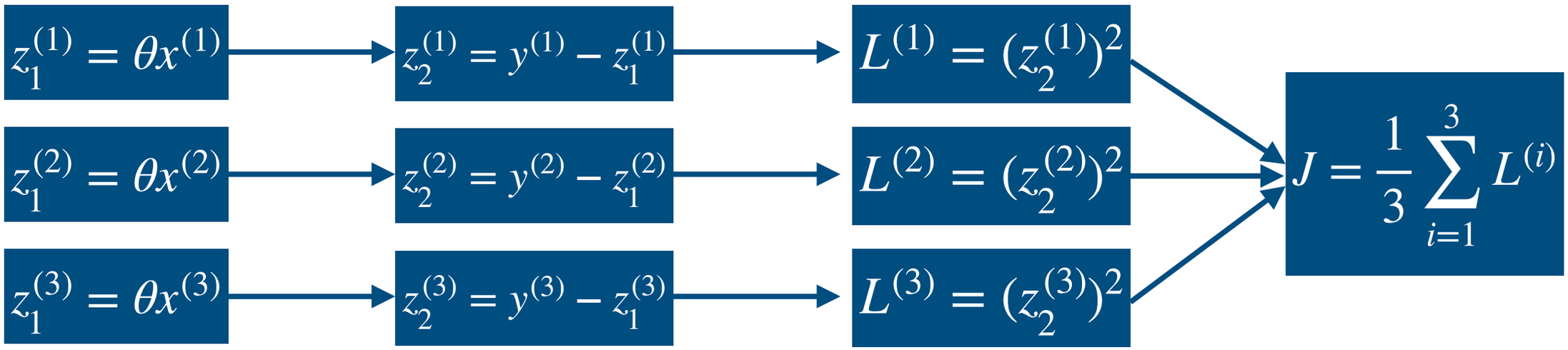
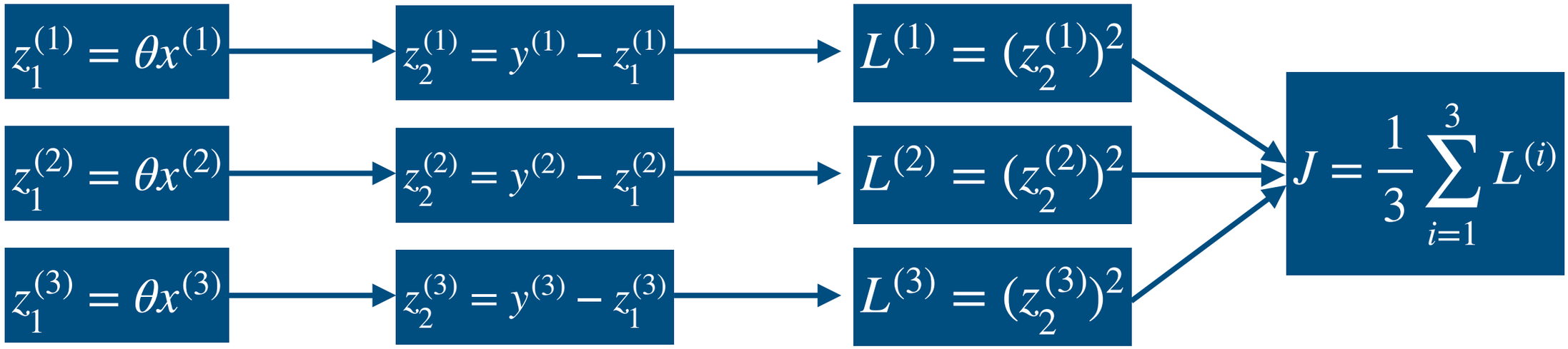
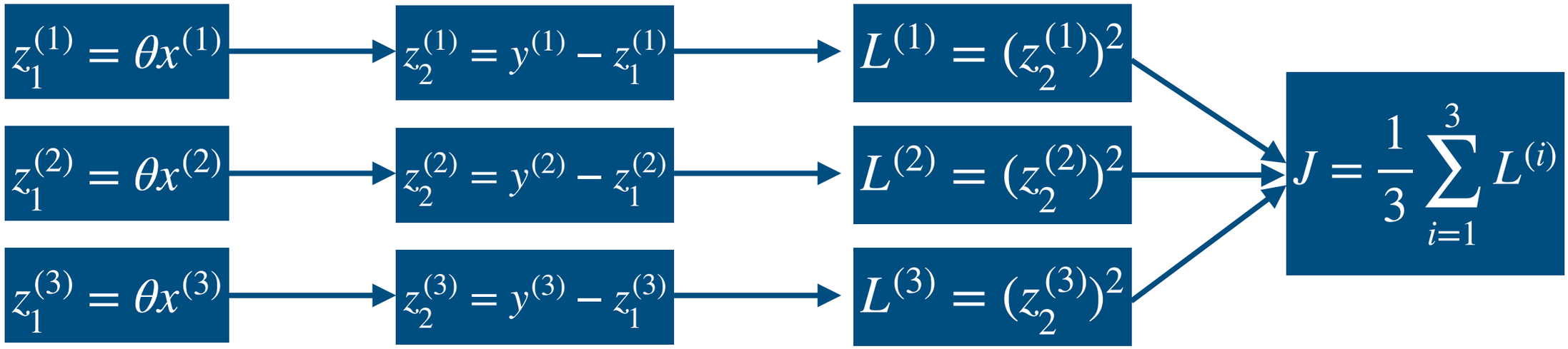
$$J(\theta) = \frac{1}{N}\sum_{i=1}^{N} L^{(i)} = \frac{1}{N}\sum_{i=1}^{N} (y^{(i)} - \theta x^{(i)})^2$$

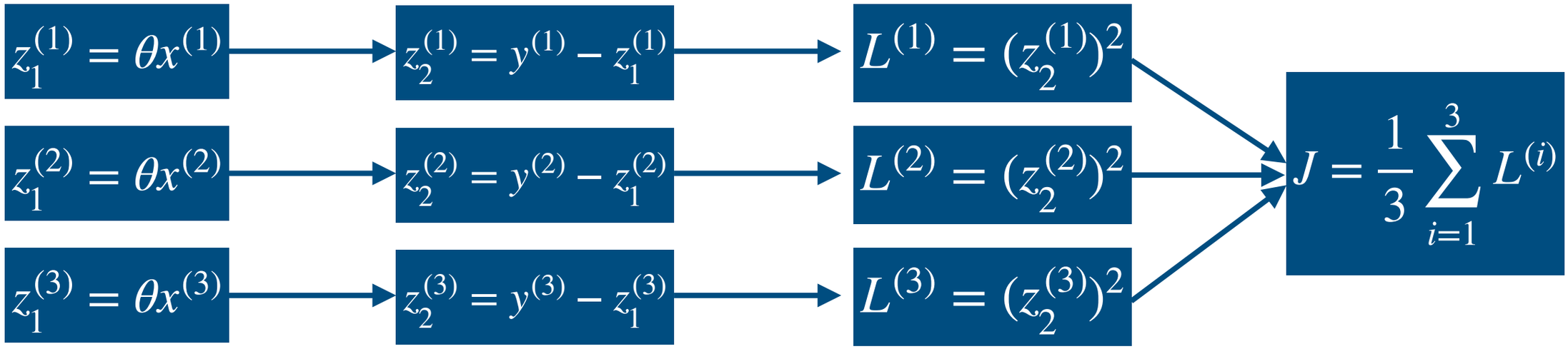




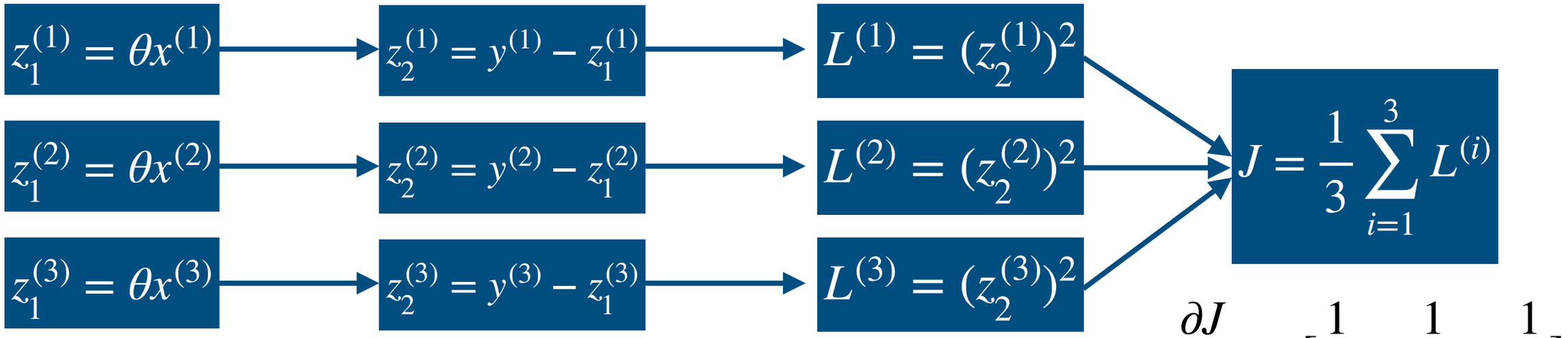
$$\frac{\partial J}{\partial \vec{L}}$$



$$\frac{\partial J}{\partial \vec{L}} = [\frac{\partial J}{\partial L^{(1)}}, \quad \frac{\partial J}{\partial L^{(2)}}, \quad \frac{\partial J}{\partial L^{(3)}}]$$



$$\frac{\partial J}{\partial \vec{L}} = [\frac{\partial J}{\partial L^{(1)}}, \quad \frac{\partial J}{\partial L^{(2)}}, \quad \frac{\partial J}{\partial L^{(3)}}] = [\frac{1}{3}, \quad \frac{1}{3}, \quad \frac{1}{3}]$$



$$\frac{\partial J}{\partial \vec{L}} = [\frac{1}{3}, \quad \frac{1}{3}, \quad \frac{1}{3}]$$

$$z_1^{(1)} = \theta x^{(1)} \rightarrow z_2^{(1)} = y^{(1)} - z_1^{(1)} \rightarrow L^{(1)} = (z_2^{(1)})^2$$

