Necessary and random minutiae in fingerprints

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Outline

Fingerprints and minutiae

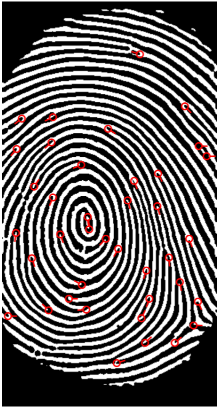
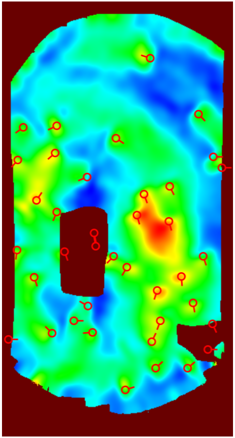


Figure: Fingerprint from Maio et al. (2002)

- **minutia** = discontinuity in the ridge line pattern
- ridge endings / bifurcations
- further discrimination in forensics
- minutiae $z_1, \ldots, z_k \in \mathfrak{X} \subseteq \mathbb{R}^2$
- region of interest $\mathfrak{X}$ (ROI)

Terminology

(a) Orientation field



(b) Ridge frequency

Synthetic fingerprints

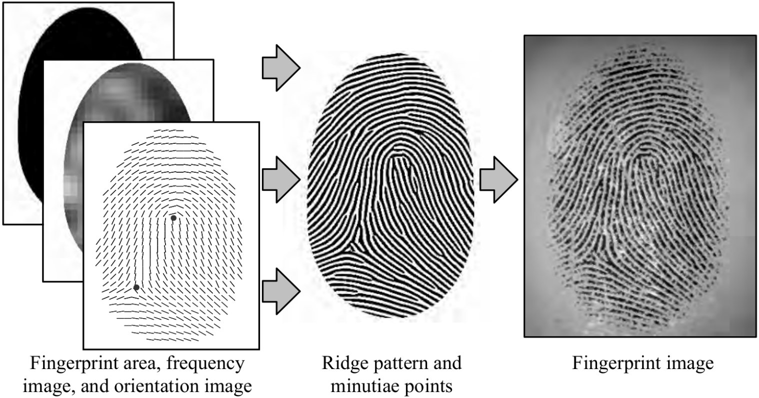


Figure: Synthetic fingerprint generation according to SFinGe (Cappelli et al. (2004)), image taken from Maltoni et al. (2009).

Synthetic fingerprints

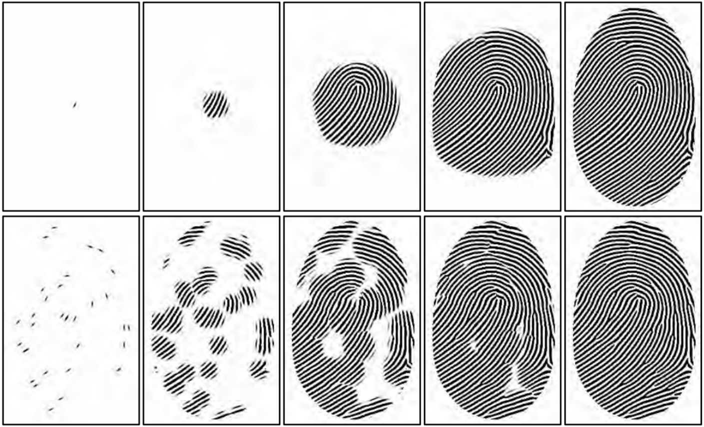
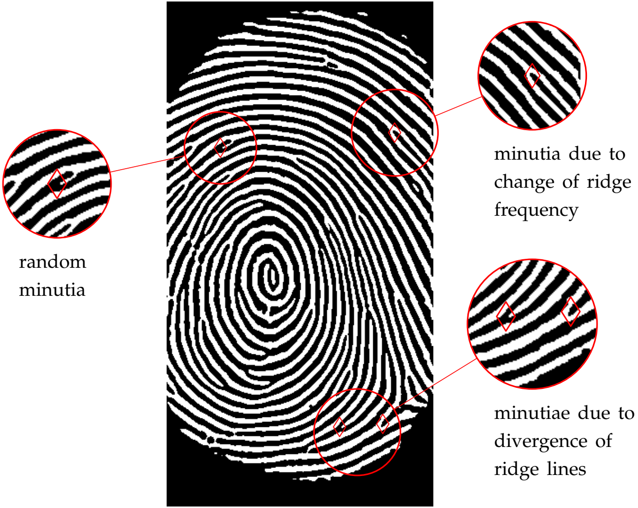


Figure: Synthetic fingerprint generation according to SFinGe (Cappelli et al. (2004)), image taken from Maltoni et al. (2009).

