

Economics 144: Project 1

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1 Introduction

The data used in this project was the total number of motor vehicles sold in the United States, which was retrieved from the Federal Reserve Bank.[1] Detailed information is provided as following table.

| Data Description | |
|-------------------|-------------------------|
| Title | Total Vehicle Sales |
| Time Period | 1976-01-01 - 2019-03-01 |
| Frequency | Monthly |
| Units | Thousands of Units |
| Seasonal Adjusted | Not |

Table 1: Source Data Information

Based on this regularly updated motor vehicle sales data, we simulated the trend and created seasonal dummies with a monthly cycle to forecast whether sales over the next two years($h = 24$) will conform to the waveform fluctuations of historical data. According to the data, about every 10 years, starting from 1981, the United States has experienced a national recession, which is reflected by a large jump in the volatility of sales. In this project, we focus on forecasting the future value of motor vehicle sales and its performance in next several recession periods based on the forecasting model we constructed.

2 Results

2.1 Modeling and Forecasting Trend

(a) Time Series Plot

The time series plot of total vehicle sales and its ACF,PACF are shown in Figure 1.

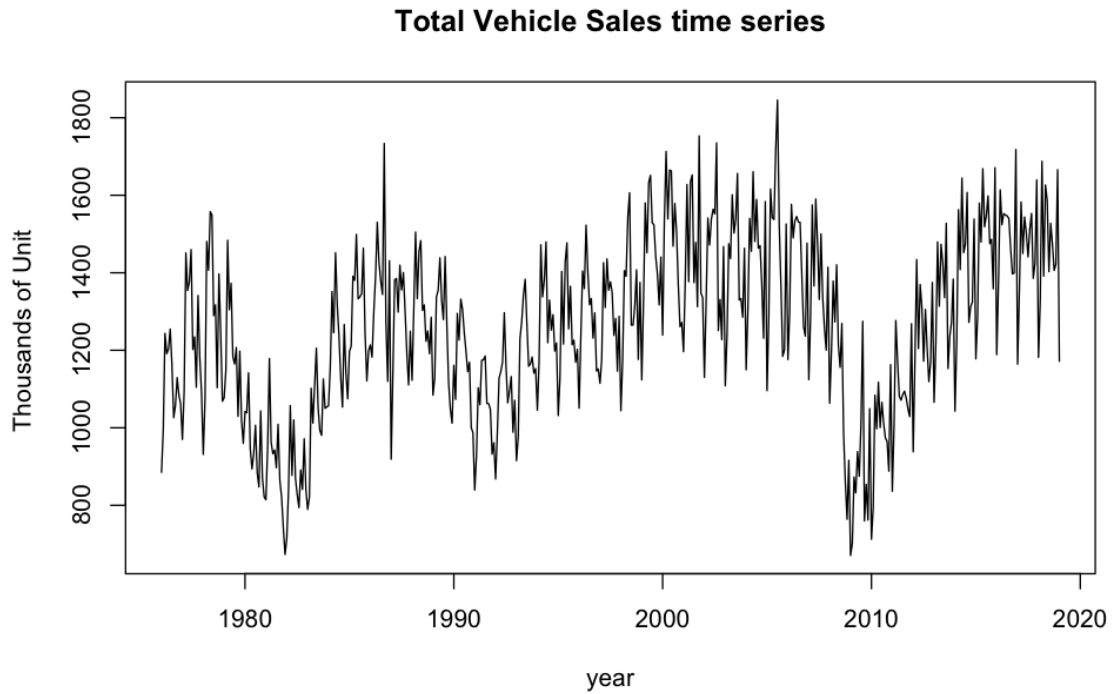


Figure 1: Time Series Display of Data Source

(b) Covariance Stationary Test

This time series is not covariance stationary. There is not constant mean or variance. It can also be seen from the ACF plot (Figure 1), that the autocovariance function is not stable.

(c) ACF and PACF

The fact that there are spikes in the PACF plot suggests that a linear model with three lags may serve as an appropriate representation of the time series.

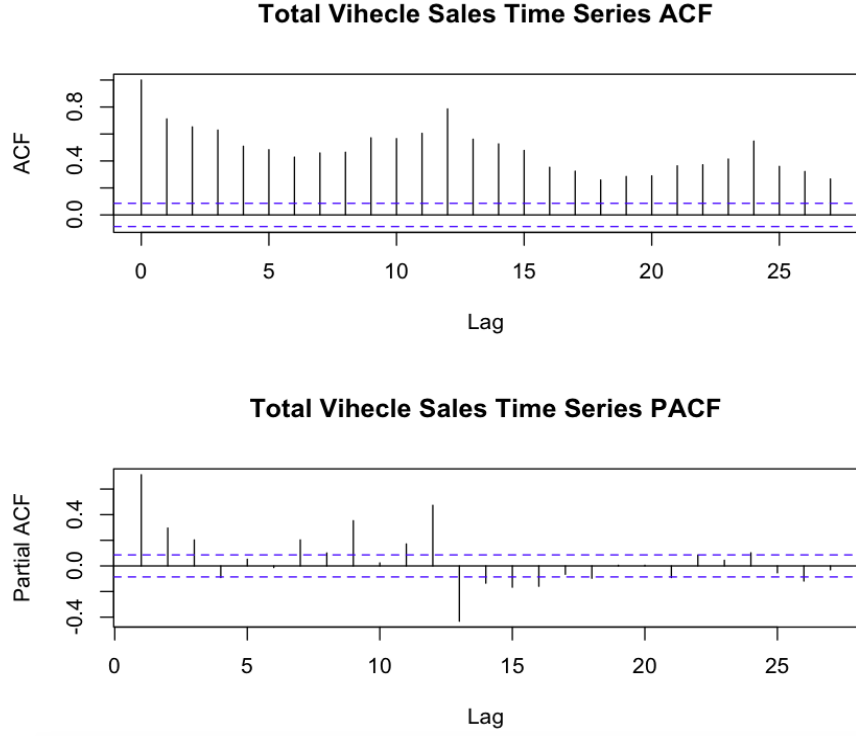


Figure 2: ACF and PACF plot of Data Source

(d) Liner and Non-linear Model

Linear with 3 lags:

