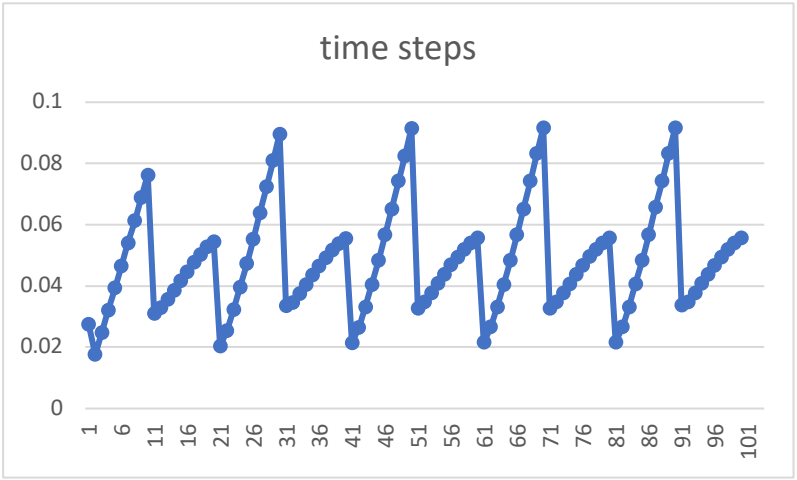
RK34 with different time steps RC Models







We can see that steps size gradually increases when the curve is smooth. When the curve takes abrupt changes, the step size decreases to adapt to the error.

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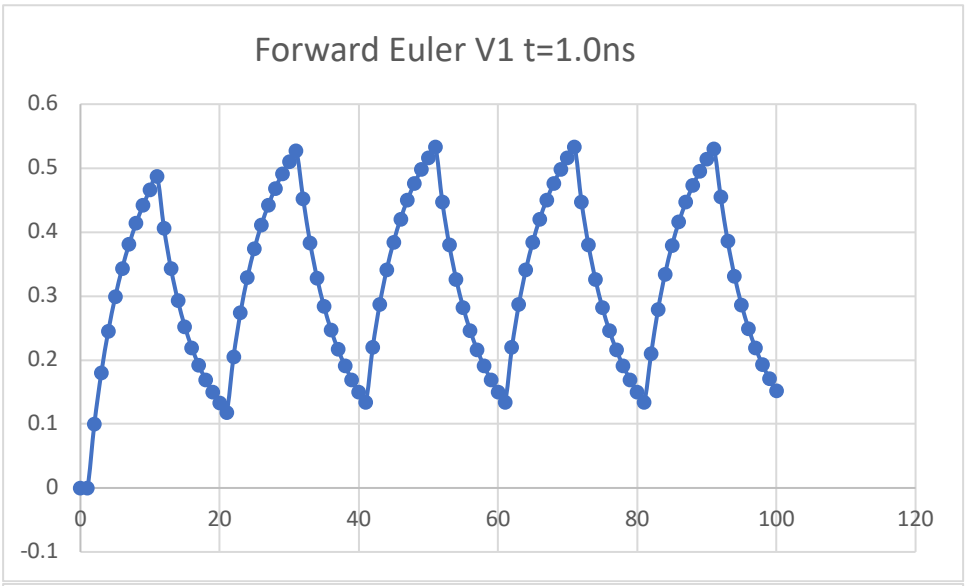
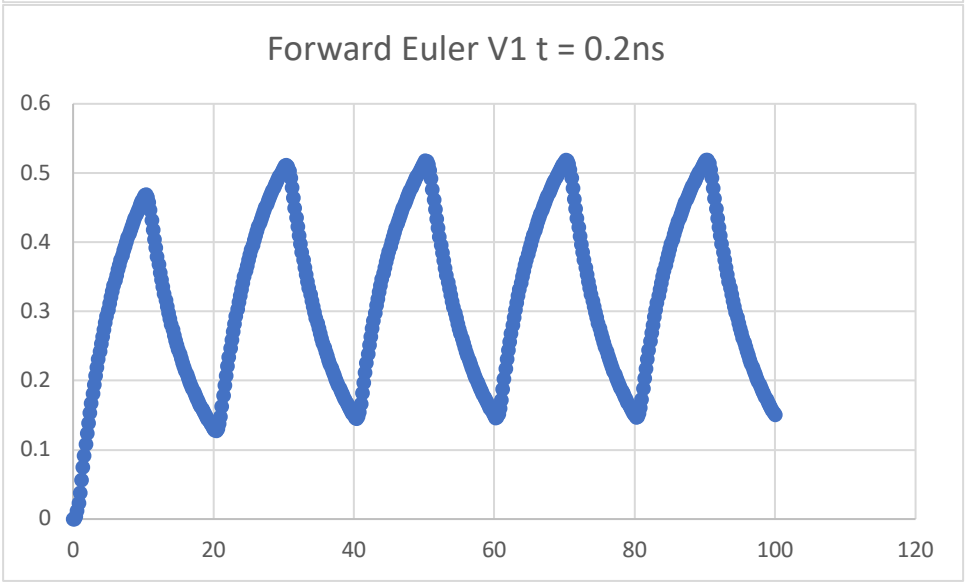
Additional Plots for all Data

