# Economics 144: Project2

Yihao Li
Claudia Huey
Sijia Hua

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## 1 Introduction

The data used in this project were the SP 500[1] and US GDP[2]. SP 500, or just the SP, is an American stock market index based on the market capitalization of 500 large companies having common stock listed on the NYSE, NASDAQ, or the Cboe BZX Exchange. GDP, or the gross domestic product, is a monetary measure of the market value of all the final goods and services produced in the United States in a period of time. Specific details pertaining to the data used can be found in the table below.

| Data Description | | |
|---|---|---|
| Title | United States GDP | S&P500 |
| Units | Billions of US Dollars | US Dollars |
| Seasonal Adjusted | Yes | No |
| Time Period | 1990-01-01 2019-01-01 | |
| Frequency | Quarterly | |

Table 1: Source Data Information

Based on these data sets, we have fit autoregressive models to both series and forcasted 12-steps ahead (h=12). We have also fit a VAR model in order to test for Granger-Causality between the two data sets. Theoretically, as the economy grows, aggregate corporate earnings should rise and vice versa, and through this project, we will see if that theory can be supported with concrete data.

## 2 Results

### 2.1 Time-series plot including the respective ACF and PACF plots.

In Figure 1, we show the GDP time series plot and corresponding ACF, PACF plot. It is obviously in an increasing trend.

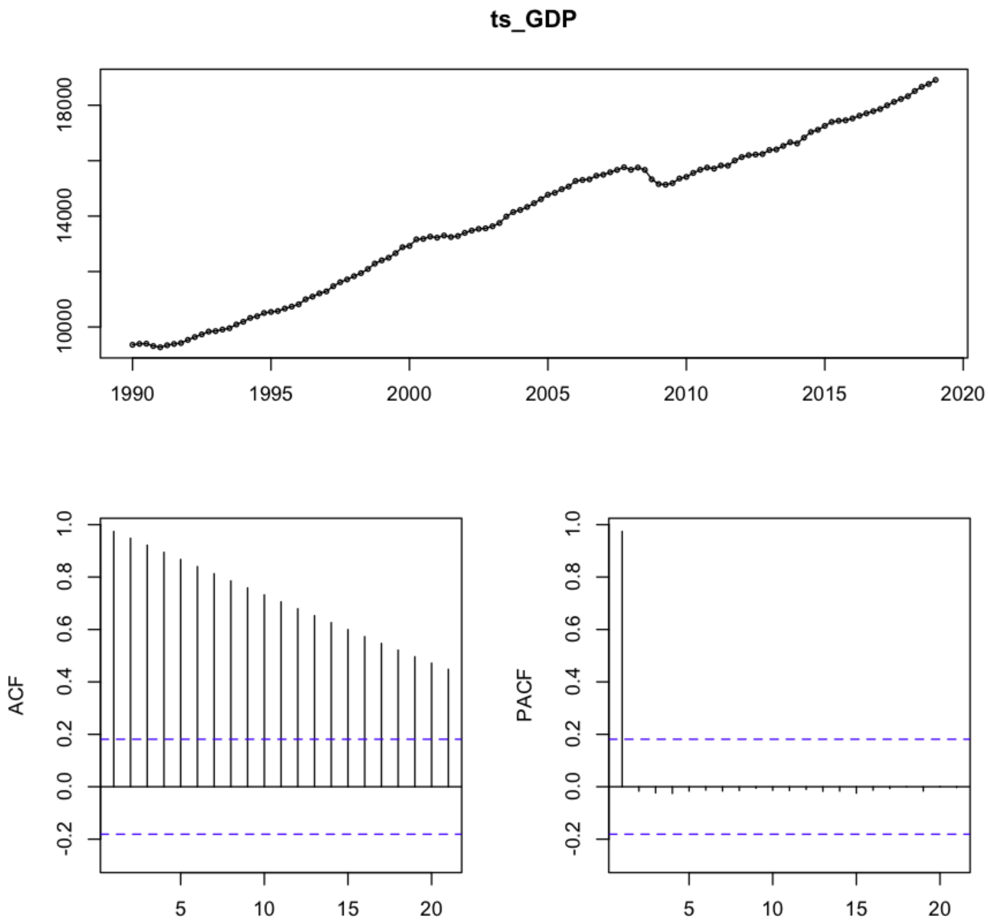


Figure 1: Time series display of GDP

In Figure 2, we show the SP500 time series plot and corresponding ACF, PACF plot. It is an increasing trend with cycle.

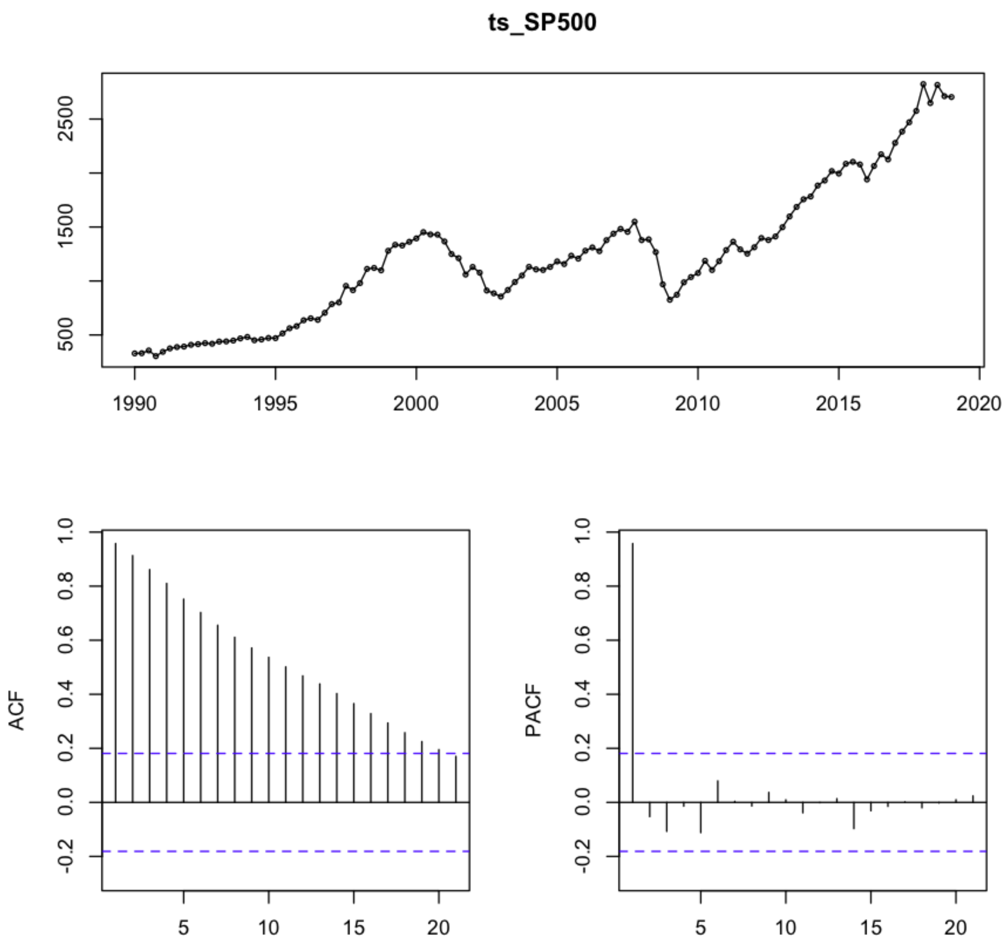


Figure 2: Time series display of SP500

## 2.2 Fit a model that includes, trend, seasonality and cyclical components.

From the chart relating to GDP in the previous section, we can see that there is a gradual decline in the ACF, and a large spike in the PACF at the fist lag. Therefore, with 1st-difference, we fit an AR(1) model and an AR(2) model as possible model candidates.

```
## Series: ts_GDP
## ARIMA(1,1,0)
##
## Coefficients:
##          ar1
##       0.6919
## s.e.  0.0666
##
## sigma^2 estimated as 6968:  log likelihood=-677.66
## AIC=1359.32   AICc=1359.43   BIC=1364.83
```

Figure 3: Summary of AR(1) fit (with 1st difference)

```
## Series: ts_GDP
## ARIMA(2,1,0) with drift
##
## Coefficients:
##          ar1     ar2    drift
##       0.3089  0.1702  82.3279
## s.e.  0.0911  0.0913  12.9879
##
## sigma^2 estimated as 5568:  log likelihood=-663.41
## AIC=1334.82   AICc=1335.18   BIC=1345.84
```

Figure 4: Summary of AR(2) fit (with 1st difference)

From the two figures above, we see that the AR(2) model has a lower BIC and AIC, so therefore we select the AR(2) model as the best fit model for GDP.

