DEEP LEARNING

Single-variable chain rule

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Single-variable Chain Rule

Notation

$$y = f(g(x)) \qquad (f \circ g)(x)$$

Derivative

$$y' = f'(g(x))g'(x) \longrightarrow u = g(x)$$

Formulation of the single-variable chain rule

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

$$y = f(g(x)) = sin(x^2):$$

1. Introduce Intermediate variable Let $u = x^2$

$$\begin{aligned} u &= x^2 && \text{(relative to definition } f(g(x)), g(x) = x^2\text{)} \\ y &= sin(u) && (y = f(u) = sin(u)) \end{aligned}$$

2. Compute derivative

$$\begin{aligned} \frac{du}{dx} &= 2x && \text{(Take derivative with respect to } x\text{)} \\ \frac{dy}{du} &= cos(u) && \text{(Take derivative with respect to } u \text{ not } x\text{)} \end{aligned}$$

$$y = f(g(x)) = sin(x^2):$$

3. Combine

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = cos(u)2x$$

4. Substitute

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = cos(x^2)2x = 2xcos(x^2)$$

Visualize the overall expression

$$\begin{array}{c} y = sin \\ \uparrow \\ u = square \\ \uparrow \\ x \end{array} \quad \begin{array}{l} \Big\} \dfrac{dy}{du} \\ \Big\} \dfrac{du}{dx} \end{array}$$

Forward differentiation from x to y

$$\frac{dy}{dx} = \frac{du}{dx}\frac{dy}{du}$$

Backward differentiation from y to x

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

Advantage

$f_4(f_3(f_2(f_1(x)))) \longrightarrow$ Easy

$$y = f(x) = ln(sin(x^3)^2):$$

1. Introduce intermediate variables.

$$\begin{array}{lcll} u_1 & = & f_1(x) & = x^3 \\ u_2 & = & f_2(u_1) & = sin(u_1) \\ u_3 & = & f_3(u_2) & = u_2^2 \\ u_4 & = & f_4(u_3) & = ln(u_3)(y = u_4) \end{array}$$

2. Compute derivatives.

$$\begin{array}{lcllcl} \frac{d}{u_x}u_1 & = & \frac{d}{x}x^3 & = & 3x^2 \\ \frac{d}{u_1}u_2 & = & \frac{d}{u_1}sin(u_1) & = & cos(u_1) \\ \frac{d}{u_2}u_3 & = & \frac{d}{u_2}u_2^2 & = & 2u_2 \\ \frac{d}{u_3}u_4 & = & \frac{d}{u_3}ln(u_3) & = & \frac{1}{u_3} \end{array}$$

3. Combine four intermediate values.

$$\frac{dy}{dx} = \frac{du_4}{dx} = \frac{du_4}{du_3}\frac{du_3}{du_2}\frac{du_2}{du_1}\frac{du_1}{dx}$$

$$= \frac{1}{u_3}2u_2cos(u_1)3x^2 = \frac{6u_2x^2cos(u_1)}{u_3}$$

4. Substitute.

$$\frac{dy}{dx} = \frac{6sin(u_1)x^2cos(x^3)}{u_2^2} = \frac{6sin(x^3)x^2cos(x^3)}{sin(u_1)^2}$$

$$= \frac{6sin(x^3)x^2cos(x^3)}{sin(x^3)^2} = \frac{6x^2cos(x^3)}{sin(x^3)}$$

Visualization

$$
\begin{array}{rcl}
y = r_4 & ln & \\
 & \uparrow & \dfrac{du_4}{du_3} \\
r_3 & square & \\
 & \uparrow & \dfrac{du_3}{du_2} \\
r_2 & sin & \\
 & \uparrow & \dfrac{du_2}{du_1} \\
r_1 & cube & \\
 & \uparrow & \dfrac{du_1}{dx} \\
 & x &
\end{array}
$$

THANK YOU

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