RC Models





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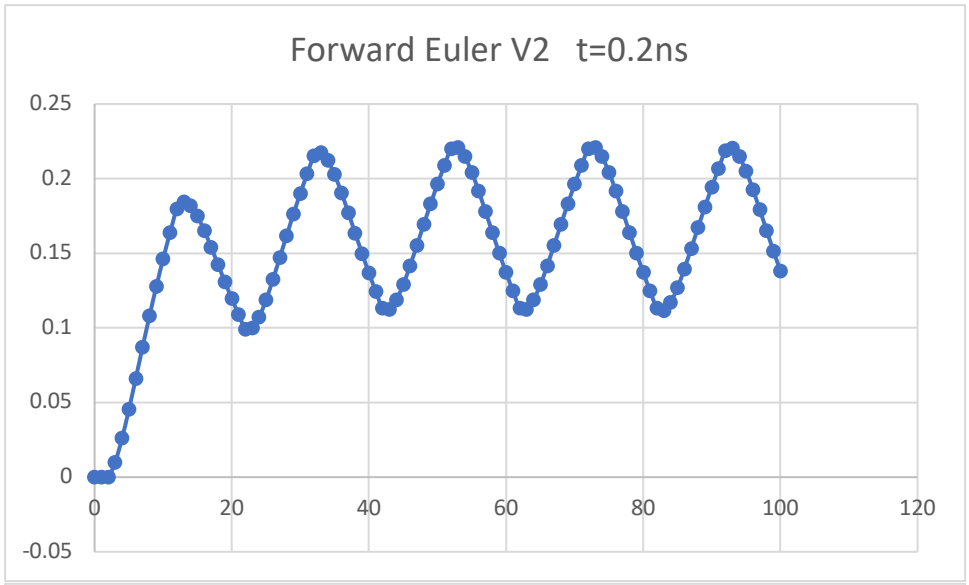