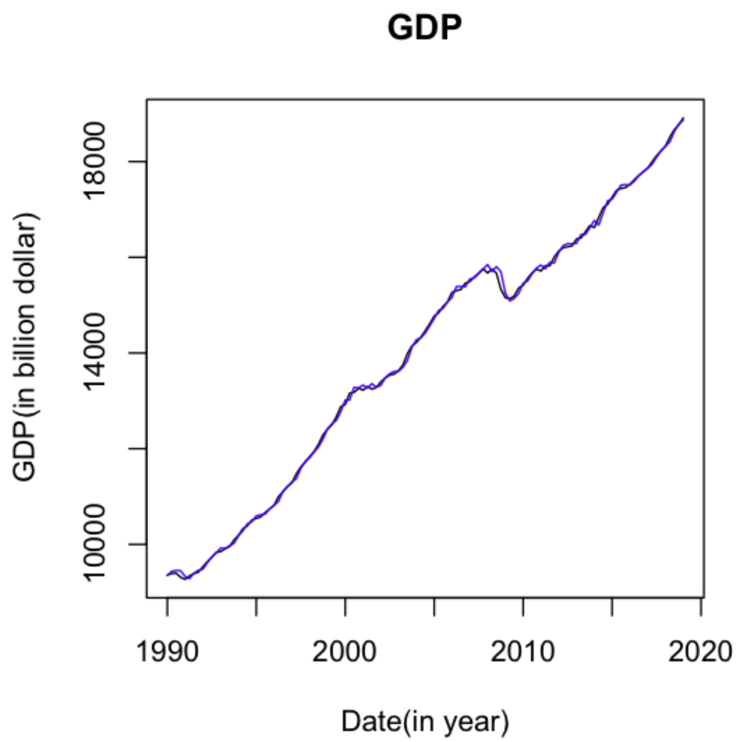


Figure 5: GDP ARIMA Fit

From the SP500 data in the previous section, we once again see that there is a gradual decline in the ACF, and a large spike in the PACF at the fist lag. Therefore, with 1st-difference, we fit an AR(1) model to the data.

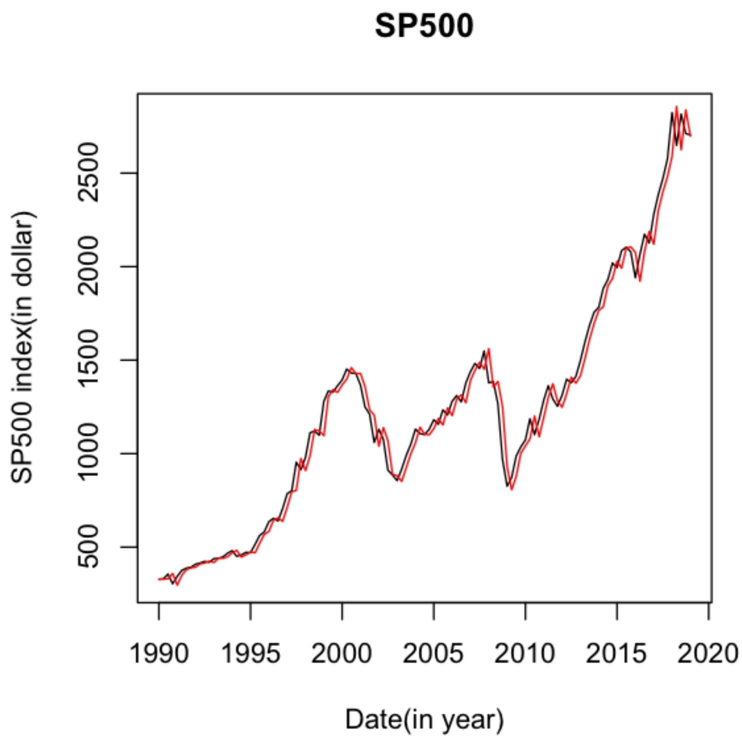


Figure 6: SP500 ARIMA Fit

## 2.3 Respective Residuals vs. Fitted Values Plot

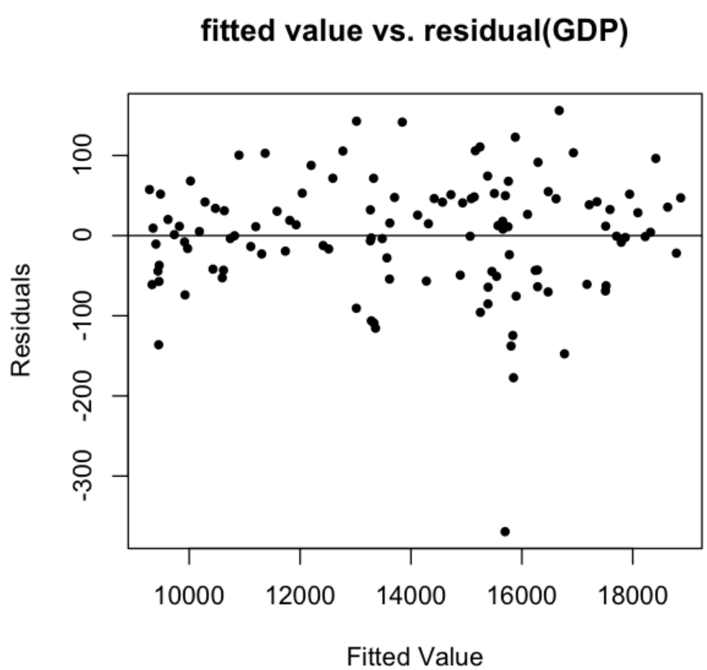


Figure 7: GDP fitted vs. residual

From the plot, we see that there is random scatter around 0. There is also an obvious outlier near (1550,-360).

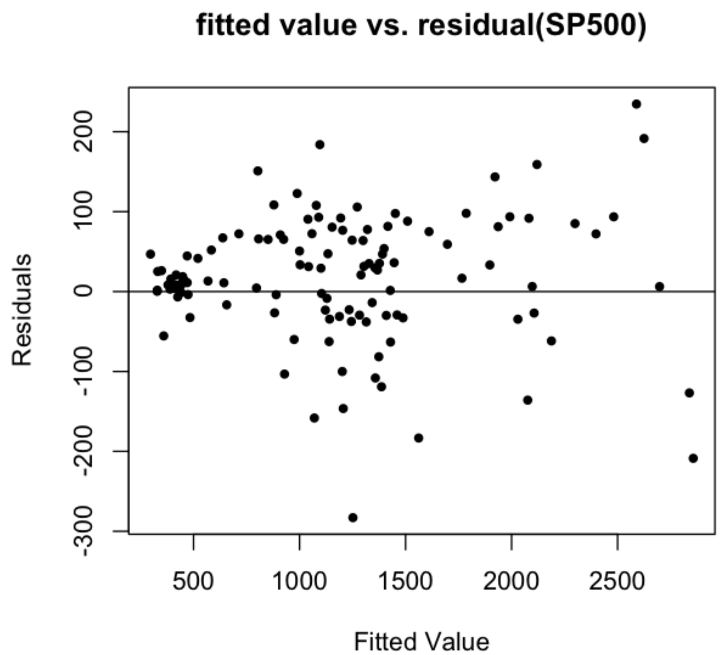


Figure 8: SP500 fitted vs. residual

From the plot we see that there is a small clump of data points near the 500s, but since they gather around 0, it does not seem very significant. There also seems to be more variation in y as x increases, but the increase is not obvious enough to be labeled as a trend. Overall, the plot seems to represent white noise pretty well.