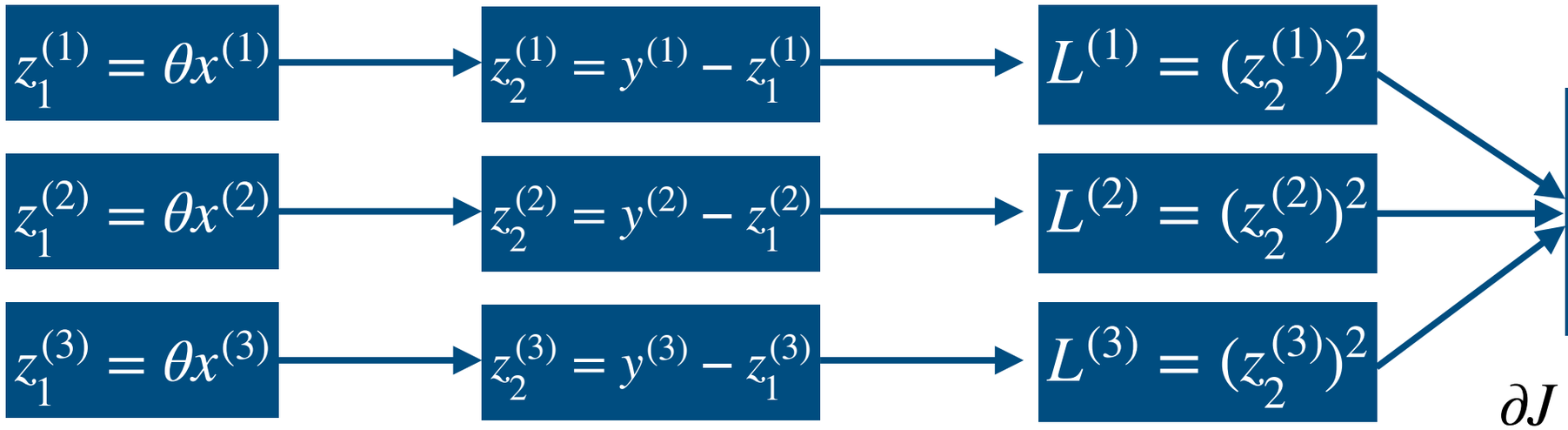
$$z_1^{(2)} = \theta x^{(2)} \rightarrow z_2^{(2)} = y^{(2)} - z_1^{(2)} \rightarrow L^{(2)} = (z_2^{(2)})^2$$

$$z_1^{(3)} = \theta x^{(3)} \rightarrow z_2^{(3)} = y^{(3)} - z_1^{(3)} \rightarrow L^{(3)} = (z_2^{(3)})^2$$

$$J = \frac{1}{3}\sum_{i=1}^{3} L^{(i)}$$

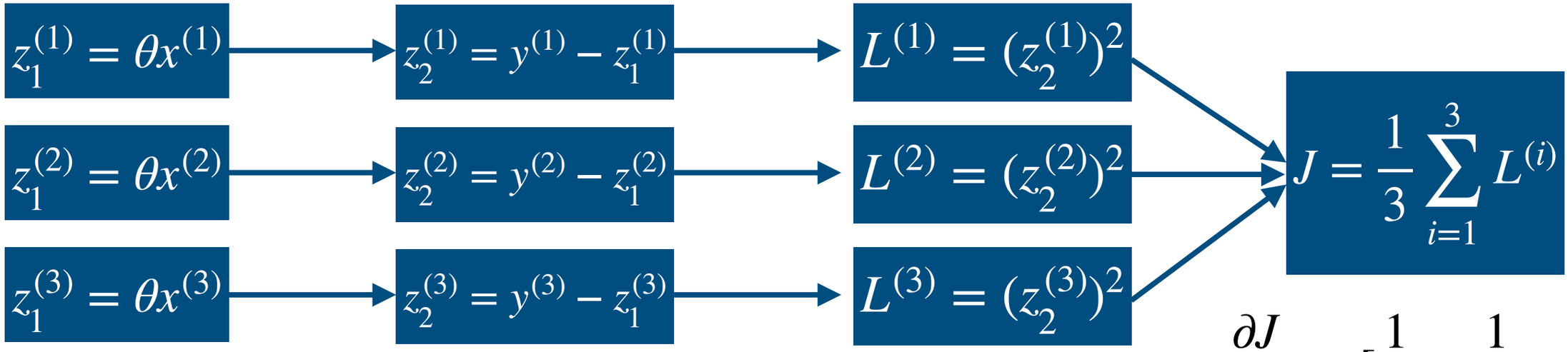
$$\frac{\partial J}{\partial \vec{L}} = [\frac{1}{3}, \quad \frac{1}{3}, \quad \frac{1}{3}]$$

$$\frac{\partial \vec{L}}{\partial \vec{z_2}}$$



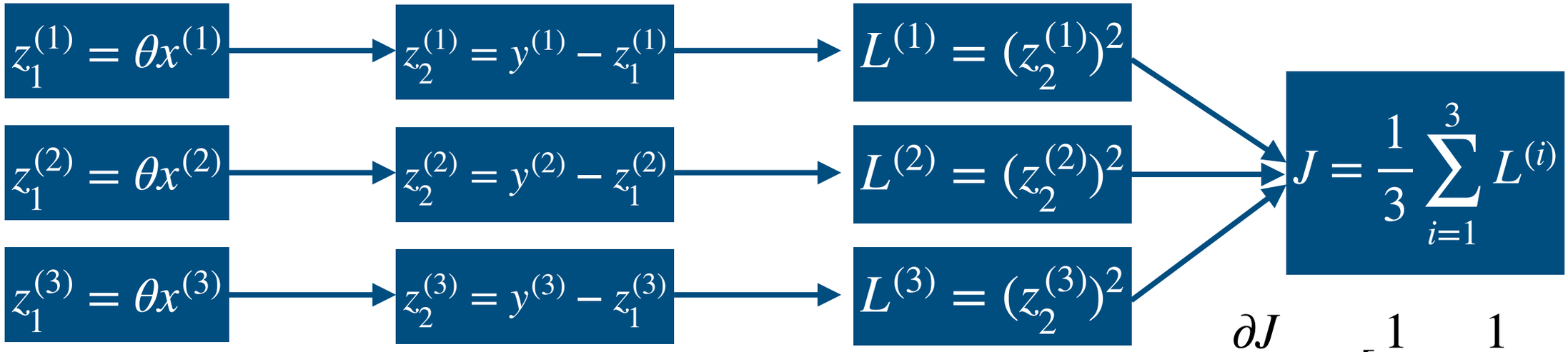
$$\frac{\partial J}{\partial \overrightarrow{L}} = [\frac{1}{3}, \quad \frac{1}{3}, \quad \frac{1}{3}]$$

$$\frac{\partial \overrightarrow{L}}{\partial \overrightarrow{z_2}} = \begin{bmatrix} \frac{\partial L^{(1)}}{\partial z_2^{(1)}} & \frac{\partial L^{(1)}}{\partial z_2^{(2)}} & \frac{\partial L^{(1)}}{\partial z_2^{(3)}} \\ \frac{\partial L^{(2)}}{\partial z_2^{(1)}} & \frac{\partial L^{(2)}}{\partial z_2^{(2)}} & \frac{\partial L^{(2)}}{\partial z_2^{(3)}} \\ \frac{\partial L^{(3)}}{\partial z_2^{(1)}} & \frac{\partial L^{(3)}}{\partial z_2^{(2)}} & \frac{\partial L^{(3)}}{\partial z_2^{(3)}} \end{bmatrix}$$



$$\frac{\partial J}{\partial \vec{L}} = [\frac{1}{3}, \quad \frac{1}{3}, \quad \frac{1}{3}]$$

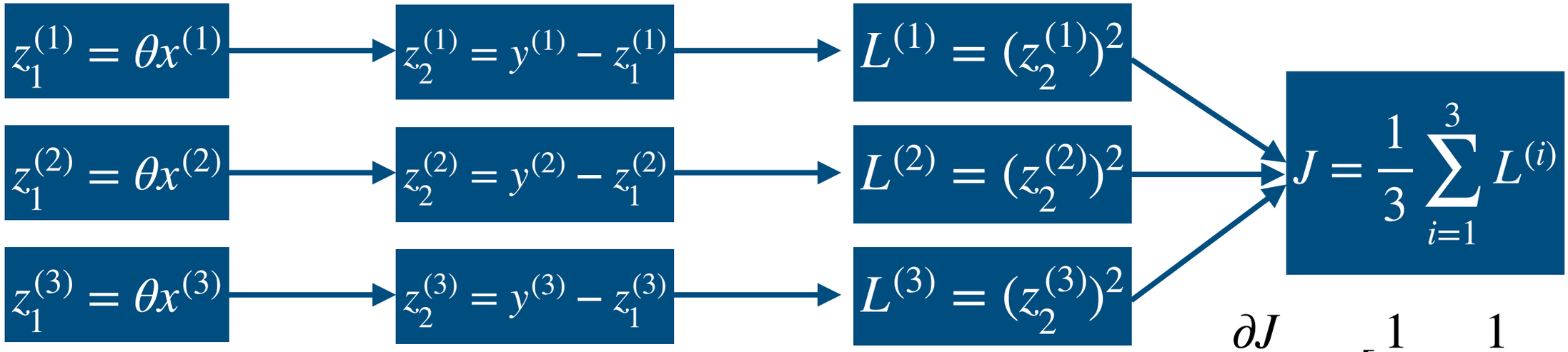
$$\frac{\partial \vec{L}}{\partial \vec{z_2}} = \begin{bmatrix} \frac{\partial L^{(1)}}{\partial z_2^{(1)}} & \frac{\partial L^{(1)}}{\partial z_2^{(2)}} & \frac{\partial L^{(1)}}{\partial z_2^{(3)}} \\ \frac{\partial L^{(2)}}{\partial z_2^{(1)}} & \frac{\partial L^{(2)}}{\partial z_2^{(2)}} & \frac{\partial L^{(2)}}{\partial z_2^{(3)}} \\ \frac{\partial L^{(3)}}{\partial z_2^{(1)}} & \frac{\partial L^{(3)}}{\partial z_2^{(2)}} & \frac{\partial L^{(3)}}{\partial z_2^{(3)}} \end{bmatrix} = \begin{bmatrix} 2z_2^{(1)} & 0 & 0 \\ 0 & 2z_2^{(2)} & 0 \\ 0 & 0 & 2z_2^{(2)} \end{bmatrix}$$



$$\frac{\partial J}{\partial \vec{L}} = [\frac{1}{3}, \quad \frac{1}{3}, \quad \frac{1}{3}]$$

$$\frac{\partial \vec{L}}{\partial \vec{z_2}} = \begin{bmatrix} \frac{\partial L^{(1)}}{\partial z_2^{(1)}} & \frac{\partial L^{(1)}}{\partial z_2^{(2)}} & \frac{\partial L^{(1)}}{\partial z_2^{(3)}} \\ \frac{\partial L^{(2)}}{\partial z_2^{(1)}} & \frac{\partial L^{(2)}}{\partial z_2^{(2)}} & \frac{\partial L^{(2)}}{\partial z_2^{(3)}} \\ \frac{\partial L^{(3)}}{\partial z_2^{(1)}} & \frac{\partial L^{(3)}}{\partial z_2^{(2)}} & \frac{\partial L^{(3)}}{\partial z_2^{(3)}} \end{bmatrix} = \begin{bmatrix} 2z_2^{(1)} & 0 & 0 \\ 0 & 2z_2^{(2)} & 0 \\ 0 & 0 & 2z_2^{(2)} \end{bmatrix}$$

