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# A Hybrid Genetic Algorithm and Sperm Swarm Optimization (HGASSO) for Multimodal Functions

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## ABSTRACT

In this paper, the authors propose a hybrid algorithm combining two different metaheuristic methods, genetic algorithms (GA) and sperm swarm optimization (SSO), for the global optimization of multimodal benchmarks functions. The proposed hybrid genetic algorithm and sperm swarm optimization (HGASSO) operates based on incorporates concepts from GA and SSO which generates individuals in a new iteration not only by crossover and mutation operations as proposed in GA but also by techniques of local search of SSO. The main idea behind this hybridization is to reduce the probability of trapping in local optimum of multi-modal problem. The algorithm is compared against GA and SSO metaheuristic optimization algorithms. The experimental results using a suite of multimodal benchmarks functions taken from the literature have evinced the superiority of the proposed HGASSO approach over the other approaches in terms of quality of results and convergence rates which obtained good results in solving the multimodal benchmarks functions including cosine, sine, and exponent in their formulation.

## KEYWORDS

Evolutionary Method, Genetic Algorithms (GA), Hybrid Algorithm, Multimodal Benchmarks Functions, Sperm Swarm Optimization (SSO), Swarm Intelligence Algorithms

## INTRODUCTION

During the last decades, there has been a huge growing in the area of evolutionary computing, which has indicated modern techniques for solving various types of optimization problems. In contrast to classical or traditional optimization approaches, which emphasize exact and accurate computation, but these approaches may fall down in obtaining the solution of a global optimum. Based on that, evolutionary computation emerged to provide more efficient and robust technique for solving complex problems (Shehadeh et al. 2018a; Fogel, 2005). “Genetic algorithm (GA)” is one of the most prevalent branches between the existing evolutionary approaches. GA is considered as an inherently discrete stochastic search approach depends on the techniques of generating genetics, natural selection, and

evolution (Shehadeh et al., 2018a; Paulinas and Ušinskas, 2007). Since this type of metaheuristics simultaneously can search and evaluate many sites in the domain of problem, which is more likely to discover the global optimum of an optimization problem. Furthermore, it is easy to implement and use, which uses a measure of performance of simple scalar that does not use or require derivative information (Shehadeh et al., 2018a).

More recently, Hisham A. Shehadeh et al. (Shehadeh et al., 2017; Shehadeh et al. 2018b, Shehadeh et al. 2018c) proposed a new metaheuristic method, namely “Sperm Swarm Optimization” (denoted as SSO). The idea of this approach is inspired by the behaviors of sperm swarm through the procedure of natural fertilization. The theory of SSO represents a solution process, which each sperm swims through the domain of multidimensional search space while the sperm’s position and velocity are constantly updated based on the previous position of the sperm, as well as the best performance of the swarm in the entire population.

In contrast with GA, SSO is an inherently continuous approach in which has various attractive features. It has memory, so the prior knowledge can be retained by all the sperms in each generation; whereas in GA, the scenario is different, which is considered as an inherently discrete approach, so the prior knowledge of the problem is discarded each iteration by reserving the best individuals and eliminating the worst individuals at each generation. To date, SSO has been successfully applied to generate the optimal solution for different continuous nonlinear functions in practice (Shehadeh et al., 2018c), but until recently it had not been hybridized to deal with multimodal problems. SSO seems particularly appropriate for multimodal tasks mainly because of the good quality of solutions and the high speed of convergence that the algorithm shows for solving different kinds of single-objective problems (Shehadeh et al., 2018c).

There are many studies that have been investigated the hybridization of evolutionary algorithms with local search. We can summarize them as follows:

Soleimani et al. proposed a hybrid approach that integrates “Particle Swarm Optimization (PSO)” with GA. The proposed approach was used to optimize problems of supply chain network. The results showed that the proposed approach has better convergence and quality of solution than GA (Soleimani & Kannan, 2015).

On the other hand, Samuel et al. suggested new optimization approach that merges the functionality of PSO and GA. The proposed approach was used to solve the scheduling problem of power generator. The results proved the efficiency of the proposed approach in finding solution for the aforementioned problem (Samuel & Rajan, 2015).

In different view, Fang et al. proposed a new hybrid approach that integrates “Artificial Fish Swarm Algorithm (AFSA)” with GA. This approach was used to solve scheduling problem of the hydrothermal systems. To prove the performance of the proposed approach, Fang et al. tested this approach on two hydrothermal systems (Fang et al., 2014).

In a different work, Kao et al. proposed a hybrid approach that integrates PSO with GA. The proposed approach was applied to solve 17 multi-modal functions. The results showed that the proposed approach has better convergence and quality of solution than other approaches (Kao & Zahara, 2008).

Gholami et al. discussed a hybrid approach that merges the functionality of PSO with GA. They used this approach to optimize bank shape problem. The result showed the performance of the approach under different scenarios (Gholami et al., 2018).

On the other hand, Gacem et al. proposed a hybrid approach that integrates the functionality of GA and PSO. The proposed approach is applied to optimize the power flow problem. The results showed that the proposed approach has better convergence and quality of solution than GA and PSO (Gacem & Benattous, 2017).

In a later study, Chaudhary et al. proposed a hybrid approach that combines the functionality of GA and “Gravitational Search Algorithm (GSA)”. They used this approach to optimize the load scheduling problem in cloud computing (Chaudhary & Kumar, 2019).

As mentioned in the aforementioned studies, the hybridization is always referred to the concept of memetic algorithm in which the procedures of two or more algorithms are merged as one procedure to solve a problem. Taking feature of the compensatory property of SSO and GA, in this paper, we propose a new method that merges evolutionary natures of SSO and GA. The proposed “Hybrid Genetic Algorithm and Sperm Swarm Optimization (HGASSO)” will be evaluated against a set of multimodal bed problems gathered from (Kao & Zahara, 2008; Surjanovic & Bingham, 2013) and the results are compared extensively with the original GA and SSO.

This paper is organized as follows: background on “Genetic Algorithm (GA)” and “Sperm Swarm Optimization (SSO)” is summarized in Sect. 2. Section 3 shows “Hybrid Genetic Algorithm and Sperm Swarm Optimization (HGASSO)”. Section 4 summarizes experimental results. Sect. 5 summarizes the discussion. This work is concluded in Sect. 6.

## **BACKGROUND ON GENETIC ALGORITHMS (GA) AND SPERM SWARM OPTIMIZATION (SSO)**

This section constructs the need for the study and gives a general background related to “Genetic Algorithm (GA)” and “Sperm Swarm Optimization (SSO)”. In addition, this section presents view in-depth at pseudocodes of these optimization algorithms and their concepts.

### **GENETIC ALGORITHMS (GA)**

GA is proposed by Holland in 1993, which refers to Biology Inspired Search (BIS) method that is inspired by Darwinian’s theory of evolution (Goldberg & Holland, 2002). GA has been successfully applied to solve different complex and nonlinear problems and proved its eligibilities over many classical optimization approaches, specifically when the problem under study has solutions of many local optimums. Each solution in GA is commonly formalized as a binary string, namely a chromosome. Usually, there is a fitness function has been used to evaluate this chromosome after the chromosome has been formalized. Upon accomplishment of the estimation, an operation called roulette wheel selection is utilized to choose pairs of better chromosomes randomly from the population to utilize such genetic operations as mutation as well as crossover that mimic natural procedure. The weak chromosome will be discarded from the population each time run. This evaluation procedure continues until the end criterion is achieved. The GA pseudocode is summarized in Algorithm 1 (Langdon & Poli, 2002).

**Algorithm 1** *Genetic Algorithm (GA)*

Begin

**Step 1:** Set the problem based on a genetic representation.

**Step 2:** Initialize population  $p(0) = x_1^0, \dots, x_N^0$ .

**Step 3:** Calculate the average fitness  $\bar{F} = \sum_i^N F(z_i) / N$  and give a normalized fitness value for each individual  $F(z_i^t) / N$ .

**Step 4:** Give every  $z_i$  a probability  $p(z_j, t)$  based on its fitness.

Based on that, select  $N$  vectors from  $P(t)$ . This provides the set  $S(t)$ .

**Step 5:** Pair values in  $S(t)$  at random forming  $N/2$  pairs. Perform crossover with probability  $p_{cross}$  to each pair. Perform mutation to form a new population  $P(t+1)$ .

**Step 6:** Set  $t = t + 1$ , go to step 3.

End.

## SPERM SWARM OPTIMIZATION (SSO)

“Sperm Swarm Optimization (SSO)” is one of the latest optimization approaches developed by Hisham Shehadeh et al. (Shehadeh et al., 2017; Shehadeh et al. 2018b, Shehadeh et al. 2018c). SSO concept is depends on a metaphor of the procedure of natural fertilization of sperm swarm. Sperms represent as candidate solutions in the SSO approach, which swim around in the multidimensional search space domain. The position of each sperm is adjusted based on the previous position and the global best solution of the swarm. The global best solution represents the closest sperm to the egg, which is called the winner. The winner is depicted in Figure 1. Sperm movement affects by many limitations of reproductive system of female. These limitations can be summarized in the following points (Shehadeh et al., 2018a; Shehadeh et al., 2017; Shehadeh et al. 2018b, Shehadeh et al. 2018c):

- pH value, which is a random number between 7–14.
- Temperature value, which is a random number between 35.1–38.5.

The sperm adjust its velocity based on three rules. These rules can be summarized as follows:

- Initial velocity of the sperm, which is affected by pH value of the area.
- Personal best solution of the sperm, which is affected by pH and temperature values of the reached location. This velocity will be adjusted in the memory just if the current location is better than the old one.
- Global best solution, which is affected by pH and temperature values of the visited location. This velocity is recorded by the closest sperm to the egg, which is called the winner.

The diagram illustrates the formula for calculating sperm velocity  $V_i(t)$ . It shows three components: 'Sperm initial velocity' (a small peak), 'Personal best solution' (a medium peak), and 'Global best solution' (a large peak). Brackets below the peaks group them into terms: the first term is  $D \cdot \text{Log}_{10}(pH\_Rand_1) \cdot V_i + \text{Log}_{10}(pH\_Rand_2) \cdot \text{Log}_{10}(\text{Temp\_Rand}_1)$ ; the second term is  $\cdot (x_{sbest_i} - x_i(t)) + \text{Log}_{10}(pH\_Rand_3) \cdot \text{Log}_{10}(\text{Temp\_Rand}_2) \cdot (x_{sgbest} - x_i(t))$ ; and the third term is the 'Global best solution' itself.

$$V_i(t) = D \cdot \text{Log}_{10}(pH\_Rand_1) \cdot V_i + \text{Log}_{10}(pH\_Rand_2) \cdot \text{Log}_{10}(\text{Temp\_Rand}_1) \\ \cdot (x_{sbest_i} - x_i(t)) + \text{Log}_{10}(pH\_Rand_3) \cdot \text{Log}_{10}(\text{Temp\_Rand}_2) \cdot (x_{sgbest} - x_i(t))$$

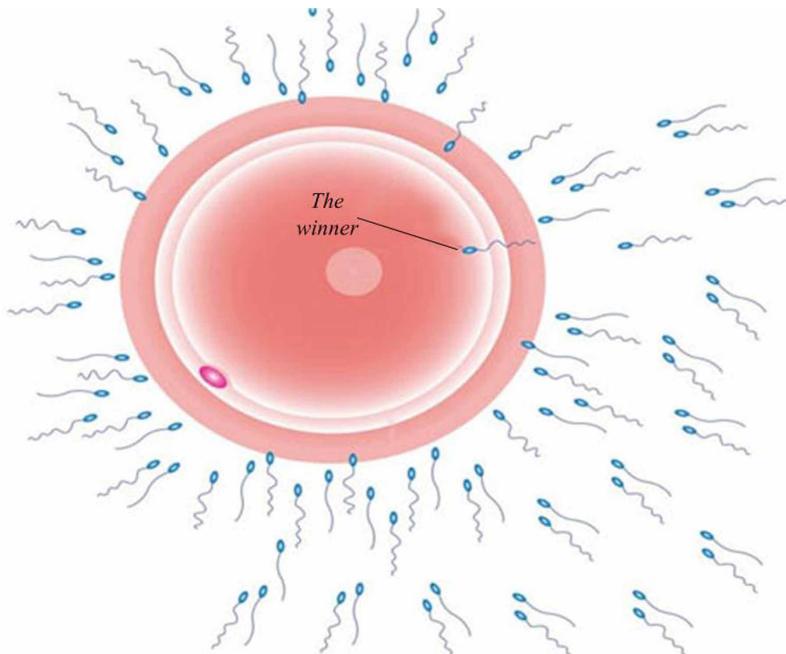
(1)

where,

- $D$  –factor of velocity damping, which is a random value between 0 and 1;
- $pH\_Rand_1$ ,  $pH\_Rand_2$ , and  $pH\_Rand_3$  – the pH value of visited area, which is a random value between 7–14;
- $Temp\_Rand_1$ ,  $Temp\_Rand_2$  – the temperature value of visited area, which is a random value between 35.1–38.5;
- $v_i$  –velocity of sperm;
- $x_i$  – current position of sperm;
- $x_{sbest}$  – personal best location of sperm  $i$ ;
- $x_{sgbest}$  – global best location of the swarm.

The current best solution can be calculated by the following formula:

Figure 1. The sperm swarm and winner



$$x_i(t) = x_i(t) + v_i(t) \quad (2)$$

The full procedure of “Sperm Swarm Optimization (SSO)” approach can be summarized as follows (Shehadeh et al., 2018b; Shehadeh et al., 2018c):

**Algorithm 2** *Sperm Swarm Optimization (SSO)*

Begin

Step 1: For the each sperm, initialize location.

**Step 2:** **for**  $i=1$ : size of population **do**

**Step 3:** evaluate the fitness for swarm

**if** obtained fitness  $> x_{sbest}$  **then**

Assign the current value as the  $x_{sbest}$

**end if**

**end for**

**Step 4:** appoint the  $x_{sgbest}$  based on the winner.

**Step 5:** **for**  $i=1$ : size of population **do**

Apply Eq(1)

Apply Eq(2)

**end for**

**Step 6:** **while** maximum criterion is not reached go to step 2

End.