Minutiae within fingerprints



Outline

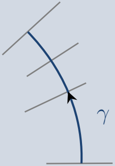
Fingerprint terminology for vector fields

Definition

For a fingerprint denote by

- $\mathfrak{X} \subseteq \mathbb{R}^2$ the **region of interest** (ROI) compact with piecewise smooth boundary,
- $\vec{F} : \mathfrak{X} \to \mathbb{S}^1$ (apart from singularities) its **orientation field**,
- $\Phi : \mathfrak{X} \to (0, \infty)$ local **ridge frequency**, i.e. for a curve $\gamma \perp \vec{F}$ holds

$$\left\lfloor \left| \int_\gamma \Phi(z)\,\mathrm{d}z \right| \right\rfloor = \text{ no. of ridges crossing } \gamma.$$



Minutiae for vector fields

Definition

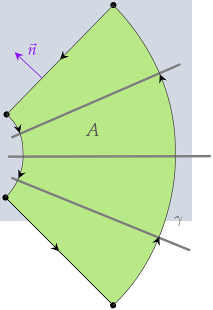
For the above considered fingerprint and

- $A \subseteq \mathfrak{X}$ with piecewise smooth boundary and without singularities,
- $\vec{n}$ outer unit normal vector on ∂A,

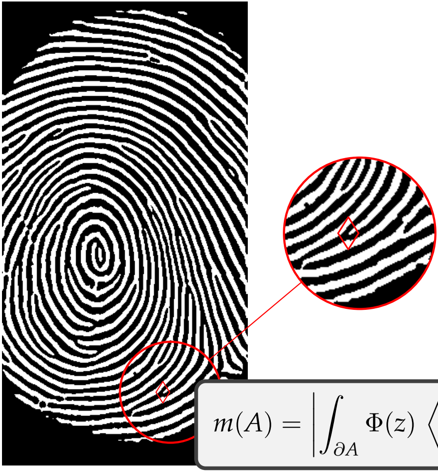
define

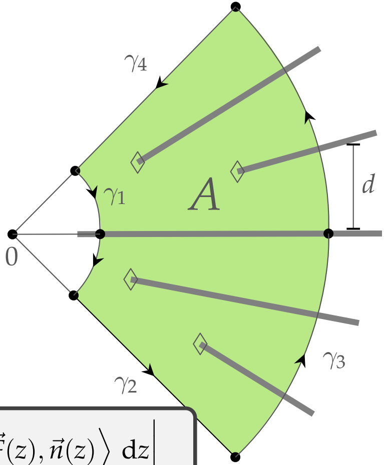
$$m(A) := \left| \int_{\partial A} \Phi(z) \left\langle \vec{F}(z), \vec{n}(z) \right\rangle \mathrm{d}z \right|$$

as the (theoretical) **number of minutiae necessary** in A.



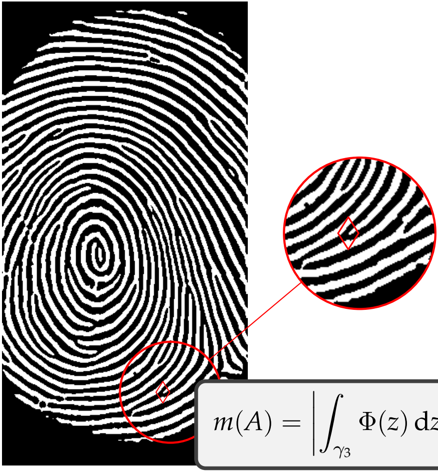
Example 1 – divergence of ridge lines

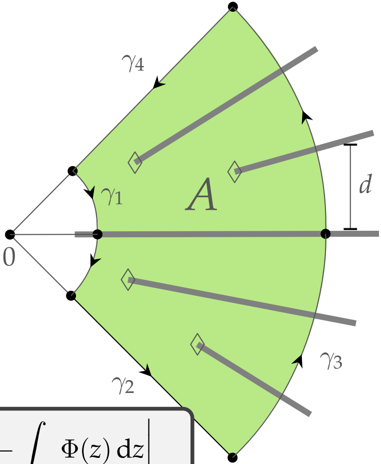




$$m(A) = \left| \int_{\partial A} \Phi(z) \left\langle \vec{F}(z), \vec{n}(z) \right\rangle \mathrm{d}z \right|$$