Let X be the Total Vehicle Sales. We modeled the data with the following equation:

$$X_t = b_0 + b_1X_{t-1} + b_2X_{t-2} + b_3X_{t-3} + u_t \quad (1)$$

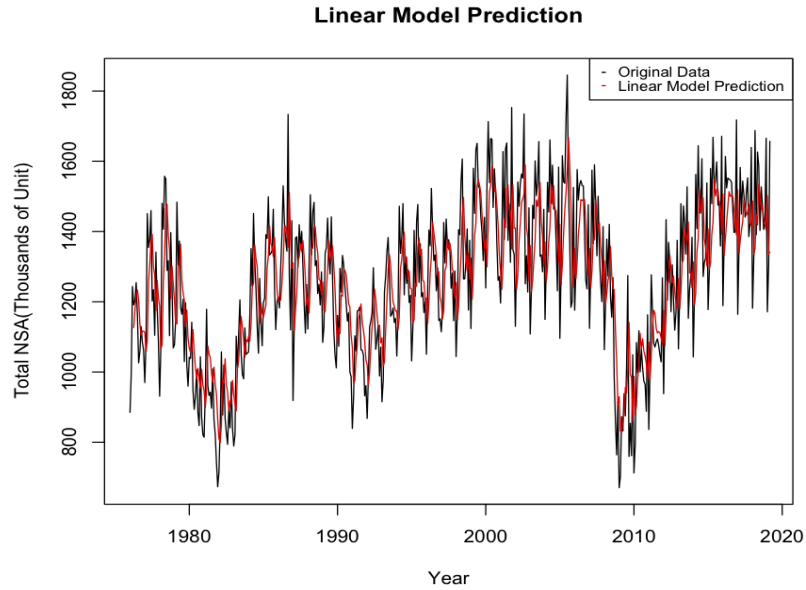


Figure 3: Original Data Plot and Linear Model Prediction

Nonlinear(exponential):

In equation(2), t is the time component, a and b are variables:

$$Y_t = e^{a+bt} \quad (2)$$

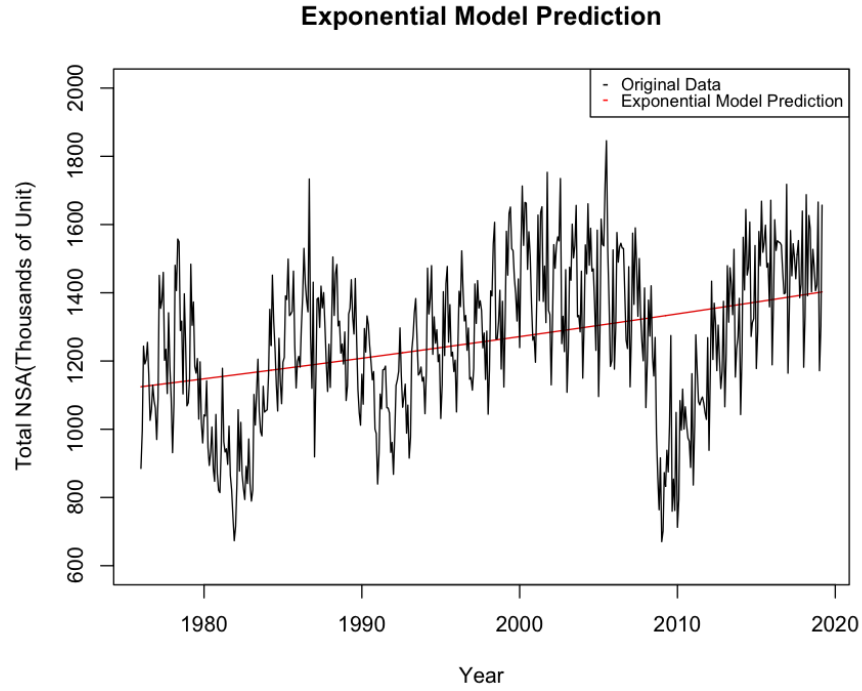


Figure 4: Original Data Plot and Exponential Model Prediction

(e) Residual and Fitted Values Plots

Linear:

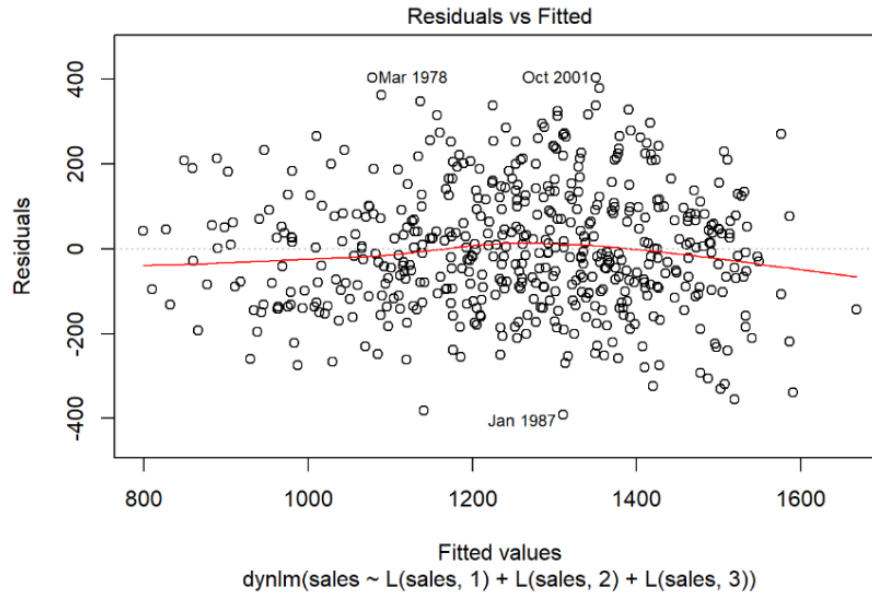


Figure 5: Residuals vs. Fitted Values for the Linear Model

Exponential:

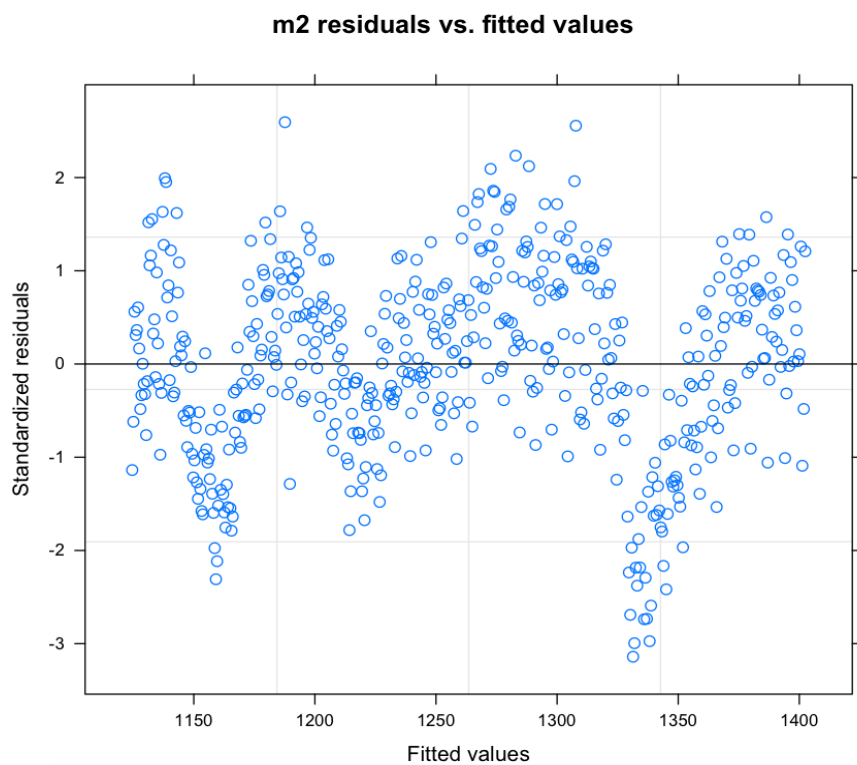


Figure 6: Residuals vs. Fitted Values for the Exponential Model

The residual plot for the linear model shows that there is almost perfectly normal scatter. The residuals for the exponential model, however, shows signs of a cyclical pattern, which suggests that the exponential model is not a good fit for this data.