## HYBRID GENETIC ALGORITHM AND SPERM SWARM OPTIMIZATION (HGASSO)

Usually, the main idea of the hybrid optimization algorithm can be considered as applying a local search approach with the genetic algorithm operations such as crossover and mutation in one procedure

(Pomar et al., 2017; Moin et al. 2015). This section summarizes the structure of our proposed algorithm (denoted by HGASSO). From the pseudocode of algorithm 3, we can notice that GA and SSO both evaluate the same initial population. The individuals of the population may be represented as sperms in the case of SSO, or as chromosomes in the case of GA. In the proposed HGASSO, the local search is applied first with all the individuals, which includes the process of selecting the global best sperm (the winner), the process of selecting the personal best solution of the sperm, and the process of applying the sperm velocity update rule (Equation 1). This is followed by evaluating a fitness function. The crossover operator of the real-coded GA comes at this stage to have the site on the procedure, which is applied to all the individuals based on a crossover probability to prepare them for the mutation operation stage. The operator of GA, called mutation is used after the crossover operation to adjust an candidate solution with a value based on a random selection in the problem's space domain. The mutation is operated depends on a mutation probability that is determined at the beginning of the procedure. The full procedure of “Hybrid Genetic Algorithm and Sperm Swarm Optimization (HGASSO)” can be summarized as follows:

**Algorithm 3** *Hybrid Genetic Algorithm and Sperm Swarm Optimization (HGASSO)*

Begin

Step 1: Initialize the population.

Step 2: apply SSO method

```

for i=1: size of population do
    evaluate the fitness for each sperm
    if obtained fitness >  $x_{sbest}$  then
        appoint the current value as the  $x_{sbest}$ 
    end if
    end for
    appoint the  $x_{sgbest}$  depends on the winner.
for i=1: size of population do
    Apply Eq(1)
    Apply Eq(2)
end for

```

Step 3: apply GA method

    Perform crossover such as two-parent crossover based  
    on a determined crossover probability

    Perform mutation based on a determined mutation  
    probability

    Perform sorting to determine the global best solution of  
    the swarm (winner)

**Step 4:** **while** maximum criterion is not reached go to step 2  
End.

In this procedure, the crossover and mutation are applied to help the local search method to skip the local minima easily. Crossover and mutation help to make diversity in the population in the range of the problem's space domain. This is very important to be applied, especially with multimodal problems that contain multi-local extremes. Based on pseudocode of Algorithm 3, the algorithm sorts the population to determine the global best solution of the swarm. Therefore, the complexity of HGASSO is equal to  $O(n \log n)$ .

## EXPERIMENT AND RESULTS

In order to compare the proposed HGASSO with both GA and SSO, we apply two assessments, including, quantitative and qualitative tests. For the quantitative test, we measure the best fitness,

standard deviation, median, maximum value and average value that are achieved by the approaches for each benchmark function. Furthermore, we adopt a statistical test, namely “one-way ANOVA (Tukey’s test)” to compare between approaches. This will be discussed later in Tables from 2 to 14, while for the qualitative test, we compare between the solution set that is achieved by our approach and the solution set that achieved by the other approaches. This refers to the concept of algorithm convergence, which is utilized to measure the ability of an algorithm to reach the optimal solution of a problem within a determined number of iterations. This will be discussed later in Figures from 12 to 22.

These assessments are measured based on several well-known multimodal benchmark functions. Benchmark model is a nonlinear problem, which is utilized to estimate and validate the metaheuristic approaches (Krzeszowski & Wiktorowicz, 2016). These functions are selected carefully as they are considered as standard bed test problem for estimating the optimization methods (Kaipa & Ghose, 2017; Adorio & Diliman, 2005; Molga & Smutnicki, 2005; Back, 1996). The listed below benchmarks functions have several local extremes and all of them are minimization models, which their results should be minimum. The following points summarize the benchmarks functions that are used in this work:

- **Rastrigin Function**

The search space domain of this function is usually estimated on the hypercube  $x_i \in [-5.12, 5.12], \forall i = 1, \dots, d$ . Rastrigin problem has multiple local minima regularly distributed in its search space domain as depicted in Figure 2. The mathematical formulation of this problem is shown in Equation 3 (Surjanovic & Bingham, 2013).

$$f(y) = 10d + \sum_{i=1}^d [y_i^2 - 10 \cos(2\pi y_i)], \quad (3)$$

- **Ackley Function:**

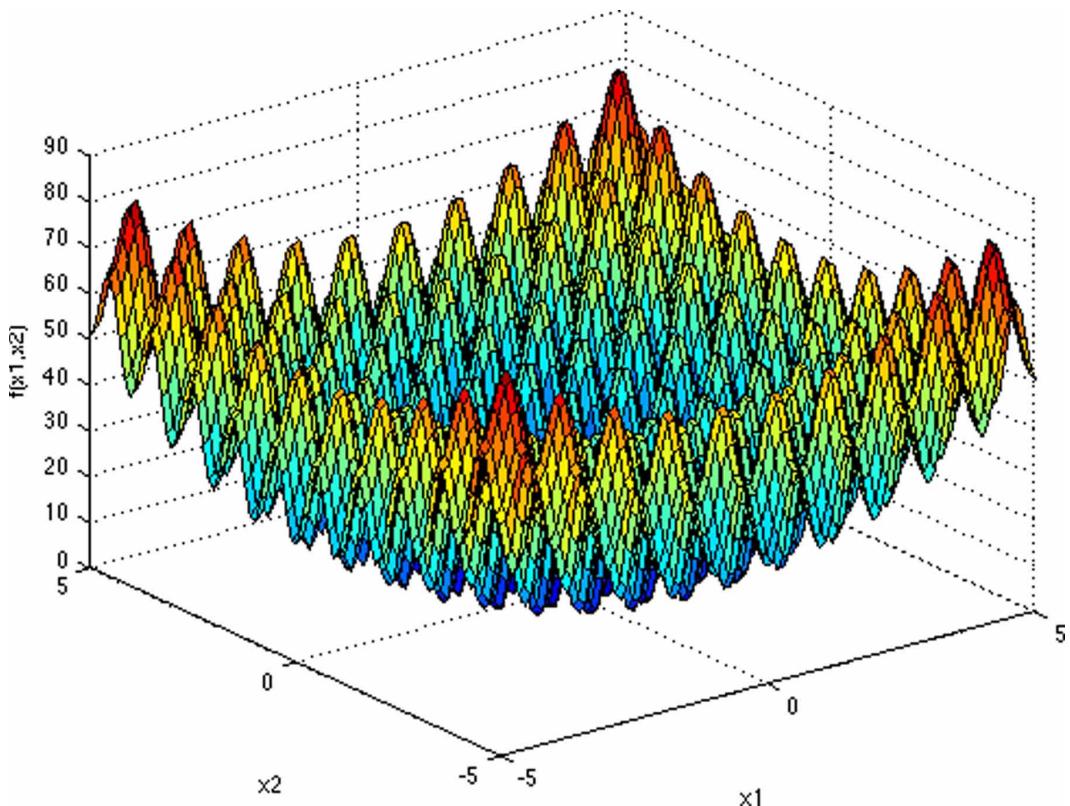
Ackley function is considered as a very complex problem in which many metaheuristics methods are trapped in one of its multiple local minima. The optimal value of this function is 0 with a single global minimum at  $(0, \dots, 0)$ , as depicted in Figure 3. The mathematical formulation of this problem is shown in Equation 4. This problem can be estimated on the hypercube  $x_i \in [-32.768, 32.768], \forall i = 1, \dots, d$ . The recommended values of the parameters  $c$ ,  $b$  and  $a$  are  $2\pi$ , 0.2, and 20 respectively (Surjanovic & Bingham, 2013).

$$f(y) = -a \exp \left( -b \sqrt{\frac{1}{d} \sum_{i=1}^d y_i^2} \right) - \exp \left( \frac{1}{d} \sum_{i=1}^d \cos(c y_i) \right) + a + \exp(1), \quad (4)$$

- **Bukin Function:**

This problem is always estimated on the rectangle  $x_1 \in [-15, -5], x_2 \in [-3, 3]$ . Bukin function has a single global minimum at  $(-10, 1)$  and many local minima with an optimal value of 0. Figure 4

**Figure 2. Rastrigin function**



shows its local extremes, which lie in a series of ridges. The mathematical formulation of this problem is shown in Equation 5 (Surjanovic & Bingham, 2013).

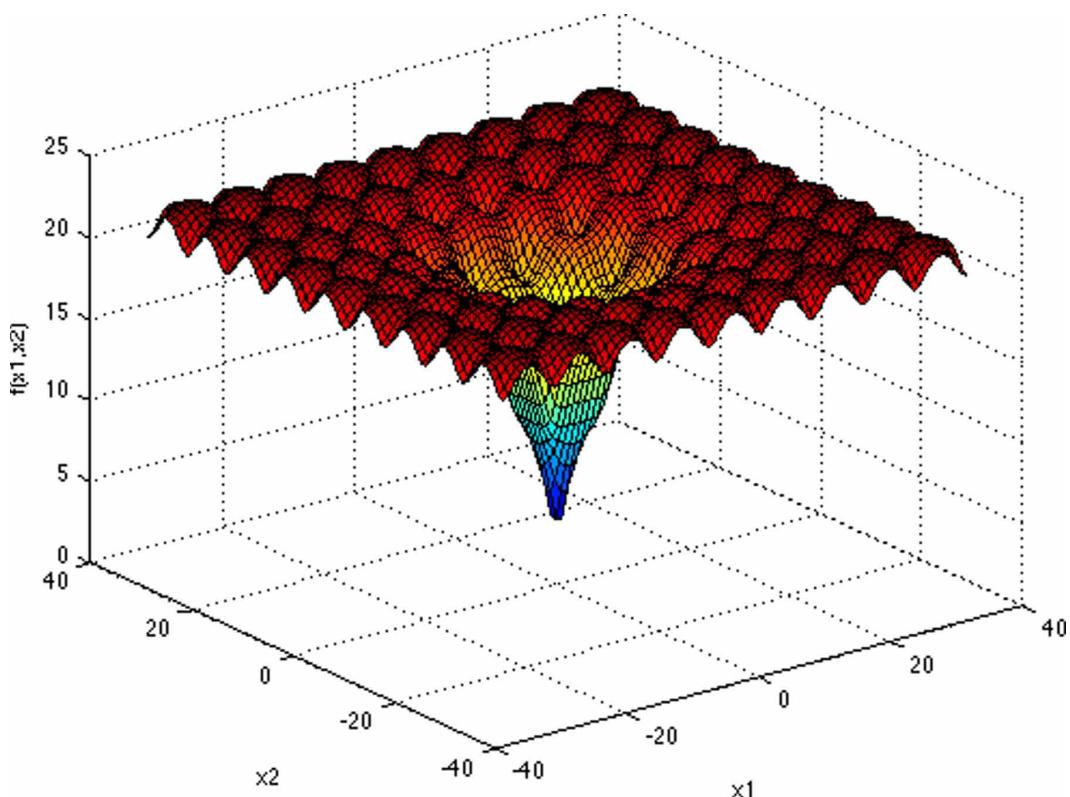
$$f(y) = 100\sqrt{|y_2 - 0.01y_1^2|} + 0.01|y_1 + 10|, \quad (5)$$

- **Griewank Function**

Griewank model has many widespread local minima in its search space domain. This problem has a global minimum at (0, 0) with an optimal value of 0. Griewank function is depicted in Figure 5, which the zoomed-in plots present its complexity. This problem is always estimated on the hypercube  $x_i \in [-600, 600]$ ,  $\forall i = 1, \dots, d$ . The mathematical formulation of this problem can be described as in Equation 6 (Surjanovic & Bingham, 2013).

$$f(y) = \sum_{i=1}^d \frac{y_i^2}{4000} - \prod_{i=1}^d \cos\left(\frac{y_i}{\sqrt{i}}\right) + 1, \quad (6)$$

**Figure 3. Ackley function**



- **Cross-In-Tray Function**

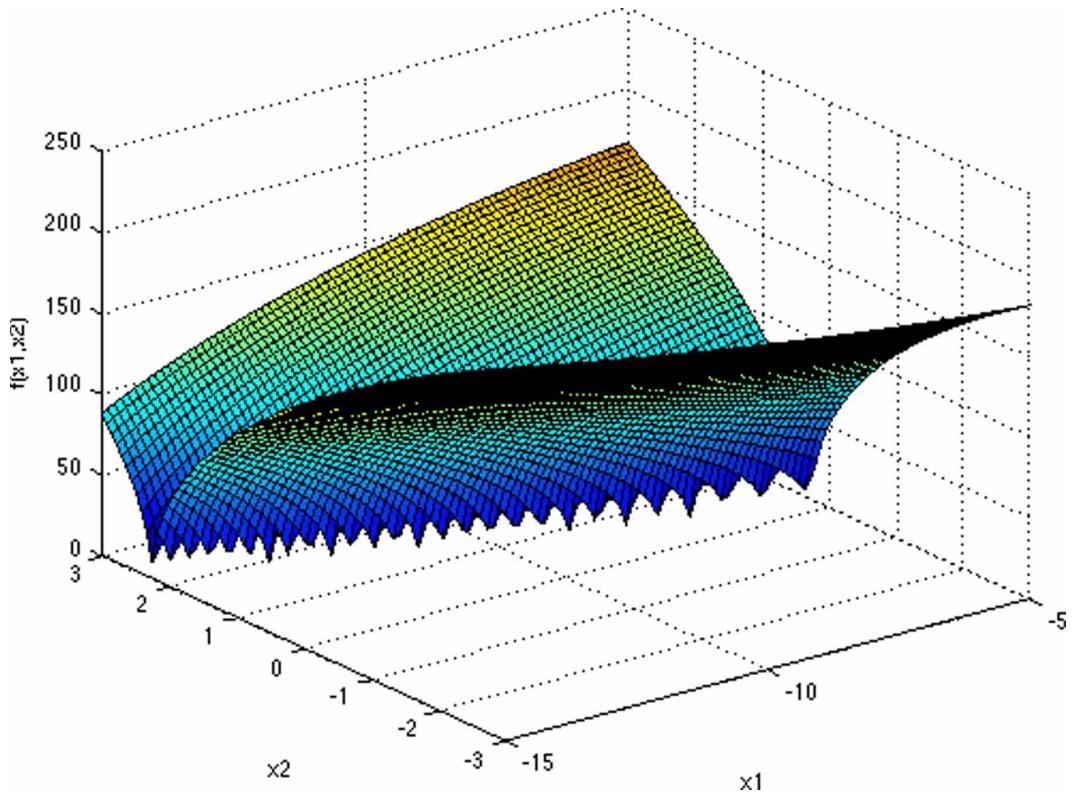
Cross-In-Tray function has an optimal value of -2.06261 with several global minima at (1.3491, -1.3491), (-1.3491, 1.3491), (-1.3491, -1.3491), and (1.3491, 1.3491). This function is depicted in Figure 6 in which the right plot shows its characteristic of a cross with its several global minima. This problem is mainly estimated on the square  $x_i \in [-10, 10]$ ,  $\forall i = 1, 2$ . The mathematical formulation of Cross-In-Tray can be represented as in Equation 7 (Surjanovic & Bingham, 2013).

$$f(y) = -0.0001 \left( \left| \sin(y_1) \sin(y_2) \exp \left( \left| 100 - \frac{\sqrt{y_1^2 + y_2^2}}{\pi} \right| \right) + 1 \right|^{0.1}, \quad (7)$$

- **Levy Function N.13**

The input values of this benchmark problem are commonly estimated on the square  $x_i \in [-10, 10]$ ,  $\forall i = 1, 2$ . This problem has a single global minimum at (1, 1) with an optimal value of 0 as depicted in Figure 7. The mathematical formulation of Levy function N.13 can be described as in Equation 8 (Surjanovic & Bingham, 2013).