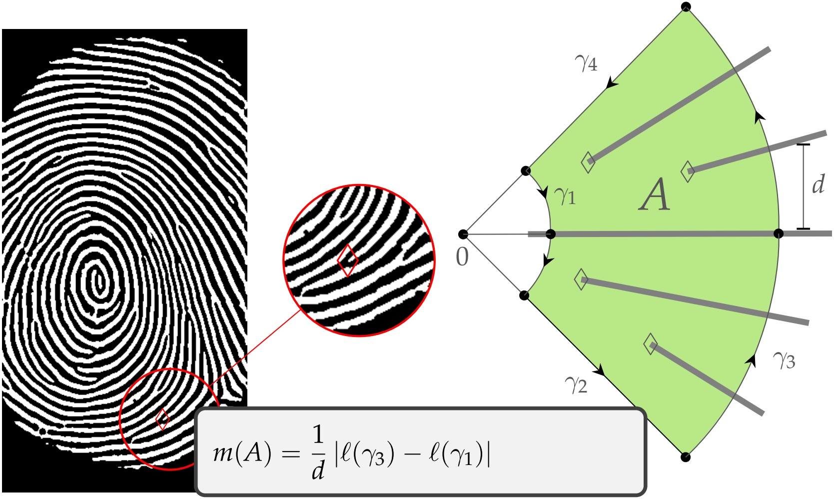
Example 1 – divergence of ridge lines





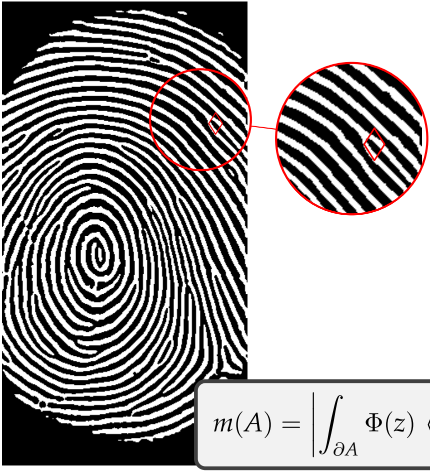
$$m(A) = \left| \int_{\gamma_3} \Phi(z)\,\mathrm{d}z - \int_{\gamma_1} \Phi(z)\,\mathrm{d}z \right|$$

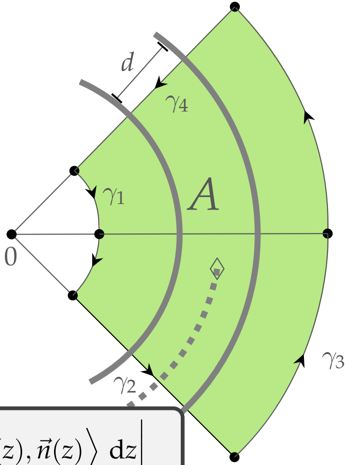
Example 1 – divergence of ridge lines



$$m(A) = \frac{1}{d}\,|\ell(\gamma_3) - \ell(\gamma_1)|$$

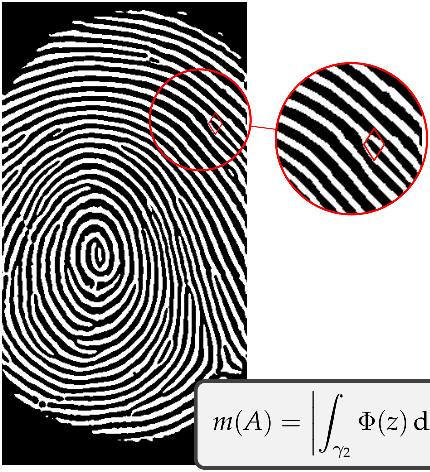
Example 2 – change of ridge frequency

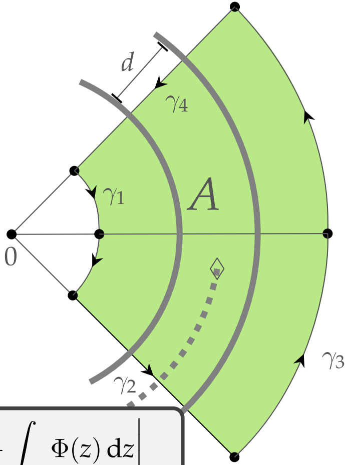




$$m(A) = \left| \int_{\partial A} \Phi(z) \left\langle \vec{F}(z), \vec{n}(z) \right\rangle \mathrm{d}z \right|$$