### 2.4 ACF and PACF of the Respective Residuals

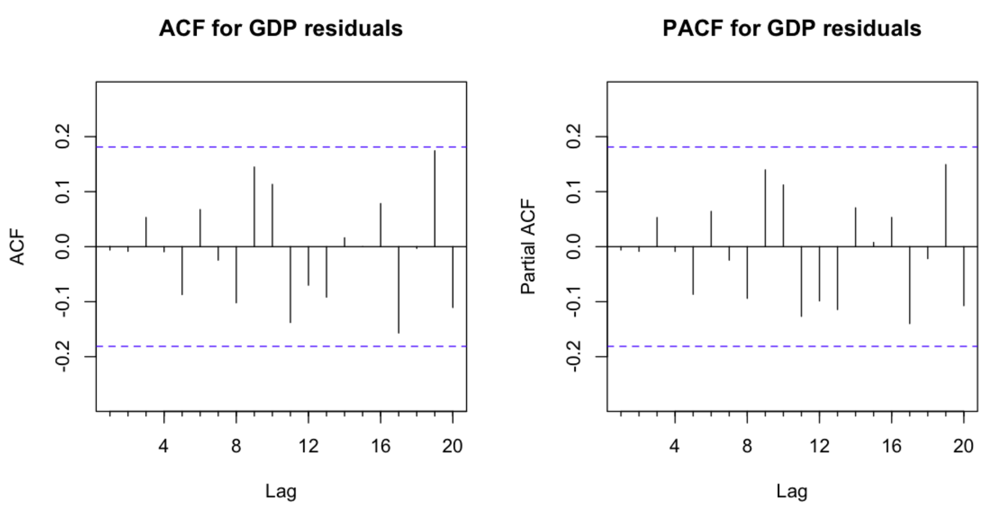


Figure 9: GDP residuals ACF PACF

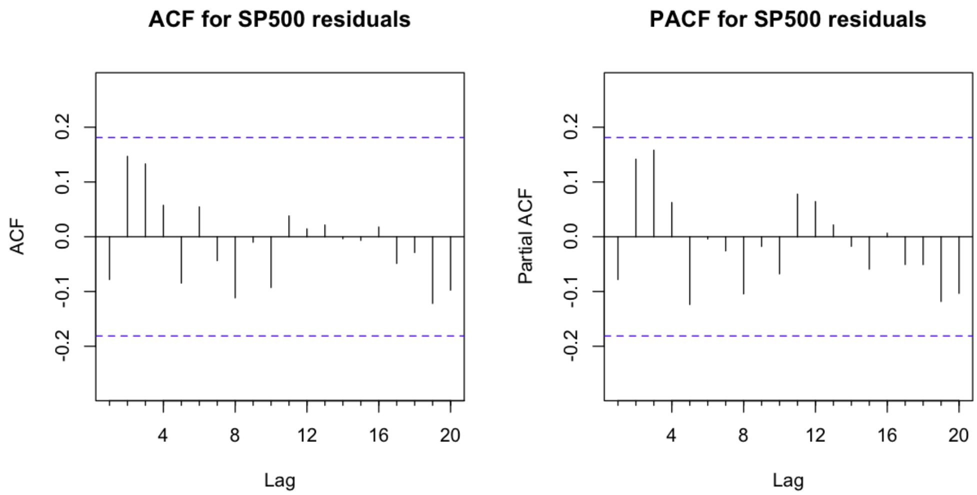


Figure 10: SP500 residual ACF PACF

There are no obvious spikes in either the ACF or PACF for both data sets, suggesting that their respective models were a good fit for the data.

### 2.5 Respective CUSUM Plot

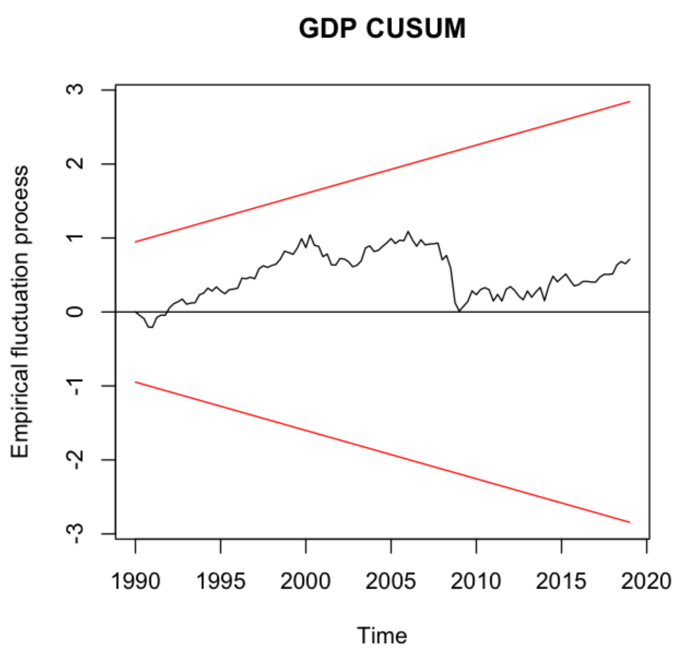


Figure 11: GDP CUSUM plot

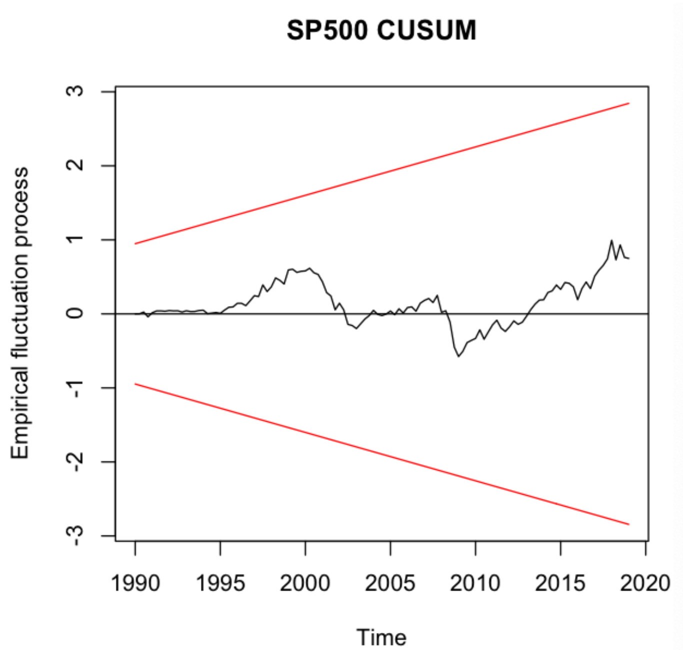


Figure 12: SP500 CUSUM plot

Neither series broke the bounds in the plots. Although GDP residuals showed a slight tendency of biasness, the outlier we noticed in the residual plot for GDP does not have enough weight to break the bounds. Both plots suggest that the models were a good fit for the data.

## 2.6 Respective Recursive Residuals Plot

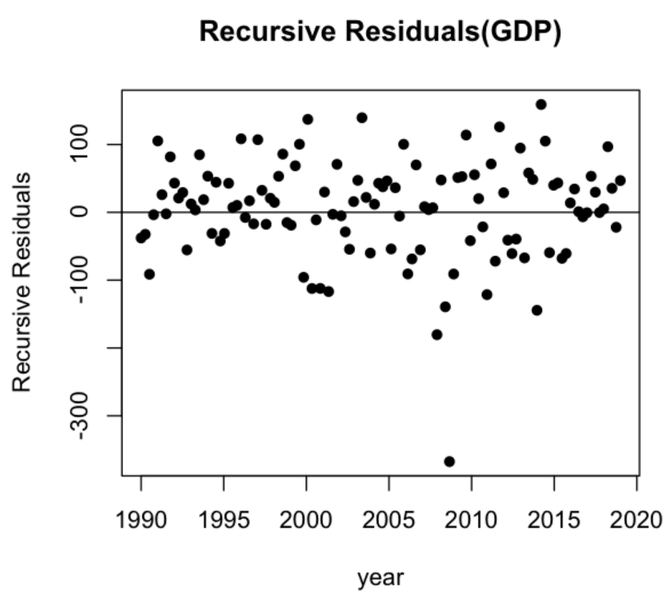


Figure 13: GDP Recursive Residual

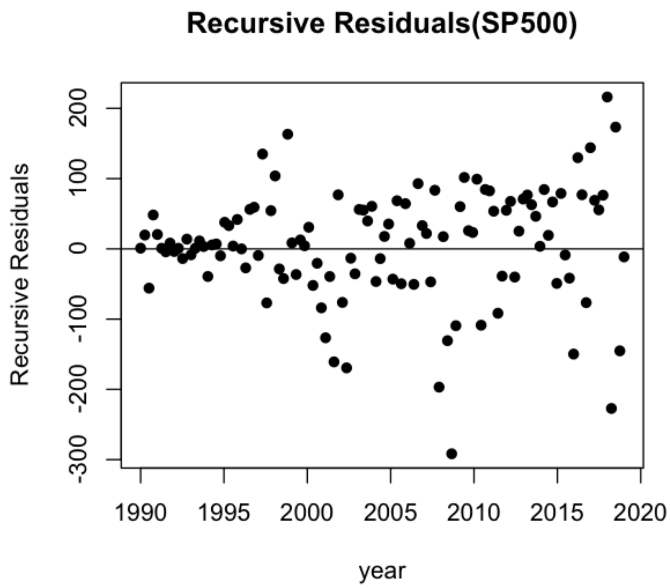


Figure 14: SP500 Recursive Residual

Unlike the normal residuals, recursive residuals form a data set conforming to normal distribution and iid by continuously reducing the number of observations. Thus, recursive residuals can better find outliers in data sets. In Figure 13 and Figure 14, the recursive residuals are randomly distributed and bouncing around 0. There is one potential outlier in GDP and SP500 respectively around 80th quarter (2010th year). However, none of these actually break the model according to CUSUM.