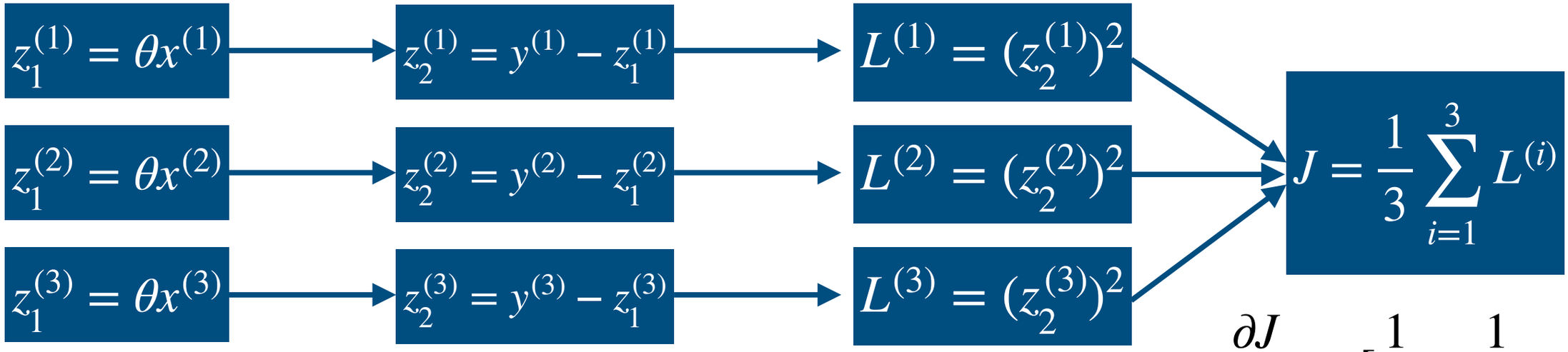
$$\frac{\partial J}{\partial \vec{z_2}} = \frac{\partial J}{\partial \vec{L}} \frac{\partial \vec{L}}{\partial \vec{z_2}}$$



$$\frac{\partial J}{\partial \vec{L}} = [\frac{1}{3}, \quad \frac{1}{3}, \quad \frac{1}{3}]$$

$$\frac{\partial \vec{L}}{\partial \vec{z_2}} = \begin{bmatrix} \frac{\partial L^{(1)}}{\partial z_2^{(1)}} & \frac{\partial L^{(1)}}{\partial z_2^{(2)}} & \frac{\partial L^{(1)}}{\partial z_2^{(3)}} \\ \frac{\partial L^{(2)}}{\partial z_2^{(1)}} & \frac{\partial L^{(2)}}{\partial z_2^{(2)}} & \frac{\partial L^{(2)}}{\partial z_2^{(3)}} \\ \frac{\partial L^{(3)}}{\partial z_2^{(1)}} & \frac{\partial L^{(3)}}{\partial z_2^{(2)}} & \frac{\partial L^{(3)}}{\partial z_2^{(3)}} \end{bmatrix} = \begin{bmatrix} 2z_2^{(1)} & 0 & 0 \\ 0 & 2z_2^{(2)} & 0 \\ 0 & 0 & 2z_2^{(2)} \end{bmatrix}$$

$$\frac{\partial J}{\partial \vec{z_2}} = \frac{\partial J}{\partial \vec{L}} \frac{\partial \vec{L}}{\partial \vec{z_2}} = [\frac{1}{3}, \quad \frac{1}{3}, \quad \frac{1}{3}] \begin{bmatrix} 2z_2^{(1)} & 0 & 0 \\ 0 & 2z_2^{(2)} & 0 \\ 0 & 0 & 2z_2^{(2)} \end{bmatrix}$$

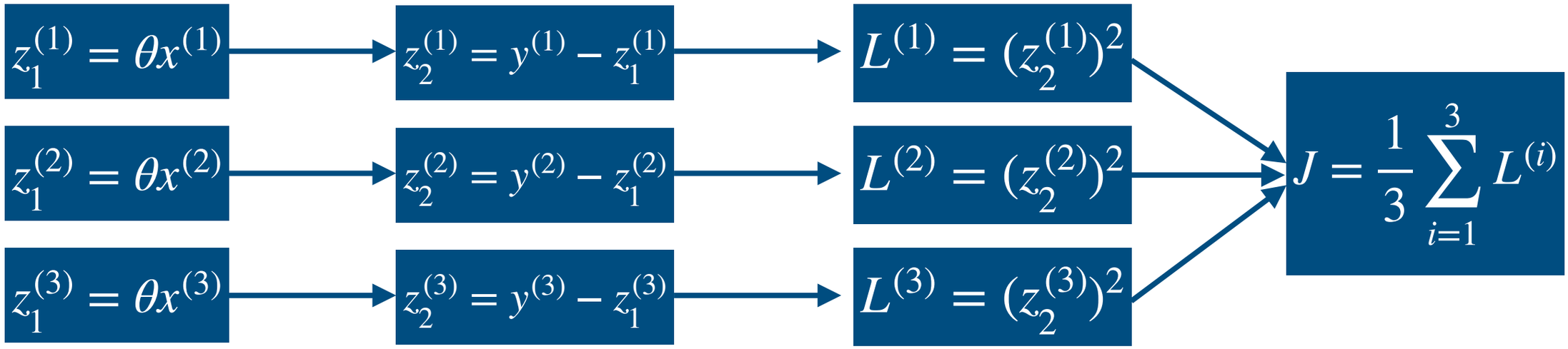


$$\frac{\partial J}{\partial \vec{L}} = [\frac{1}{3}, \quad \frac{1}{3}, \quad \frac{1}{3}]$$

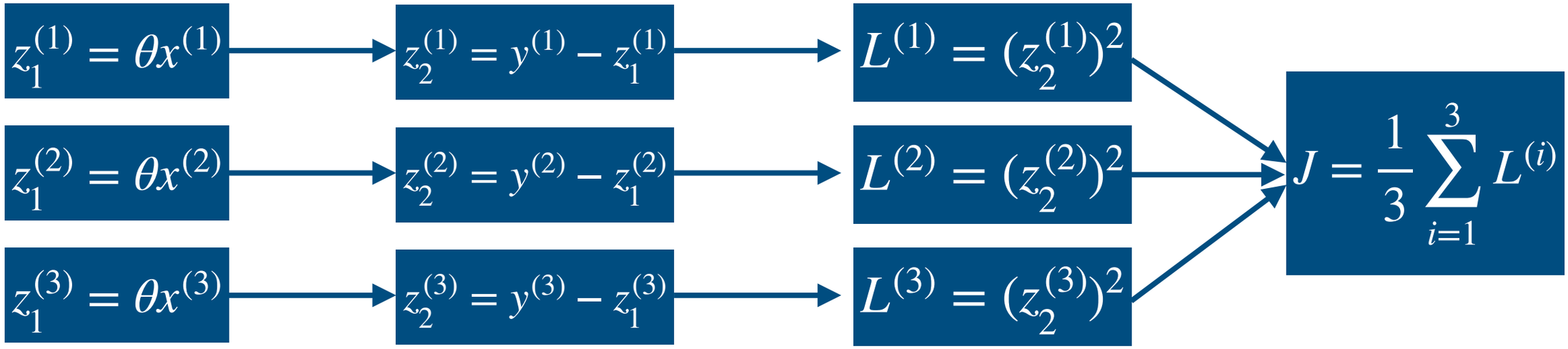
$$\frac{\partial \vec{L}}{\partial \vec{z_2}} = \begin{bmatrix} \frac{\partial L^{(1)}}{\partial z_2^{(1)}} & \frac{\partial L^{(1)}}{\partial z_2^{(2)}} & \frac{\partial L^{(1)}}{\partial z_2^{(3)}} \\ \frac{\partial L^{(2)}}{\partial z_2^{(1)}} & \frac{\partial L^{(2)}}{\partial z_2^{(2)}} & \frac{\partial L^{(2)}}{\partial z_2^{(3)}} \\ \frac{\partial L^{(3)}}{\partial z_2^{(1)}} & \frac{\partial L^{(3)}}{\partial z_2^{(2)}} & \frac{\partial L^{(3)}}{\partial z_2^{(3)}} \end{bmatrix} = \begin{bmatrix} 2z_2^{(1)} & 0 & 0 \\ 0 & 2z_2^{(2)} & 0 \\ 0 & 0 & 2z_2^{(2)} \end{bmatrix}$$

$$\frac{\partial J}{\partial \vec{z_2}} = \frac{\partial J}{\partial \vec{L}} \frac{\partial \vec{L}}{\partial \vec{z_2}} = [\frac{1}{3}, \quad \frac{1}{3}, \quad \frac{1}{3}] \begin{bmatrix} 2z_2^{(1)} & 0 & 0 \\ 0 & 2z_2^{(2)} & 0 \\ 0 & 0 & 2z_2^{(2)} \end{bmatrix}$$

$$= [\frac{1}{3}2z_2^{(1)}, \quad \frac{1}{3}2z_2^{(2)}, \quad \frac{1}{3}2z_2^{(3)}]$$

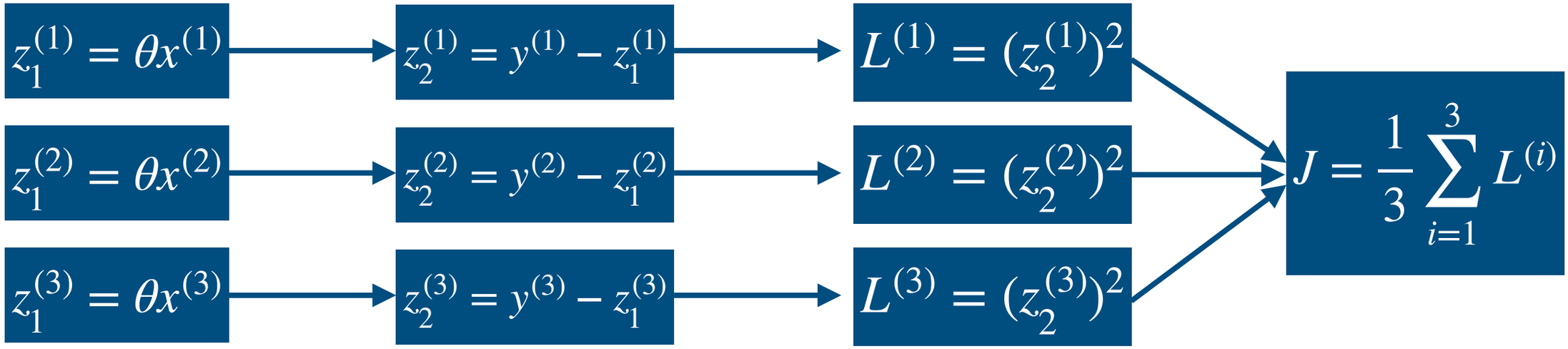


$$\frac{\partial J}{\partial \overrightarrow{z_2}} = [\frac{1}{3}2z_2^{(1)}, \quad \frac{1}{3}2z_2^{(2)}, \quad \frac{1}{3}2z_2^{(3)}]$$