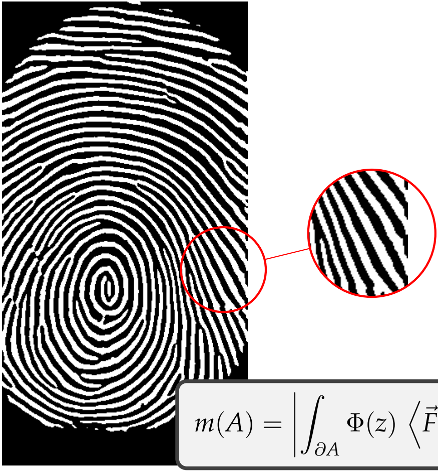
Example 2 – change of ridge frequency

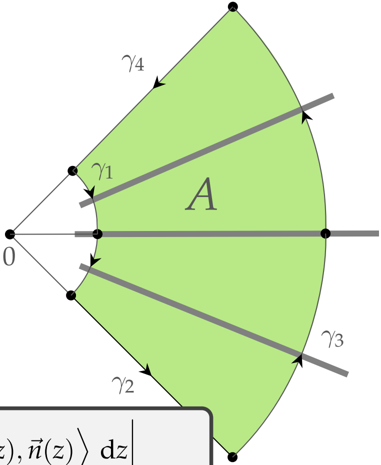




$$m(A) = \left| \int_{\gamma_2} \Phi(z)\,\mathrm{d}z - \int_{\gamma_4} \Phi(z)\,\mathrm{d}z \right|$$

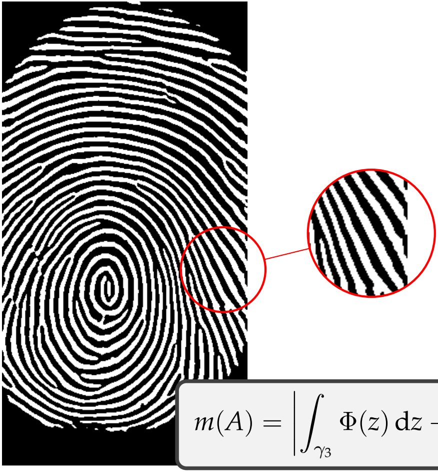
Example 3 – field divergence vs. ridge divergence

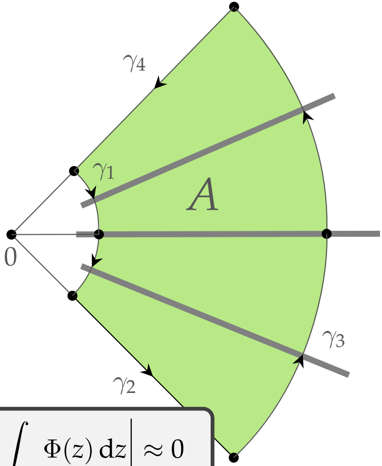




$$m(A) = \left| \int_{\partial A} \Phi(z) \left\langle \vec{F}(z), \vec{n}(z) \right\rangle \mathrm{d}z \right|$$

Example 3 – field divergence vs. ridge divergence





$$m(A) = \left| \int_{\gamma_3} \Phi(z)\,\mathrm{d}z - \int_{\gamma_1} \Phi(z)\,\mathrm{d}z \right| \approx 0$$

Minutiae divergence formula

Theorem

Consider a fingerprint with

- $A \subseteq \mathfrak{X}$ *without singularities, piecewise smooth boundary,*
- *orientation field* $\vec{F} : A \to \mathbb{S}^1$ *smooth,*
- *local ridge frequency* $\Phi : A \to (0, \infty)$ *smooth.*

Then,

$$m(A) = \left| \underbrace{\int_A \Phi(z) \operatorname{div} \vec{F}(z) \,\mathrm{d}z}_{\text{field divergence}} + \underbrace{\int_A \left\langle \nabla\Phi(z), \vec{F}(z) \right\rangle \,\mathrm{d}z}_{\text{ridge divergence}} \right|.$$

Minutiae divergence formula

Theorem

Consider a fingerprint with

- $A \subseteq \mathfrak{X}$ *without singularities, piecewise smooth boundary, star-shaped w.r.t.* z_0*,*
- *orientation field* $\vec{F} : A \to \mathbb{S}^1$ *smooth,*
- *local ridge frequency* $\Phi : A \to (0, \infty)$ *smooth.*