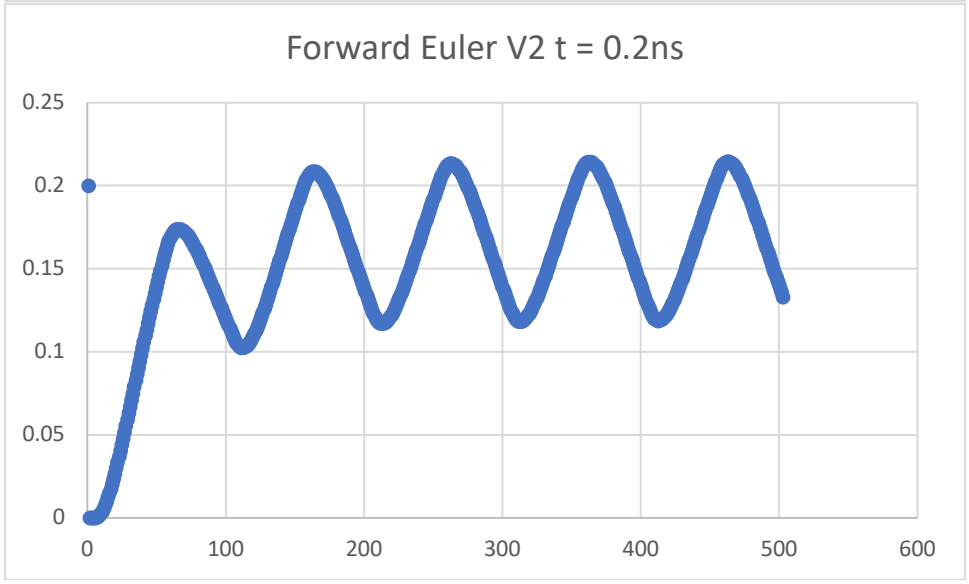

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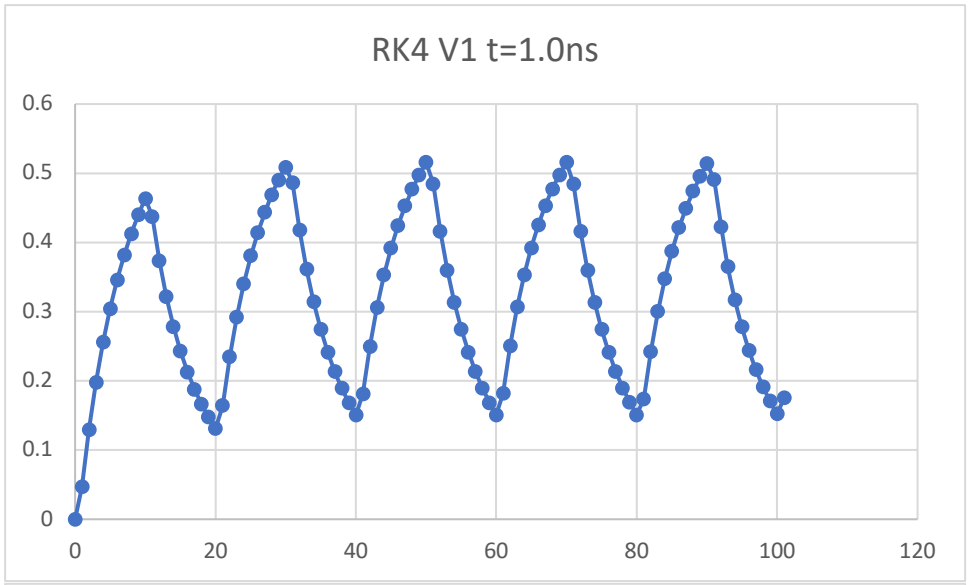