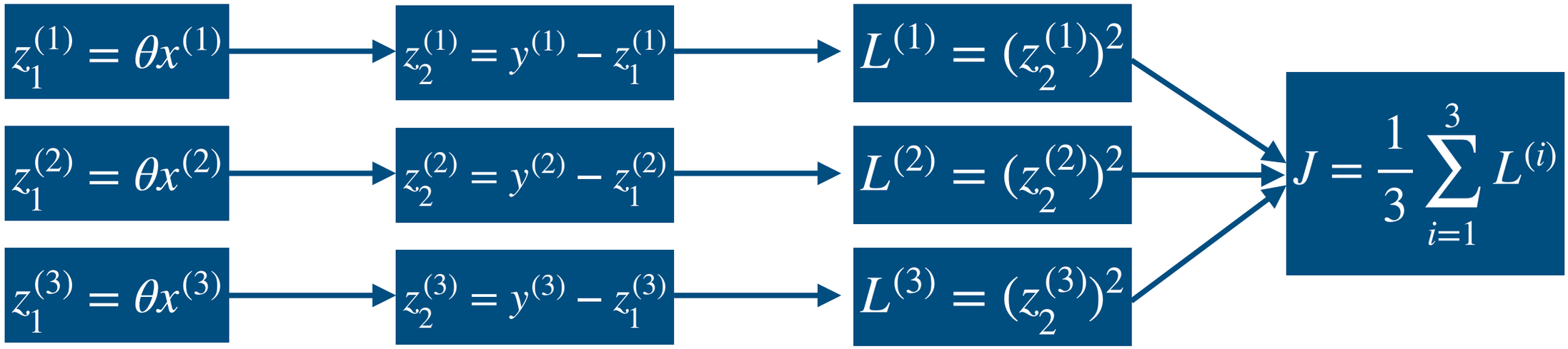
$$\frac{\partial J}{\partial \overrightarrow{z_2}} = [\frac{1}{3}2z_2^{(1)}, \quad \frac{1}{3}2z_2^{(2)}, \quad \frac{1}{3}2z_2^{(3)}]$$

$$\frac{\partial \overrightarrow{z_2}}{\partial \overrightarrow{z_1}}$$



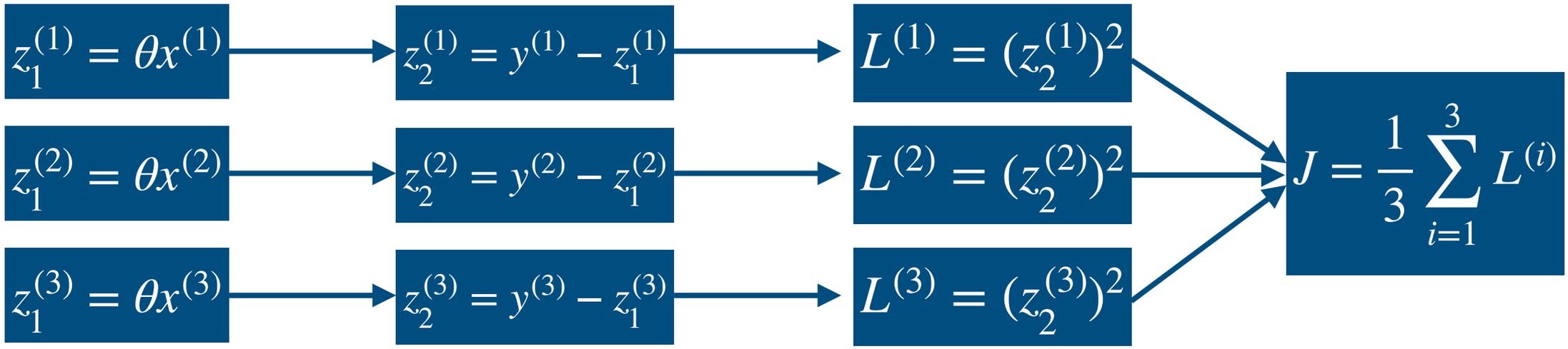
$$\frac{\partial J}{\partial \overrightarrow{z_2}} = [\frac{1}{3}2z_2^{(1)}, \quad \frac{1}{3}2z_2^{(2)}, \quad \frac{1}{3}2z_2^{(3)}]$$

$$\frac{\partial \overrightarrow{z_2}}{\partial \overrightarrow{z_1}} = \begin{bmatrix} \frac{\partial z_2^{(1)}}{\partial z_1^{(1)}} & \frac{\partial z_2^{(1)}}{\partial z_1^{(2)}} & \frac{\partial z_2^{(1)}}{\partial z_1^{(3)}} \\ \frac{\partial z_2^{(2)}}{\partial z_1^{(1)}} & \frac{\partial z_2^{(2)}}{\partial z_1^{(2)}} & \frac{\partial z_2^{(2)}}{\partial z_1^{(3)}} \\ \frac{\partial z_2^{(3)}}{\partial z_1^{(1)}} & \frac{\partial z_2^{(3)}}{\partial z_1^{(2)}} & \frac{\partial z_2^{(3)}}{\partial z_1^{(3)}} \end{bmatrix}$$



$$\frac{\partial J}{\partial \overrightarrow{z_2}} = [\frac{1}{3}2z_2^{(1)}, \quad \frac{1}{3}2z_2^{(2)}, \quad \frac{1}{3}2z_2^{(3)}]$$

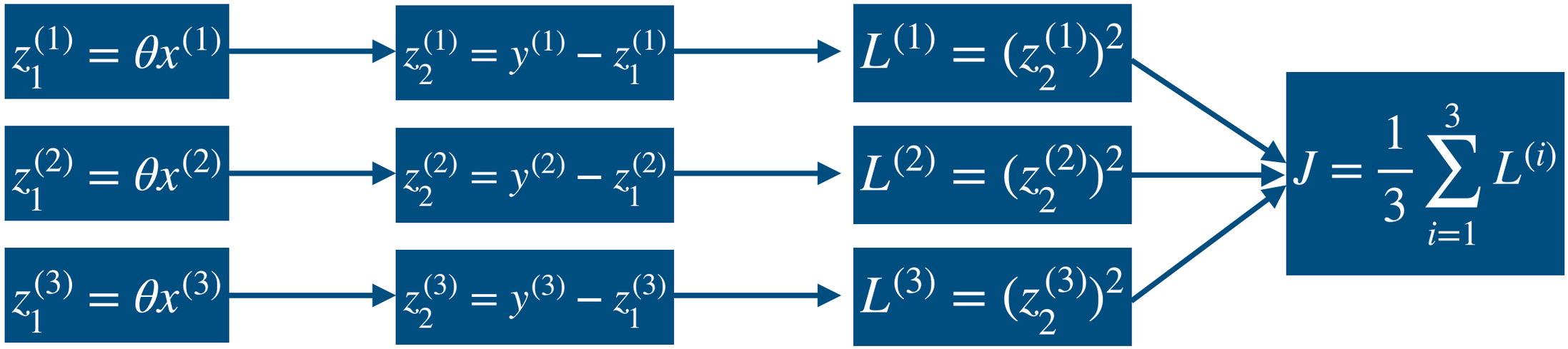
$$\frac{\partial \overrightarrow{z_2}}{\partial \overrightarrow{z_1}} = \begin{bmatrix} \frac{\partial z_2^{(1)}}{\partial z_1^{(1)}} & \frac{\partial z_2^{(1)}}{\partial z_1^{(2)}} & \frac{\partial z_2^{(1)}}{\partial z_1^{(3)}} \\ \frac{\partial z_2^{(2)}}{\partial z_1^{(1)}} & \frac{\partial z_2^{(2)}}{\partial z_1^{(2)}} & \frac{\partial z_2^{(2)}}{\partial z_1^{(3)}} \\ \frac{\partial z_2^{(3)}}{\partial z_1^{(1)}} & \frac{\partial z_2^{(3)}}{\partial z_1^{(2)}} & \frac{\partial z_2^{(3)}}{\partial z_1^{(3)}} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$



$$\frac{\partial J}{\partial \overrightarrow{z_2}} = [\frac{1}{3}2z_2^{(1)}, \quad \frac{1}{3}2z_2^{(2)}, \quad \frac{1}{3}2z_2^{(3)}]$$

$$\frac{\partial \overrightarrow{z_2}}{\partial \overrightarrow{z_1}} = \begin{bmatrix} \frac{\partial z_2^{(1)}}{\partial z_1^{(1)}} & \frac{\partial z_2^{(1)}}{\partial z_1^{(2)}} & \frac{\partial z_2^{(1)}}{\partial z_1^{(3)}} \\ \frac{\partial z_2^{(2)}}{\partial z_1^{(1)}} & \frac{\partial z_2^{(2)}}{\partial z_1^{(2)}} & \frac{\partial z_2^{(2)}}{\partial z_1^{(3)}} \\ \frac{\partial z_2^{(3)}}{\partial z_1^{(1)}} & \frac{\partial z_2^{(3)}}{\partial z_1^{(2)}} & \frac{\partial z_2^{(3)}}{\partial z_1^{(3)}} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\frac{\partial J}{\partial \overrightarrow{z_1}} = \frac{\partial J}{\partial \overrightarrow{z_2}} \frac{\partial \overrightarrow{z_2}}{\partial \overrightarrow{z_1}} = [\frac{1}{3}2z_2^{(1)}, \quad \frac{1}{3}2z_2^{(2)}, \quad \frac{1}{3}2z_2^{(3)}] \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

