In [2]:

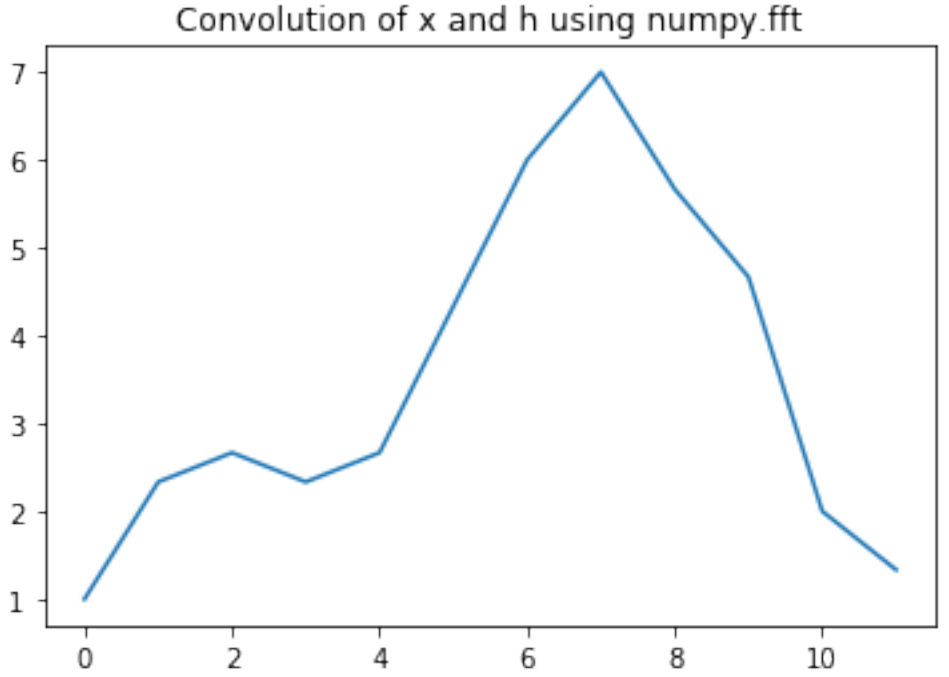
```
#!/usr/bin/python3
# Samantha Hong sh974
# ECE 4250 Assignment 2
import numpy as np
import math
import time
import scipy
from scipy.io import wavfile
from scipy import signal
import matplotlib.pyplot as plt
```

1. Convolutions

In [3]:

```
x = np.array([3,4,1,2,5,6,7,8,2,4])
h = np.array([1.0/3, 1.0/3, 1.0/3])
fft_size = x.size + h.size - 1

X = np.fft.fft(x, fft_size) # x in the frequency domain
H = np.fft.fft(h, fft_size) # h in the frequency domain
conv_freq = np.multiply(X,H) # convolving in frequency domain
result = np.fft.ifft(conv_freq) # returning to time domain

print(conv_freq)

plt.plot(result)
plt.title("Convolution of x and h using numpy.fft")
plt.show()
```

```
[ 4.20000000e+01+0.00000000e+00j -1.34728614e+01+5.1
8739261e+00j
  1.00000000e+00-6.35085296e+00j -1.33333333e+00-6.6
6666667e-01j
  1.66533454e-16-3.84592537e-16j -1.93805238e-01+1.1
4594072e+00j
 -2.00000000e+00+0.00000000e+00j -1.93805238e-01-1.1
4594072e+00j
  1.66533454e-16+3.84592537e-16j -1.33333333e+00+6.6
6666667e-01j
  1.00000000e+00+6.35085296e+00j -1.34728614e+01-5.1
8739261e+00j]

/usr/local/lib/python2.7/site-packages/numpy/core/nu
meric.py:501: ComplexWarning: Casting complex values
to real discards the imaginary part
  return array(a, dtype, copy=False, order=order)
```