- (f) Histograms
Linear:

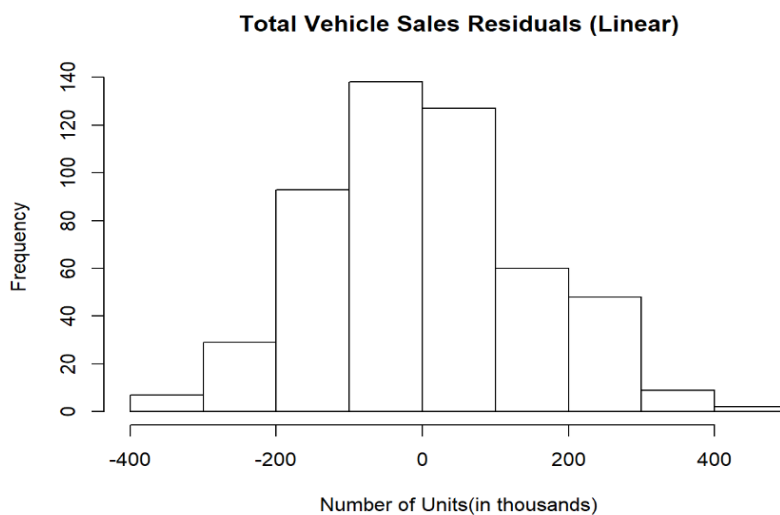


Figure 7: Histogram for residual of the Linear Model

Exponential:

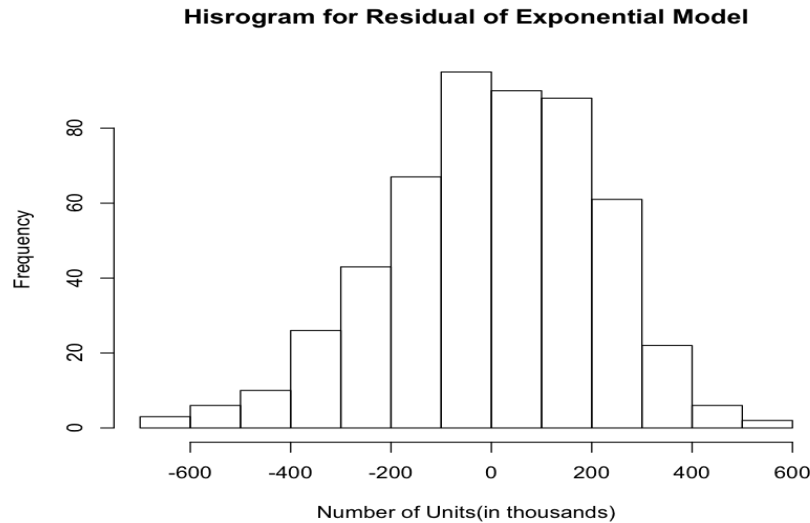


Figure 8: Histogram for residual of the Exponential Model

The histogram from the linear model is relatively normally distributed. The histogram from the exponential model is also relatively normally distributed, but there seems to be a slight left skew as well. When comparing the two histograms side by side, the residuals of the linear model seem to be closer to normal distribution than the residuals of the exponential model.

- (g) The R-squared value from the linear model is around 0.5709, which is high enough to be considered acceptable. The F statistic is also small enough so that the null hypothesis stating that all the coefficients are equal to 0 can be rejected at any significance level. In our case, we chose a significant level of 0.01. For the exponential model, all the t-values are far greater than any usually accepted confidence interval, so we can reject the null hypothesis that any coefficient is 0 as well.

- (h) AIC and BIC

The AIC for the linear model is 6608.876 and the BIC is 6634.341, while the AIC for the exponential model is 7002.632 with a BIC of 7015.376. Both the AIC and the BIC agree that the linear model is a better fit.

- (i) Forecasting

In forecast function, only $h=519$ can be used, otherwise there would be Error in variables. We can zoom in the forecasting with a smaller step size ($h=129$) by using the code below instead of 101st and 102rd lines:

```
1 quartz("linear model(m1) forecasting")
2 plot(forecast(m1,h = 519,xlim=c(1976,2030)))
```

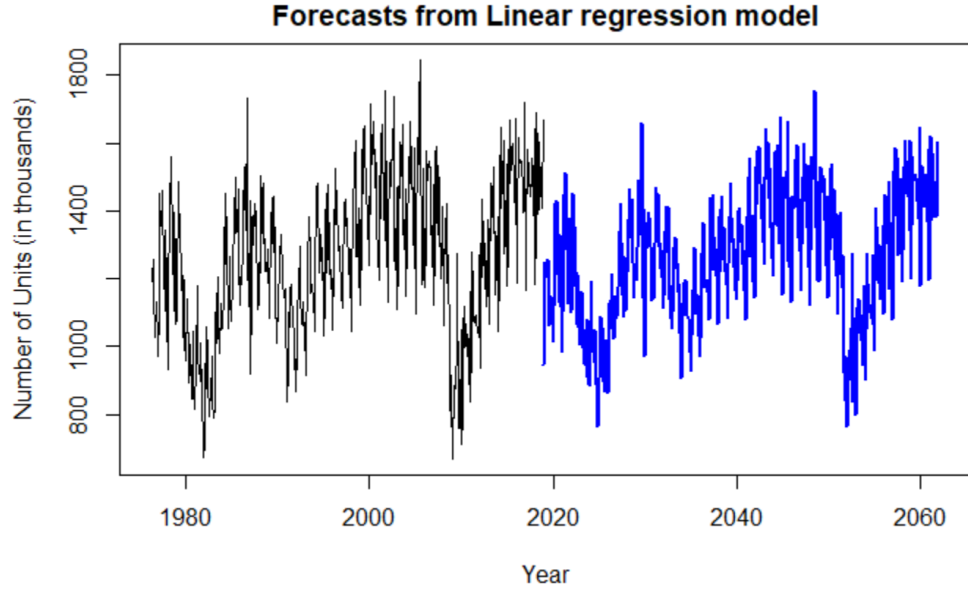


Figure 9: Forecast for the Linear Regression Model

2.2 Modeling and Forecasting Seasonality

(a) Constructing and Testing Seasonal Dummies

We constructed a set of seasonal dummies in order to represent monthly seasonality through the 519×12 matrix shown below.

$$D_{i1} = \{0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, \dots\}$$

$$D_{i2} = \{0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, \dots\}$$

...

$$D_{i11} = \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, \dots\}$$

$$D_{i12} = \{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \dots\}$$

Where as the row index i , from 1 to 519, indicates the number of entries, the column index j , from 1 to 12, indicates the 12 months in a year.