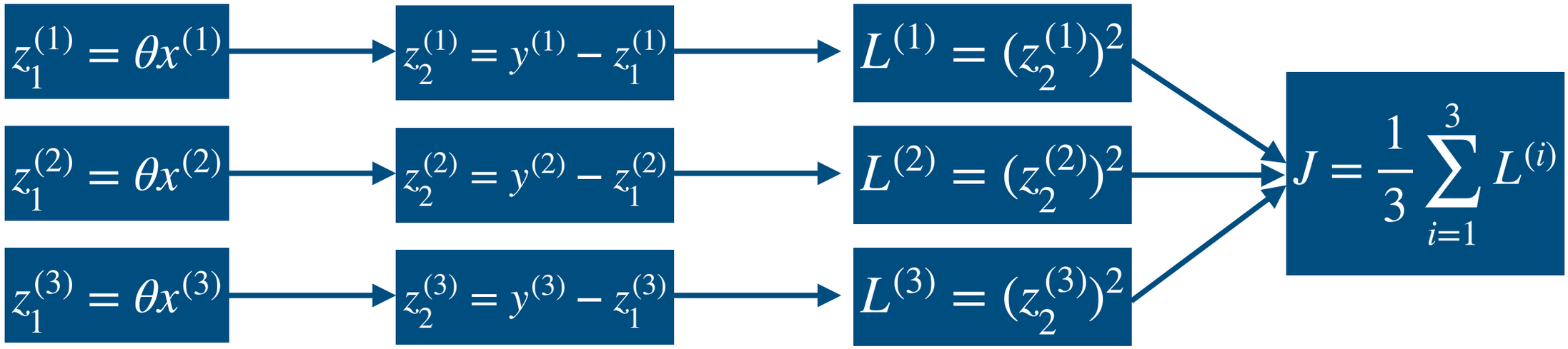


$$\frac{\partial J}{\partial \overrightarrow{z_2}} = [\frac{1}{3}2z_2^{(1)}, \quad \frac{1}{3}2z_2^{(2)}, \quad \frac{1}{3}2z_2^{(3)}]$$

$$\frac{\partial \overrightarrow{z_2}}{\partial \overrightarrow{z_1}} = \begin{bmatrix} \frac{\partial z_2^{(1)}}{\partial z_1^{(1)}} & \frac{\partial z_2^{(1)}}{\partial z_1^{(2)}} & \frac{\partial z_2^{(1)}}{\partial z_1^{(3)}} \\ \frac{\partial z_2^{(2)}}{\partial z_1^{(1)}} & \frac{\partial z_2^{(2)}}{\partial z_1^{(2)}} & \frac{\partial z_2^{(2)}}{\partial z_1^{(3)}} \\ \frac{\partial z_2^{(3)}}{\partial z_1^{(1)}} & \frac{\partial z_2^{(3)}}{\partial z_1^{(2)}} & \frac{\partial z_2^{(3)}}{\partial z_1^{(3)}} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\frac{\partial J}{\partial \overrightarrow{z_1}} = \frac{\partial J}{\partial \overrightarrow{z_2}} \frac{\partial \overrightarrow{z_2}}{\partial \overrightarrow{z_1}} = [\frac{1}{3}2z_2^{(1)}, \quad \frac{1}{3}2z_2^{(2)}, \quad \frac{1}{3}2z_2^{(3)}] \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$= [-\frac{1}{3}2z_2^{(1)}, \quad -\frac{1}{3}2z_2^{(2)}, \quad -\frac{1}{3}2z_2^{(3)}]$$



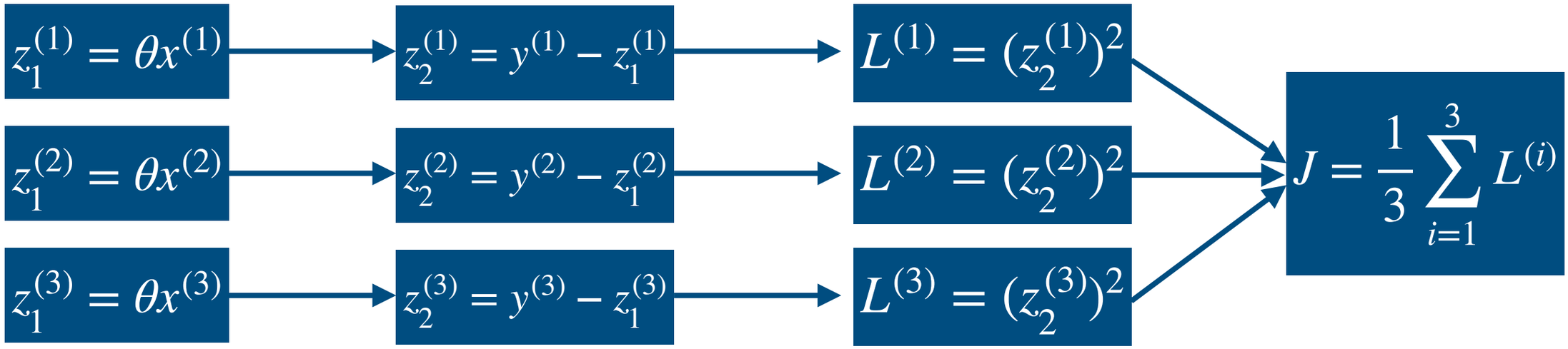
$$\frac{\partial J}{\partial \overrightarrow{z_2}} = [\frac{1}{3}2z_2^{(1)}, \quad \frac{1}{3}2z_2^{(2)}, \quad \frac{1}{3}2z_2^{(3)}]$$

$$\frac{\partial \overrightarrow{z_2}}{\partial \overrightarrow{z_1}} = \begin{bmatrix} \frac{\partial z_2^{(1)}}{\partial z_1^{(1)}} & \frac{\partial z_2^{(1)}}{\partial z_1^{(2)}} & \frac{\partial z_2^{(1)}}{\partial z_1^{(3)}} \\ \frac{\partial z_2^{(2)}}{\partial z_1^{(1)}} & \frac{\partial z_2^{(2)}}{\partial z_1^{(2)}} & \frac{\partial z_2^{(2)}}{\partial z_1^{(3)}} \\ \frac{\partial z_2^{(3)}}{\partial z_1^{(1)}} & \frac{\partial z_2^{(3)}}{\partial z_1^{(2)}} & \frac{\partial z_2^{(3)}}{\partial z_1^{(3)}} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

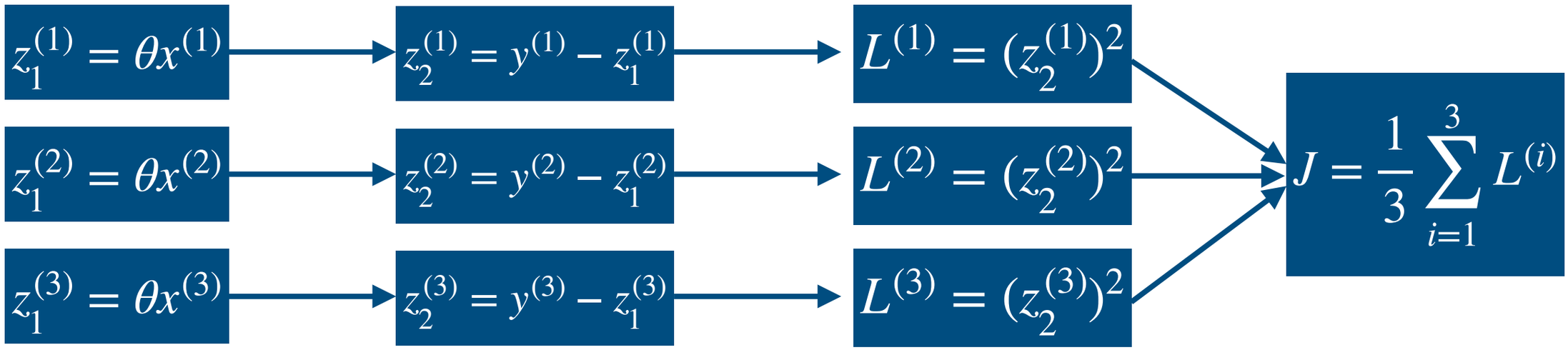
$$\frac{\partial J}{\partial \overrightarrow{z_1}} = \frac{\partial J}{\partial \overrightarrow{z_2}} \frac{\partial \overrightarrow{z_2}}{\partial \overrightarrow{z_1}} = [\frac{1}{3}2z_2^{(1)}, \quad \frac{1}{3}2z_2^{(2)}, \quad \frac{1}{3}2z_2^{(3)}] \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$= [-\frac{1}{3}2z_2^{(1)}, \quad -\frac{1}{3}2z_2^{(2)}, \quad -\frac{1}{3}2z_2^{(3)}]$$

$$= [-\frac{1}{3}2(y^{(1)} - z^{(1)}), \quad -\frac{1}{3}2(y^{(2)} - z^{(2)}), \quad -\frac{1}{3}2(y^{(3)} - z^{(3)})]$$

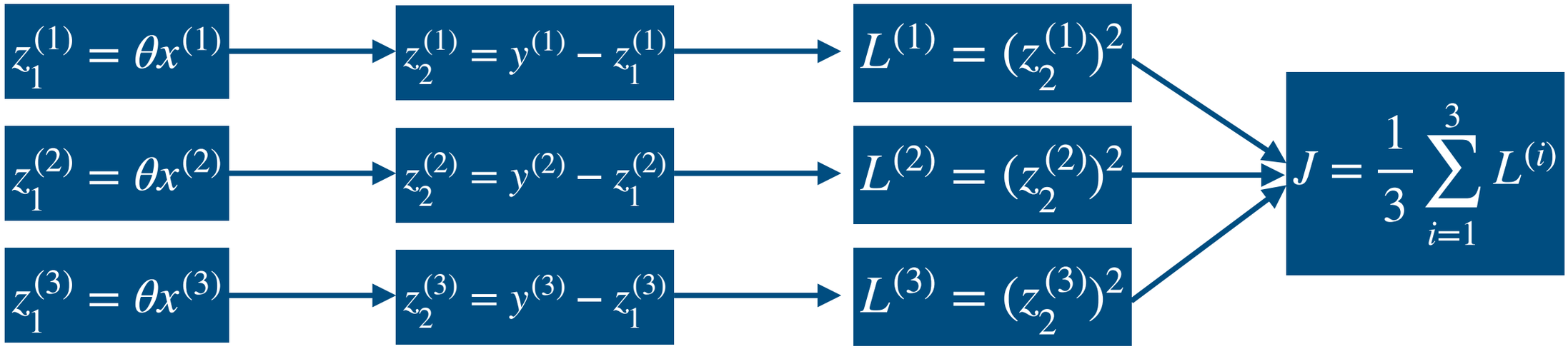


$$\frac{\partial J}{\partial \overrightarrow{z_1}} = [-\frac{1}{3}2(y^{(1)} - z^{(1)}), \quad -\frac{1}{3}2(y^{(2)} - z^{(2)}), \quad -\frac{1}{3}2(y^{(3)} - z^{(3)})]$$