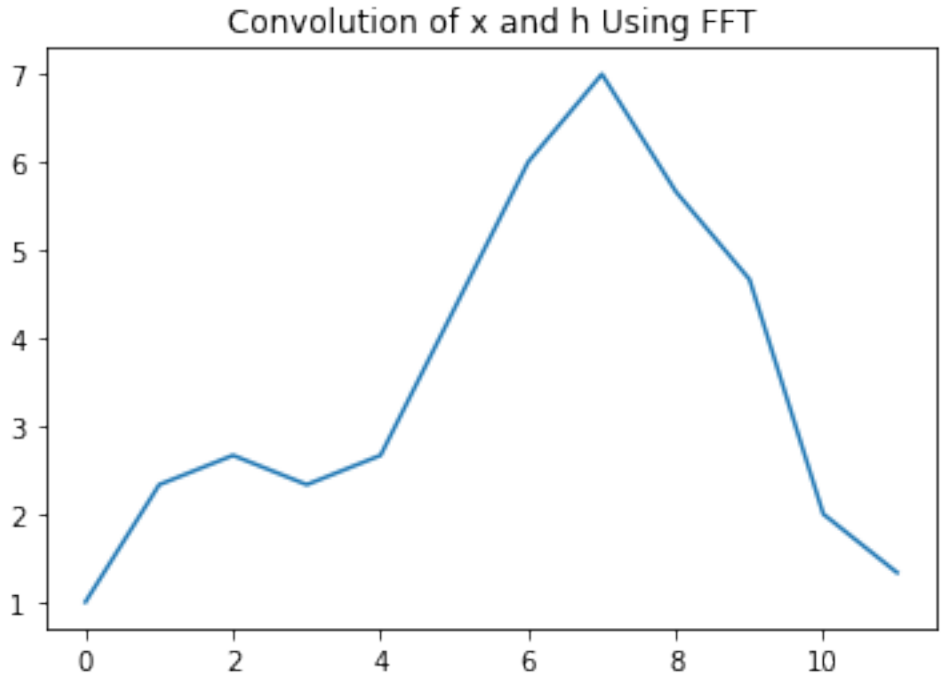


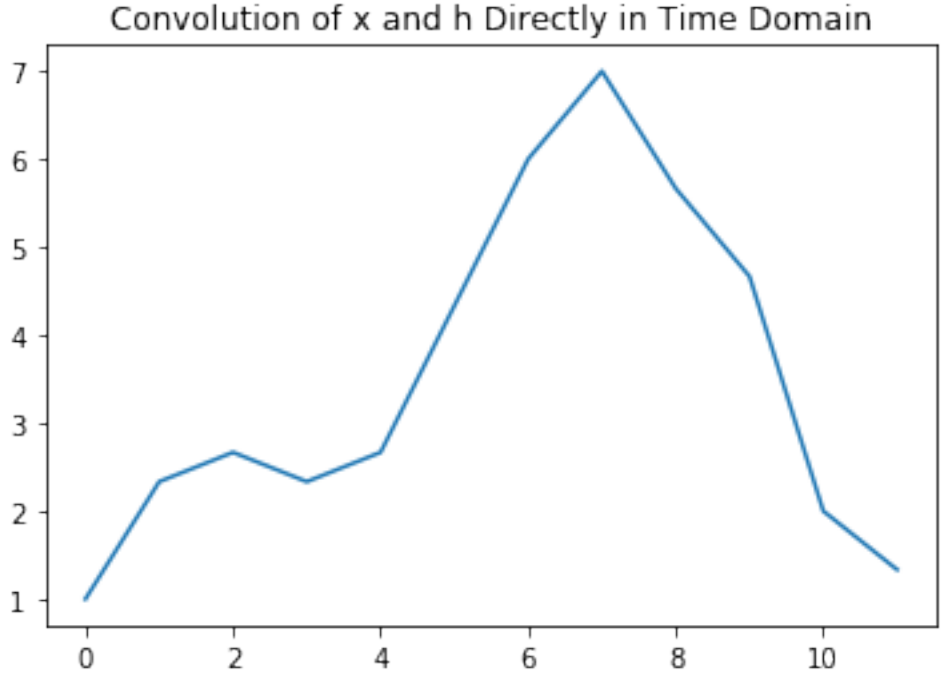
In [4]:

```
# my convolution function from A1
def conv(x, h):
    conv_size = x.size + h.size - 1
    y = np.zeros(conv_size)
    for i in range(h.size):
        for j in range(x.size):
            y[j + i] = y[j + i] + h.item(i) * x.item(j)

    return y

# fft method
def fft_conv(x, h):
    fft_size = x.size + h.size - 1

    X = np.fft.fft(x, fft_size) # x in the frequency domain
    H = np.fft.fft(h, fft_size) # h in the frequency domain
    conv_freq = np.multiply(X,H) # convolving in frequency domai
n
    result = np.fft.ifft(conv_freq) # returning to time domain
    return result
```