**Figure 4.** Bukin function



$$f(y) = \sin^2(3\pi y_1) + (y_1 - 1)^2 [1 + \sin^2(3\pi y_2)] + (y_2 - 1)^2 [1 + \sin^2(2\pi y_2)], \quad (8)$$

### • Branin

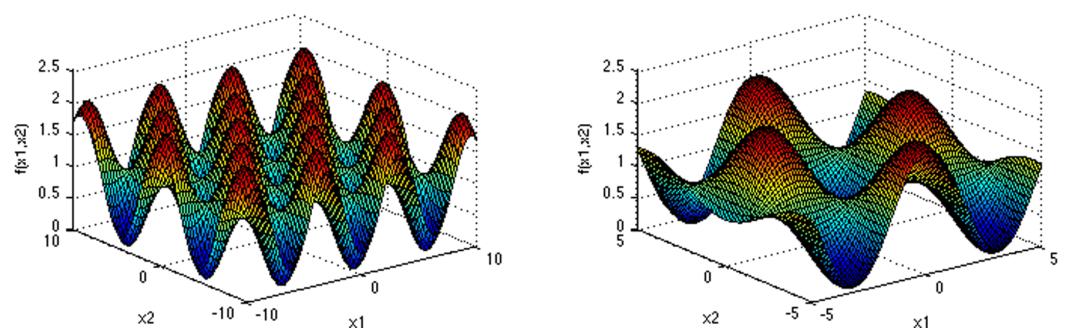
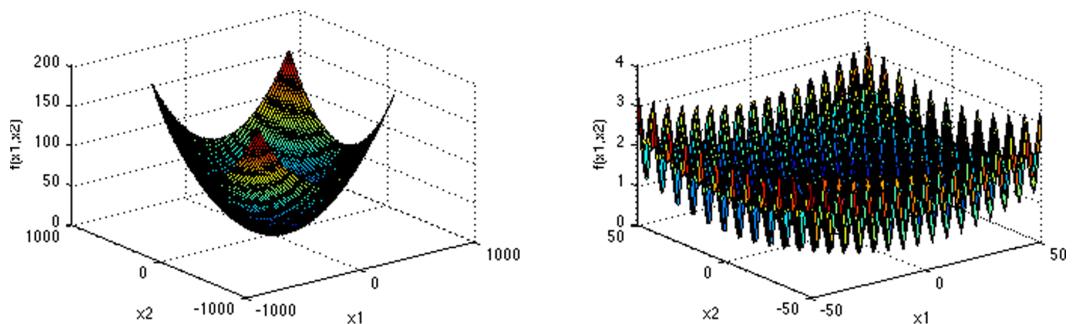
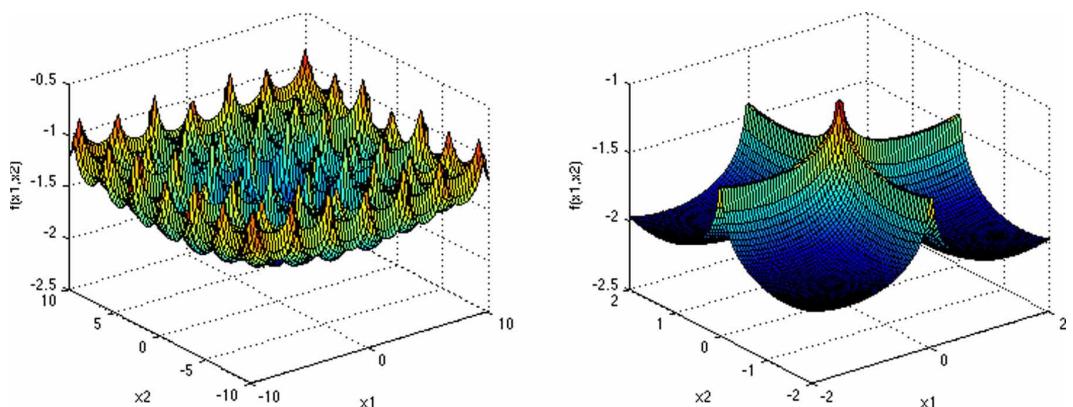
This benchmark is usually estimated on the square  $x_1 \in [-5, 10]$ ,  $x_2 \in [0, 15]$ , as shown in Figure 8. The optimal value of this function is 0.39789 with three local minima values at (-3.14, 12.275), (3.14, 2.275), and (9.42478, 2.475). The mathematical representation of Branin can be represented as in Equation 9 (Surjanovic & Bingham, 2013).

$$f(y) = \frac{1}{51.95} \left[ \left( \bar{y}_2 - \frac{5.1\bar{y}_1^2}{4\pi^2} + \frac{5\bar{y}_1}{\pi} - 6 \right)^2 + \left( 10 - \frac{10}{8\pi} \right) \cos(\bar{y}_1) - 44.81 \right], \quad (9)$$

Where,

$$\bar{y}_1 = 15\bar{y}_1 - 5, \quad (9.1)$$

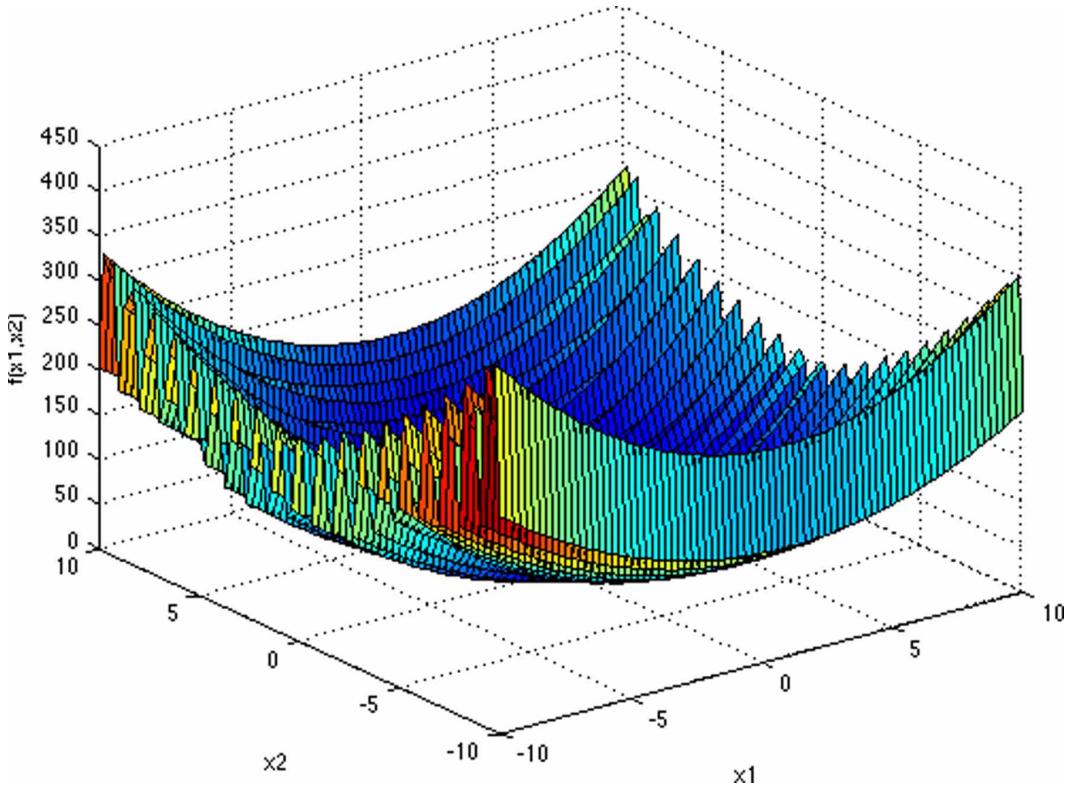
$$\bar{y}_2 = 15\bar{y}_2 \quad (9.2)$$

**Figure 5. Griewank function****Figure 6. Cross-In-Tray function**

- **Easom (ES)**

This benchmark is mainly estimated on the square  $x_i \in [-100, 100]$ ,  $\forall i = 1, 2$ , as shown in Figure 9. The Easom problem has several local minima with a single global minima at  $(3.14, 3.14)$ . The optimal value of this function is -1. The mathematical formulation of Easom can be described as in Equation 10 (Surjanovic & Bingham, 2013).

Figure 7. Levy function N.13



$$f(y) = -\cos(y_1) \cos(y_2) \exp(-(y_1 - \pi)^2 - (y_2 - \pi)^2), \quad (10)$$

- **Goldstein and Price (GP)**

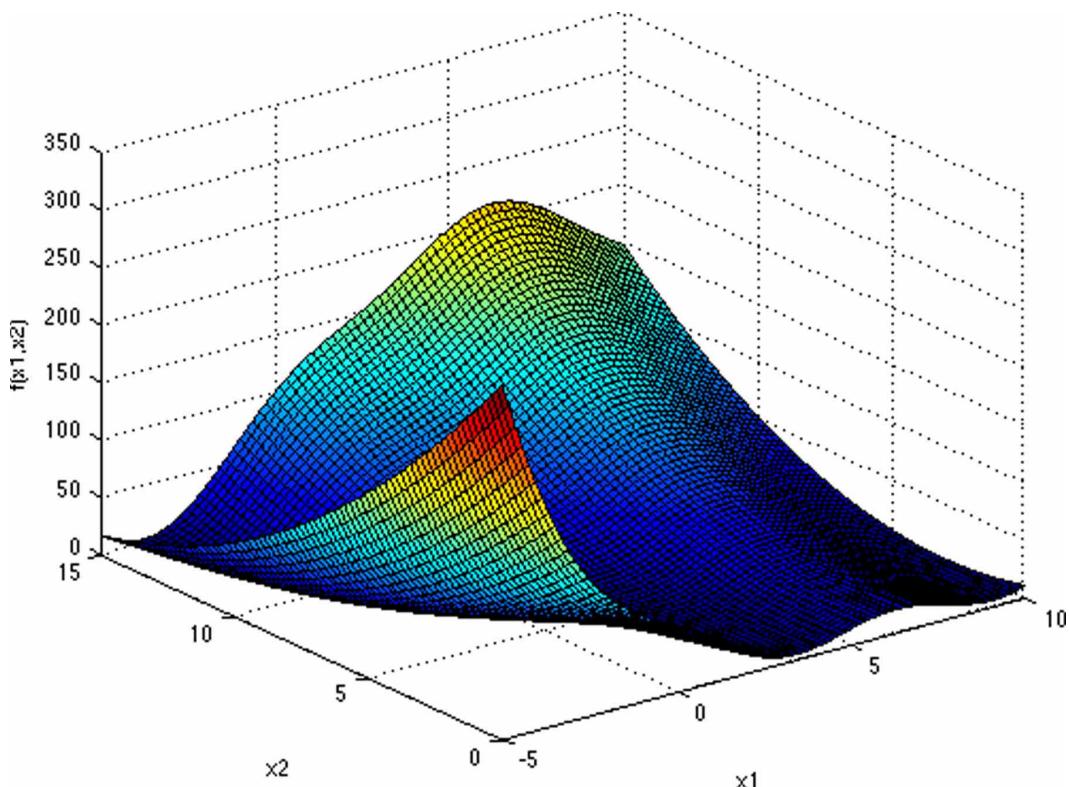
The model is mainly estimated on the square  $x_i \in [-2, 2]$ ,  $\forall i = 1, 2$ . The optimal value of this function is 3 with a single global minimum at  $(0, -1)$  as shown in Figure 10. The mathematical formulation of this problem can be described as in Equation 11 (Surjanovic & Bingham, 2013).

$$f(y) = \left[ 1 + (y_1 + y_2 + 1)^2 (19 - 14y_1 + 3y_1^2 - 14y_2 + 6y_1y_2 + 3y_2^2) \right] \times \left[ 30 + (2y_1 - 3y_2)^2 (18 - 32y_1 + 12y_1^2 + 48y_2 - 36y_1y_2 + 27y_2^2) \right], \quad (11)$$

- **Shubert**

The model is mainly evaluated on the square  $x_i \in [-10, 10]$ ,  $\forall i = 1, 2$ , although this may be restricted to the square  $x_i \in [-5.12, 5.12]$ ,  $\forall i = 1, 2$ . The optimal value of this benchmark is -186.7309, as shown in Figure 11, which the zoomed-in plot presents its multiple local minima. The mathematical formulation of this problem can be described as in Equation 12 (Surjanovic & Bingham, 2013).