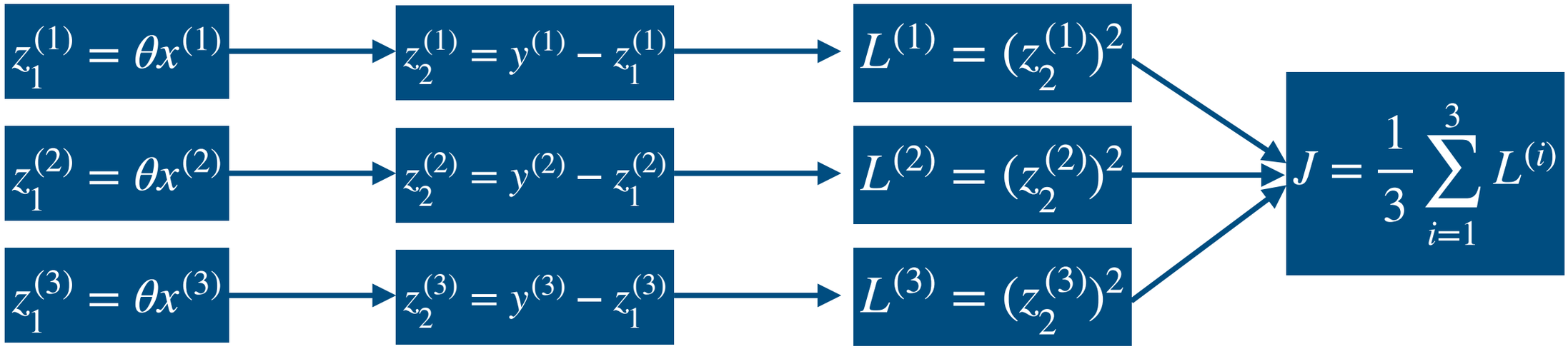
$$\frac{\partial J}{\partial \overrightarrow{z_1}} = [-\frac{1}{3}2(y^{(1)} - z^{(1)}), \quad -\frac{1}{3}2(y^{(2)} - z^{(2)}), \quad -\frac{1}{3}2(y^{(3)} - z^{(3)})]$$

$$\frac{\partial \overrightarrow{z_1}}{\partial \theta} = \begin{bmatrix} \frac{\partial z_1^{(1)}}{\partial \theta} \\ \frac{\partial z_2^{(2)}}{\partial \theta} \\ \frac{\partial z_3^{(3)}}{\partial \theta} \end{bmatrix}$$



$$\frac{\partial J}{\partial \overrightarrow{z_1}} = [-\frac{1}{3}2(y^{(1)} - z^{(1)}), \quad -\frac{1}{3}2(y^{(2)} - z^{(2)}), \quad -\frac{1}{3}2(y^{(3)} - z^{(3)})]$$

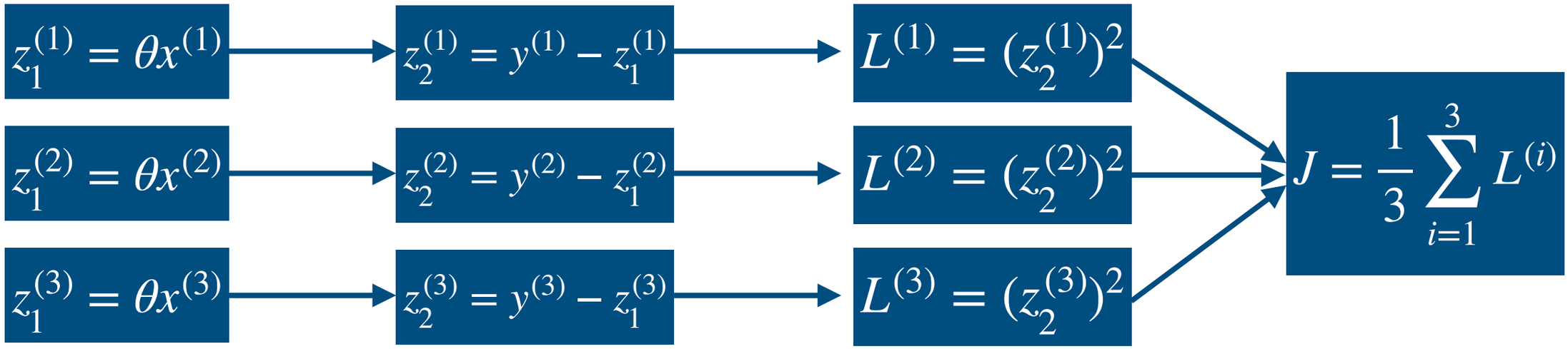
$$\frac{\partial \overrightarrow{z_1}}{\partial \theta} = \begin{bmatrix} \frac{\partial z_1^{(1)}}{\partial \theta} \\ \frac{\partial z_2^{(2)}}{\partial \theta} \\ \frac{\partial z_3^{(3)}}{\partial \theta} \end{bmatrix} = \begin{bmatrix} x^{(1)} \\ x^{(2)} \\ x^{(3)} \end{bmatrix}$$



$$\frac{\partial J}{\partial \overrightarrow{z_1}} = [-\frac{1}{3}2(y^{(1)} - z^{(1)}), \quad -\frac{1}{3}2(y^{(2)} - z^{(2)}), \quad -\frac{1}{3}2(y^{(3)} - z^{(3)})]$$

$$\frac{\partial \overrightarrow{z_1}}{\partial \theta} = \begin{bmatrix} \frac{\partial z_1^{(1)}}{\partial \theta} \\ \frac{\partial z_2^{(2)}}{\partial \theta} \\ \frac{\partial z_3^{(3)}}{\partial \theta} \end{bmatrix} = \begin{bmatrix} x^{(1)} \\ x^{(2)} \\ x^{(3)} \end{bmatrix}$$

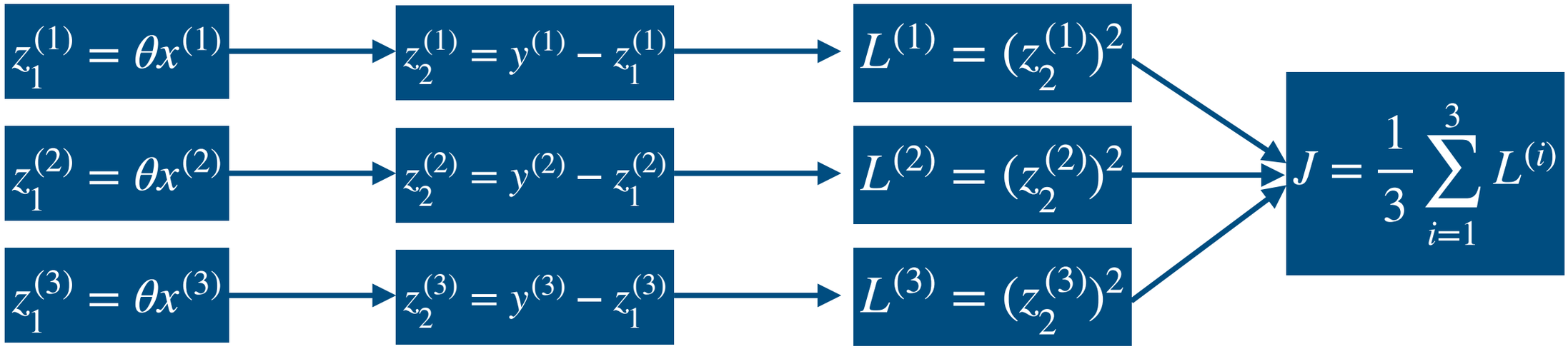
$$\frac{\partial J}{\partial \theta} = \frac{\partial J}{\partial \overrightarrow{z_1}} \frac{\partial \overrightarrow{z_1}}{\partial \theta}$$



$$\frac{\partial J}{\partial \overrightarrow{z_1}} = [-\frac{1}{3}2(y^{(1)} - z^{(1)}), \quad -\frac{1}{3}2(y^{(2)} - z^{(2)}), \quad -\frac{1}{3}2(y^{(3)} - z^{(3)})]$$

$$\frac{\partial \overrightarrow{z_1}}{\partial \theta} = \begin{bmatrix} \frac{\partial z_1^{(1)}}{\partial \theta} \\ \frac{\partial z_2^{(2)}}{\partial \theta} \\ \frac{\partial z_3^{(3)}}{\partial \theta} \end{bmatrix} = \begin{bmatrix} x^{(1)} \\ x^{(2)} \\ x^{(3)} \end{bmatrix}$$

$$\frac{\partial J}{\partial \theta} = \frac{\partial J}{\partial \overrightarrow{z_1}} \frac{\partial \overrightarrow{z_1}}{\partial \theta} = [-\frac{1}{3}2(y^{(1)} - z^{(1)}), \quad -\frac{1}{3}2(y^{(2)} - z^{(2)}), \quad -\frac{1}{3}2(y^{(3)} - z^{(3)})] \begin{bmatrix} x^{(1)} \\ x^{(2)} \\ x^{(3)} \end{bmatrix}$$

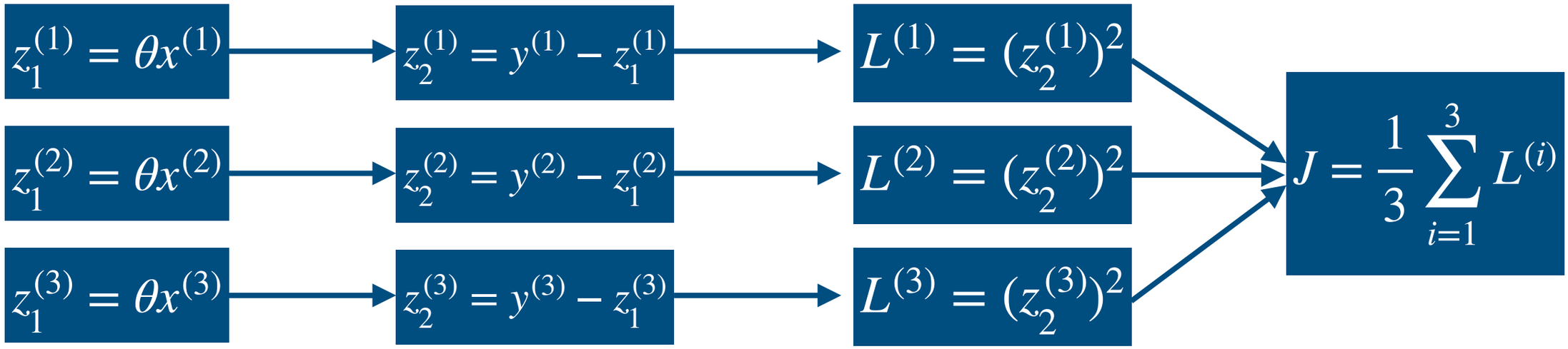


$$\frac{\partial J}{\partial \overrightarrow{z_1}} = [-\frac{1}{3}2(y^{(1)} - z^{(1)}), \quad -\frac{1}{3}2(y^{(2)} - z^{(2)}), \quad -\frac{1}{3}2(y^{(3)} - z^{(3)})]$$

$$\frac{\partial \overrightarrow{z_1}}{\partial \theta} = \begin{bmatrix} \frac{\partial z_1^{(1)}}{\partial \theta} \\ \frac{\partial z_2^{(2)}}{\partial \theta} \\ \frac{\partial z_3^{(3)}}{\partial \theta} \end{bmatrix} = \begin{bmatrix} x^{(1)} \\ x^{(2)} \\ x^{(3)} \end{bmatrix}$$

$$\frac{\partial J}{\partial \theta} = \frac{\partial J}{\partial \overrightarrow{z_1}} \frac{\partial \overrightarrow{z_1}}{\partial \theta} = [-\frac{1}{3}2(y^{(1)} - z^{(1)}), \quad -\frac{1}{3}2(y^{(2)} - z^{(2)}), \quad -\frac{1}{3}2(y^{(3)} - z^{(3)})] \begin{bmatrix} x^{(1)} \\ x^{(2)} \\ x^{(3)} \end{bmatrix}$$

$$= -\frac{1}{3}2x^{(1)}(y^{(1)} - z^{(1)}) - \frac{1}{3}2x^{(2)}(y^{(2)} - z^{(2)}) - \frac{1}{3}2x^{(3)}(y^{(3)} - z^{(3)})$$