From Figure 10, it is observed that the residual plot does not look random; there are several outliers that are in different years at top and bottom. The normal Q-Q plot, which is a normal probability plot, provides a relatively straight line with no deviation points. This shows that the residuals are almost normally distributed. The Scale-Location graph does not look random as well. In the residuals vs. Leverage graph, there are no leverage points shown.

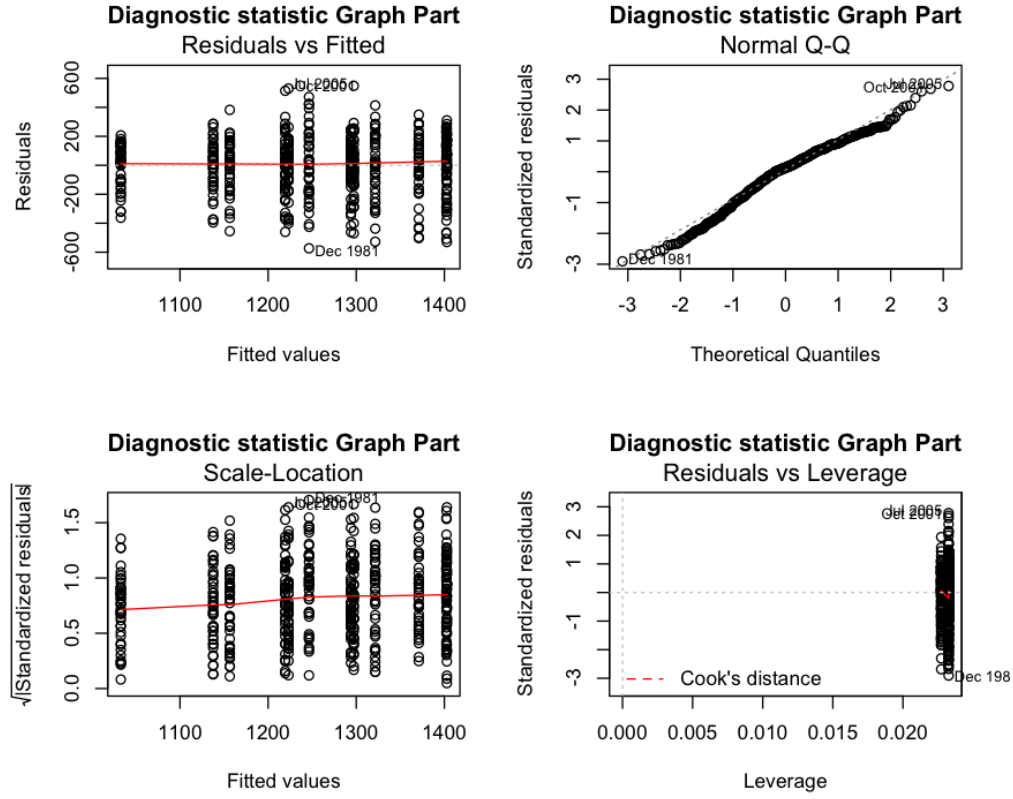


Figure 10: Diagnostic statistic Graph Part

Figure 11 provides specific information on this model, which consists purely of seasonal dummies. The variables `dummies[3]`, `dummies[10]`, and `dummies[11]` are not significant in this model; while `dummies[12]` is NA. It can be interpreted that `dummies[12]` is linearly related to the other variables.


```

> summary(m3)

Time series regression with "ts" data:
Start = 1976(1), End = 2019(3)

Call:
dynlm(formula = sales ~ dummies[, 1] + dummies[, 2] + dummies[,
3] + dummies[, 4] + dummies[, 5] + dummies[, 6] + dummies[,
7] + dummies[, 8] + dummies[, 9] + dummies[, 10] + dummies[,
11] + dummies[, 12])

Residuals:
    Min       1Q   Median       3Q      Max
-573.20 -117.66   25.13  141.75  548.54

Coefficients: (1 not defined because of singularities)
              Estimate Std. Error t value Pr(>|t|)
(Intercept)   1137.64     30.44   37.376 < 2e-16 ***
dummies[, 1]    108.75     43.05    2.526 0.011825 *
dummies[, 2]   -104.82     42.80   -2.449 0.014659 *
dummies[, 3]     18.86     42.80    0.441 0.659732
dummies[, 4]    265.18     42.80    6.196 1.20e-09 ***
dummies[, 5]    156.33     43.05    3.632 0.000310 ***
dummies[, 6]    264.73     43.05    6.150 1.57e-09 ***
dummies[, 7]    233.33     43.05    5.421 9.20e-08 ***
dummies[, 8]    159.52     43.05    3.706 0.000234 ***
dummies[, 9]    183.90     43.05    4.272 2.31e-05 ***
dummies[, 10]    81.76     43.05    1.899 0.058087 .
dummies[, 11]    85.50     43.05    1.986 0.047541 *
dummies[, 12]      NA         NA      NA      NA
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 199.6 on 507 degrees of freedom
Multiple R-squared:  0.2317,    Adjusted R-squared:  0.2151
F-statistic: 13.9 on 11 and 507 DF,  p-value: < 2.2e-16

```

Figure 11: Diagnostic statistic Numerical Part

(b) Seasonal Factor Plot

Figure 13 is the plot of the coefficients in the pure seasonal dummy model we constructed in (a). The intercept seems to have the largest value, which may affect our prediction, but the intercept is not considered a seasonal factor. Besides the intercept, March and May are in the peak of dummies, while after May, the dummies' influence decays until November in the same year.

Note: actually we considered to set intercept = 0 to obtain a pure seasonal dummy model and we obtained adjusted R-squared 0.99 with all variables significant. However, it behaves as if the value of the removed intercept is equally assigned to each variable. Due to the possible issue of biased prediction, we decided to leave intercept in this pure seasonal dummy model.

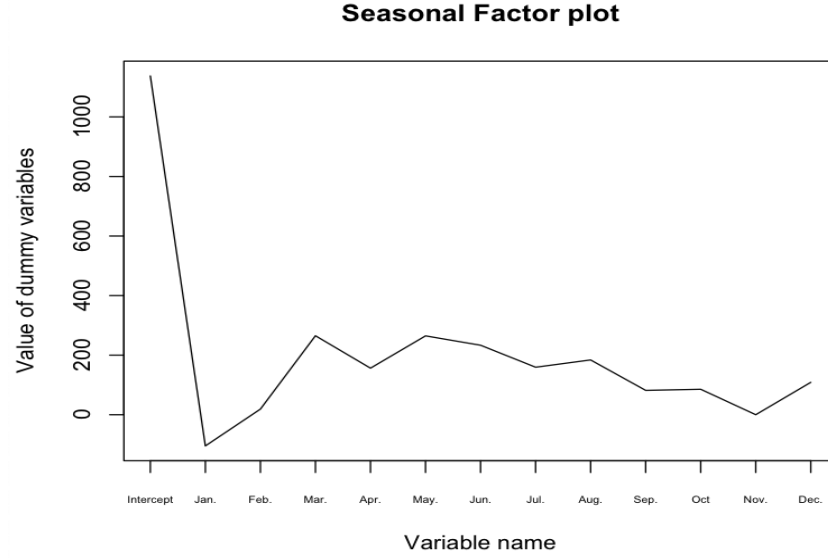


Figure 12: Seasonal Factor Plot

(c) Model Improvement

After combining the previous linear model(equation(1)) with the pure seasonal dummies, the model was improved by omitting dummies[10] due to its lack of significance, which can be seen in Figure 14. As the intercept is not statistically significant (as shown in Figure 15), we chose to delete the intercept factor for achieving a higher Adjusted R Square($\sim 99.4\%$) in Figure 16. We chose this as the full model to discuss in the following subsections.