## 2.7 For your model, discuss the associated diagnostic statistics.

In Figure 15, the ARIMA(2,1,0) fit for GDP has the smallest AIC and BIC compared to ARIMA model with other orders. Mean Error is around 0.5, which indicates this is generally a good fit with a small number of outliers. Mean Absolute Error is aronud 50, so for each point forecast this is a distance of 50 from the true value.

```
> summary(fit_GDP)
Series: ts_GDP
ARIMA(2,1,0) with drift

Coefficients:
         ar1     ar2    drift
      0.3089  0.1702  82.3279
s.e.  0.0911  0.0913  12.9879

sigma^2 estimated as 5568:  log likelihood=-663.41
AIC=1334.82   AICc=1335.18   BIC=1345.84

Training set error measures:
                    ME     RMSE      MAE          MPE      MAPE      MASE         ACF1
Training set 0.4570117 73.33312 54.10485 -0.001459078 0.3920863 0.1463298 -0.006050874
```

Figure 15: GDP ARIMA Model Summary

In Figure 16, the ARIMA(1,1,0) fit for SP500 has the smallest AIC and BIC compared to ARIMA model with other orders. Mean Error is around 17.6, which indicates there may be some outliers. Mean Absolute Error is aronud 61, which tells us for each point forecast this is a distance of about 60 from the true value.

```
> summary(fit_SP500_1)
Series: ts_SP500
ARIMA(1,1,0)

Coefficients:
         ar1
      0.1325
s.e.  0.0916

sigma^2 estimated as 6724:  log likelihood=-675.28
AIC=1354.57   AICc=1354.67   BIC=1360.07

Training set error measures:
                   ME     RMSE      MAE      MPE     MAPE      MASE        ACF1
Training set 17.60292 81.29654 61.09371 1.369713 5.185108 0.3674704 -0.07813973
>
```