Then,

$$m(A) = \left| \underbrace{\Phi(z_0) \operatorname{div} \vec{F}(z_0)}_{\text{field divergence}} + \underbrace{\left\langle \nabla\Phi(z_0), \vec{F}(z_0) \right\rangle}_{\text{ridge divergence}} \right| \cdot |A| + o(|A|) \ (\operatorname{diam}(A) \to 0).$$

Theory vs. practice

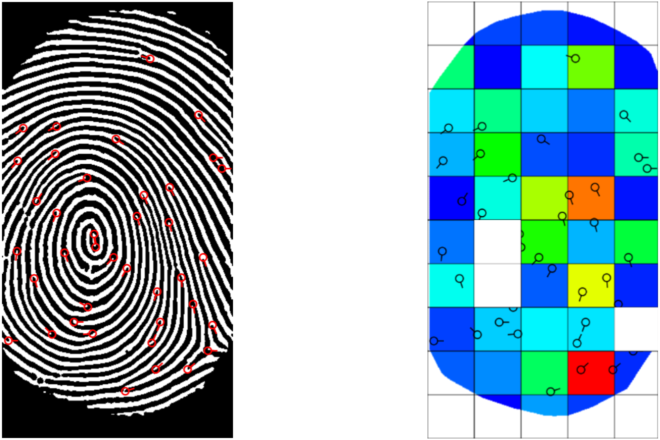


Figure: Fingerprint (left) and computed numbers $m(A)$ for a partition into 50 patches (red $\approx 2-3$, yellow $\approx 1-2$, green ≈ 1, blue ≈ 0)

But wait, there is more ...

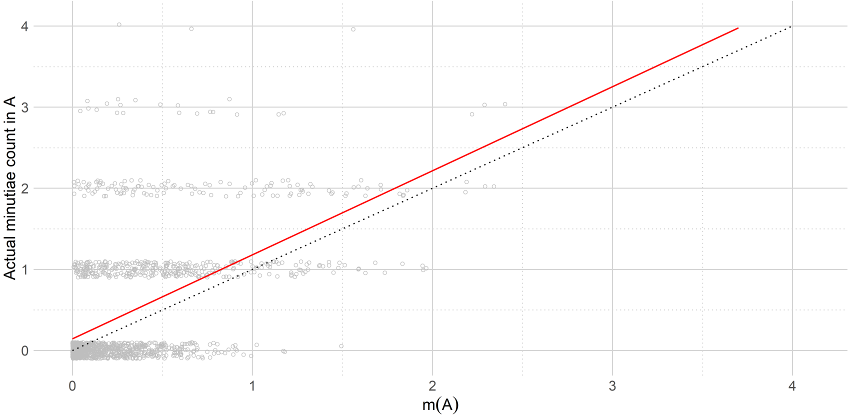
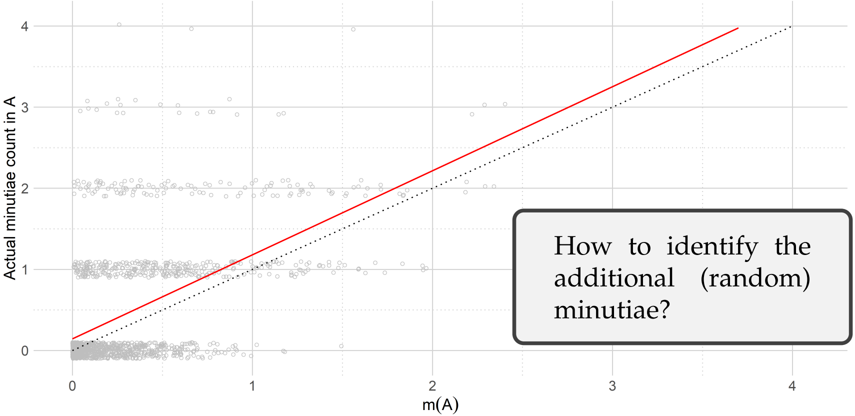


Figure: $m(A)$ vs. actual minutiae count in patches A from 20 fingerprints of (Maio et al., 2002, DB 1), Poisson regression line (red)

But wait, there is more . . .



How to identify the additional (random) minutiae?