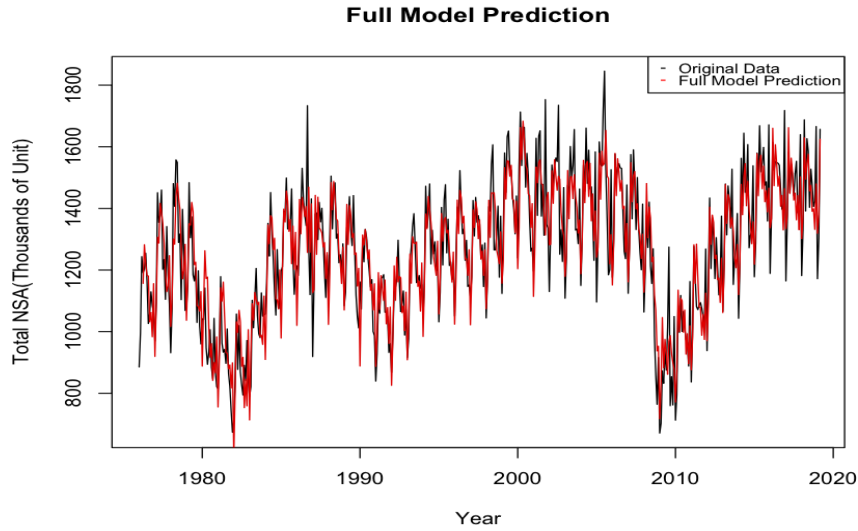


Figure 13: Original Data Plot and Full Model Prediction

In the fitted value vs. residual graph of the full model(Figure 17), the residual bounce randomly around 0, but with several outliers. The relationship of variables in the full model is reasonable. There is a horizontal bond formed by residuals, which indicates that the variances of error terms are nearly equal.

```

Time series regression with "ts" data:
Start = 1976(4), End = 2019(3)

Call:
dynlm(formula = sales ~ L(sales, 1) + L(sales, 2) + L(sales,
3) + dummies[, 1] + dummies[, 2] + dummies[, 3] + dummies[,
4] + dummies[, 5] + dummies[, 6] + dummies[, 7] + dummies[,
8] + dummies[, 9] + dummies[, 10] + dummies[, 11] + dummies[,
12])

Residuals:
    Min       1Q   Median       3Q      Max
-319.14  -57.41   -0.59    57.59   472.61

Coefficients: (1 not defined because of singularities)
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  -25.29461    34.02512  -0.743  0.457582
L(sales, 1)    0.42256     0.04215  10.025 < 2e-16 ***
L(sales, 2)    0.16592     0.04559   3.639 0.000302 ***
L(sales, 3)    0.33579     0.04215   7.967 1.10e-14 ***
dummies[, 1]  178.55970    22.15284   8.060 5.63e-15 ***
dummies[, 2]  -64.60630    22.17471  -2.914 0.003734 **
dummies[, 3]  158.86352    23.85563   6.659 7.25e-11 ***
dummies[, 4]  351.07968    22.38719  15.682 < 2e-16 ***
dummies[, 5]  191.92740    27.02307   7.102 4.23e-12 ***
dummies[, 6]  261.88907    25.32209  10.342 < 2e-16 ***
dummies[, 7]  119.90869    22.23442   5.393 1.07e-07 ***
dummies[, 8]   75.95543    23.26541   3.265 0.001171 **
dummies[, 9]  100.33223    22.03220   4.554 6.62e-06 ***
dummies[, 10]  10.67633    21.78840   0.490 0.624348
dummies[, 11]  78.31920    22.28799   3.514 0.000481 ***
dummies[, 12]      NA         NA      NA      NA
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 100.4 on 501 degrees of freedom
Multiple R-squared:  0.8065,    Adjusted R-squared:  0.801
F-statistic: 149.1 on 14 and 501 DF,  p-value: < 2.2e-16

```

Figure 14: Summary of Model consists of linear and all seasonal dummies

```

> summary(m6)

Time series regression with "ts" data:
Start = 1976(4), End = 2019(3)

Call:
dynlm(formula = sales ~ L(sales, 1) + L(sales, 2) + L(sales,
3) + dummies[, 1] + dummies[, 2] + dummies[, 3] + dummies[,
4] + dummies[, 5] + dummies[, 6] + dummies[, 7] + dummies[,
8] + dummies[, 9] + dummies[, 11] + dummies[, 12])

Residuals:
    Min       1Q   Median       3Q      Max
-319.14  -57.41   -0.59    57.59   472.61

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  -14.61827    35.49186  -0.412 0.680606
L(sales, 1)    0.42256     0.04215  10.025 < 2e-16 ***
L(sales, 2)    0.16592     0.04559   3.639 0.000302 ***
L(sales, 3)    0.33579     0.04215   7.967 1.10e-14 ***
dummies[, 1]  167.88336    22.33965   7.515 2.64e-13 ***
dummies[, 2]  -75.28263    22.10448  -3.406 0.000713 ***
dummies[, 3]  148.18719    24.14428   6.138 1.70e-09 ***
dummies[, 4]  340.40335    22.72503  14.979 < 2e-16 ***
dummies[, 5]  181.25107    26.21396   6.914 1.44e-11 ***
dummies[, 6]  251.21274    24.78626  10.135 < 2e-16 ***
dummies[, 7]  109.23236    21.89750   4.988 8.41e-07 ***
dummies[, 8]   65.27910    22.73138   2.872 0.004255 **
dummies[, 9]   89.65589    21.97008   4.081 5.22e-05 ***
dummies[, 11]  67.64287    22.29306   3.034 0.002537 **
dummies[, 12] -10.67633    21.78840  -0.490 0.624348
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 100.4 on 501 degrees of freedom
Multiple R-squared:  0.8065,    Adjusted R-squared:  0.801
F-statistic: 149.1 on 14 and 501 DF,  p-value: < 2.2e-16