Figure 16: SP500 ARIMA Model Summary

## 2.8 Use your model to forecast 12-steps ahead. Your forecast should include the respective error bands

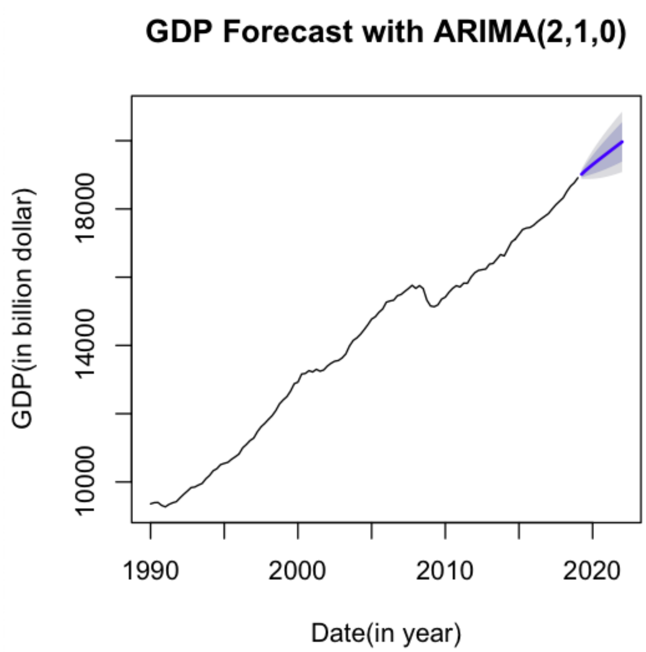


Figure 17: GDP Forecast

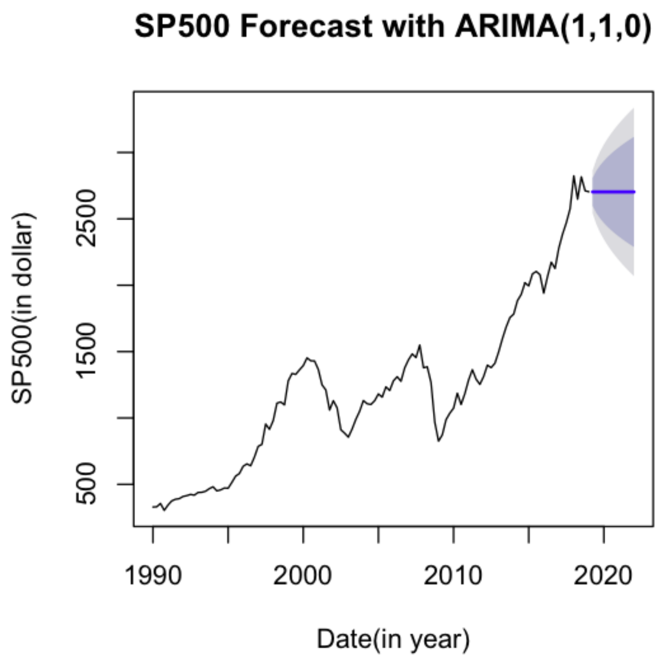


Figure 18: SP500 Forecast

## 2.9 VAR model Fit Result

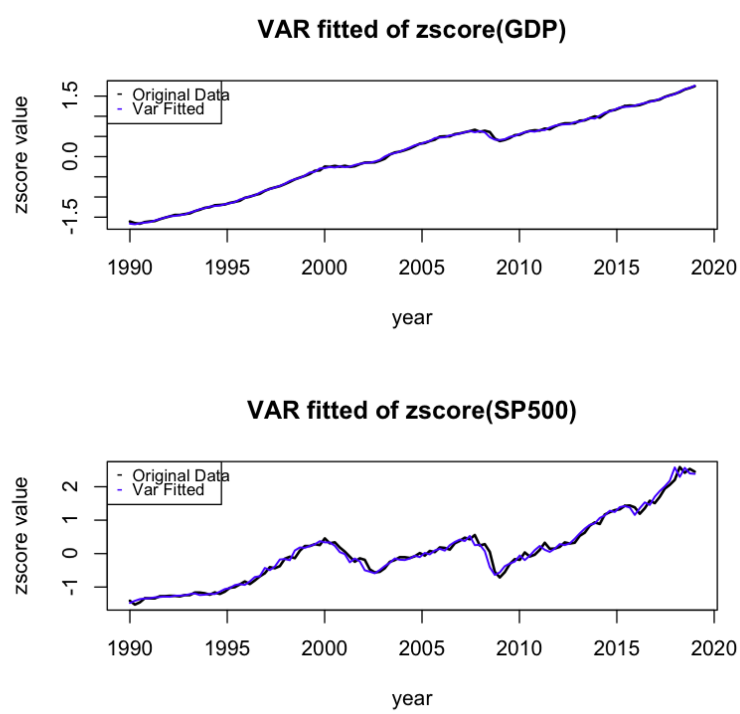


Figure 19: Z-score VAR Fitted

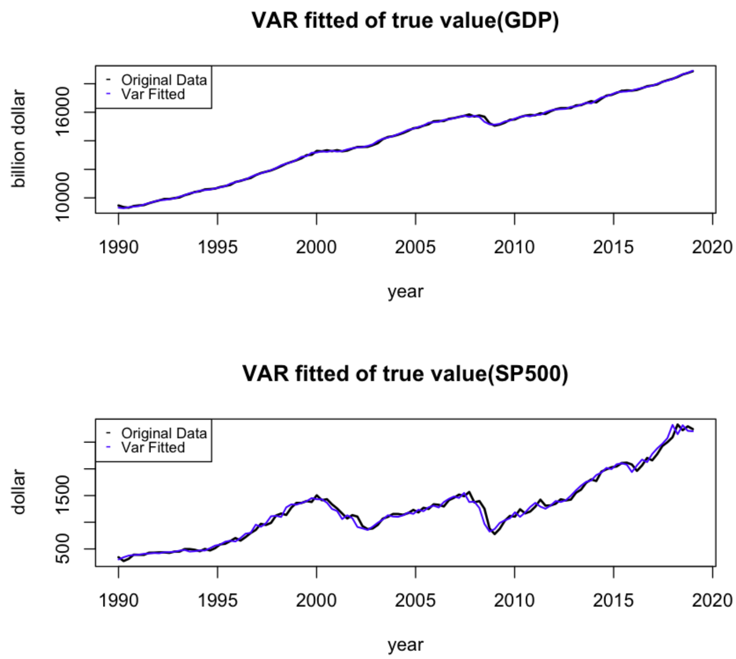


Figure 20: True Value VAR Fitted

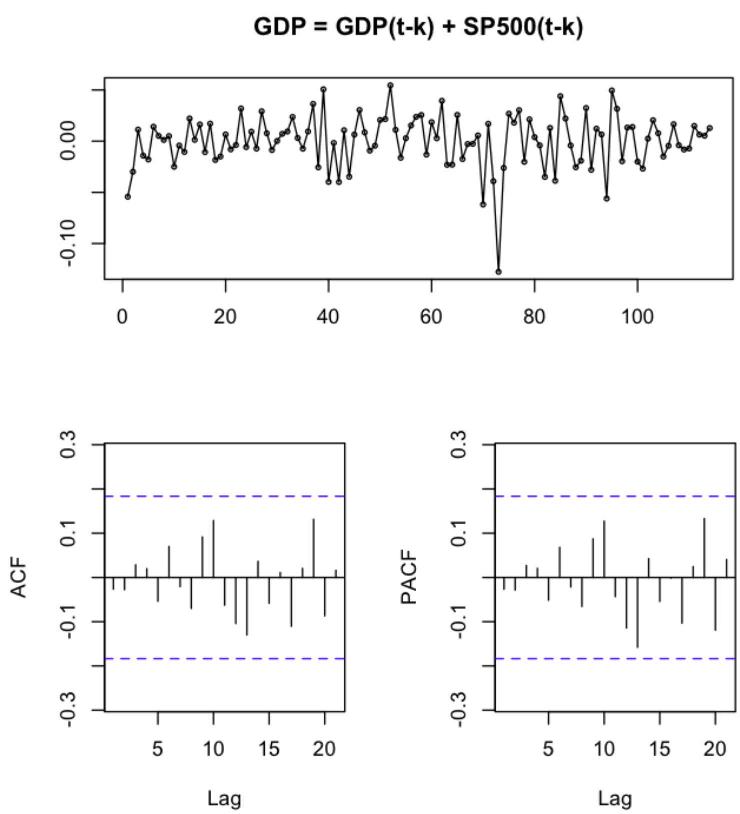


Figure 21: GDP Residual Summary

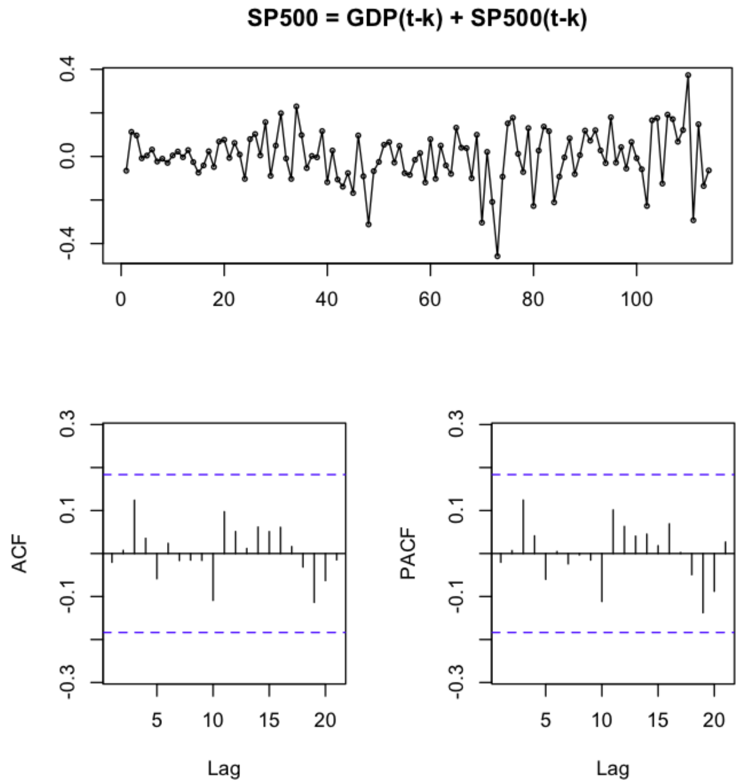


Figure 22: SP500 Residual Summary

From our VAR select, we have fit a VAR(3) model to the data. From the ACF and PACF plots shown above, we see that there are no spikes in any of the plots, and that the series seem to resemble white noise. Therefore, we can conclude that the VAR(3) model is a good fit for the data.