Figure: $m(A)$ vs. actual minutiae count in patches A from 20 fingerprints of (Maio et al., 2002, DB 1), Poisson regression line (red)

Outline

Separating minutiae

Consider minutiae pattern $\zeta = \{z_1, z_2, \dots, z_k\}$. Assume

- $\zeta = \eta \,\dot{\cup}\, \xi$ is sample from $Z = \mathrm{H} \,\dot{\cup}\, \Xi$ where
- necessary minutiae H – contain fingerprint information, i.e. depends on

$$\mu(z) := \left| \Phi(z) \operatorname{div} \vec{F}(z) + \left\langle \nabla\Phi(z), \vec{F}(z) \right\rangle \right|,$$

  and incorporates discrete structure of the finger/ image, etc.
- random minutiae Ξ – no further information (completely random),
- latent variable $W = (w_1, w_2, \dots, w_k) \in \{0,1\}^k$ where $w_i = \mathbf{1}\{z_i \in \eta\}$.

A model for minutiae

- dependence on μ as above
- minutiae can only occur on ridges

  $\rightsquigarrow$ distance at least one interridge distance (hard core distance h)
- beyond that, minutiae tend to locally repel each other on $r > h$ (Chen and Moon (2006))

H $\sim$ Strauss-hard core

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$\Bigg\}$ H $\sim$ Strauss-hard core

$$f_{\mathrm{H}}(\eta) = \alpha \left(\prod_{z_i \in \eta} \beta \mu(z_i) \right) \gamma^{s_r(\eta)} \mathbf{1}\left(d_{\min}(\eta) > h\right),$$

$$s_r(\eta) := \sum_{z_i \neq z_j \in \eta} \mathbf{1}\left(\|z_i - z_j\| \leq r \right), \qquad d_{\min}(\eta) := \min_{z_i \neq z_j \in \eta} \|z_i - z_j\|$$

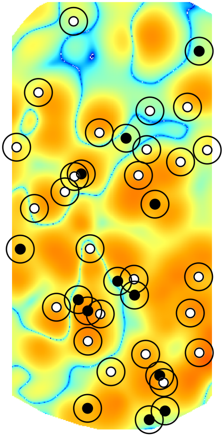
A model for minutiae

- random minutiae: no further assumptions on the process
  $\rightsquigarrow \Xi \sim \text{Poisson}(\lambda)$

$$g_\Xi(\xi) = \mathrm{e}^{(1-\lambda)|\mathfrak{X}|} \left(\prod_{z_i \in \xi} \lambda \right)$$

- if labels W are known, we have

$$h_Z(\zeta) = f_H(\eta) \cdot g_\Xi(\xi)$$



Possible approaches

- Maximum-Likelihood:

$$\sup_{\substack{\beta,\gamma,\lambda \\ \eta\subseteq\zeta}} \left(\alpha \left(\prod_{z_i\in\eta} \beta\mu(z_i)\right) \gamma^{s_r(\eta)} \mathbf{1}\left(d_{\min}(\eta) > h\right)\right) \left(\mathrm{e}^{(1-\lambda)|\mathfrak{X}|} \lambda^{k-|\eta|}\right)$$

$\rightsquigarrow$ high dimensional mixed-integer program due to unknown labels W,

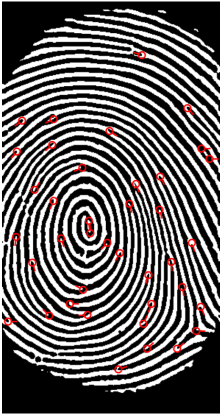
$\rightsquigarrow$ unique solution reasonable?

- Bayesian approach using MCMC:

$\rightsquigarrow$ intractable normalising constant $\alpha = \alpha(\beta, \gamma)$ (sampling is expensive),

$\rightsquigarrow$ implementation of a **mi**nutiae **se**parating **al**gorithm (MiSeal)

MiSeal on simulated data



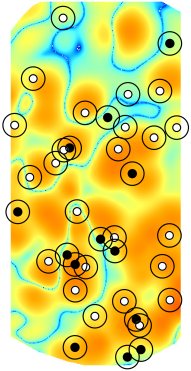


Figure: Fingerprint (left) and simulated point pattern (right) from its divergence field (background, log-scale).