```

Figure 15: Summary of Model after deleting dummy[,10]

```
> summary(m7_2)

Time series regression with "ts" data:
Start = 1976(4), End = 2019(3)

Call:
dynlm(formula = sales ~ 0 + L(sales, 1) + L(sales, 2) + L(sales,
3) + dummies[, 1] + dummies[, 2] + dummies[, 3] + dummies[,
4] + +dummies[, 5] + dummies[, 6] + dummies[, 7] + dummies[,
8] + dummies[, 9] + dummies[, 11])

Residuals:
    Min       1Q   Median       3Q      Max
-313.45  -59.12   -0.55   58.39  476.79

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
L(sales, 1)    0.41984    0.04143   10.133 < 2e-16 ***
L(sales, 2)    0.16428    0.04540    3.618 0.000326 ***
L(sales, 3)    0.32676    0.04016    8.137 3.20e-15 ***
dummies[, 1]  169.37297   18.36935    9.220 < 2e-16 ***
dummies[, 2]  -73.60386   18.40033   -4.000 7.28e-05 ***
dummies[, 3]  148.70132   19.89414    7.475 3.46e-13 ***
dummies[, 4]  341.89132   18.69957   18.283 < 2e-16 ***
dummies[, 5]  181.61772   23.20702    7.826 2.99e-14 ***
dummies[, 6]  252.81482   21.87948   11.555 < 2e-16 ***
dummies[, 7]  113.16045   19.06294    5.936 5.44e-09 ***
dummies[, 8]   68.37058   19.98664    3.421 0.000675 ***
dummies[, 9]   93.47349   18.94710    4.933 1.10e-06 ***
dummies[, 11]  70.21827   18.93665    3.708 0.000232 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 100.2 on 503 degrees of freedom
Multiple R-squared:  0.994,    Adjusted R-squared:  0.9939
F-statistic: 6429 on 13 and 503 DF,  p-value: < 2.2e-16
```

Figure 16: Summary of model after deleting dummy[,10] and intercept

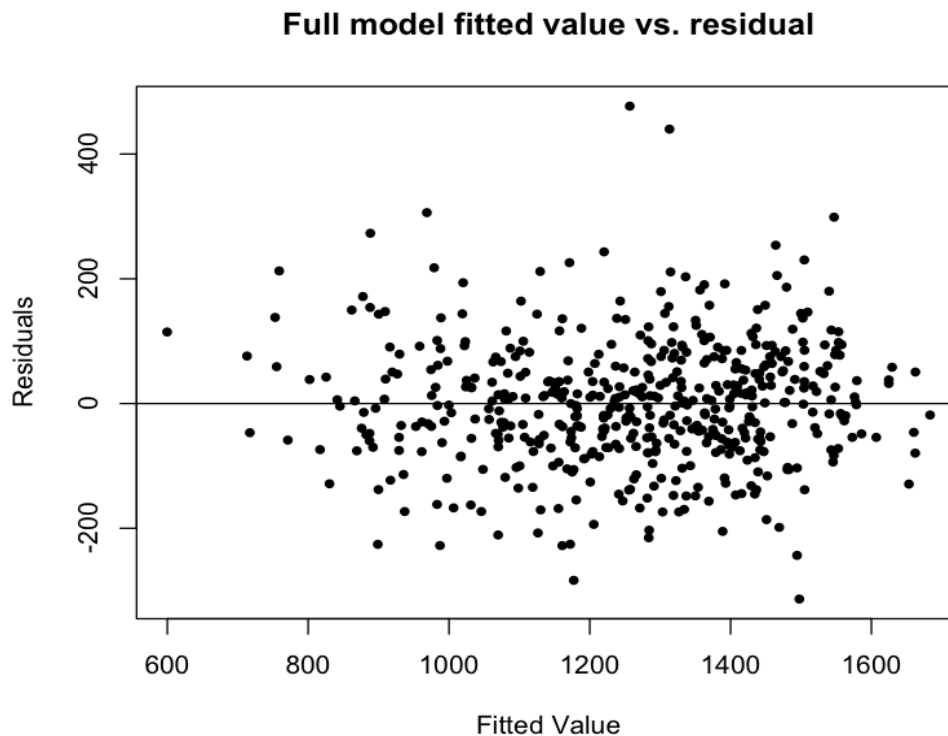


Figure 17: Full model Residuals vs. Fitted Value Plot

(d) Interpret Summary Statistic and Error Matrics

The final statistic result is in Figure 16, all variables are of significance and with a high Adjusted R Square. By look at table 2, even though they improved compared to previous models as we discussed above, AIC, BIC and MSE all perform poorly in this full model(Because we haven't discuss cycles in lectures).

| Error Metrics for full model | |
|------------------------------|----------|
| AIC | 6234.185 |
| BIC | 6293.63 |
| MSE | 9794.861 |

Table 2: Error Metrics for full model

(e) Full Model Forecasting

In forecast function, only h=519 can be used, otherwise there would be Error in variables. We can focus on the forecasting with a smaller step size (h=129) by using the code below instead of 195th and 196th lines:

```
1 quartz()  
2 plot(forecast(m7,h = 519,xlim=c(1976,2030)))
```

Forecasts from Linear regression model

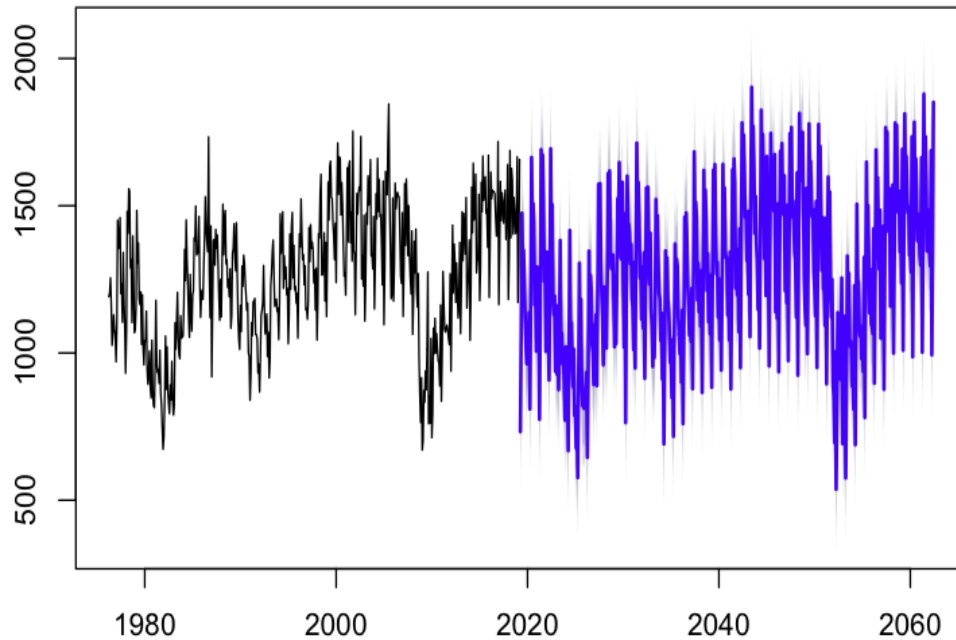


Figure 18: Forecasting with full model with h=519

2.3 Conclusions and Future Work

With the addition of seasonal dummies, the R-squared value improved from the 0.5709 seen in the original linear model, to a very impressive 0.9939. In other words, the new linear model is a near perfect fit for our time series. However, by looking at the ACF and PACF of the residuals shown in the figure below, we notice that there are still spikes in the plots. In other words, there is still room for improvement.