Figure 8. Branin function



$$f(y) = \left( \sum_{i=1}^5 i \cos((i+1)y_1 + i) \right) \left( \sum_{i=1}^5 i \cos((ii+1)y_2 + i) \right), \quad (12)$$

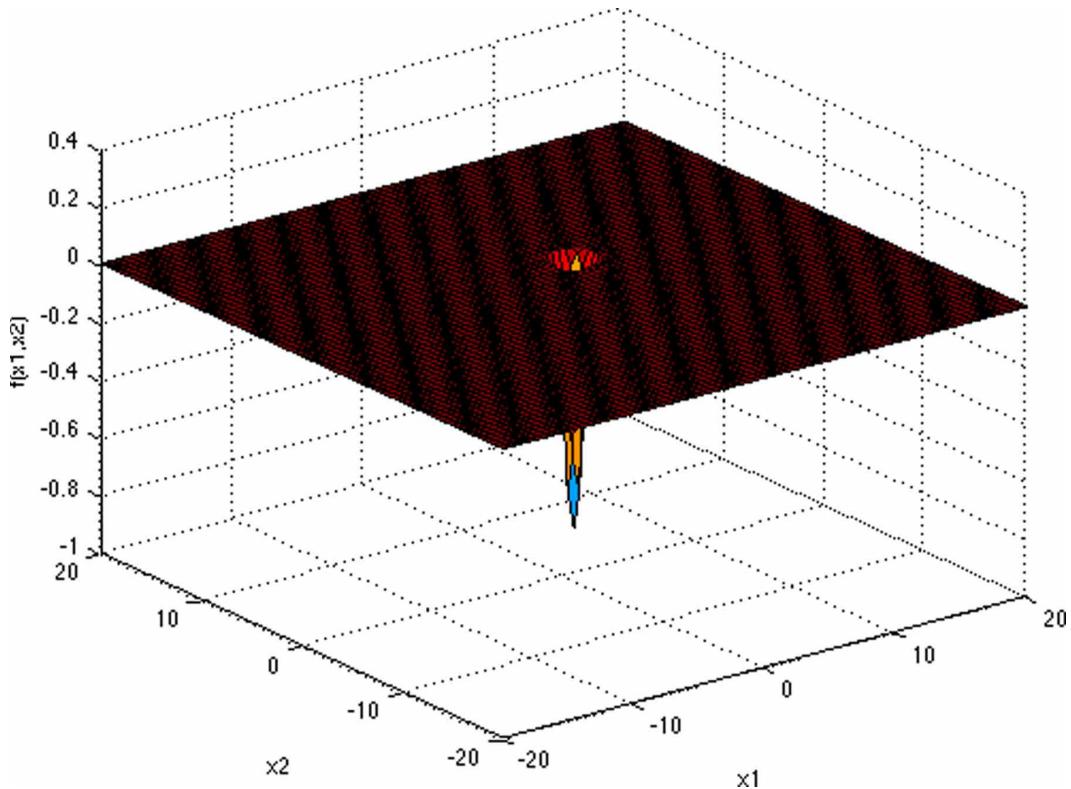
- **Shekel**

Shekel function is usually evaluated on 4 dimensions search space domain, which has multiple local minima and 4 global minima values. This benchmark is mainly evaluated on the hypercube  $x_i \in [0, 10]$ ,  $\forall i = 1, 2, 3, 4$ . The optimal value of this function is mainly less or equal to -10.5 in which is changed depends on the value of  $m$  and  $x$ . The mathematical formulation of this problem can be represented as in Equation 13 (Surjanovic & Bingham, 2013).

$$f(y) = \sum_{i=1}^m \left( \sum_{j=1}^4 (y_j - c_{ji})^2 + \beta_i \right)^{-1}, \quad (13)$$

Where,

Figure 9. Easom problem



$$\beta = \frac{1}{10} (1, 2, 2, 4, 4, 6, 3, 7, 5, 5)^T, \quad (13.1)$$

$$c = \begin{pmatrix} 4.0 & 1.0 & 8.0 & 6.0 & 3.0 & 2.0 & 5.0 & 8.0 & 6.0 & 7.0 \\ 4.0 & 1.0 & 8.0 & 6.0 & 7.0 & 9.0 & 3.0 & 1.0 & 2.0 & 3.6 \\ 4.0 & 1.0 & 8.0 & 6.0 & 3.0 & 2.0 & 5.0 & 8.0 & 6.0 & 7.0 \\ 4.0 & 1.0 & 8.0 & 6.0 & 7.0 & 9.0 & 3.0 & 1.0 & 2.0 & 3.6 \end{pmatrix}, \quad (13.2)$$

The MatLab 2017a has been utilized to implement the proposed approach. All of the tests are carried out on a computer with 4 GB RAM and Intel dual-core CPU T3200 running Windows 7. The parameters of all the algorithms have been standardized during the evaluation of the proposed benchmark problems, as recommended in (Shehadeh et al., 2018b; Shehadeh et al., 2018c; Saravanan & Sachithanandam, 2001) to compare the result of HGASSO against the results of GA and SSO. The parameters of HGASSO, SSO, and GA are listed in Table 1, which the parameters of SSO are set as follows;  $D$  is a velocity damping factor, which sets in the range of (0, 1);  $pH_{Rand_1}$ ,  $pH_{Rand_2}$ , and  $pH_{Rand_3}$  are the potential of Hydrogen values of the visited location, which set in the range of (7, 14).  $Temp_{Rand_1}$  and  $Temp_{Rand_2}$  are the temperature parameters of the visited location, which set in the range of (35.1, 38.5) (Shehadeh et al., 2018b; Shehadeh et al., 2018c). The parameters of GA

Figure 10. Goldstein and Price (GP) function

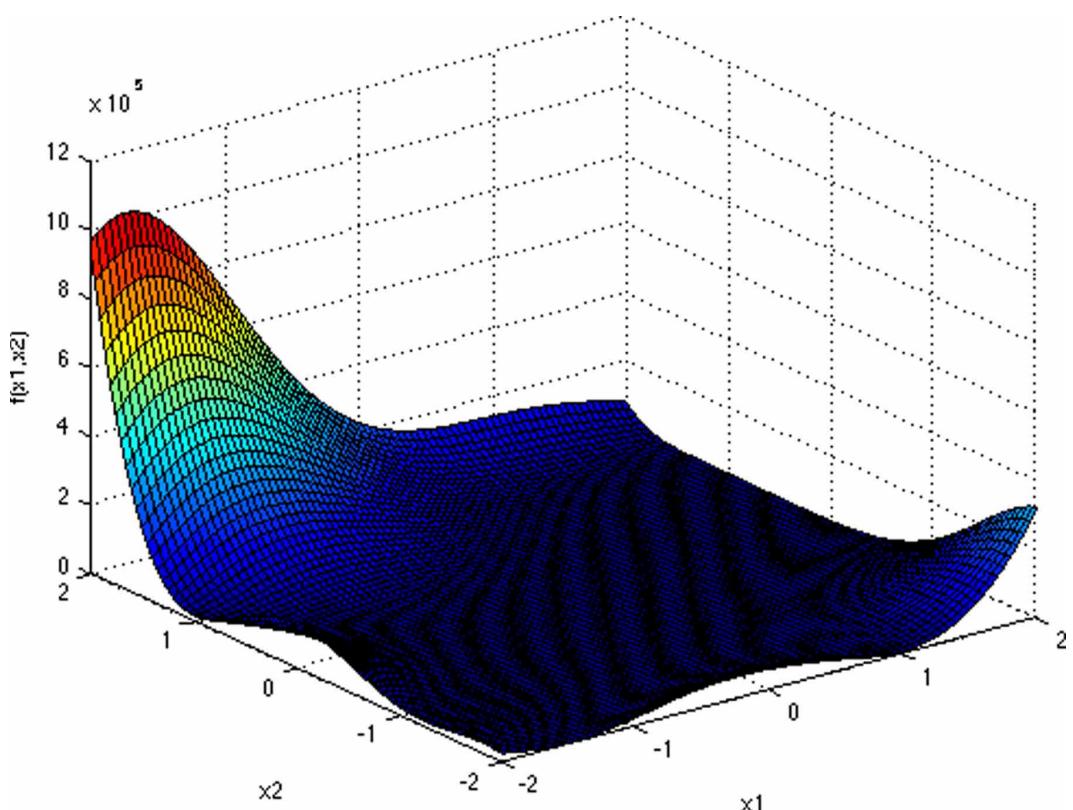
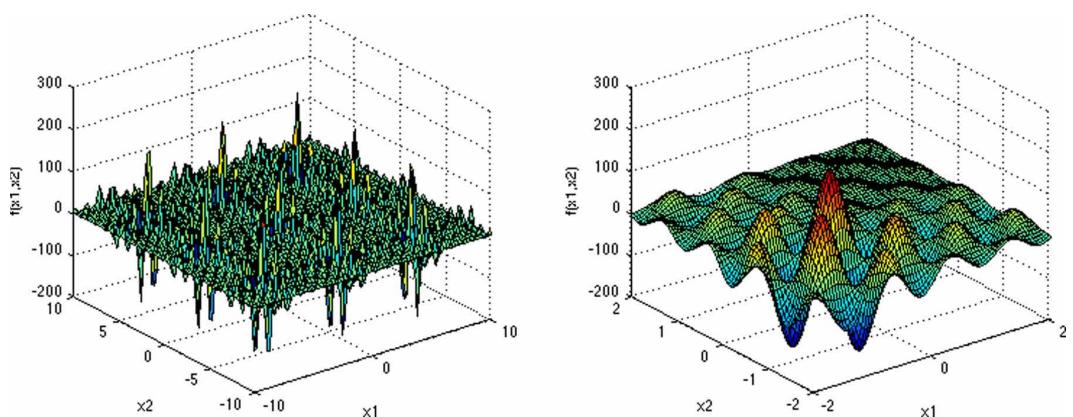


Figure 11. Shubert function



are set as follows; crossover and mutation are set to 0.8 and 0.05 respectively as in (Shehadeh et al., 2018c; Saravanan & Sachithanandam, 2001).

Overall, every test function is evaluated in a total of ten-time runs. The experimental outcomes for HGASSO, SSO, and GA are presented in Tables from 2 to 12. Each table shows the number of iterations, the best fitness value of each benchmark problem, the average value, median value, maximum value, and the standard deviation. The algorithm convergences of HGASSO, SSO, and GA

are depicted in Figures from 12 to 22. We follow the standard strategy of comparing the proposed approach against GA and SSO in which the number of generations has been changed through the evaluation. This scenario of evolution is used to estimate many well-known approaches, such as in (Shehadeh et al., 2018c; Banharnsakun et al, 2011; Sathya & Radhika, 2013; Rbouh & Imrani, 2014; Shi & Eberhart, 1999).

Overall, the proposed HGASSO algorithm obtained the best fitness for Rastrigin, Ackley, Bukin, Griewank, Levy function N.13, and Shekel benchmark functions, followed by SSO and GA as outlined in Table 13. However, for Easom and Shubert test functions, the proposed HGASSO attained the same rank with SSO. In addition, for Cross-in-Tray, Branin, and Goldstein and Price (GP) test functions, the proposed HGASSO attained the same rank of best fitness value with SSO and GA. For the average best of ten-runs on the proposed benchmarks test functions, the HGASSO is the best on Rastrigin, Ackley, Bukin, Griewank, Levy function N.13, Branin, Easom (ES), Goldstein and Price (GP), and Shubert test functions. However, for Cross-in-Tray and Shekel test functions, the proposed HGASSO attained the same rank of best average with SSO. This shows that the proposed HGASSO has a better convergence than the other algorithms to explore the search space domain. Particularly, in solving the very complex benchmarks functions that contain exponent, cosine, sine, summation, and multiplications of multiple variables in their formulation. This is clear in the algorithm convergence measurement as depicted in the prior Figures from 12 to 22 in which HGASSO has the ability more than other approaches in skipping the multiple local extremes of the problems.

Table 14 presents the statistical analysis using “one-way ANOVA (Tukey’s test)” for all benchmarks functions when the population size for all the approaches is set to 1000. As outlined in the

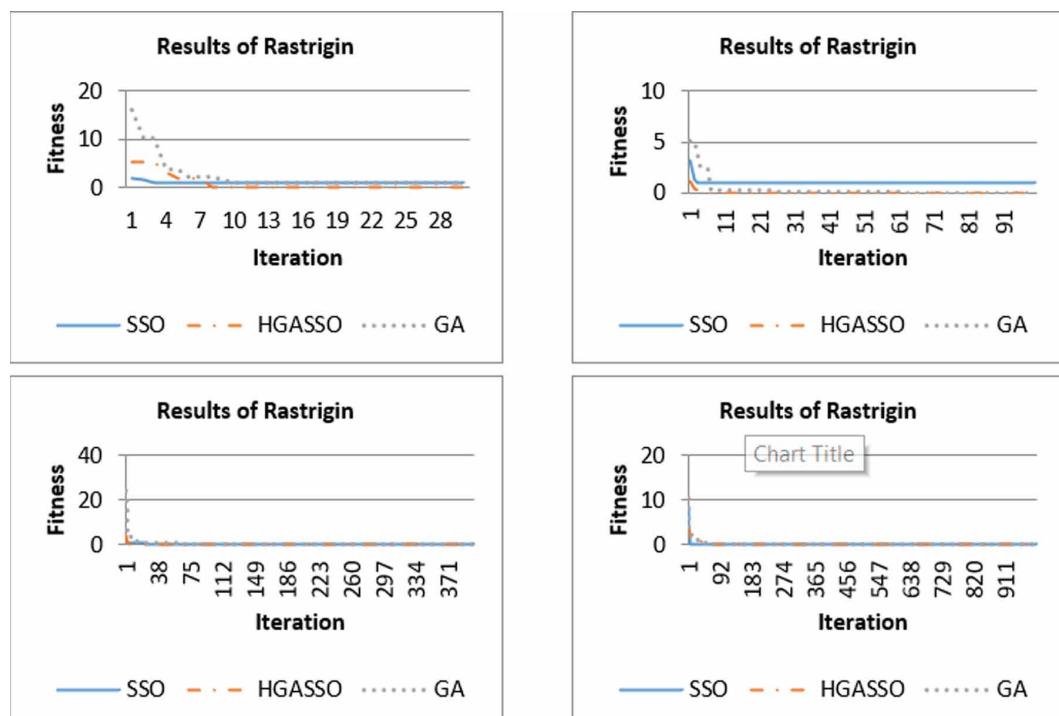
**Table 1. Parameters of the algorithms**

Parameters	Value
<b>HGASSO</b>	
Crossover probability	0.8
Probability of mutation	0.05
velocity damping factor (D)	Rand (0, 1)
Temperature	Rand (35.5, 38.5)
pH	Rand (7, 14)
Population sizes	20
Numbers of generations/iterations	30, 100, 400, and 1000
<b>SSO</b>	
velocity damping factor (D)	Rand (0, 1)
Temperature	Rand (35.5, 38.5)
pH	Rand (7, 14)
Population sizes	20
Numbers of generations/iterations	30, 100, 400, and 1000
<b>GA</b>	
Probability of mutation	0.05
Crossover probability	0.8
Population sizes	20
Numbers of generations/iterations	30, 100, 400, and 1000

**Table 2.** Comparison of experimental results between HGASSO, SSO, and GA in terms of mean, standard deviation, maximum value, median, and best fitness for the Rastrigin test function. The bold font with the highlighted background represents the best fitness and mean for the respective benchmark function