MiSeal on simulated data

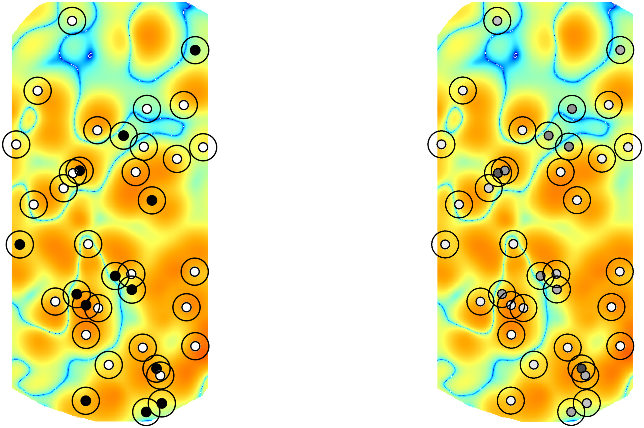


Figure: True labels (left) and posterior probabilities of being necessary (right)

MiSeal on real data

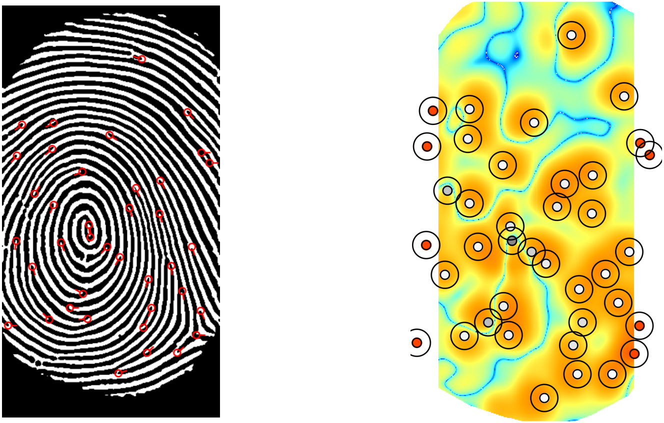


Figure: Minutiae pattern (l.) and posterior probabilities of being necessary (r.)

Application – characteristic minutiae

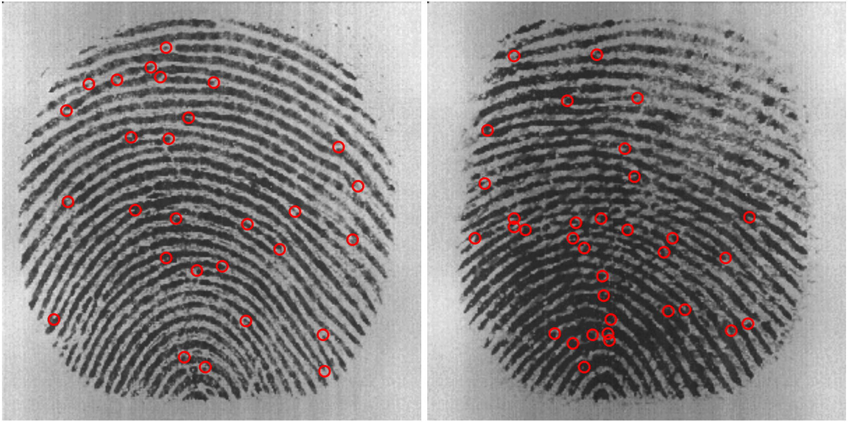


Figure: Two similar fingerprints from yet different persons.

Application – characteristic minutiae

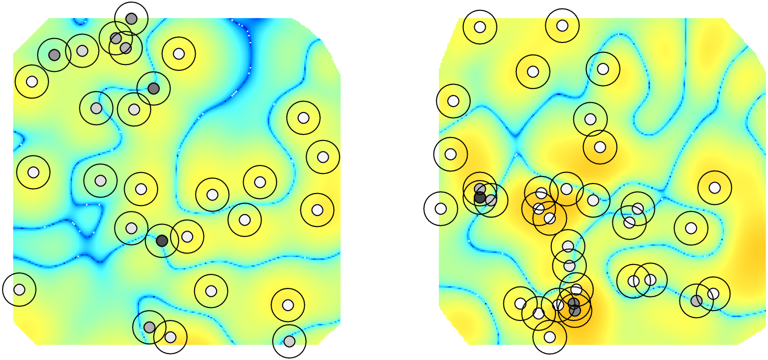
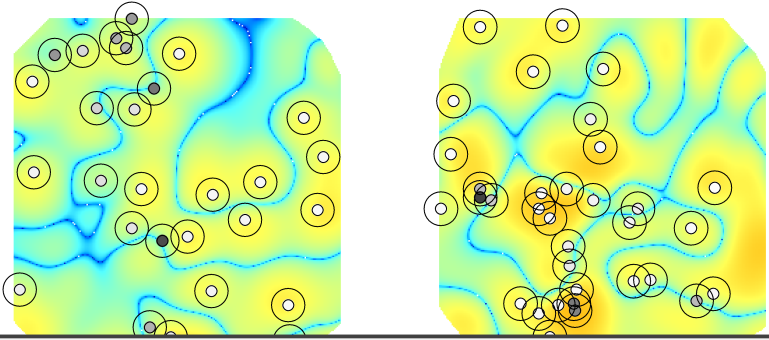


Figure: Corresponding marginal posterior probabilities (greyscale) and intensity (background).