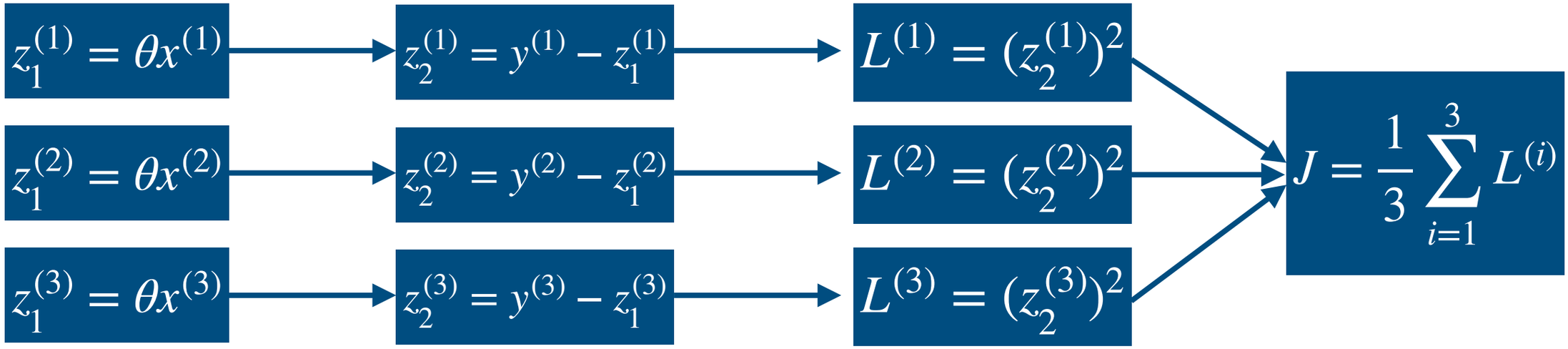
$$\frac{\partial J}{\partial \overrightarrow{z_1}} = [-\frac{1}{3}2(y^{(1)} - z^{(1)}), \quad -\frac{1}{3}2(y^{(2)} - z^{(2)}), \quad -\frac{1}{3}2(y^{(3)} - z^{(3)})]$$

$$\frac{\partial \overrightarrow{z_1}}{\partial \theta} = \begin{bmatrix} \frac{\partial z_1^{(1)}}{\partial \theta} \\ \frac{\partial z_2^{(2)}}{\partial \theta} \\ \frac{\partial z_3^{(3)}}{\partial \theta} \end{bmatrix} = \begin{bmatrix} x^{(1)} \\ x^{(2)} \\ x^{(3)} \end{bmatrix}$$

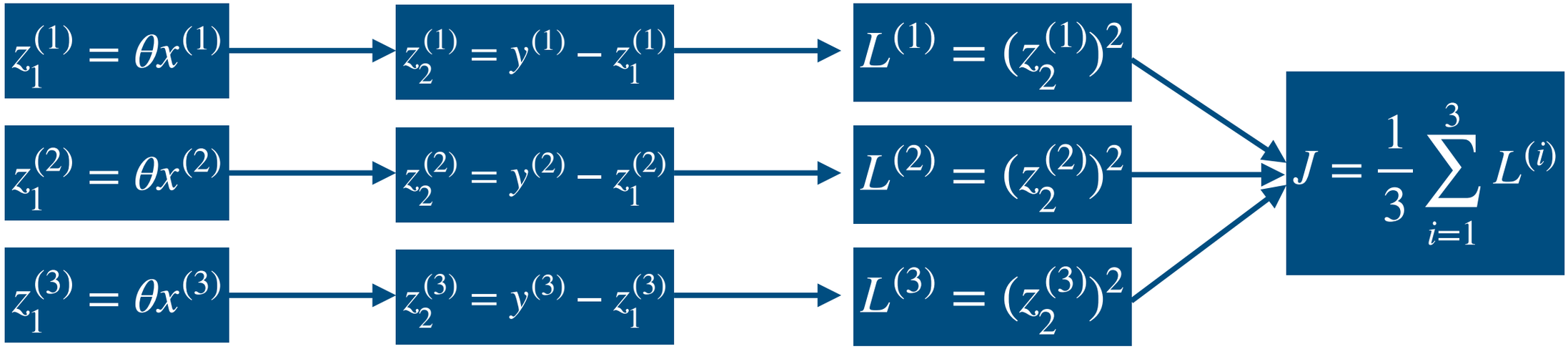
$$\frac{\partial J}{\partial \theta} = \frac{\partial J}{\partial \overrightarrow{z_1}} \frac{\partial \overrightarrow{z_1}}{\partial \theta} = [-\frac{1}{3}2(y^{(1)} - z^{(1)}), \quad -\frac{1}{3}2(y^{(2)} - z^{(2)}), \quad -\frac{1}{3}2(y^{(3)} - z^{(3)})] \begin{bmatrix} x^{(1)} \\ x^{(2)} \\ x^{(3)} \end{bmatrix}$$

$$= -\frac{1}{3}2x^{(1)}(y^{(1)} - z^{(1)}) - \frac{1}{3}2x^{(2)}(y^{(2)} - z^{(2)}) - \frac{1}{3}2x^{(3)}(y^{(3)} - z^{(3)})$$

$$= \frac{1}{3}\sum_{i=1}^{3}(-2x^{(i)}(y^{(i)} - z^{(i)}))$$

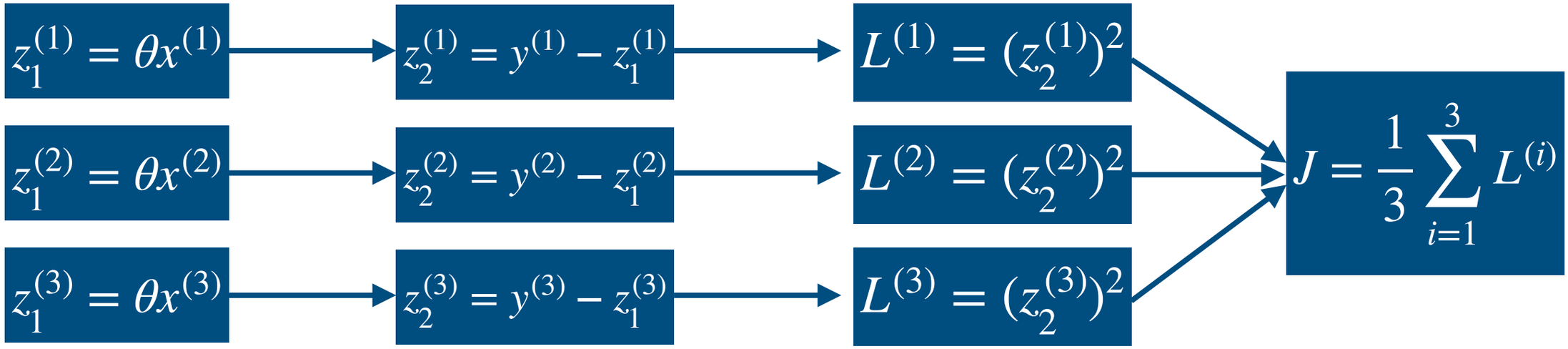


$$\frac{\partial J}{\partial \theta} = \frac{\partial J}{\partial \vec{L}} \frac{\partial \vec{L}}{\partial \vec{z_2}} \frac{\partial \vec{z_2}}{\partial \vec{z_1}} \frac{\partial \vec{z_1}}{\partial \theta}$$



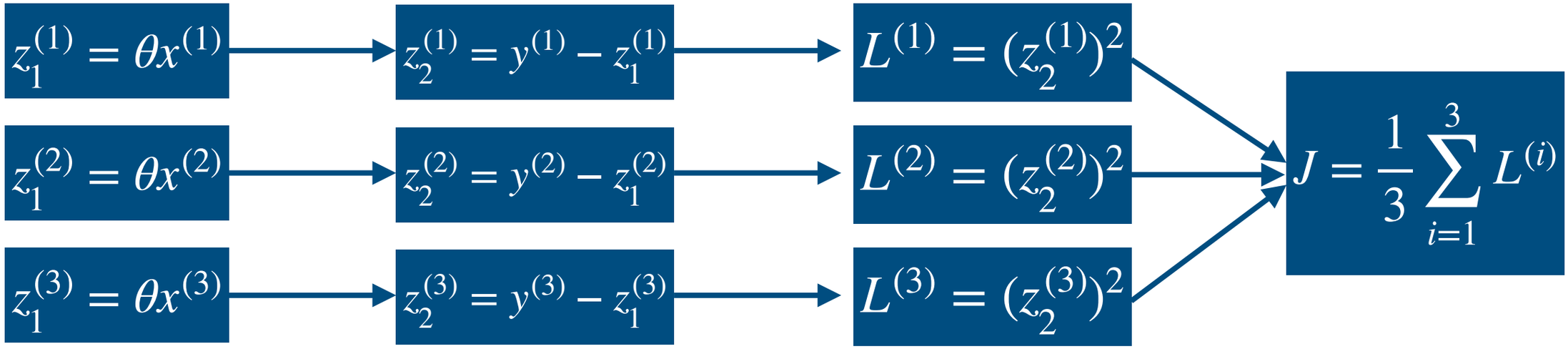
$$\frac{\partial J}{\partial \theta} = \frac{\partial J}{\partial \vec{L}} \frac{\partial \vec{L}}{\partial \vec{z_2}} \frac{\partial \vec{z_2}}{\partial \vec{z_1}} \frac{\partial \vec{z_1}}{\partial \theta}$$

$$\left[\frac{\partial J}{\partial L^{(1)}}, \frac{\partial J}{\partial L^{(2)}}, \frac{\partial J}{\partial L^{(3)}}\right] \begin{bmatrix} \frac{\partial L^{(1)}}{\partial z_2^{(1)}} & \frac{\partial L^{(1)}}{\partial z_2^{(2)}} & \frac{\partial L^{(1)}}{\partial z_2^{(3)}} \\ \frac{\partial L^{(2)}}{\partial z_2^{(1)}} & \frac{\partial L^{(2)}}{\partial z_2^{(2)}} & \frac{\partial L^{(2)}}{\partial z_2^{(3)}} \\ \frac{\partial L^{(3)}}{\partial z_2^{(1)}} & \frac{\partial L^{(3)}}{\partial z_2^{(2)}} & \frac{\partial L^{(3)}}{\partial z_2^{(3)}} \end{bmatrix} \begin{bmatrix} \frac{\partial z_2^{(1)}}{\partial z_1^{(1)}} & \frac{\partial z_2^{(1)}}{\partial z_1^{(2)}} & \frac{\partial z_2^{(1)}}{\partial z_1^{(3)}} \\ \frac{\partial z_2^{(2)}}{\partial z_1^{(1)}} & \frac{\partial z_2^{(2)}}{\partial z_1^{(2)}} & \frac{\partial z_2^{(2)}}{\partial z_1^{(3)}} \\ \frac{\partial z_2^{(3)}}{\partial z_1^{(1)}} & \frac{\partial z_2^{(3)}}{\partial z_1^{(2)}} & \frac{\partial z_2^{(3)}}{\partial z_1^{(3)}} \end{bmatrix} \begin{bmatrix} \frac{\partial z_1^{(1)}}{\partial \theta} \\ \frac{\partial z_2^{(2)}}{\partial \theta} \\ \frac{\partial z_3^{(3)}}{\partial \theta} \end{bmatrix}$$



$$\frac{\partial J}{\partial \theta} = \frac{\partial J}{\partial \vec{L}} \frac{\partial \vec{L}}{\partial \vec{z_2}} \frac{\partial \vec{z_2}}{\partial \vec{z_1}} \frac{\partial \vec{z_1}}{\partial \theta}$$