Algorithm	Iteration no.	Average, mean	Std.	Max.	Median	Best fitness
<b>Function: Rastrigin</b>						
HGASSO	30	0.83069	2.3583	5.354263043	0.00056906	6.499e-05
	100	0.02177	0.38374	1.22085478	9.77392e-07	9.7739e-07
	400	0.018452	0.24184	4.345719212	1.42109e-14	1.4211e-14
	1000	0.046841	0.27594	4.191540893	0	0
SSO	30	1.058	0.224	2.0155	0.99520	0.99496
	100	1.023	0.22607	3.248725968	0.994959057	0.029416
	400	0.074	0.2228	1.180860894	0.020193641	0.010194
	1000	0.053	0.27557	8.073771956	0	0
GA	30	7.598	3.4325	15.98928286	0.99589	0.99496
	100	1.6349	0.87803	5.100138537	1.060852	0.99496
	400	0.76901	1.6062	24.12894	0.087708	0.017129
	1000	0.29101	0.50583	10.4	6.14716e-06	6.1472e-06

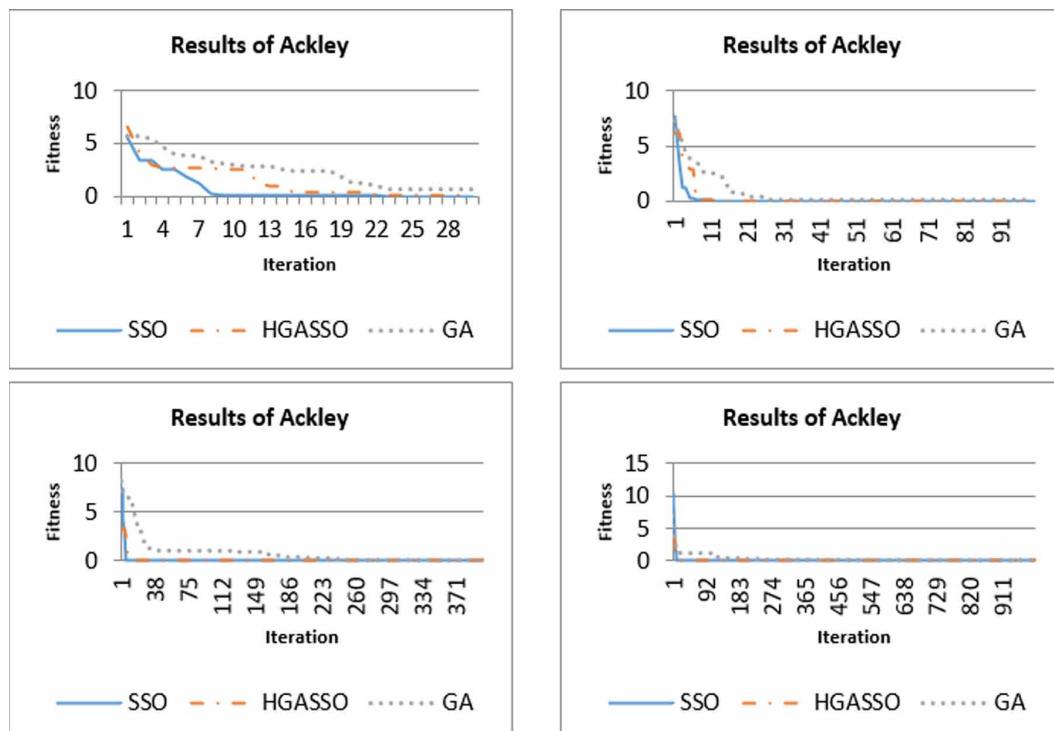
**Figure 12.** Comparison of experimental results between HGASSO, SSO, and GA in term of convergence for the Rastrigin test function



**Table 3.** Comparison of experimental results between HGASSO, SSO, and GA in terms of mean, standard deviation, maximum value, median, and best fitness for the Ackley test function. The bold font with the highlighted background represents the best fitness and mean for the respective benchmark function

Algorithm	Iteration no.	Average, mean	Std.	Max.	Median	Best fitness
<b>Function: Ackley</b>						
<b>HGASSO</b>	30	<b>0.33743</b>	0.99988	5.670564	0.4076E-05	<b>2.4366E-08</b>
	100	<b>0.0736</b>	0.64297	7.352609216	0.114E-11	<b>8.0824E-14</b>
	400	<b>0.017362</b>	0.2218	8.111862635	3.991E-13	<b>1.8649E-15</b>
	1000	<b>0.0054678</b>	0.12183	6.923460765	6.91E-16	<b>6.9165E-16</b>
<b>SSO</b>	30	0.45821	0.99405	5.516434	0.001582	2.9937E-08
	100	0.18659	0.50742	7.669341	1.52E-11	9.8412E-14
	400	0.023298	0.24924	7.347074	8.8E-15	2.6645E-15
	1000	0.0078659	0.15351	10.4051	8.88E-16	8.8818E-16
<b>GA</b>	30	3.1013	1.9973	6.623962	2.34069	1.7499
	100	1.951	1.2934	6.162792	1.6357	1.2311
	400	1.4138	0.76135	3.379967	1.5219	1.2312
	1000	1.7328	0.44703	3.332738	1.8725	1.6505

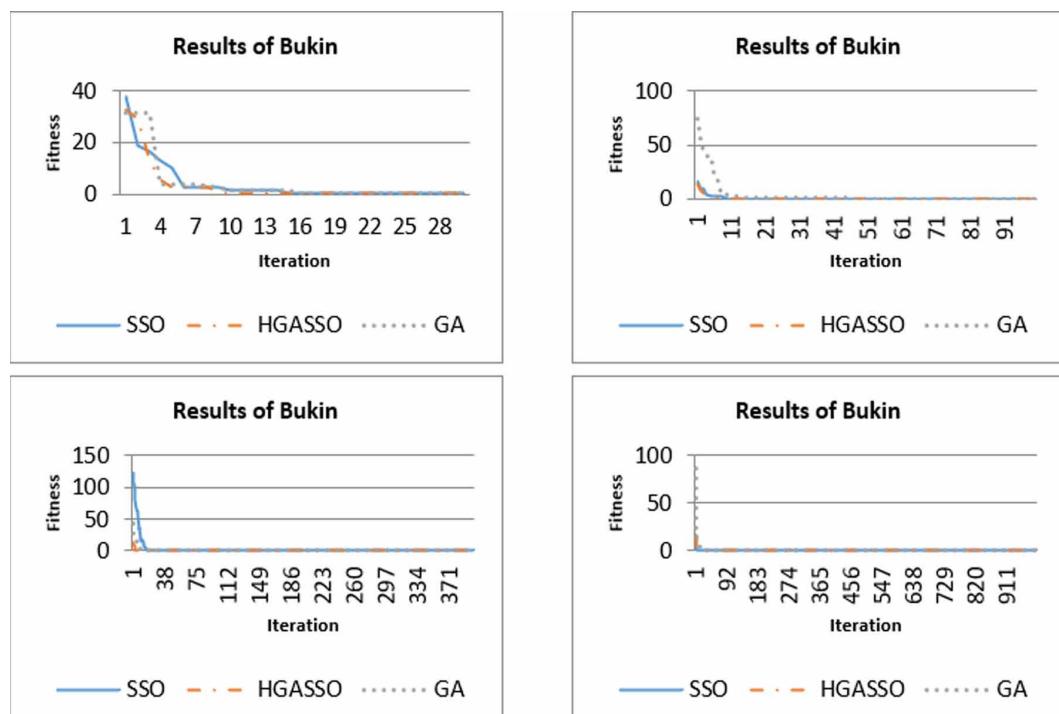
**Figure 13.** Comparison of experimental results between HGASSO, SSO, and GA in term of convergence for the Ackley test function



**Table 4.** Comparison of experimental results between HGASSO, SSO, and GA in terms of mean, standard deviation, maximum value, median, and best fitness for the Bukin test function. The bold font with the highlighted background represents the best fitness and mean for the respective benchmark function

Algorithm	Iteration no.	Average, mean	Std.	Max.	Median	Best fitness
<b>Function: Bukin</b>						
<b>HGASSO</b>	30	<b>3.1538</b>	9.4634	32.4804	0.09003	<b>0.04691</b>
	100	<b>0.4906</b>	2.306	14.23453	0.065994	<b>0.065994</b>
	400	<b>0.14783</b>	0.9582	12.58949	0.0507	<b>0.044777</b>
	1000	<b>0.10513</b>	0.6923	16.2696	0.053808	<b>0.053808</b>
<b>SSO</b>	30	34.488	7.9943	38.08249	0.431059	0.09003
	100	5.6346	2.1636	16.89265	0.126685	0.12669
	400	1.9536	0.67845	122.5681	0.044777	0.0447
	1000	0.5872	0.73588	16.41006	0.055188	0.055188
<b>GA</b>	30	13.5755	9.1456	31.43089	0.838493	0.199
	100	8.6237	11.7059	74.92827	0.237401	0.13858
	400	1.8721	3.1999	42.25904	0.058669	0.058669
	1000	0.54051	3.1527	86.18405	0.140932	0.0714093

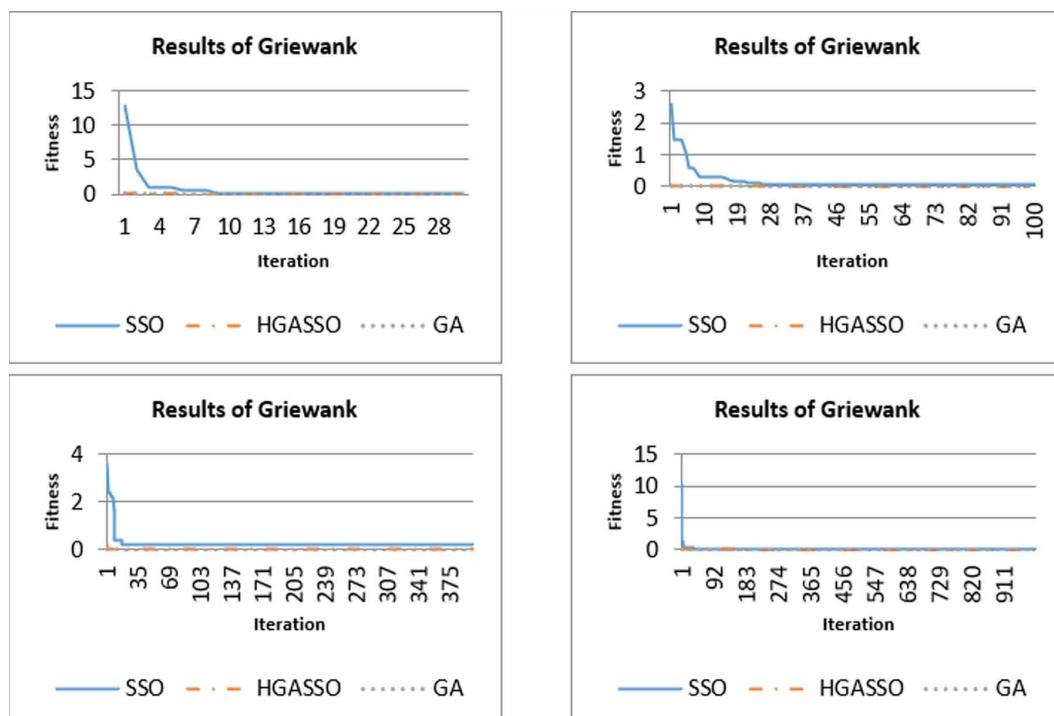
**Figure 14.** Comparison of experimental results between HGASSO, SSO, and GA in term of convergence for the Bukin test function



**Table 5.** Comparison of experimental results between HGASSO, SSO, and GA in terms of mean, standard deviation, maximum value, median, and best fitness for the Griewank test function. The bold font with the highlighted background represents the best fitness and mean for the respective benchmark function

Algorithm	Iteration no.	Average, mean	Std.	Max.	Median	Best fitness
<b>Function: Griewank</b>						
<b>HGASSO</b>	30	<b>0.0724772</b>	0.016421	0.025229	6.31E-05	<b>6.2751e-05</b>
	100	<b>0.012366</b>	0.0080656	0.039221	0.007408	<b>0.007391</b>
	400	<b>0.0057372</b>	0.01106	0.193659	0.009865	<b>2.1959e-05</b>
	1000	<b>0.0011059</b>	0.0060654	0.076761	1.558E-06	<b>5.3584e-08</b>
<b>SSO</b>	30	5.0707	2.366	12.94653	0.034655	0.01972
	100	2.5189	0.36012	2.60039	0.081556	0.081362
	400	0.75265	0.20696	3.587809	0.206964	0.20696
	1000	0.34638	0.33338	10.38389	0.13664	0.13664
<b>GA</b>	30	0.098954	0.015128	0.062387	0.013702	0.013702
	100	0.037079	0.0016401	0.016512	0.007396	0.007396
	400	0.012626	0.00048227	0.010224	0.007396	0.007396
	1000	0.010697	0.0054164	0.096065	0.007396	0.0073962

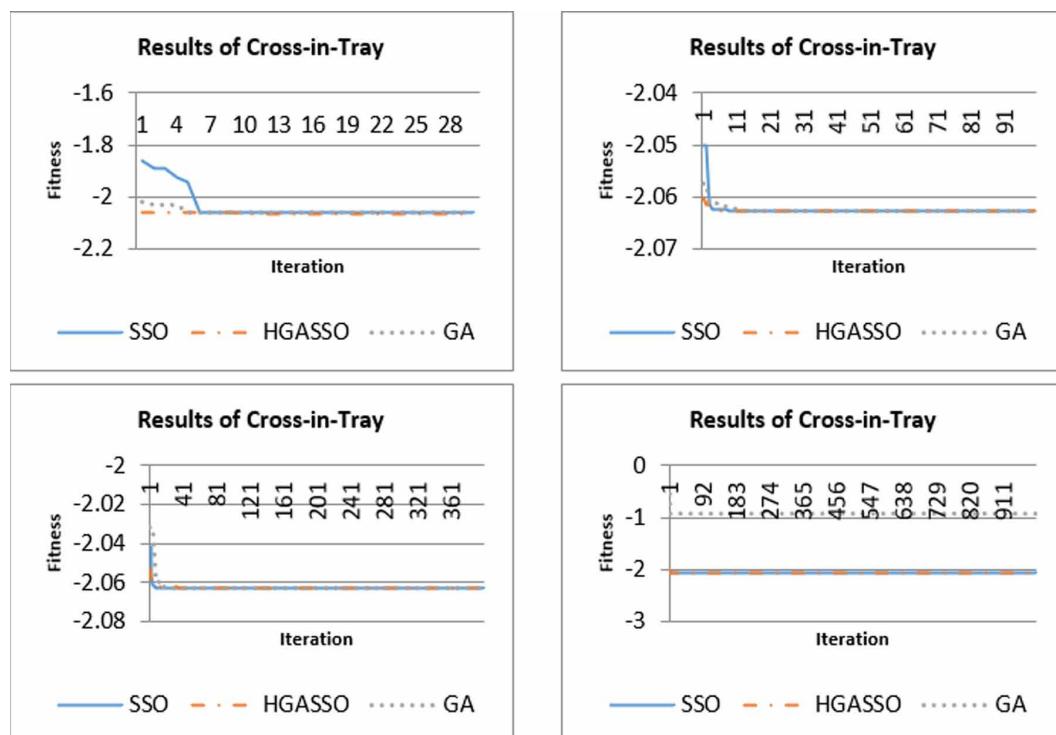
**Figure 15.** Comparison of experimental results between HGASSO, SSO, and GA in term of convergence for the Griewank test function