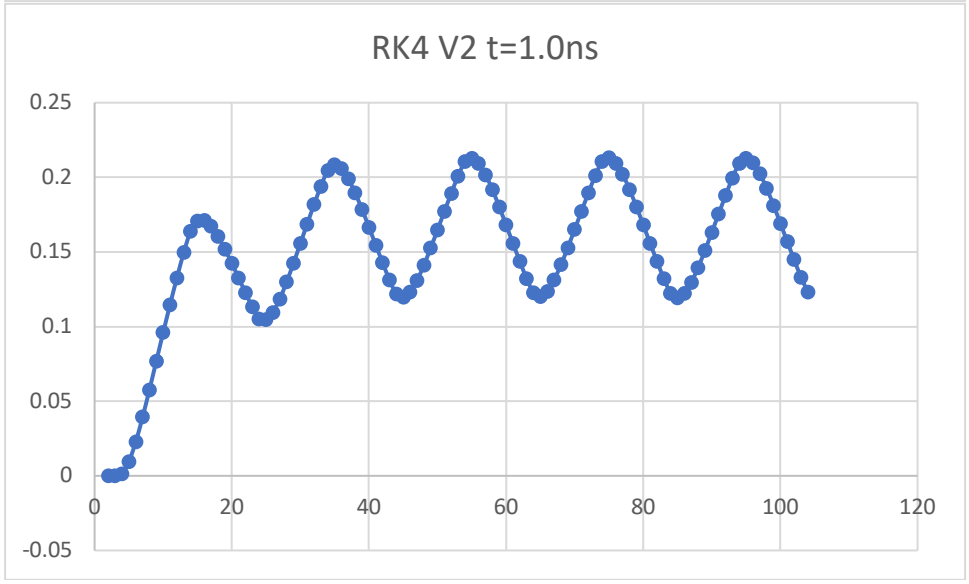

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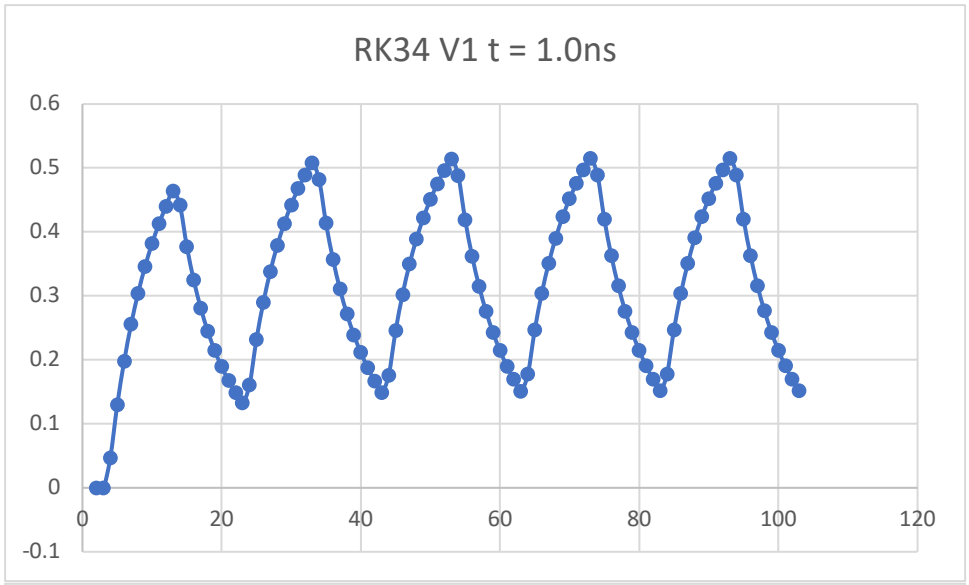