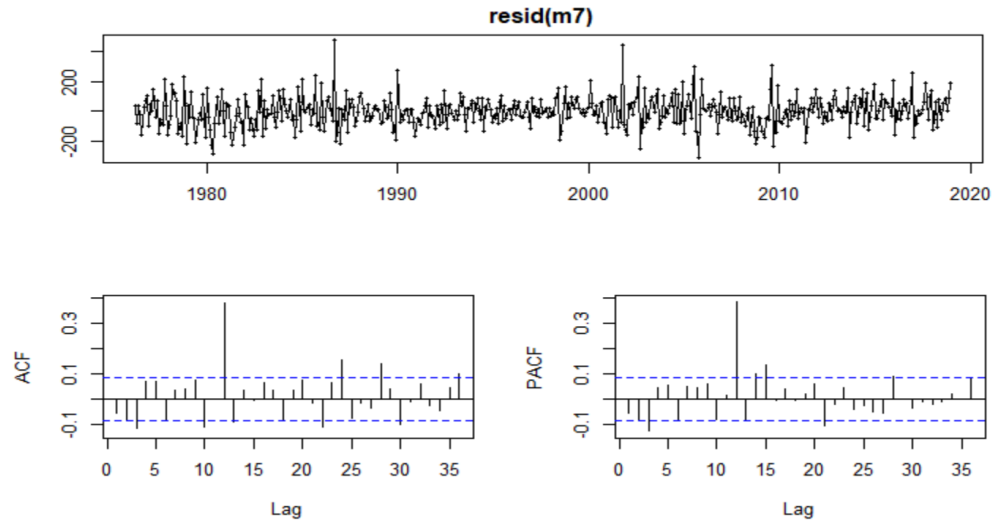


Figure 19: ACF and PACF for the Seasonal Model

When plotting the original time series, the first thing that was noticed was the shape of the trend; in particular, the cycles seen throughout the data. When fitting the exponential model, the cyclical shape was seen in the residual plot as well. Even in the residuals for the seasonally adjusted model, instead of taking on the form of white noise, there is a lingering trace of the original shape. In future works, we may add a cyclical attribute to our model to account for it, and consider using an ARIMA model.

References

- [1] U.S. Bureau of Economic Analysis. Total vehicle sales [totalnsa]. April 22, 2019. retrieved from FRED, Federal Reserve Bank of St. Louis <https://fred.stlouisfed.org/series/TOTALNSA>.

3 R Source code

```
1 ## R SETUP
2 library(lubridate)
3 library(zoo)
4 library(dynlm)
5 library(forecast)
6 library(minpack.lm)
7 library(numbers)
8 #read in the data
9 Data = read.csv("TOTALNSA.csv", header=TRUE, sep=",")
10
11 #decode the date
12 Data$DATE = as.Date(Data$DATE,"%Y-%m-%d")
13 ts_TOTALNSA = ts(data = Data$TOTALNSA,1976,2019,freq=12)
14
15 #II.1
16 #(a) Show a time-series plot of your data.
17 quartz()
18 plot(ts_TOTALNSA,main="Total Vehicle Sales time series",xlab="year",ylab="Thousands of
   Unit ")
19
20 #(b) Does your plot in (a) suggest that the data are covariance stationary? Explain your
   answer.
21 #The data points are not covariance stationary because they are not sharing the same mean
   .
22
23 #(c) Plot and discuss the ACF and PACF of your data.
24
25 Sales_acf<-acf(Data$TOTALNSA)
26 Sales_pacf<-pacf(Data$TOTALNSA)
27 quartz()
28 par(mfrow=c(2,1))
29 plot(Sales_acf,main="Total Vihecle Sales Time Series ACF")
30 plot(Sales_pacf,main="Total Vihecle Sales Time Series PACF")
31
32
33 #(d) Fit a linear and nonlinear (e.g., polynomial, exponential, quadratic + periodic, etc
   .) model to your series.
34 # In one window, show both figures of the original times series plot with the respective
   fit.
35
36 #Linear(3 Lag):
37 sales = ts(Data$TOTALNSA, start = 1976, frequency = 12)
38 m1=dynlm(sales~L(sales, 1)+L(sales, 2)+L(sales, 3))
39 t1<-seq(1976, 2019,length=length(m1$fit))
40 quartz("Linear")
41 plot(sales, main="Linear Model Prediction", ylab = "Total NSA(Thousands of Unit)", xlab =
   "Year", lwd = 1, xlim = c(1976,2019))
42 lines(m1$fitted.values,col = "red3", lwd = 1)
43 legend("topright",pch= c("-", "-"),legend=c("Original Data", "Linear Model Prediction"),
   col=c("black", "red"), cex=0.8)
44 summary(m1)
45
```



```

46 AIC(m1)
47 BIC(m1)
48
49 #Exponential:
50 t2 = seq(2917,2019,length = length(sales))
51 ds = data.frame(x = t, y = sales)
52 m2 = nlsLM(y ~ exp(a+b*t),data = ds, start = list(a = 0, b = 0))
53 quartz("plots exp")
54 plot(x = Data$DATE, y= m2$m$fitted()[1:519], type = "l", main="Exponential Model
    Prediction",col = "red3",ylab = "Total NSA(Thousands of Unit)", xlab = "Year", ylim =
    c(600,2000))
55 lines(x = Data$DATE, y = sales, lwd = 1)
56 legend("topright",pch= c("-", "-"),legend=c("Original Data", "Exponential Model Prediction
    "),col=c("black", "red"), cex=0.8)
57
58 quartz("Exponential Model residuals vs. fitted values")
59 plot(m2)
60 summary(m2)
61
62 #(e) For each model, plot the respective residuals vs. fitted values and discuss your
    observations.
63
64 #for linear:
65 quartz("e_linear")
66 plot(x = m1$fitted.values, y = m1$residuals, type = "p", pch = 20, main = "fitted value
    vs. residual",xlab = "Fitted Value", ylab = "Residuals")
67 abline(a = 0, b = 0)
68
69 #for Exponential:
70 quartz("e_Exp")
71 plot(x = m2$m$fitted(), y = m2$m$resid(), type = "p", pch = 20, main = "fitted value vs.
    residual", xlab = "Fitted Value", ylab = "Residuals")
72 abline(a = 0, b = 0)
73
74 #There's no obvious shape and generally symmetrically distributed around 0 in both graph
    plotting
75 #Thus the variables is linearly related and generally homoscedastic.
76
77 #(f) For each model, plot a histogram of the residuals and discuss your observations.
78 quartz("m1 residuals histogram")
79 hist(m1$residuals,main="Hisrogram for Residual of Linear Model",xlab="Number of Units(in
    thousands)")
80 quartz("m2 residuals histogram")
81 hist(m2$m$resid(),main="Hisrogram for Residual of Exponential Model",xlab="Number of
    Units(in thousands)")
82 #From the histogram, the residuals has a relatively good normal distribution
83
84 #(g) For each model, discuss the associated diagnostic statistics (R2 , tdistribution,
    Fdistribution, etc.)
85 summary(m1)
86 #Multiple R-squared: 0.5734, Adjusted R-squared: 0.5709 which is quite acceptable
87 #t-distribution: all the t-value is far greater than any usually accepted confidence
    interval,
88 # so we can reject the null hypothesis that any coefficient is 0