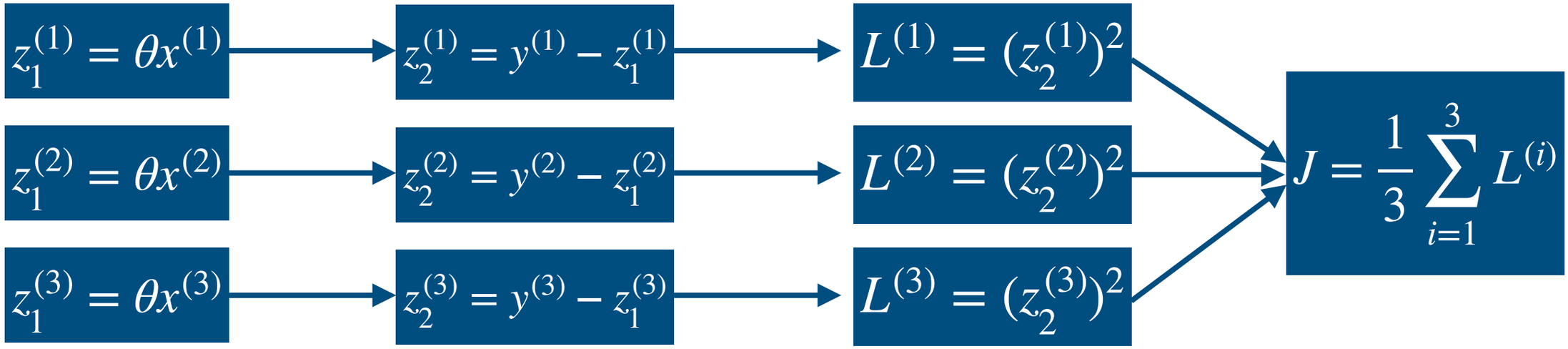
$$[\frac{\partial J}{\partial L^{(1)}}, \frac{\partial J}{\partial L^{(2)}}, \frac{\partial J}{\partial L^{(3)}}] \begin{bmatrix} \frac{\partial L^{(1)}}{\partial z_2^{(1)}} & 0 & 0 \\ 0 & \frac{\partial L^{(2)}}{\partial z_2^{(2)}} & 0 \\ 0 & 0 & \frac{\partial L^{(3)}}{\partial z_2^{(3)}} \end{bmatrix} \begin{bmatrix} \frac{\partial z_2^{(1)}}{\partial z_1^{(1)}} & 0 & 0 \\ 0 & \frac{\partial z_2^{(2)}}{\partial z_1^{(2)}} & 0 \\ 0 & 0 & \frac{\partial z_2^{(3)}}{\partial z_1^{(3)}} \end{bmatrix} \begin{bmatrix} \frac{\partial z_1^{(1)}}{\partial \theta} \\ \frac{\partial z_2^{(2)}}{\partial \theta} \\ \frac{\partial z_3^{(3)}}{\partial \theta} \end{bmatrix}$$



$$\frac{\partial J}{\partial \theta} = \frac{\partial J}{\partial \vec{L}} \frac{\partial \vec{L}}{\partial \vec{z_2}} \frac{\partial \vec{z_2}}{\partial \vec{z_1}} \frac{\partial \vec{z_1}}{\partial \theta}$$

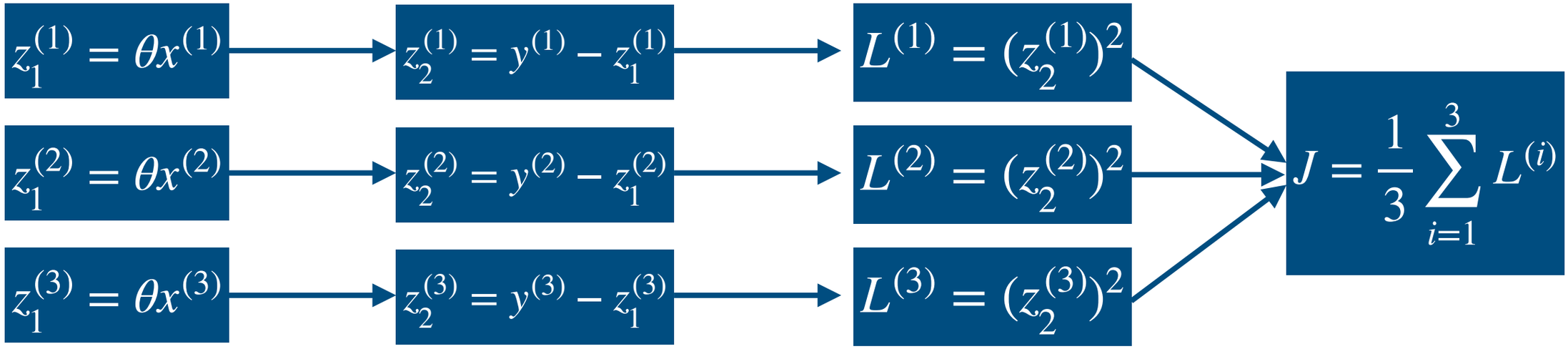
$$\frac{\partial J}{\partial \theta} = np.sum(\begin{bmatrix} \frac{\partial J}{\partial L^{(1)}} \\ \frac{\partial J}{\partial L^{(2)}} \\ \frac{\partial J}{\partial L^{(3)}} \end{bmatrix} * \begin{bmatrix} \frac{\partial L^{(1)}}{\partial z_2^{(1)}} \\ \frac{\partial L^{(2)}}{\partial z_2^{(2)}} \\ \frac{\partial L^{(3)}}{\partial z_2^{(3)}} \end{bmatrix} * \begin{bmatrix} \frac{\partial z_2^{(1)}}{\partial z_1^{(1)}} \\ \frac{\partial z_2^{(2)}}{\partial z_1^{(2)}} \\ \frac{\partial z_2^{(3)}}{\partial z_1^{(3)}} \end{bmatrix} * \begin{bmatrix} \frac{\partial z_1^{(1)}}{\partial \theta} \\ \frac{\partial z_2^{(2)}}{\partial \theta} \\ \frac{\partial z_3^{(3)}}{\partial \theta} \end{bmatrix})$$

$$\frac{\partial J}{\partial \theta} = np.sum(dL * dZ2 * dZ1 * d\Theta)$$



$$\frac{\partial J}{\partial \theta} = \frac{\partial J}{\partial \vec{L}} \frac{\partial \vec{L}}{\partial \vec{z_2}} \frac{\partial \vec{z_2}}{\partial \vec{z_1}} \frac{\partial \vec{z_1}}{\partial \theta}$$

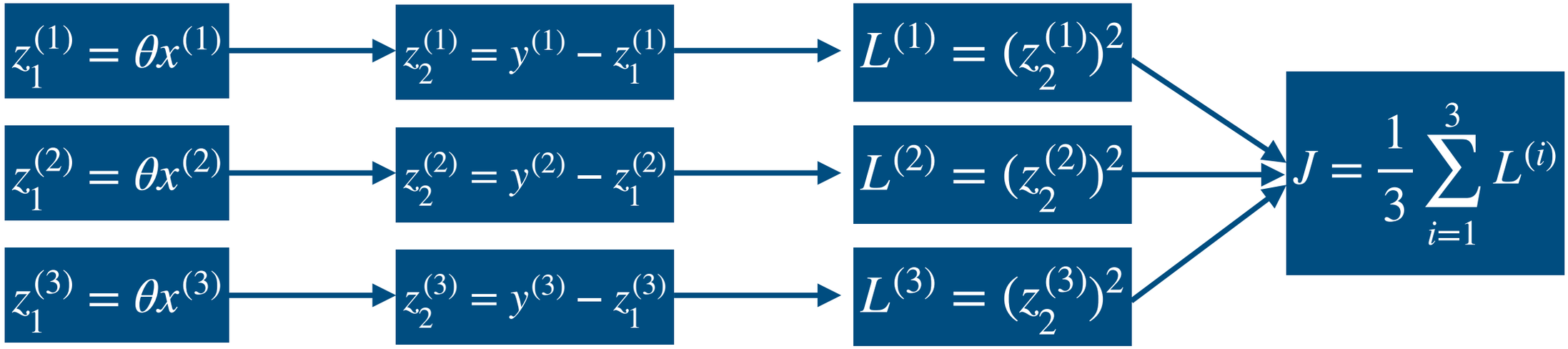
$$\theta = \theta - \alpha \frac{\partial J}{\partial \theta}$$



$$\frac{\partial J}{\partial \theta} = \frac{\partial J}{\partial \vec{L}} \frac{\partial \vec{L}}{\partial \vec{z_2}} \frac{\partial \vec{z_2}}{\partial \vec{z_1}} \frac{\partial \vec{z_1}}{\partial \theta}$$

$$\theta = \theta - \alpha \frac{\partial J}{\partial \theta}$$

$$= \theta - \alpha \frac{\partial J}{\partial \vec{L}} \frac{\partial \vec{L}}{\partial \vec{z_2}} \frac{\partial \vec{z_2}}{\partial \vec{z_1}} \frac{\partial \vec{z_1}}{\partial \theta}$$



$$\frac{\partial J}{\partial \theta} = \frac{\partial J}{\partial \vec{L}} \frac{\partial \vec{L}}{\partial \vec{z_2}} \frac{\partial \vec{z_2}}{\partial \vec{z_1}} \frac{\partial \vec{z_1}}{\partial \theta}$$

$$\theta = \theta - \alpha \frac{\partial J}{\partial \theta}$$

$$= \theta - \alpha * np\,.\,sum(dL * dZ2 * dZ1 * d\Theta)$$