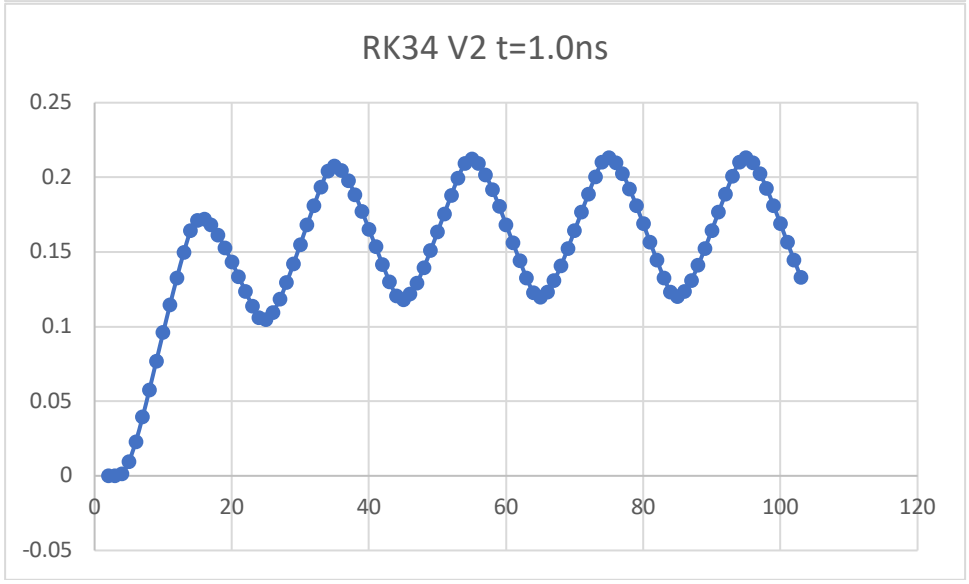

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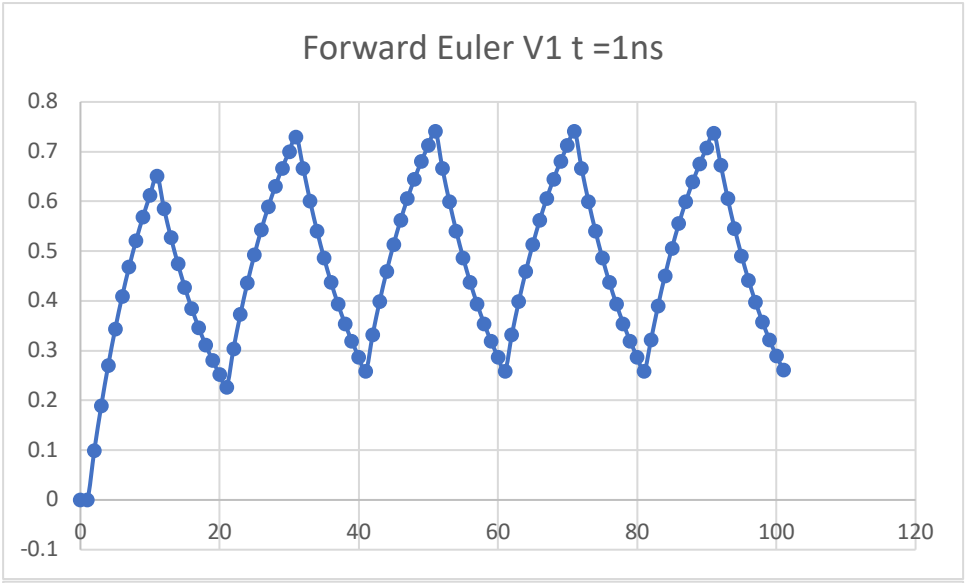
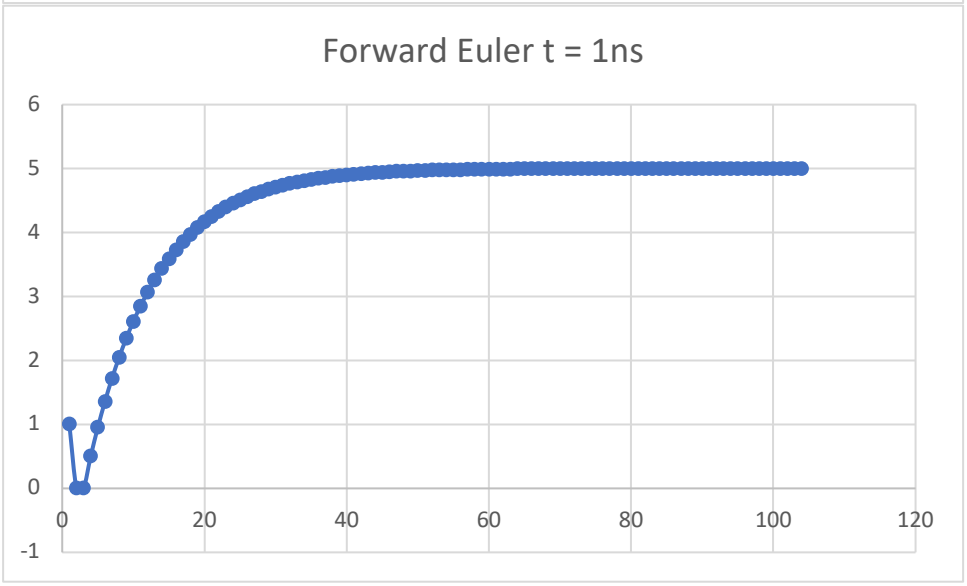
EKV Models