### 2.10 Respective Impulse Response Functions Plot

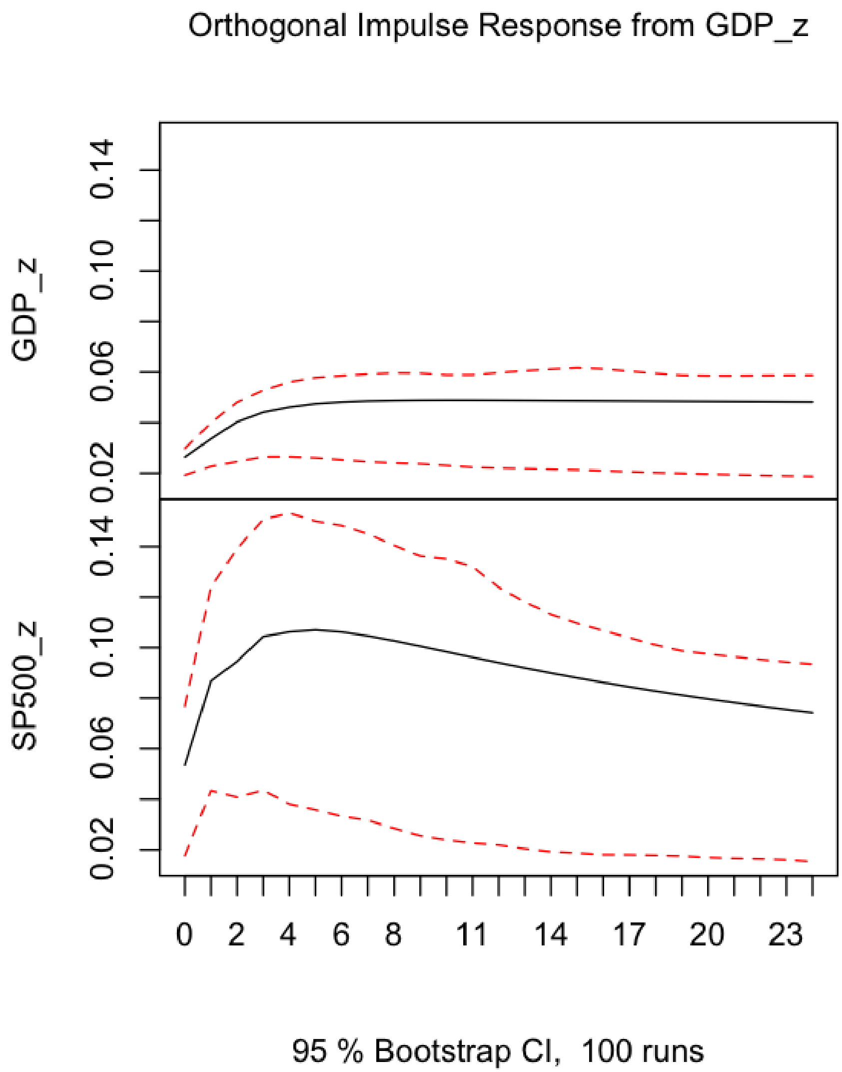


Figure 23: IRF from GDP

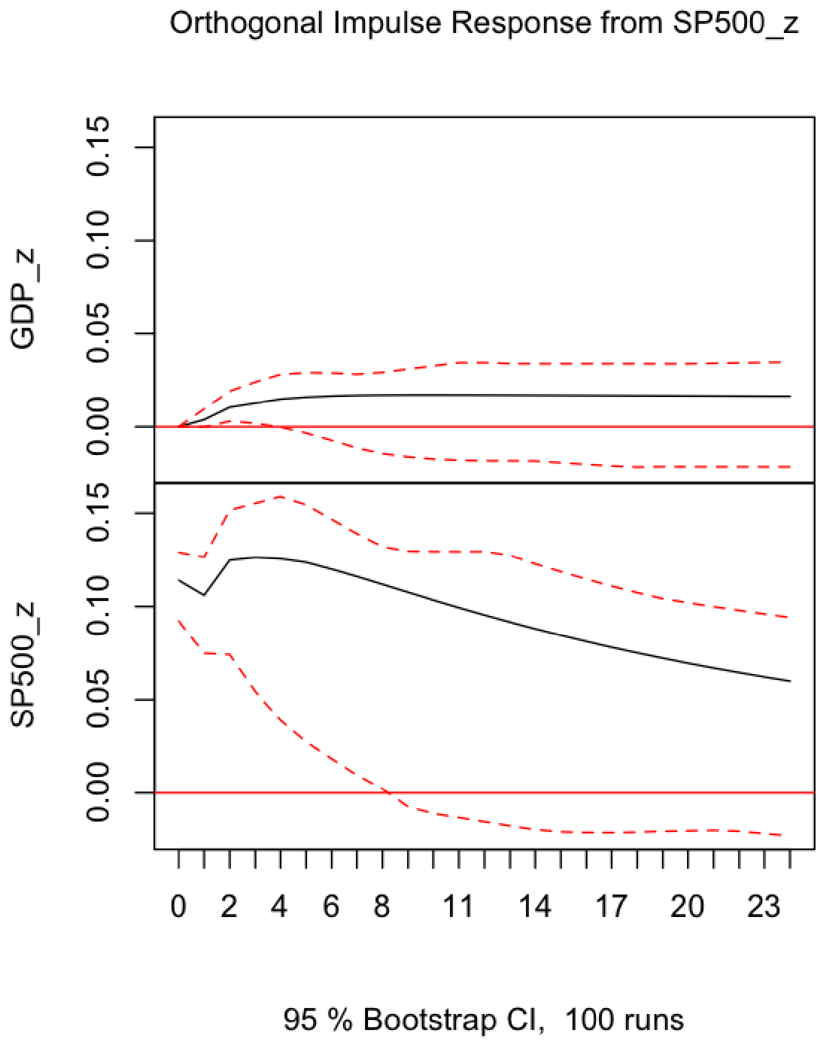


Figure 24: IRF from SP500

From the impluse response functions, we see that a shock in GDP will barely affect GDP, much like how a shock in GDP will barely affect SP500. A shock from SP500 on GDP, however, has a much larger affect, as we see a sharp rise in GDP, and then a gradual decent. A similar case can be seen for the affects on SP500 from a shock in itself. There is a slight but sharp increase and a relatively steep decent back down. One should also note that the error bounds for these two plots are much larger than the others, suggesting that there is a larger factor of unpredictability at play.

### 2.11 Granger-Causality test on variables

```
> grangertest(GDP_z,SP500_z, order = 3)
Granger causality test

Model 1: SP500_z ~ Lags(SP500_z, 1:3) + Lags(GDP_z, 1:3)
Model 2: SP500_z ~ Lags(SP500_z, 1:3)
  Res.Df Df      F  Pr(>F)
1    107
2    110 -3 3.0557 0.03155 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Figure 25: Granger-test Result

With a significance level of 5% (alpha = 0.05), we reject the null that states that there is no causality between the two variables. From the test, we can see that GDP can be used to predict SP500.

### 2.12 Use VAR model to Forecast

Since our data set includes 29 years, we choose to forecast 24 steps to further observe their future trends.

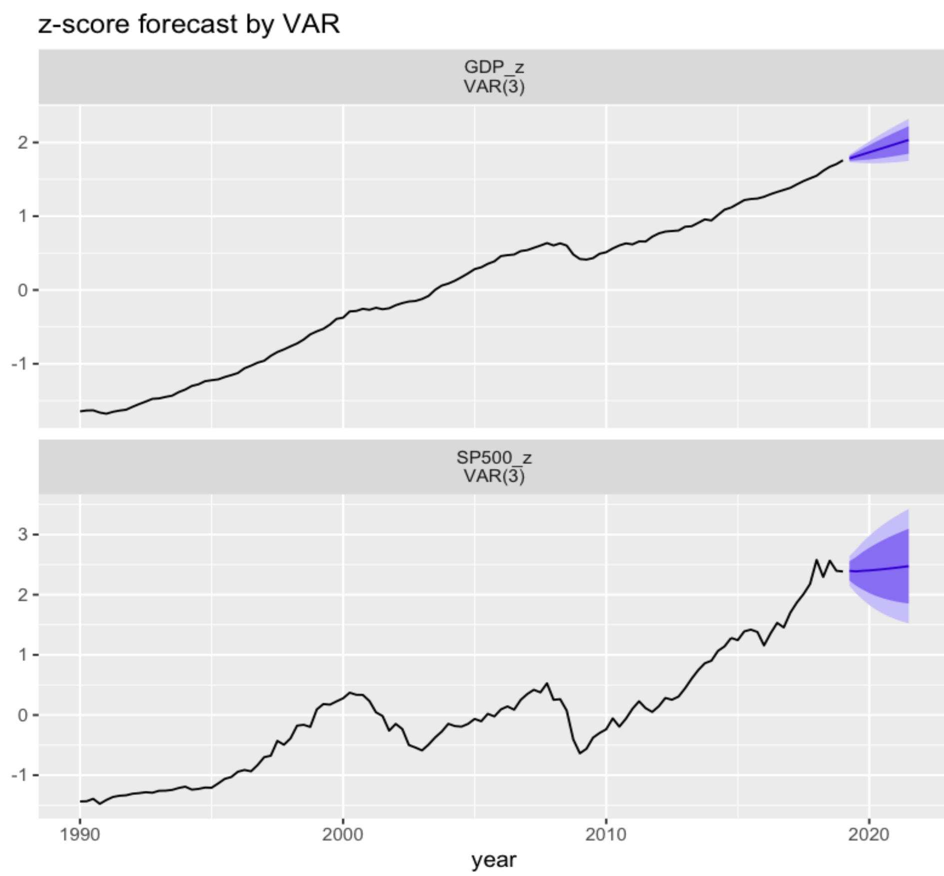


Figure 26: Z-score Forecast by VAR

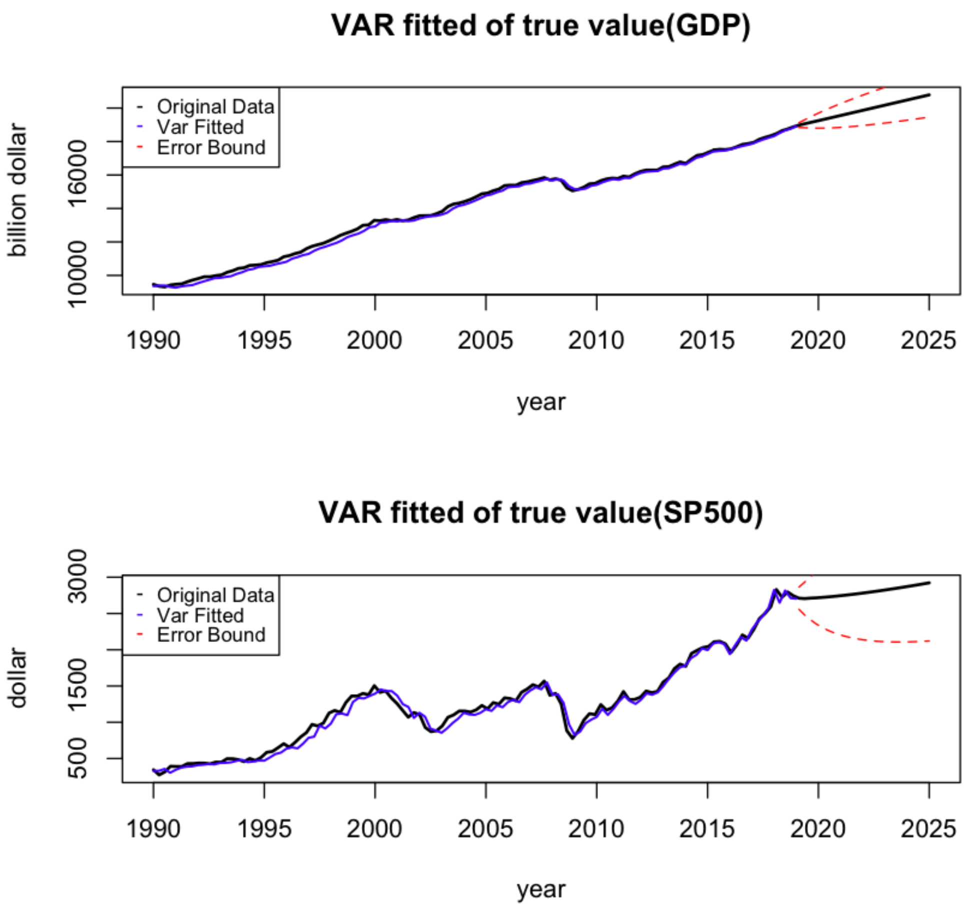


Figure 27: True Value Forecast by VAR

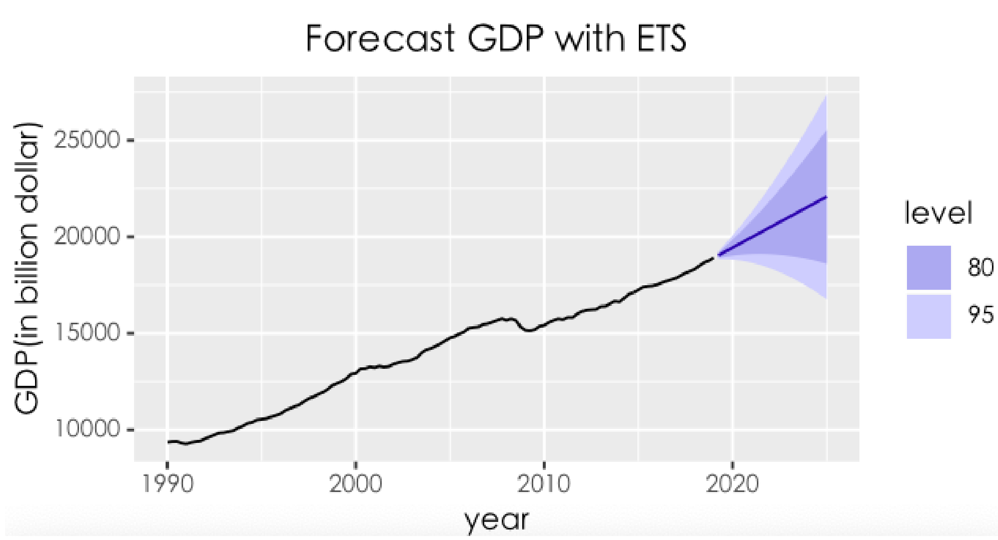


Figure 28: GDP ETS Forecast

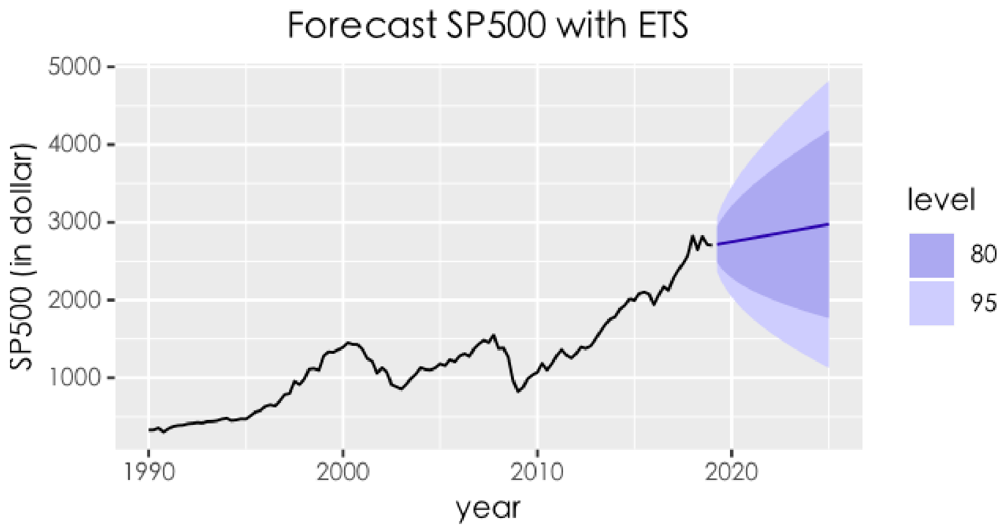


Figure 29: SP500 ETS Forecast

Both forecasts seem to very similar. Both VAR and ARIMA predict a steady increase in GDP and a flatter/slower increase for SP500. From the bounds we can see that perhaps VAR has a much more conservative prediction, as it seems to predict slightly lower numbers. We also tried forecasting with ETS as well, and from the figures we can see that the plots there were also quite similar in shape.

# 3 Conclusions and Future Work

By looking at the Forecast Result from VAR, ARIMA, and ETS, it is hard to tell which model provides a better prediction. The forecasts for GDP are all slowly rising, while SP500 is relatively stable. Considering the interaction between two data sets, the VAR model is a better and more reasonable model. According to the Granger Test shown above, GDP can be used to forecast SP500. In the Impulse Response Functions, we see that GDP has a large and long impact on SP500 that last more than 20 quarters.

Certainly, the prediction of SP500 and GDP is not very detailed, but only used to understand the relative trend in the future. More accurate numerical predictions require us to add more dummy variables, such as relevant industry news and political current events.

## References

[1] Yahoo! Finance. Sp 500 ($^gspc$). $May30, 2019. SNP - SNPRealTimePrice.CurrencyinUSDhttps: //finance.yahoo.com/quote/$

[2] U.S. Bureau of Economic Analysis. Real gross domestic product. May 30, 2019. retrieved from FRED, Federal Reserve Bank of St. Louis https://fred.stlouisfed.org/series/TOTALNSA.

# 4 R Source code

