**Introduction**

In this lab, we implemented a mini SPICE program using ordinary differential equations to perform nodal analysis. The goal is to build a transient simulation of circuits into ODE. In this mini SPICE program, we only implemented a few simple circuit elements: resistor, capacitor, current source with the EKV large-signal model.

**Implementation**

First step is to construct generic ODE solvers. Three methods are implemented: Forward Euler method, Runge-Kutta methods for RK4 and RK34.

Forward Euler:



Runge Kutta without time adaptation:



Runge Kutta with time adaptation:



**Testing and Validation**

The error estimator used to validate the result is the L1 error. On each iteration, the compute error helper will calculate the ground truth and compare with the calculated results by the solver. Each solver has its own testing function helper to compute the ground truth and do the comparison.

**Results and Analysis**

Below

|  |  |  |  |
| --- | --- | --- | --- |
| Forward Euler | | | |
| Time | Result | Truth | Error |
| 0 | 1 | 2 | 0.00% |
| 1 | 5 | 6.19463 | 19.28% |
| 2 | 11.40216 | 14.84392 | 23.19% |
| 3 | 25.51321 | 33.67717 | 24.24% |
| 4 | 56.84931 | 75.33896 | 24.54% |
| 5 | 126.55478 | 167.90591 | 24.63% |

|  |  |  |  |
| --- | --- | --- | --- |
| RK4 |  |  |  |
| Time | Result | Truth | Error |
| 0 | 2 | 2 | 0.00% |
| 1 | 6.20104 | 6.19463 | 0.10% |
| 2 | 14.86248 | 14.84392 | 0.13% |
| 3 | 33.72135 | 33.67717 | 0.13% |
| 4 | 75.43917 | 75.33896 | 0.13% |
| 5 | 168.13017 | 167.90591 | 0.13% |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| RK34 |  |  |  |  |
| Time | Step | Result | Truth | Error |
| 0 | 0 | 2 | 2 | 0.00% |
| 1 | 0.15537 | 6.20104 | 6.19463 | 0.10% |
| 2 | 0.07336 | 14.84781 | 14.84392 | 0.03% |
| 3 | 0.07381 | 33.67953 | 33.67717 | 0.01% |
| 4 | 0.07395 | 75.3404 | 75.33896 | 0.00% |
| 5 | 0.07398 | 167.90678 | 167.90591 | 0.00% |

From the tables above we can see that forward Euler’s method actually diverges, while RK34 has quadratic convergence.

Some plots are selected to illustrate interesting results. Additional plots for all data plotting are included at the end of the report.

**RC Model and EKV Model from different estimation methods**

From the plots above we see that there is not much difference in the estimation from the different solvers. Among the slight difference, Runge Kutta methods produce similar results, while Euler methods has deviates more on the abrupt curves.

**RK34 with different time steps RC Models**

We can see that steps size gradually increases when the curve is smooth. When the curve takes abrupt changes, the step size decreases to adapt to the error.

**Additional Plots for all Data**

**RC Models**

**EKV Models**