```

```

89 #F-statistic: 229.4 on 3 and 512 DF, p-value: < 2.2e-16 which is quite acceptable
90 summary(m2)
91 #t-distribution: all the t-value is far greater than any usually accepted confidence
    interval,
92 # so we can reject the null hypothesis that any coefficient is 0
93 #The other two data are not provided
94
95 #(h) Select a trend model using AIC and one using BIC (show the values obtained from each
    criterion). Do the selected models agree?
96 AIC(m1,m2)
97 BIC(m1,m2)
98 #Because of the trend, both model have extremely large AIC and BIC, but linear model
    performs relatively better
99
100 #(i) Use your preferred model to forecast h-steps (at least 16) ahead. Your forecast
    should include the respective uncertainty prediction interval. Depending on your data
    , h will be in days, months, years, etc.
101 quartz("m1 forecasting")
102 plot(forecast(m1,h = 519))
103
104 ##II.2
105 #(a) Construct and test (by looking at the diagnostic statistics) a model with a full set
    of seasonal dummies.
106 #construct the dummy set, each column is a dummy variable
107
108 # Construct seasonal dummies
109 dummies = matrix(nrow = length(Data$TOTALNSA), ncol = 12)
110 for (i in 1:12)
111 {
112     for(j in 1:519)
113     {
114         dummies[j,i] = as.integer(mod(j,12) == i-1)
115     }
116 }
117 # Construct model with full seasonal dummies
118 m3=dynlm(sales~dummies[,1]+dummies[,2]+dummies[,3]+dummies[,4]+dummies[,5]+dummies[,6]+
    dummies[,7]+dummies[,8]+dummies[,9]+dummies[,10]+dummies[,11]+dummies[,12])
119 #Diagnostic Statistics
120 quartz("Model diagnostic statistic with seasonal dummies")
121 par(mfrow=c(2,2))
122 plot(m3,main="Diagnostic statistic Graph Part")
123 # numerical diagnostic statistic
124 summary(m3)
125 AIC(m3)
126
127 #(b) Plot the estimated seasonal factors and interpret your plot.
128 coef_sdumy <-m3$coefficients
129 # rearrange order
130 Dec_sdumy<-coef_sdumy[2]
131 coef_sdumy <- coef_sdumy[-c(2)]
132 coef_sdumy[12]<-0 # change na to 0
133 coef_sdumy <-c(coef_sdumy,Dec_sdumy)
134 x<-c(1:13) # x-axis
135 # x-axis name

```

```

136 x_name <- c("Intercept","Jan.","Feb.","Mar.","Apr.","May.","Jun.","Jul.","Aug.","Sep.","
    Oct","Nov.","Dec.")
137 # plot seasonal factor
138 quartz("seasonal dummies plot")
139 plot(x,coef_sdumy,xaxt="none",type="l",main="Seasonal Factor plot",ylab="Value of dummy
    variables",xlab="Variable name")
140 axis(1, at=1:13, labels=x_name, cex.axis=0.5)
141
142 #(c)
143 m5=dynlm(sales~L(sales, 1)+L(sales, 2)+L(sales, 3)+dummies[,1]+dummies[,2]+dummies[,3]+
    dummies[,4]++dummies[,5]+dummies[,6]+dummies[,7]+dummies[,8]+dummies[,9]+dummies
    [,10]+dummies[,11]+dummies[,12])
144 summary(m5)
145 AIC(m5)
146
147 #have dummies[,10](insignificant) removed
148 m6=dynlm(sales~L(sales, 1)+L(sales, 2)+L(sales, 3)+dummies[,1]+dummies[,2]+dummies[,3]+
    dummies[,4]++dummies[,5]+dummies[,6]+dummies[,7]+dummies[,8]+dummies[,9]+dummies
    [,11]+dummies[,12])
149 summary(m6)
150 AIC(m6)
151
152 #remove insignificant factor
153 #Alex's note:
154 #I hesitate for a while if we should delete the intercept,
155 #for it's a linear model. If I keep it
156 m7_1=dynlm(sales~L(sales, 1)+L(sales, 2)+L(sales, 3)+dummies[,1]+dummies[,2]+dummies[,3]+
    dummies[,4]++dummies[,5]+dummies[,6]+dummies[,7]+dummies[,8]+dummies[,9]+dummies
    [,11])
157 summary(m7_1)
158 AIC(m7_1)
159
160 #But if I removed the intercept the R^2 is greatly improved(.99 R^2...)
161 m7_2=dynlm(sales~0+L(sales, 1)+L(sales, 2)+L(sales, 3)+dummies[,1]+dummies[,2]+dummies
    [,3]+dummies[,4]++dummies[,5]+dummies[,6]+dummies[,7]+dummies[,8]+dummies[,9]+dummies
    [,11])
162 summary(m7_2)
163 AIC(m7_2)
164
165 #For the sake of .99 R^2, I choose the one without intercept as the full model
166 m7 = m7_2
167
168 #plot original data and predction by the full model
169 quartz("Linear")
170 plot(sales, main="Full Model Prediction", ylab = "Total NSA(Thousands of Unit)", xlab = "
    Year", lwd = 1, xlim = c(1976,2019))
171 lines(m7_2$fitted.values,col = "red3", lwd = 1)
172 legend("topright",pch= c("-", "-"),legend=c("Original Data", "Full Model Prediction"),col=
    c("black", "red"), cex=0.8)
173
174
175 #for residuals vs. fitted values
176 quartz("Residuals vs. fitted values for full model")

```

```

177 plot(x = m7_2$fitted.values, y = m7$residuals, type = "p", pch = 20, main = "Full model
    fitted value vs. residual", xlab = "Fitted Value", ylab = "Residuals")
178 abline(a = 0, b = 0)
179 #There's no obvious shape and generally symmetrically distributed around 0 in both graph
    plotting
180 #Thus the variables not linearly correlated and generally homoscedastic.
181
182 #(d) Interpret the respective summary statistics including the error metrics of your full
    model.
183 #Very Impressive R^2, t, and f
184 summary(m7_2)
185 #However we are pretty bad at this(Because we haven't discuss cycles in lectures)
186 AIC(m7)
187 BIC(m7)
188 #Find mse is 9794.861, behave poorly
189 mean((sales - m7_2$fitted.values)^2)
190
191
192 #(e)Use the full model to forecast h-steps (at least 16) ahead. Your forecast should
    include the respective prediction interval.
193
194 # In forecast function, only h=519 can be used, otherwise there would be Error in
    variables
195 quartz()
196 plot(forecast(m7,h = 519))
197
198 #III
199 #As what I mentioned in II.2.(d), we have a horrible AIC and BIC,
200 #which is not only because of we have great amount of variables(it's overfitted for sure)
201 #but we also have a very stochastic trend, see:
202 plot(stl(ts(Data$TOTALNSA,freq = 12), s.window = "periodic"))
203 #as what is discussed in OH, we need more techniques to dealing with the cycles pattern

```