```
library(Hmisc)
library("varhandle")
library(timeSeries)
library(lubridate)
library(zoo)
library(dynlm)
library(forecast)
library(dplyr)
library(vars)
##### 29year sp & gdp #########
#read in the csv file
gdp29 <- read.csv('gdp29y.csv', header = TRUE, sep = ",")
sp29 <- read.csv('sp29y.csv', header = TRUE, sep = ",")

#pick the every close index of SP500
sp29_close <- sp29[c(1,5)]
colnames(sp29_close) <- c("DATE","SP500")

#data set using for the project
data <- merge(gdp29,sp29_close)
colnames(data)<- c("DATE","GDP","SP500")
data$DATE = as.Date(data$DATE,"%Y-%m-%d")

# time series data from orginal datasets
ts_GDP <- ts(data$GDP,1990,2019,frequency = 4)
ts_SP500 <- ts(data$SP500,1990,2019,frequency = 4)

#(a) Produce a time-series plot of your data including the respective ACF and PACF plots.
quartz("ts_GDP")
tsdisplay(ts_GDP)#AR(1)
quartz("ts_SP500")
tsdisplay(ts_SP500)#AR(1)
#actually both plot is strong  A R

#(b) Fit a model that includes, trend, seasonality and cyclical components. Make sure to discuss your model in detail.
#Alex: I use auto.ARIMA here, we might need to "pretend" have tried a lot of models:)

## fit with auto.ARIMA
fit_GDP <- auto.ARIMA(ts_GDP)
fit_GDP#ARIMA(2,1,0)??? with drift
quartz()
plot(x = data$DATE, y = ts_GDP, xlab = "Date(in year)", ylab = "GDP(in billion dollar)", main = "GDP", type = "l")
lines(x = data$DATE, y = fit_GDP$fitted, col = "blue")

#Emma: The autoARIMA process is suspect. I prefer using ARIMA(1,1,0).
# fit_SP500 <- auto.ARIMA(ts_SP500)
# fit_SP500 #ARIMA(0,1,0) with no AR???
# quartz()
# plot(x = data$DATE, y = ts_SP500, xlab = "Date(in year)", ylab = "SP500 index(in dollar)", main = "SP500", type = "l")
# lines(x = data$DATE, y = fit_SP500$fitted, col = "blue")

#I...personally think this one makes slightly better prediction, at least intuitively from the graph
fit_SP500_1 <- ARIMA(ts_SP500, order = c(1, 1, 0))
quartz()
plot(data$DATE, ts_SP500, xlab = "Date(in year)", ylab = "SP500 index(in dollar)", main = "SP500", type = "l")
lines(data$DATE,fit_SP500_1$fitted, col = "red")

#(c) Plot the respective residuals vs. fitted values and discuss your observations.
#for GDP:
quartz("e_GDP")
#transform it into vector to remove the quarterly frequency will remove horrible data notation
plot(x = c(fit_GDP$fitted), y = c(fit_GDP$residuals), type = "p", pch = 20, main = "fitted value vs. residual(GDP)",xlab = "Fitted Value", ylab = "Residuals")
abline(a = 0, b = 0)

#for SP500:
quartz("e_SP500") # fit with ARIMA(1,1,0)
plot(x = c(fit_SP500_1$fitted), y = c(fit_SP500_1$residuals), type = "p", pch = 20, main = "fitted value vs. residual(SP500)",xlab = "Fitted Value", ylab = "Residuals")
abline(a = 0, b = 0)

#(d) Plot the ACF and PACF of the respective residuals and interpret the plots.
quartz("tsdisplay for GDP residuals")
par(mfcol=c(1,2))
Acf(fit_GDP$residuals, main = "ACF for GDP residuals")
Pacf(fit_GDP$residuals, main = "PACF for GDP residuals")
tsdisplay(fit_GDP$residuals)

quartz("tsdisplay for SP500 residuals")
par(mfcol=c(1,2))
Acf(fit_SP500_1$residuals, main = "ACF for SP500 residuals")
Pacf(fit_SP500_1$residuals, main = "PACF for SP500 residuals")
tsdisplay(fit_SP500_1$residuals)

#(e) Plot the respective CUSUM and interpret the plot.
quartz("CUSUM-GDP")
plot(efp(fit_GDP$residuals ~ 1, type = "Rec-CUSUM"), main = "GDP CUSUM")

quartz("CUSUM-SP500")
plot(efp(fit_SP500_1$residuals ~ 1, type = "Rec-CUSUM"), main = "SP500 CUSUM")

#(f) Plot the respective Recursive Residuals and interpret the plot.
recursive_GDP <- recresid(fit_GDP$res ~ 1)
quartz("Recursive Residuals_GDP")
plot(recursive_GDP, pch = 16, ylab = "Recursive Residuals", main = "Recursive Residuals(GDP)")

recursive_SP500 <- recresid(fit_SP500_1$res ~ 1)
quartz("Recursive Residuals_GDP")
plot(recursive_SP500, pch = 16, ylab = "Recursive Residuals", main = "Recursive Residuals(SP500)")