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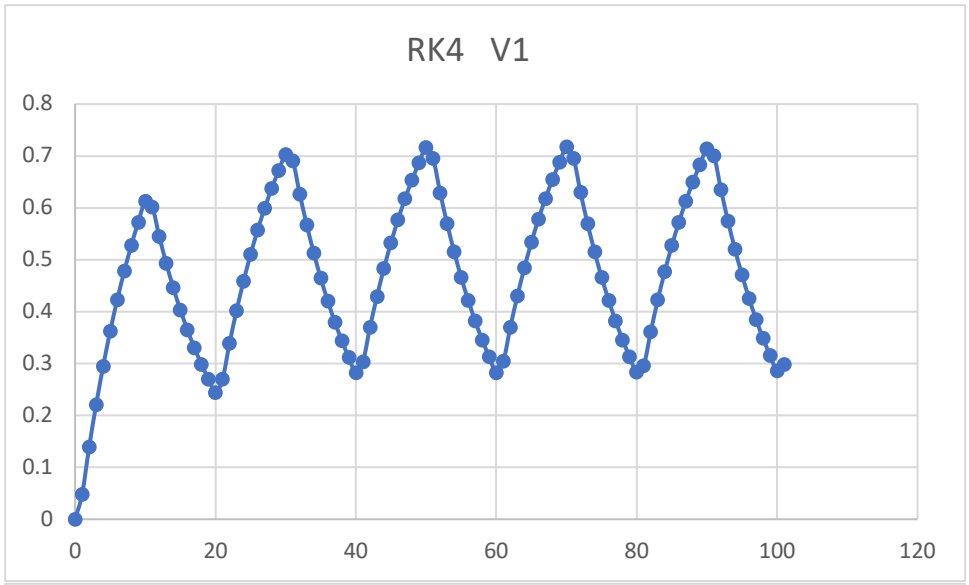

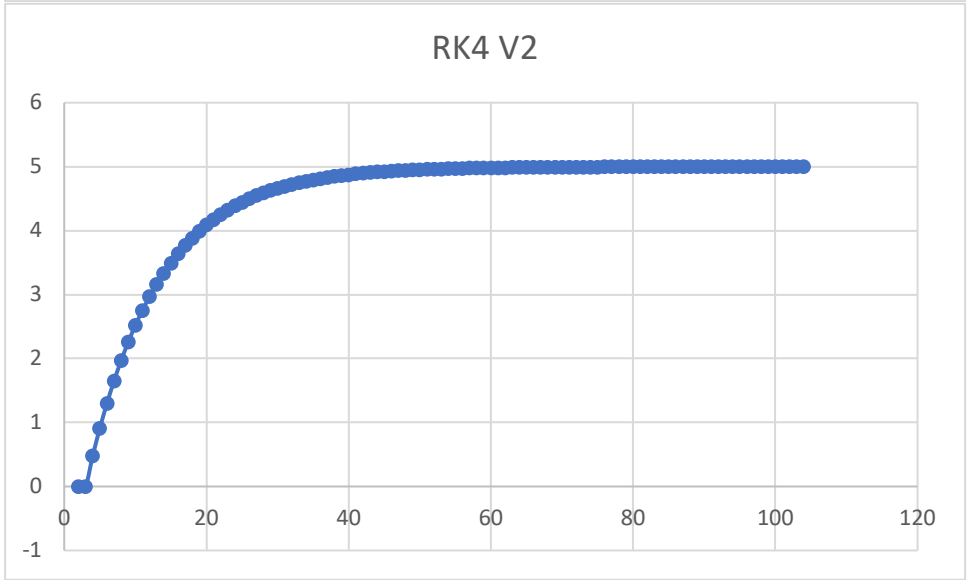

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