**Table 6.** Comparison of experimental results between HGASSO, SSO, and GA in terms of mean, standard deviation, maximum value, median, and best fitness for the Cross-in-Tray test function. The bold font with the highlighted background represents the best fitness and mean for the respective benchmark function

Algorithm	Iteration no.	Average, mean	Std.	Max.	Median	Best fitness
<b>Function: Cross-in-Tray</b>						
<b>HGASSO</b>	30	<b>-2.0624</b>	0.0065705	-2.06036	-2.06261	<b>-2.0626</b>
	100	<b>-2.0626</b>	0.0019581	-2.06004	-2.06261	<b>-2.0626</b>
	400	<b>-2.0626</b>	0.00065859	-2.05285	-2.06261	<b>-2.0626</b>
	1000	<b>-2.0626</b>	0.001813	-2.006	-2.06261	<b>-2.0626</b>
<b>SSO</b>	30	-1.9725	0.061039	-1.86304	-2.06261	<b>-2.0626</b>
	100	-2.0314	0.0017321	-2.05025	-2.06261	<b>-2.0626</b>
	400	<b>-2.0626</b>	0.0014933	-2.04194	-2.06261	<b>-2.0626</b>
	1000	<b>-2.0626</b>	0.0020119	-2.00025	-2.06261	<b>-2.0626</b>
<b>GA</b>	30	-2.0324	0.011546	-2.01877	-2.06261	<b>-2.0626</b>
	100	-2.0547	0.00077371	-2.05743	-2.06261	<b>-2.0626</b>
	400	-2.0601	0.0036624	-2.03175	-2.06261	<b>-2.0626</b>
	1000	-2.0621	0.00082917	-2.04536	-2.06261	<b>-2.0626</b>

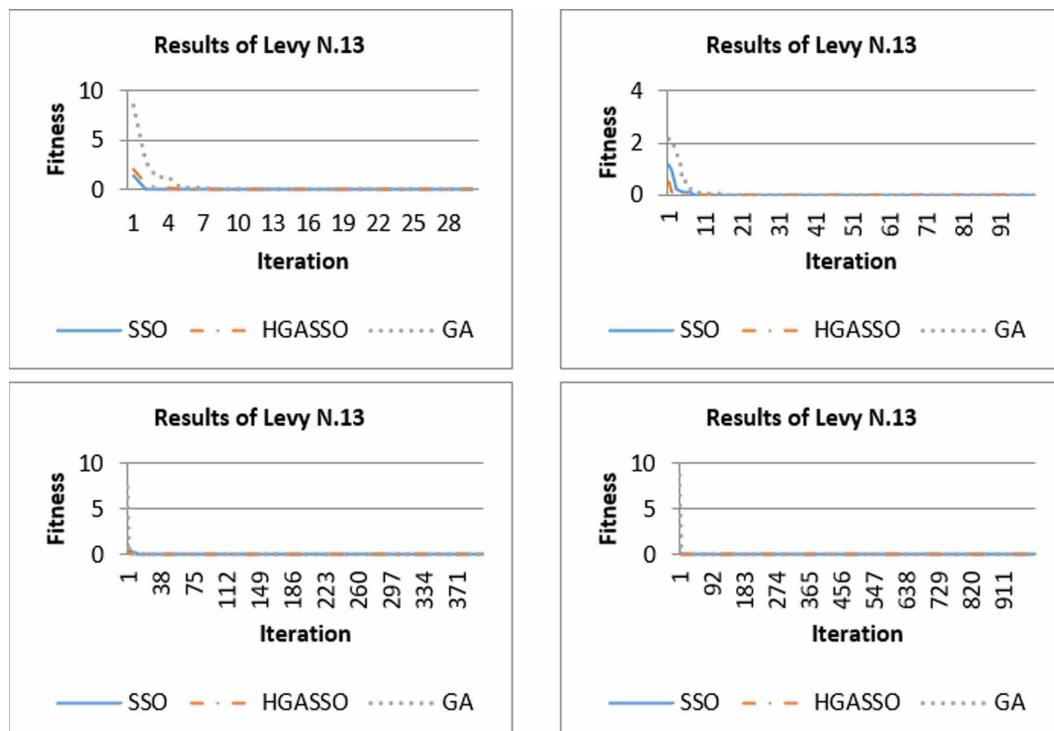
**Figure 16.** Comparison of experimental results between HGASSO, SSO, and GA in term of convergence for the Cross-in-Tray test function



**Table 7.** Comparison of experimental results between HGASSO, SSO, and GA in terms of mean, standard deviation, maximum value, median, and best fitness for the Levy N.13 test function. The bold font with the highlighted background represents the best fitness and mean for the respective benchmark function

Algorithm	Iteration no.	Average, mean	Std.	Max.	Median	Best fitness
<b>Function: Levy function N.13</b>						
<b>HGASSO</b>	30	<b>0.11263</b>	0.48914	2.112283	1.67E-06	<b>1.0054e-06</b>
	100	<b>0.0098208</b>	0.1288	0.53653	7.18E-16	<b>1.3498e-31</b>
	400	<b>0.0040293</b>	0.060411	0.984788	2.21E-15	<b>1.3498e-31</b>
	1000	<b>0.00040945</b>	0.011655	0.213259	1.35E-31	<b>1.3498e-31</b>
<b>SSO</b>	30	1.0284	0.14993	1.453477	0.01309	0.0056401
	100	0.40176	0.16547	1.148953	9.39E-23	1.335E-31
	400	0.20985	0.043645	1.297072	1.349E-31	1.335E-31
	1000	1.0284	0.14993	1.092091	2.034E-30	2.0345e-30
<b>GA</b>	30	2.9547	1.5963	8.523651	0.014209	1.955e-05
	100	0.88172	0.36938	2.160539	3.98E-11	3.9844e-11
	400	0.31151	0.37344	7.442983	4.47E-10	4.4668e-13
	1000	0.099797	0.37253	8.727115	1.35E-31	1.3490e-31

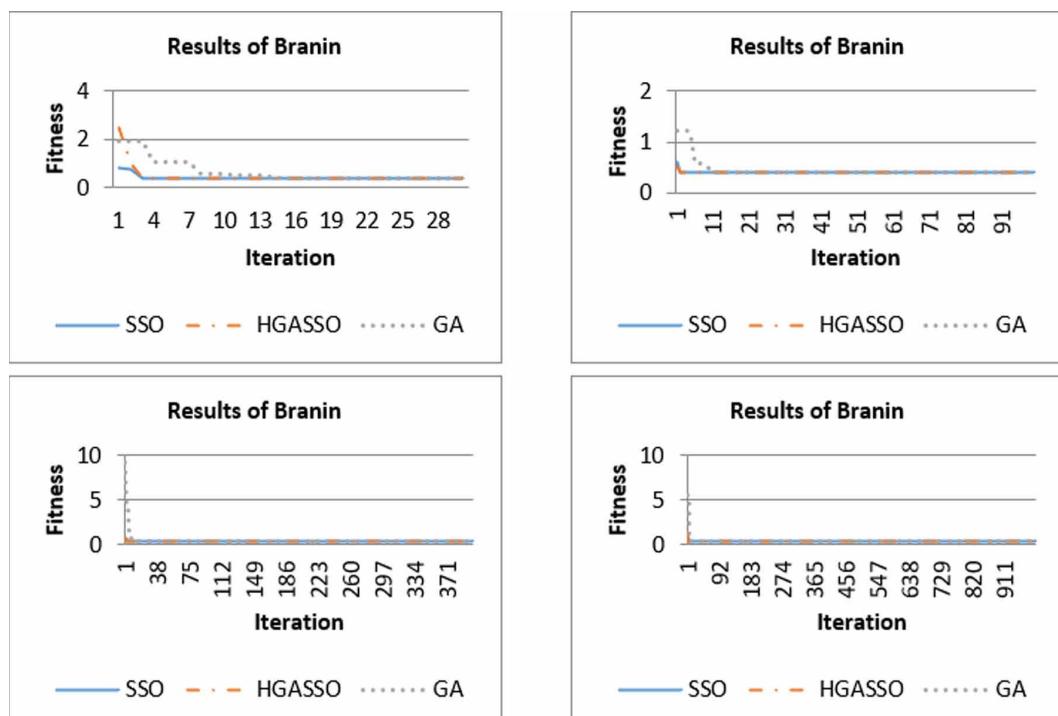
**Figure 17.** Comparison of experimental results between HGASSO, SSO, and GA in term of convergence for the Levy N.13 test function



**Table 8.** Comparison of experimental results between HGASSO, SSO, and GA in terms of mean, standard deviation, maximum value, median, and best fitness for the Branin test function. The bold font with the highlighted background represents the best fitness and mean for the respective benchmark function

Algorithm	Iteration no.	Average, mean	Std.	Max.	Median	Best fitness
<b>Function: Branin</b>						
HGASSO	30	<b>0.49065</b>	0.53949	2.479416	0.397887	<b>0.39789</b>
	100	<b>0.40034</b>	0.085994	0.521464	0.397887	<b>0.39789</b>
	400	<b>0.39964</b>	0.037387	0.748338	0.397887	<b>0.39789</b>
	1000	<b>0.39884</b>	0.031369	1.321689	0.397887	<b>0.39789</b>
SSO	30	1.4434	0.093386	0.807697	0.397887	<b>0.39789</b>
	100	1.1274	0.020624	0.605062	0.397887	<b>0.39789</b>
	400	0.6506	0.030288	0.80531	0.397887	<b>0.39789</b>
	1000	0.4296	0.030804	1.221217	0.397887	<b>0.39789</b>
GA	30	6.0451	0.46534	1.912022	0.415508	<b>0.39789</b>
	100	1.9106	0.17979	1.233101	0.397887	<b>0.39789</b>
	400	1.2753	0.56752	9.194571	0.397887	<b>0.39789</b>
	1000	0.57151	0.27957	5.506512	0.397887	<b>0.39789</b>

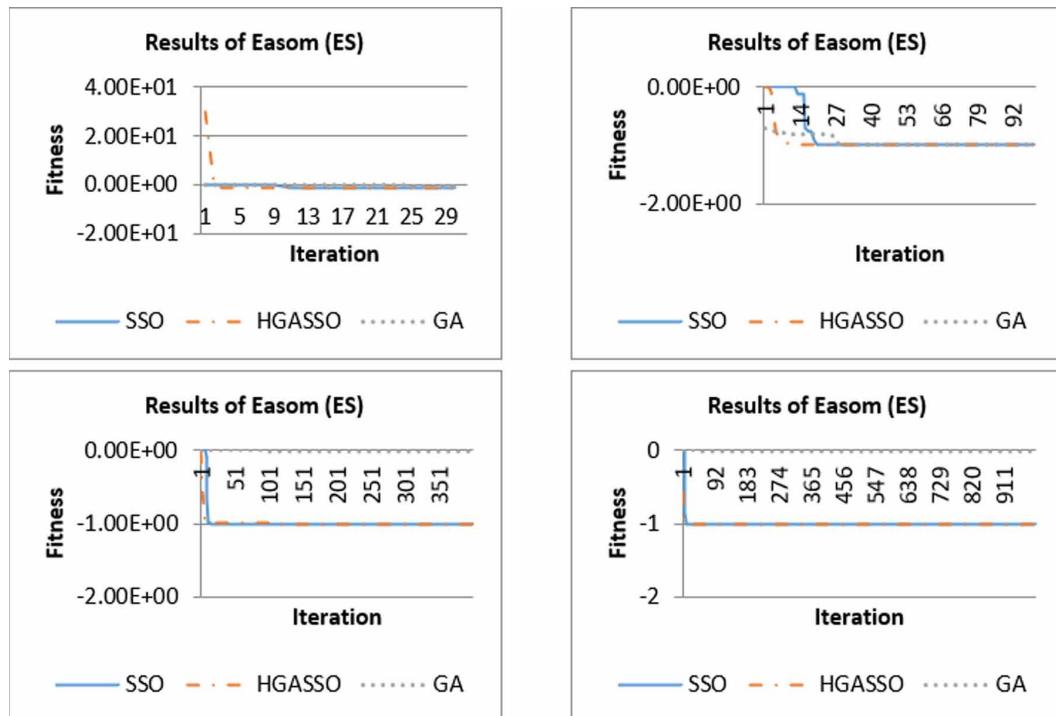
Figure 18. Comparison of experimental results between HGASSO, SSO, and GA in term of convergence for the Branin test function



**Table 9.** Comparison of experimental results between HGASSO, SSO, and GA in terms of mean, standard deviation, maximum value, median, and best fitness for the Easom (ES) test function. The bold font with the highlighted background represents the best fitness and mean for the respective benchmark function

Algorithm	Iteration no.	Average, mean	Std.	Max.	Median	Best fitness
<b>Function: Easom (ES)</b>						
<b>HGASSO</b>	30	<b>-0.93629</b>	0.067942	-0.57854	-1	<b>-1</b>
	100	<b>-0.98936</b>	0.19349	-0.00023	-1	<b>-1</b>
	400	<b>-0.99574</b>	0.061802	-0.00932	-1	<b>-1</b>
	1000	<b>-0.9996</b>	0.013206	-0.58761	-1	<b>-1</b>
<b>SSO</b>	30	-0.45032	0.45255	-1.30E-80	-9.96E-01	<b>-1</b>
	100	-0.78718	0.34859	-3.0E-171	-1	<b>-1</b>
	400	-0.96241	0.14033	-6.55E-84	-1	<b>-1</b>
	1000	-0.98494	0.080968	0	-1	<b>-1</b>
<b>GA</b>	30	-2.0138e-05	2.6482e-05	-4.38E-07	-1.24E-05	-8.1095e-05
	100	-7.1524e-05	0.086161	-0.70954	-0.99288	-8.1101e-05
	400	-7.9285e-05	2.2619e-05	-3.68E-13	-8.11E-05	-8.11e-05
	1000	-0.93195	9.2674e-06	-1.12E-07	-8.11E-05	-0.99291