#(g) For your model, discuss the associated diagnostic statistics.
summary(fit_GDP)
summary(fit_SP500_1)
```

```


#(h) Use your model to forecast 12-steps ahead. Your forecast should include the respective error bands.
quartz()
plot(forecast(fit_GDP, h = 12), main = "GDP Forecast with ARIMA(2,1,0)", xlab = "Date(in year)", ylab = "GDP(in billion dollar)")
quartz()
plot(forecast(fit_SP500_1, h = 12), main = "SP500 Forecast with ARIMA(1,1,0)", xlab = "Date(in year)", ylab = "SP500(in dollar)")#...this ARIMA (1,1,0) behaves weird

#(i) Fit an appropriate VAR model using your two variables. Make sure to show the relevant plots and discuss your results from the fit.
# z-score
GDP_z <- (data$GDP - mean(data$GDP)) / sd(data$GDP)
SP500_z <- (data$SP500 - mean(data$SP500)) / sd(data$SP500)
data_z<- data_frame(GDP_z,SP500_z)
data_z <- ts(data_z, 1990, 2019, frequency = 4) # convert to time series

VARselect(data_z) # order = 3
var_model <-VAR(data_z, p = 3)
summary(var_model)

# construct x-axis for this data set
x_ts <- seq(1990, 2019, length = length(var_model$varresult$GDP_z$fitted.values))
# plot VAR fitted values
quartz()
par(mfcol = c(2, 1))
plot(x_ts, var_model$varresult$GDP_z$fitted.values, type = 'l', lwd = 2, main = "VAR fitted of zscore(GDP)", xlab = "year", ylab = "zscore value")
lines(x_ts, data_z[4:117,1], col = "blue", lwd = 1.5)
legend("topleft",pch= c("-","-"),legend=c("Original Data", "Var Fitted"),col=c("black", "blue"),  cex=0.8)
plot(x_ts, var_model$varresult$SP500_z$fitted.values, type = 'l', lwd = 2, main = "VAR fitted of zscore(SP500)", xlab = "year", ylab = "zscore value")
lines(x_ts, data_z[4:117,2], col = "blue", lwd = 1.5)
legend("topleft",pch= c("-","-"),legend=c("Original Data", "Var Fitted"),col=c("black", "blue"),  cex=0.8)
# plot VAR fitted values(after transfering zscore to true value)
quartz()
par(mfcol = c(2, 1))
plot(x_ts, var_model$varresult$GDP_z$fitted.values * sd(ts_GDP) + mean(ts_GDP), type = 'l', lwd = 2, main = "VAR fitted of true value(GDP)", xlab = "year", ylab = "billion dollar")
lines(x_ts, data[4:117,2], col = "blue", lwd = 1.5)
legend("topleft",pch= c("-","-"),legend=c("Original Data", "Var Fitted"),col=c("black", "blue"),  cex=0.8)
plot(x_ts, var_model$varresult$SP500_z$fitted.values* sd(ts_SP500) + mean(ts_SP500), type = 'l', lwd = 2, main = "VAR fitted of true value(SP500)", xlab = "year", ylab = "dollar")
lines(x_ts, data[4:117,3], col = "blue", lwd = 1.5)
legend("topleft",pch= c("-","-"),legend=c("Original Data", "Var Fitted"),col=c("black", "blue"),  cex=0.8)

# Look at ACF and PACf
quartz()
tsdisplay(residuals(var_model)[,1], main = "GDP = GDP(t-k) + SP500(t-k)")
quartz()
tsdisplay(residuals(var_model)[,2], main ="SP500 = GDP(t-k) + SP500(t-k)")

#(j) Compute, plot, and interpret the respective impulse response functions.
irf(var_model)
quartz()
plot(irf(var_model, n.ahead = 24))#this n.ahead need to be modified then


#(k) Perform a Granger-Causality test on your variables and discuss your results from the test.
grangertest(SP500_z,GDP_z, order = 3) # not significant
grangertest(GDP_z,SP500_z, order = 3) # significant
#Granger test kind of accept that change in GDP is a cause of change in SP500

#(l) Use your VAR model to forecast 12-steps ahead. Your forecast should include the respective error bands.
#Comment on the differences between the two forecasts (VAR vs. ARIMA).
var.predict <- predict(object = var_model, n.ahead = 24)

# construct a dataframe that contains fitted value + predicted value
var_fullgdp <- c(var_model$varresult$GDP_z$fitted.values, var.predict$fcst$GDP_z[,1])
var_fullsp500 <- c(var_model$varresult$SP500_z$fitted.values, var.predict$fcst$SP500_z[,1])
var_fullgdp <- var_fullgdp * sd(ts_GDP) + mean(ts_GDP)
var_fullsp500 <- var_fullsp500 * sd(ts_SP500) + mean(ts_SP500)
var_full <- data.frame(var_fullgdp, var_fullsp500)

# forecast zscore
quartz()
forecast(var_model) %>%
  autoplot() +
  xlab("year") +
  ggtitle("z-score forecast by VAR")
#at least it behaves better in SP500

# plot var model fitted value + prediction compared to real data
xfull_ts <- seq(1990, 2025, length = length(var_fullgdp))
xfit_ts <- seq(1990, 2019, length = length(data$GDP))
xpre_ts <- seq(2019, 2025, length = 24)

quartz()
par(mfcol = c(2, 1))
plot(xfull_ts, var_fullgdp, type = 'l', lwd = 2, main = "VAR fitted of true value(GDP)", xlab = "year", ylab = "billion dollar")
lines(xfit_ts, data[1:117,2], col = "blue", lwd = 1.5)
lines(xfull_ts[115:138], var.predict$fcst$GDP_z[,2] * sd(ts_GDP) + mean(ts_GDP), col = "red", lty = 2)
lines(xfull_ts[115:138], var.predict$fcst$GDP_z[,3] * sd(ts_GDP) + mean(ts_GDP), col = "red", lty = 2)
legend("topleft",pch= c("-","-","-"),legend=c("Original Data", "Var Fitted", "Error Bound"),col=c("black", "blue", "Red"),  cex=0.8)

plot(xfull_ts, var_fullsp500, type = 'l', lwd = 2, main = "VAR fitted of true value(SP500)", xlab = "year", ylab = "dollar")
lines(xfit_ts, data[1:117,3], col = "blue", lwd = 1.5)
lines(xfull_ts[115:138], var.predict$fcst$SP500_z[,2] * sd(ts_SP500) + mean(ts_SP500), col = "red", lty = 2)
lines(xfull_ts[115:138], var.predict$fcst$SP500_z [,3] * sd(ts_SP500) + mean(ts_SP500), col = "red", lty = 2)
legend("topleft",pch= c("-","-","-"),legend=c("Original Data", "Var Fitted", "Error Bound"),col=c("black", "blue", "Red"),  cex=0.8)

## forecast with ARIMA
quartz()
plot(forecast(fit_GDP,h=24),shadecols="oldstyle")
quartz()
plot(forecast(fit_SP500_1,h=24),shadecols="oldstyle")

## forecast with ETS
quartz()
```

```r
ts_GDP %>% forecast(h = 24) %>%
  autoplot() +
  xlab("year") +
  ylab("GDP(in billion dollar)")+
  ggtitle('Forecast GDP with ETS') +
  theme(text = element_text(family = "STHeiti"))+
  theme(plot.title = element_text(hjust = 0.5))

quartz()
ts_SP500 %>% forecast(h = 24) %>%
  autoplot() +
  xlab("year") +
  ylab("SP500 (in dollar)")+
  ggtitle('Forecast SP500 with ETS') +
  theme(text = element_text(family = "STHeiti"))+
  theme(plot.title = element_text(hjust = 0.5))




##*************************************************************
#not quite good tries:)
##test for the relationship btw log-return and growthrate(grangertest failed)

# try logreturn
#SP500_logreturn<-diff(log(data$SP500))
#GDP_logreturn<-diff(log(data$GDP))
#grangertest(SP500_logreturn, GDP_logreturn, order=3) #fail

# try growth rate
#growth_GDP<-(data$GDP - Lag(data$GDP,1))/Lag(data$GDP,1)*100
#growth_SP500<-(data$SP500 - Lag(data$SP500,1))/Lag(data$SP500,1)*100

#grangertest(growth_GDP,growth_SP500,order=1)  #fail
#grangertest(growth_SP500,growth_GDP,order=4)  #fail

# omit NAs for test correlation
#growth_GDP_nonan<-na.omit(growth_GDP)
#growth_SP500_nonan<-na.omit(growth_SP500)
#cor(growth_GDP_nonan,growth_SP500_nonan) # 0.4372168, not very correlated
```