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ECE 4250
April 27, 2020

Assignment 4

Problem Set

Question 1

- Two images $x_1(m,n)$ and $x_2(m,n)$ have unnormalized histograms h_1 and h_2. Give a condition under which one can determine these image histograms from the unnormalized histogram of $x_1 + x_2$. Describe the procedure you would implement to compute the individual histograms.

  To determine each histogram, the condition is for x_1 to have the same intensity value across its pixels. However, x_2 can be anything. To compute the histograms, we can take the combined histogram and subtract the total number of pixels from x_1 (the image that has all constant intensity values). Whatever is left is the histogram for x_2.

- Give a single integer-valued intensity transformation for spreading the intensities of an input image so lowest intensity is 0 and highest intensity is $L-1$.

  Let X be the image matrix, and $m_1 = min(X)$ and $m_2 = max(X)$.

  Let G be the new image.

$$G(i,k) = \frac{(X(i,k) - m_1)(L-1)}{m_2 - m_1}$$

Question 2

Consider following 2D image:

$$cos(2\pi\mu_0 m/M + 2\pi\nu_0 n/N),$$

where $m, m_0 \in [M]$ and $n, n_0 \in [N]$. Derive the DFT.

$$\begin{aligned} f(m,n) &= cos(2\pi\mu_0 m/M + 2\pi\nu_0 n/N) \\ &= \frac{1}{2}\left[e^{j(2\pi\mu_0 m/M + 2\pi\nu_0 n/N)} + e^{-j(2\pi\mu_0 m/M + 2\pi\nu_0 n/N)}\right] \end{aligned}$$

$$\begin{aligned} F\{e^{j(2\pi\mu_0 m/M + 2\pi\nu_0 n/N)}\} &= \iint e^{j(2\pi\mu_0 m/M + 2\pi\nu_0 n/N)} e^{-j2\pi(mu+nv)}\,\mathrm{d}m\mathrm{d}n \\ &= \int e^{j2\pi\mu_0 m/M} e^{-j2\pi mu}\,\mathrm{d}m \int e^{j2\pi\nu_0 n/N} e^{-j2\pi nv}\,\mathrm{d}m \\ &= \delta\left(u-\mu_0, v-\nu_0\right) \end{aligned}$$

$$f(m,n) \leftrightarrow F(u,v) = \frac{1}{2}\left[\delta(u-\mu_0, v-\nu_0) + \delta(u+\mu_0, v+\nu_0)\right]$$

Question 3

Consider a 3×3 spatial mask that averages the four closest neighbors of a pixel (m, n), but excludes the pixel itself from the average.

- Express the filter as a 2D array (in spatial coordinates).
- Derive the 2D discrete Fourier transform of this filter.
- Is this a high or low pass filter? Explain your answer.

$$g(m,n) = \frac{1}{4}\left[f(m+1,n) + f(m-1,n) + f(m,n+1) + f(m,n-1)\right]$$

$$\begin{aligned}
G(i,k) &= \frac{1}{4}\left[F(i,k)e^{j2\pi i/M} + F(i,k)e^{-j2\pi i/M} + F(i,k)e^{j2\pi k/N} + F(i,k)e^{-j2\pi k/N}\right] \\
&= F(i,k)\left[\frac{1}{4}\left(e^{j2\pi i/M} + e^{-j2\pi i/M} + e^{j2\pi k/N} + e^{-j2\pi k/N}\right)\right] \\
&= F(i,k)H(i,k)
\end{aligned}$$

$$H(i,k) = \frac{1}{4}\left(e^{j2\pi i/M} + e^{-j2\pi i/M} + e^{j2\pi k/N} + e^{-j2\pi k/N}\right)$$

This is a low pass filter because $H(0,0) = \frac{1}{4}(4) = 1$. As (i,k) increases, $H(i,k)$ decreases.