**Figure 19.** Comparison of experimental results between HGASSO, SSO, and GA in term of convergence for the Easom (ES) test function



**Table 10.** Comparison of experimental results between HGASSO, SSO, and GA in terms of mean, standard deviation, maximum value, median, and best fitness for the Goldstein and Price (GP) test function. The bold font with the highlighted background represents the best fitness and mean for the respective benchmark function

Algorithm	Iteration no.	Average, mean	Std.	Max.	Median	Best fitness
<b>Function: Goldstein and Price (GP)</b>						
HGASSO	30	<b>7.9669</b>	13994.6643	59.90158	3.009872	<b>3</b>
	100	<b>5.0281</b>	24078.6216	90.36963	3	<b>3</b>
	400	<b>3.3042</b>	3045.3292	67.17167	3	<b>3</b>
	1000	<b>4.2432</b>	610.4686	1154.375	3	<b>3</b>
SSO	30	346.1439	0.18953	75.95701	3	<b>3</b>
	100	710.5087	1.2212	15.27657	3.000677	<b>3</b>
	400	91.9061	1.4806	31.75424	3	<b>3</b>
	1000	13.9474	0.048025	4.277457	3	<b>3</b>
GA	30	18301161.1854	0.023637	64052.63	5.059146	<b>3</b>
	100	1264107.5031	93246.1569	689269.2	3	<b>3</b>
	400	69346.2974	0.18467	3.999575	3	<b>3</b>
	1000	77245.4291	2580.6349	81517.88	3	<b>3</b>

**Figure 20.** Comparison of experimental results between HGASSO, SSO, and GA in term of convergence for the Goldstein and Price (GP) test function

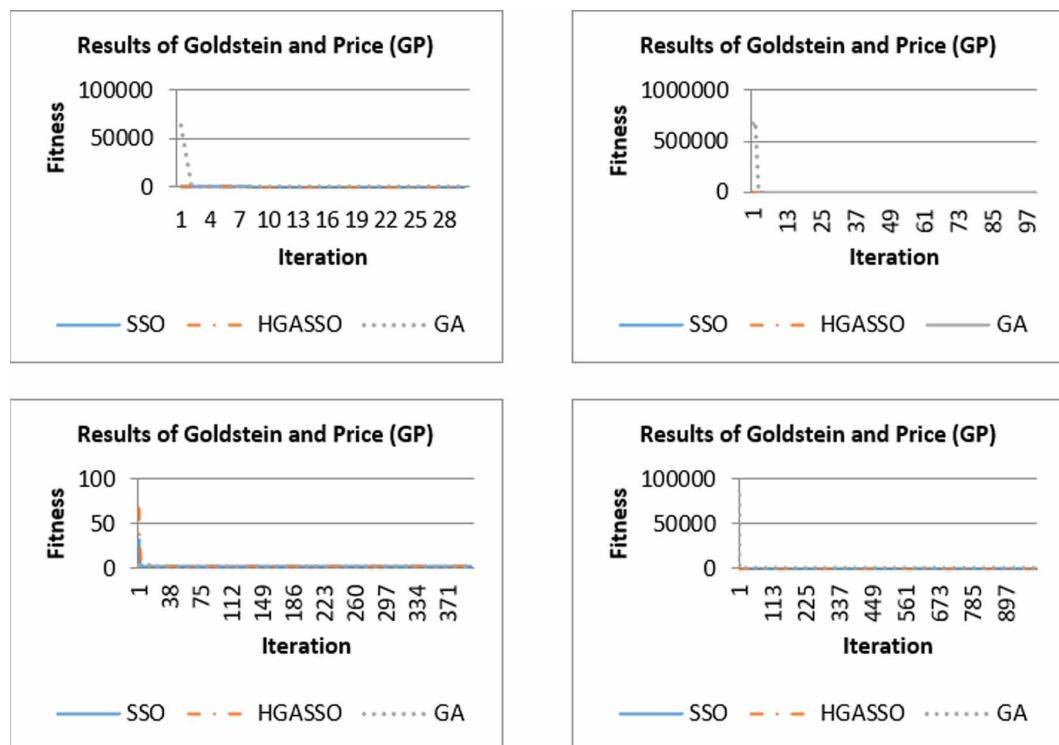
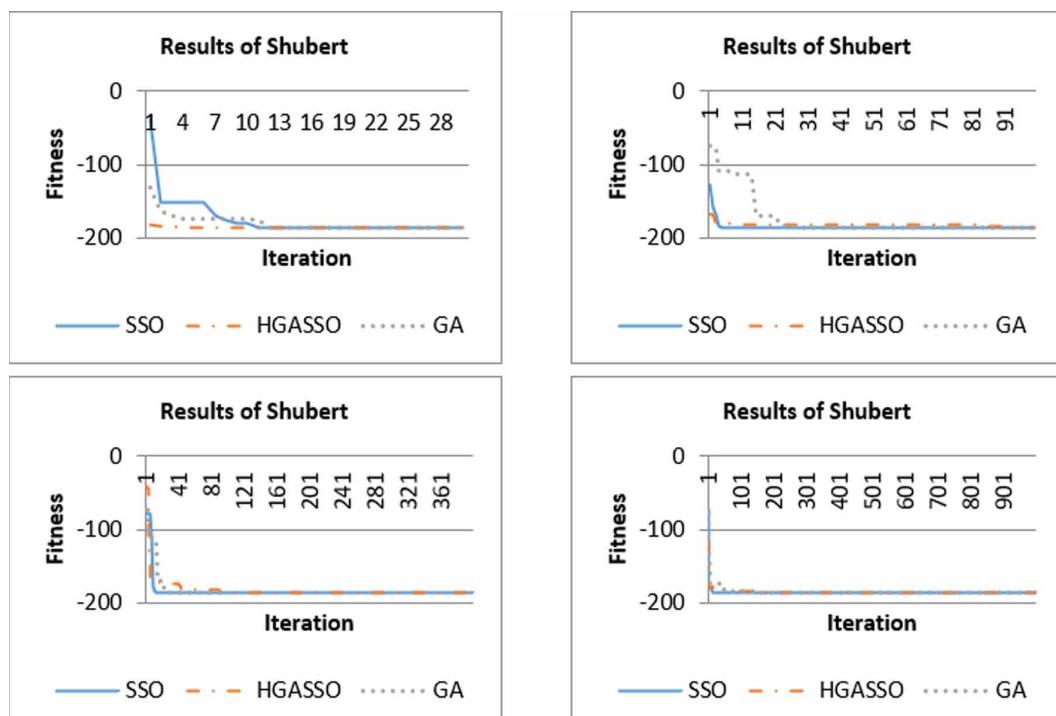


Table 11. Comparison of experimental results between HGASSO, SSO, and GA in terms of mean, standard deviation, maximum value, median, and best fitness for the Shubert test function. The bold font with the highlighted background represents the best fitness and mean for the respective benchmark function

Algorithm	Iteration no.	Average, mean	Std.	Max.	Median	Best fitness
<b>Function: Shubert</b>						
HGASSO	30	<b>-172.1084</b>	7.2895	-181.491	-186.711	<b>-186.7309</b>
	100	<b>-175.6017</b>	4.5234	-167.469	-186.926	<b>-186.7309</b>
	400	<b>-183.1624</b>	15.3322	-39.1642	-186.7	<b>-186.7309</b>
	1000	<b>-186.2046</b>	2.6593	-114.26	-186.707	<b>-186.7309</b>
SSO	30	-83.8316	28.3472	-38.3897	-186.728	<b>-186.7309</b>
	100	-167.5589	6.8631	-126.795	-186.731	<b>-186.7309</b>
	400	-179.9879	15.2599	-71.217	-186.731	<b>-186.7309</b>
	1000	-181.9694	6.2583	-73.0234	-186.731	<b>-186.7309</b>
GA	30	-100.130	11.0655	-132.217	-186.411	-186.1020
	100	-164.6632	28.7466	-75.3547	-185.645	-186.5432
	400	-175.7541	22.8959	-26.1109	-186.731	<b>-186.7309</b>
	1000	-185.0282	5.742	-79.429	-186.731	<b>-186.7309</b>

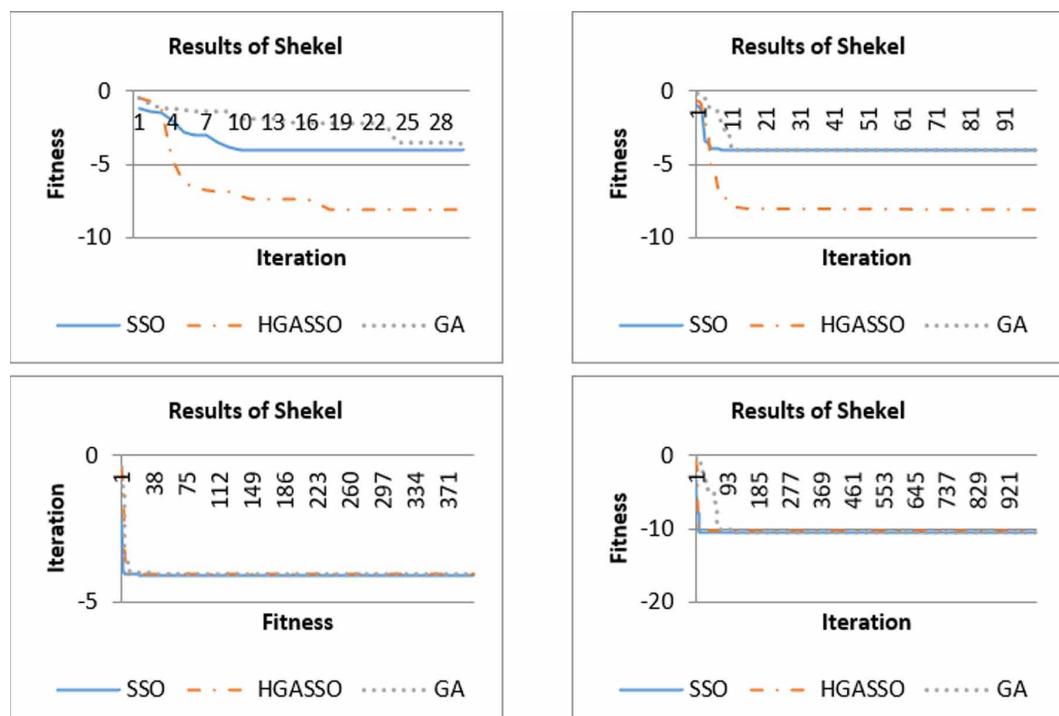
Figure 21. Comparison of experimental results between HGASSO, SSO, and GA in term of convergence for the Shubert test function



**Table 12.** Comparison of experimental results between HGASSO, SSO, and GA in terms of mean, standard deviation, maximum value, median, and best fitness for the Shekel test function. The bold font with the highlighted background represents the best fitness and mean for the respective benchmark function

Algorithm	Iteration no.	Average, mean	Std.	Max.	Median	Best fitness
<b>Function: Shekel</b>						
<b>HGASSO</b>	30	<b>-6.8296</b>	2.1785	-0.47689	-7.37539	<b>-8.109</b>
	100	<b>-7.7206</b>	1.336	-0.70059	-8.06945	<b>-8.1013</b>
	400	<b>-5.0144</b>	0.29884	-0.42681	-4.04711	<b>-8.6471</b>
	1000	-10.1939	0.54918	-0.4664	-10.2329	<b>-10.5</b>
<b>SSO</b>	30	-3.0108	0.88317	-1.20946	-4.06778	-4.0699
	100	-3.8949	0.40467	-0.97569	-4.0687	-4.066
	400	-4.0436	0.097992	-2.14548	-4.0687	-4.0687
	1000	<b>-10.4368</b>	0.3629	-3.40547	-10.5	<b>-10.5</b>
<b>GA</b>	30	-1.8848	0.8852	-0.48731	-2.19278	-3.5802
	100	-3.6846	0.85376	-0.19777	-4.0617	-4.0617
	400	-3.9919	0.32832	-0.37066	-4.05631	-4.0775
	1000	-10.246	0.96724	-0.30462	-10.5317	<b>-10.5</b>

**Figure 22.** Comparison of experimental results between HGASSO, SSO, and GA in term of convergence for the Shekel test function



**Table 13.** Ranking the algorithms from best to worse obtained fitness value

Function	The approaches are ranked from the best-obtained fitness value to the worse obtained fitness value
Rastrigin	HGASSO, SSO, GA
Ackley	HGASSO, SSO, GA
Bukin	HGASSO, SSO, GA
Griewank	HGASSO, GA, SSO
Cross-in-Tray	All in the same rank
Levy function N.13	HGASSO, SSO, GA
Branin	All in the same rank
Easom	HGASSO and SSO in the first rank, followed by GA
Goldstein and Price (GP)	All in the same rank
Shubert	HGASSO and SSO in the first rank, followed by GA
Shekel	HGASSO, SSO, GA

Table 14, HGASSO significantly outperforms GA in solving the majority of benchmarks functions. This is clear in the *p*-Value and mean difference between HGASSO and the other approaches, which is significant at the 0.05 level.

At the end, comparisons between SSO, GA, and HGASSO can be summarized as in Table 15 (Shehadeh et al., 2017; Shehadeh et al., 2018c).

## DISCUSSION

Multimodal problems include a set of mathematical operations in their evaluation, such as cosine, sine, exponent, and multiplications of multiple variables. Furthermore, these problems mainly have several local extremes, which many optimization algorithms are trapped in one of their several local minima. Based on that, many local search approaches have been hybridized with evolutionary algorithms to skip these local minima easily until obtaining the optimal solution of global minima. The concept of the hybrid algorithm is usually used to combine the concept of the evolutionary method with the concept of local search in one procedure. SSO method is a promising optimization approach that is inspired by sperm motility. This method is suitable for solving variant types of continuous nonlinear functions, but until recently it had not been hybridized with evolutionary algorithms to deal with multimodal problems.

In this paper, HGASSO, SSO, and GA have been evaluated on multimodal test bed suites. Standard metrics are calculated in the experiments, namely the best fitness value, mean value, median, maximum value, standard deviation and the algorithm convergence. In addition, the statistical test, called “one-way ANOVA (Tukey’s test)” is adopted in this study to compare between approaches. For each approach evaluated in the experiment, the maximum iteration for each benchmark function is set to 30, 100, 400, and 1000 respectively. The evaluation is reaccredited 10 times for each benchmark test function to ensure the convergence of the results.