In [5]:

```
# plotting both methods of convolution to compare output
plt.plot(fft_conv(x, h))
plt.title("Convolution of x and h Using FFT")
plt.show()

plt.plot(conv(x, h))
plt.title("Convolution of x and h Directly in Time Domain")
plt.show()
```



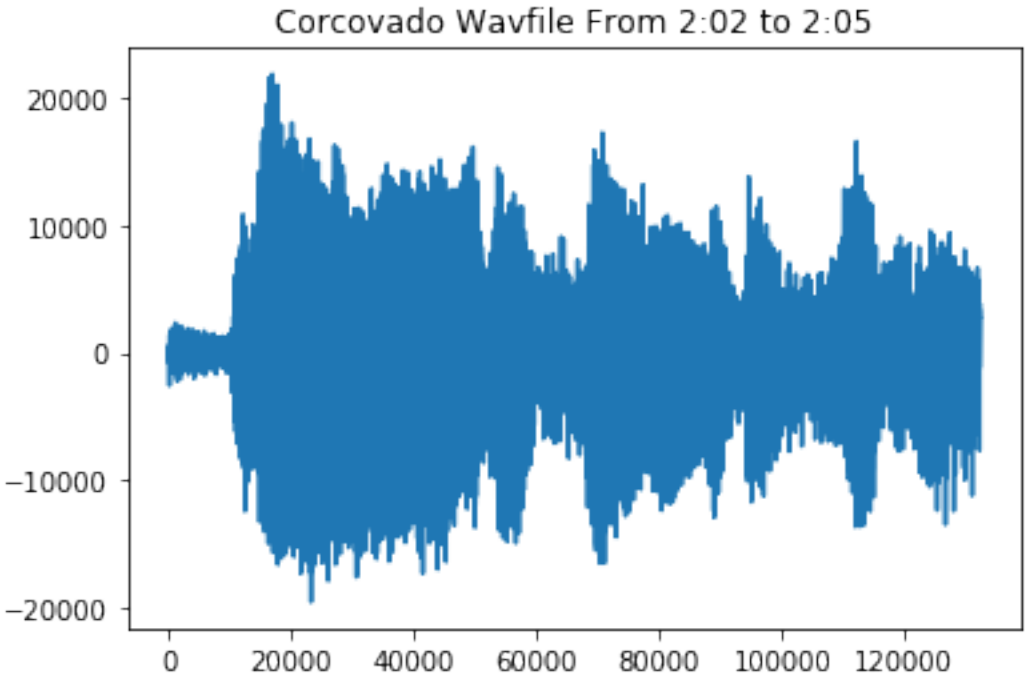


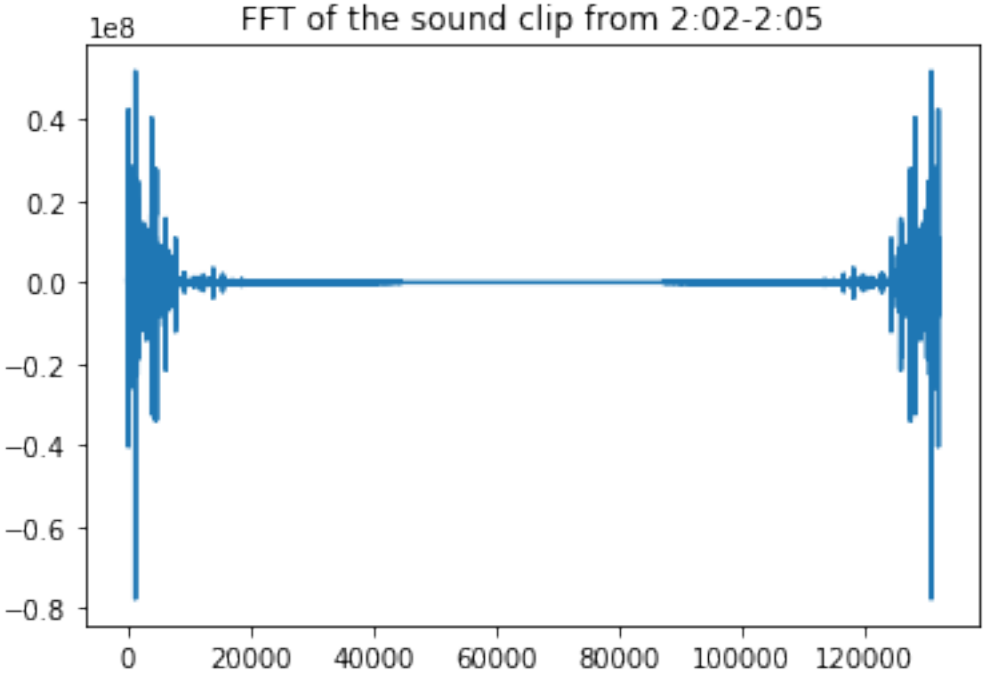
2. Fourier Transforms

Part A

In [6]:

```
corcovado_rate, corcovado_data = wavfile.read('./HW1-Files/Corco
vado.wav')

# converting to float and cropping from 2:02 to 2:05
# corcovado wavfile length: 4 min 14 sec

corcovado = np.array([i[1] for i in corcovado_data]).astype(comp
lex)
signal = corcovado[122*44100:125*44100]
plt.plot(signal)
plt.title('Corcovado Wavfile From 2:02 to 2:05')
plt.show()


# taking the fft of this sound clip
corcovado_fft = np.fft.fft(signal)
plt.plot(corcovado_fft)
plt.title("FFT of the sound clip from 2:02-2:05")
plt.show()
```