Application – characteristic minutiae



by deletion of characteristic minutiae, different fingers become more similar in comparison to randomly deleting the same number

Outline

Conclusion and discussion

Summary

- model for a quantitative forecast of numbers of minutiae
- MCMC algorithm (MiSeal) that allows for separating an inhomogeneous and a homogeneous process
- posterior probabilities give rise to characterictiy

  $\rightsquigarrow$ by deletion of these minutiae fingerprints of different fingers become more simular in comparison to randomly deleting the same number

Outlook

- improvement of matching algorithms
- generation of synthetic fingerprints

References I

Cappelli, R., Maio, D., and Maltoni, D. (2004). SFinGe: an approach to synthetic fingerprint generation. In *International Workshop on Biometric Technologies (BT2004)*, pages 147–154.

Chen, J. and Moon, Y.-S. (2006). A statistical study on the fingerprint minutiae distribution. In *2006 IEEE International Conference on Acoustics Speech and Signal Processing Proceedings*, volume 2, pages II–II. IEEE.

Maio, D., Maltoni, D., Cappelli, R., Wayman, J. L., and Jain, A. K. (2002). FVC2002: Second fingerprint verification competition. In *Object recognition supported by user interaction for service robots*, volume 3, pages 811–814. IEEE.

Maltoni, D., Maio, D., Jain, A. K., and Prabhakar, S. (2009). *Handbook of fingerprint recognition*. Springer Science & Business Media.

MiSeal on simulated data

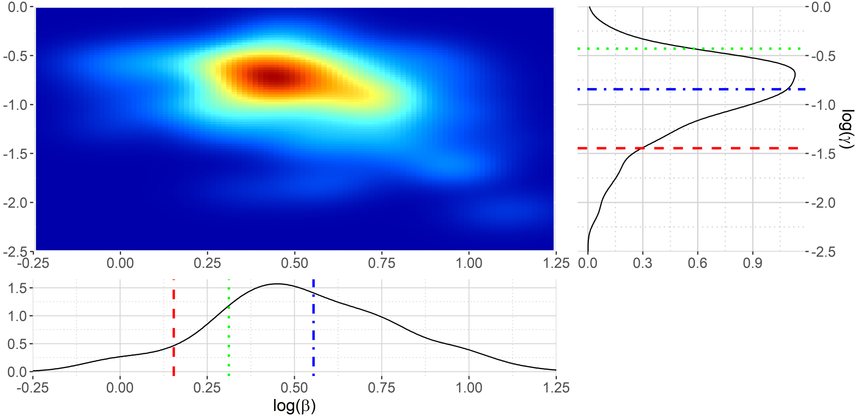


Figure: Posterior distribution of (β, γ) and its marginals; true value (dashed red), PMLE (dotted green) and posterior mean (dotted-dashed blue).

MiSeal on real data

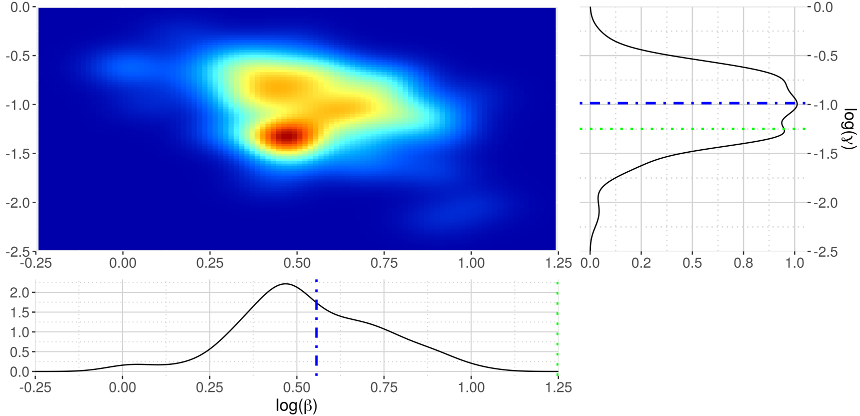


Figure: Posterior distribution of (β, γ) and its marginals; PMLE (dotted green) and posterior mean (dotted-dashed blue).