Overall, the proposed HGASSO algorithm outperformed both SSO and GA algorithms in solving benchmarks functions that include the cosine, sine, and exponent in their evaluation. HGASSO outperformed SSO and GA in solving the Rastrigin, Ackley, Bukin, Griewank, Levy function N.13, and Shekel multimodal functions, and obtained better fitness value than GA of Easom and Shubert multimodal functions. Additionally, the high-quality performance of the proposed HGASSO was

**Table 14. Analysis of “one-way ANOVA (Tukey’s test)” between SSO, GA and HGASSO for all benchmarks functions**

Benchmark function	Algorithm (I)	Algorithm (J)	Mean Difference (I-J)	p-Value
Rastrigin	HGASSO	GA	-0.244169	0.043378
		SSO	-0.006159	<0.001*
Ackley	HGASSO	GA	-1.7273322	<0.001*
		SSO	-0.0023981	<0.001*
Bukin	HGASSO	GA	-0.43538	<0.001*
		SSO	-0.48207	0.794144
Griewank	HGASSO	GA	-0.0095911	<0.001*
		SSO	-0.3452741	0
Cross-in-Tray	HGASSO	GA	-0.0005	0.240279
		SSO	0	0.654482
Levy function N.13	HGASSO	GA	-0.09938755	0.096145
		SSO	-1.02799055	0.720205
Branin	HGASSO	GA	-0.17267	0.127417
		SSO	-0.03076	0.197382
Easom	HGASSO	GA	-0.06765	0
		SSO	-0.01466	0.012605
Goldstein and Price (GP)	HGASSO	GA	-77241.1859	0.288343
		SSO	-9.7042	0.122608
Shubert	HGASSO	GA	-1.1764	<0.001*
		SSO	-4.2352	0.363168
Shekel	HGASSO	GA	0.0521	0.017848
		SSO	0.2429	<0.001*

\*Mean Difference is significant at the 0.05 level.

reflected on the metrics of convergence in which attained a typical convergence regarding to these benchmark functions. This proves that the proposed HGASSO has a better convergence than the other algorithms to explore the domain of search space and to skip the multiple local minima easily.

## CONCLUSION

In this article, a hybrid approach consists of two methods has been proposed to solve multimodal benchmark functions. This method integrates the concept of discreet procedures of the evolutionary method that is originally inspired by GA with the concept of local search of SSO. The proposed approach is validated in two ways. First, we used the standard multimodal problems that are currently adopted in the evolutionary single objective optimization community. The fitness value, average, maximum value, median value and standard deviation have been chosen to compare between the proposed algorithm and other algorithms based on a set of generation sizes, such as 30, 100, 400, and 1000. Furthermore, a statistical test, called “one-way ANOVA (Tukey’s test)” is used in this study to compare between approaches. Second, the algorithm convergence of each algorithm for

**Table 15. Comparison between SSO, GA, and HGASSO (Shehadeh et al., 2017; Shehadeh et al., 2018c)**

<b>Criteria of comparison</b>	<b>SSO</b>	<b>GA</b>	<b>HGASSO</b>
Type of procedure	Continuous procedure.	Discrete procedure.	Hybrid procedure.
Type of a metaphor	Mimics the motility of swarm of sperm through the natural fertilization procedure of an egg.	Mimics the theory of evolution applied to biology that proposed by Darwin. This procedure simulates the structure of the chromosome and its evolution.	Mimics the theory of evolution applied to biology that proposed by Darwin as well as it uses the local search procedure of SSO.
Sorting the population	Population will not be sorted.	Population needs ranking and selection.	The population will be sorted at the end of the procedure to determine the global best solution.
Use crossover operation	No need for crossover Operation.	Apply crossover operation.	Apply crossover operation.
Use mutation operation	No need for mutation.	Apply mutation operation.	Apply mutation operation.
Best solution effects Population	The value of winner (the best solution) is used as a reference value for the swarm to modify their velocities.	Treat with every individual independently.	Apply a local search strategy of SSO to enhance the results.

each benchmark function has been drawn, after that, compared based on different generation sizes. The results of these comparisons demonstrated that HGASSO synthesizes the merits of both SSO and GA, and it is yet effective model and simple to handle various types of continuous optimization problems. Moreover, the experimental results show that the proposed HGASSO is a viable alternative since HGASSO has a good ability to solve the multimodal benchmark functions, which is superior to the other approaches in the ability to reaching the global optimum easily.

The benchmarks models in this study may have their limitations, which other metrics in real experiments may affect the outcomes of the studies. Hence, in the future, the proposed HGASSO should be implemented to solve real problems that utilize cosine, sine, and exponent in there mathematical modeling such as minimizing propagation loss and interference of wireless, telecommunication and mobile networks (Shehadeh et al., 2020; Hamdan et al., 2015; Hamdan, 2018), engineering applications (Jayabarathi et al., 2018), designing the optimal spacing and length of Yagi–Uda antenna (Baumgartner et al., 2017). Furthermore, in the future, the proposed HGASSO should be compared against a set of well-known approaches, such as “Chaotic Gravitational Search Algorithm (CGSA)” (Rather & Bala, 2020), “Grey Wolf Optimizer (GWO)” (Mirjalili et al., 2014), and “Sine Cosine Algorithm (SCA)” (Mirjalili et al., 2016).

## REFERENCES

- Adorio, E. P., & Diliman, U. (2005). Mvf-multivariate test functions library in c for unconstrained global optimization. *Quezon City, Metro Manila, Philippines*, 4, 100–104.
- Back, T. (1996). *Evolutionary algorithms in theory and practice: evolution strategies, evolutionary programming, genetic algorithms*. Oxford University Press. doi:10.1093/oso/9780195099713.001.0001
- Banharaksakun, A., Achalakul, T., & Sirinaovakul, B. (2011). The best-so-far selection in artificial bee colony algorithm. *Applied Soft Computing*, 11(2), 2888–2901. doi:10.1016/j.asoc.2010.11.025
- Baumgartner, P., Bauernfeind, T., Biro, O., Hackl, A., Magele, C., Renhart, W., & Torchio, R. (2017). Multi-objective optimization of Yagi-Uda antenna applying enhanced firefly algorithm with adaptive cost function. *IEEE Transactions on Magnetics*, 54(3), 1–4. doi:10.1109/TMAG.2017.2764319
- Chaudhary, D., & Kumar, B. (2019). Cost optimized hybrid genetic-gravitational search algorithm for load scheduling in cloud computing. *Applied Soft Computing*, 83, 105627. doi:10.1016/j.asoc.2019.105627
- Fang, N., Zhou, J., Zhang, R., Liu, Y., & Zhang, Y. (2014). A hybrid of real coded genetic algorithm and artificial fish swarm algorithm for short-term optimal hydrothermal scheduling. *International Journal of Electrical Power & Energy Systems*, 62, 617–629. doi:10.1016/j.ijepes.2014.05.017
- Fogel, D. B. (2005). *Evolutionary Computation: Toward a New Philosophy of Machine Intelligence* (3rd ed.). Wiley-IEEE Press. doi:10.1002/0471749214
- Gacem, A., & Benattous, D. (2017). Hybrid genetic algorithm and particle swarm for optimal power flow with non-smooth fuel cost functions. *International Journal of System Assurance Engineering and Management*, 8(1), 146–153. doi:10.1007/s13198-014-0312-8
- Gholami, A., Bonakdari, H., Ebtehaj, I., Mohammadian, M., Gharabaghi, B., & Khodashenas, S. R. (2018). Uncertainty analysis of intelligent model of hybrid genetic algorithm and particle swarm optimization with ANFIS to predict threshold bank profile shape based on digital laser approach sensing. *Measurement*, 121, 294–303. doi:10.1016/j.measurement.2018.02.070
- Goldberg, D. E., & Holland, J. H. (1988). Genetic algorithms and machine learning. *Machine Learning*, 3(2/3), 95–99. doi:10.1023/A:1022602019183
- Hamdan, M., Bani-Yaseen, M., & Shehadeh, H. A. (2018). Multi-objective optimization modeling for the impacts of 2.4-GHz ISM band interference on IEEE 802.15. 4 health sensors. In L. Ismail & L. Zhang (Eds.), *Information Innovation Technology in Smart Cities* (pp. 317–330). Springer. doi:10.1007/978-981-10-1741-4\_21
- Hamdan, M., Yassein, M. B., & Shehadeh, H. A. (2015). Multi-objective optimization modeling of interference in home health care sensors. *11th international conference on innovations in information technology (IIT)*, 219–224. doi:10.1109/INNOVATIONS.2015.7381543
- Jayabarathi, T., Raghunathan, T., & Gandomi, A. H. (2018). The bat algorithm, variants and some practical engineering applications: a review. In *Nature-Inspired Algorithms and Applied Optimization*. Springer. doi:10.1007/978-3-319-67669-2\_14
- Kaipa, K. N., & Ghose, D. (2017). Multimodal function optimization. In K. N. Kaipa & D. Ghose (Eds.), *Book: Glowworm Swarm Optimization* (pp. 91–131). Springer. doi:10.1007/978-3-319-51595-3\_4
- Kao, Y. T., & Zahara, E. (2008). A hybrid genetic algorithm and particle swarm optimization for multimodal functions. *Applied Soft Computing*, 8(2), 849–857. doi:10.1016/j.asoc.2007.07.002
- Krzeszowski, T., & Wiktorowicz, K. (2016). Evaluation of selected fuzzy particle swarm optimization algorithms. In *Federated Conference on Computer Science and Information Systems (FedCSIS)*. IEEE. doi:10.15439/2016F206
- Langdon, W. B., & Poli, R. (2002). *Foundations of Genetic Programming* (3rd ed.). Springer Science & Business Media. doi:10.1007/978-3-662-04726-2
- Mirjalili, S. (2016). SCA: A sine cosine algorithm for solving optimization problems. *Knowledge-Based Systems*, 96, 120–133. doi:10.1016/j.knosys.2015.12.022

- Mirjalili, S., Mirjalili, S. M., & Lewis, A. (2014). Grey wolf optimizer. *Advances in Engineering Software*, 69, 46–61. doi:10.1016/j.advengsoft.2013.12.007
- Moin, N. H., Chung Sin, O., & Omar, M. (2015). Hybrid genetic algorithm with multiparents crossover for job shop scheduling problems. *Mathematical Problems in Engineering*, 12, 1–12. doi:10.1155/2015/210680
- Molga, M., & Smutnicki, C. (2005). *Test functions for optimization needs*. Majid Vafaei Jahan Web. <https://www.vafaeijahan.com/en/wp-content/uploads/2012/02/Test-functions-for-optimization-needs.pdf>
- Paulinas, M., & Ušinskas, A. (2007). A survey of genetic algorithms applications for image enhancement and segmentation. *Inf. Technol. Information Technology and Control*, 36, 278–284. <https://itc.ktu.lt/index.php/ITC/article/view/11886>
- Pomar, L. A., Pulido, E. C., & Roa, J. D. T. (2017). A hybrid genetic algorithm and particle swarm optimization for flow shop scheduling problems. In *4th Workshop on Engineering Applications*, WEA. Springer.
- Rather, S. A., & Bala, P. S. (2020). Swarm-based chaotic gravitational search algorithm for solving mechanical engineering design problems. *World Journal of Engineering*, 17(1), 97–114. doi:10.1108/WJE-09-2019-0254
- Rbouh, I., & El Imrani, A. A. (2014). Hurricane Search algorithm a new model for function optimization. In *2014 5th International Conference on Information and Communication Systems (ICICS)*. IEEE. doi:10.1109/IACS.2014.6841947
- Samuel, G. G., & Rajan, C. C. A. (2015). Hybrid: Particle swarm optimization–genetic algorithm and particle swarm optimization–shuffled frog leaping algorithm for long-term generator maintenance scheduling. *International Journal of Electrical Power & Energy Systems*, 65, 432–442. doi:10.1016/j.ijepes.2014.10.042
- Saravanan, R., & Sachithanandam, M. (2001). Genetic algorithm (GA) for multivariable surface grinding process optimisation using a multi-objective function model. *International Journal of Advanced Manufacturing Technology*, 17(5), 330–338. doi:10.1007/s001700170167
- Sathy, S. S., & Radhika, M. V. (2013). Convergence of nomadic genetic algorithm on benchmark mathematical functions. *Applied Soft Computing*, 13(5), 2759–2766. doi:10.1016/j.asoc.2012.11.011
- Shehadeh, H. A., Ahmedy, I., & Idris, M. Y. I. (2018). Sperm swarm optimization algorithm for optimizing wireless sensor network challenges. *Proceedings of the ACM International Conference on Communications and Broadband Networking (ICCBN)*, 53–59. doi:10.1145/3193092.3193100
- Shehadeh, H. A., Ahmedy, I., & Idris, M. Y. I. (2018). Empirical study of sperm swarm optimization algorithm. In *Proceedings of SAI Intelligent Systems Conference*. Springer. doi:10.1007/978-3-030-01057-7\_80
- Shehadeh, H. A., Idna Idris, M. Y., Ahmedy, I., Ramli, R., & Mohamed Noor, N. (2018). The Multi-objective optimization algorithm based on sperm fertilization procedure (MOSFP) method for solving wireless sensor networks optimization problems in smart grid applications. *Energies*, 11(1), 97. doi:10.3390/en11010097
- Shehadeh, H. A., Idris, M. Y. I., Ahmedy, I., & Hassen, H. R. (2020). Optimal placement of near ground VHF/UHF radio communication network as a multi objective problem. *Wireless Personal Communications*, 110(3), 1169–1197. doi:10.1007/s11277-019-06780-6
- Shehadeh, H. A., Idris, M., & Ahmedy, I. (2017). Multi-objective optimization algorithm based on sperm fertilization procedure (MOSFP). *Symmetry*, 9(10), 241. doi:10.3390/sym9100241
- Shi, Y., & Eberhart, R. C. (1999). Empirical study of particle swarm optimization. *IEEE Proceedings of the 1999 Congress on Evolutionary Computation-CEC99*, 1945–1950. doi: doi:10.1109/CEC.1999.785511
- Soleimani, H., & Kannan, G. A. (2015). A hybrid particle swarm optimization and genetic algorithm for closed-loop supply chain network design in large-scale networks. *Applied Mathematical Modelling*, 39(14), 3990–4012. doi:10.1016/j.apm.2014.12.016
- Surjanovic, S., & Bingham, D. (2013). *Virtual library of simulation experiments: test functions and datasets*. <https://www.sfu.ca/~ssurjano/optimization.html>

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