Corcovado Wavfile From 2:02 to 2:05
20000
10000
0
−10000
−20000
0
20000
40000
60000
80000
100000
120000

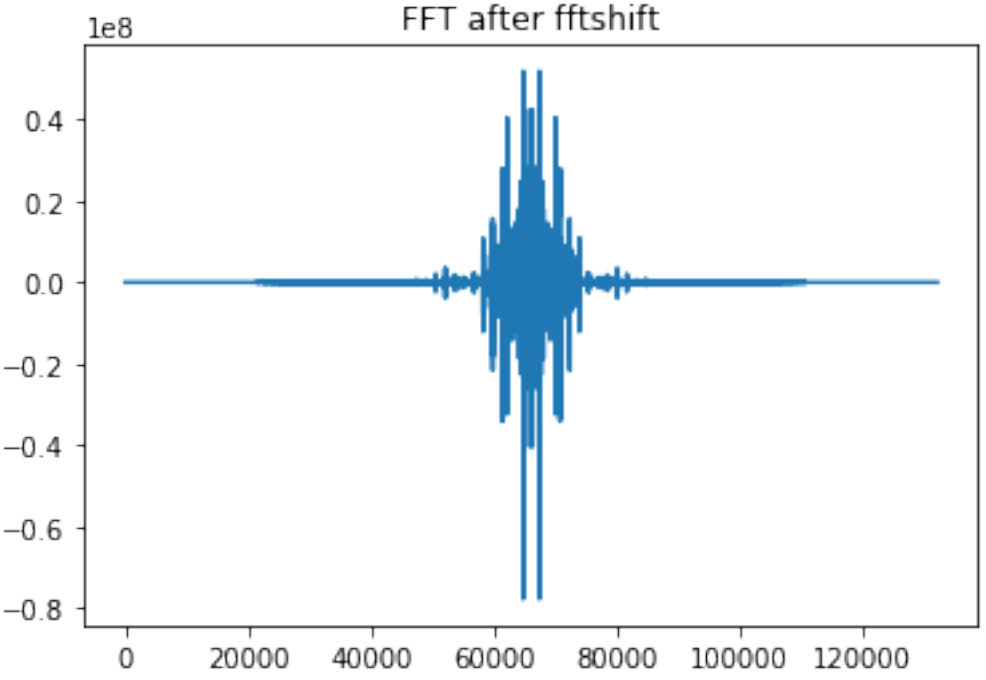

FFT of the sound clip from 2:02-2:05
1e8
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
0
20000
40000
60000
80000
100000
120000

Part B

In [7]:

```
print("DC component is the 0 Hz term. ")
print("is equivalent to the average of all the samples in the wi
ndow. ")
print("The DC component of this is " + str(np.absolute(corcovado
_fft[0])))

fft_shift = np.fft.fftshift(corcovado_fft)
plt.plot(fft_shift)
plt.title("FFT after fftshift")
plt.show()
```

```
DC component is the 0 Hz term.
is equivalent to the average of all the samples in t
he window.
The DC component of this is 483436.0
```



Part C

In [8]:

```
idx = np.argmax(np.abs(corcovado_fft))
freq = abs(idx * 44100 / len(corcovado_fft))

print("The frequency (Hz) of the saxophone is ", freq)
```

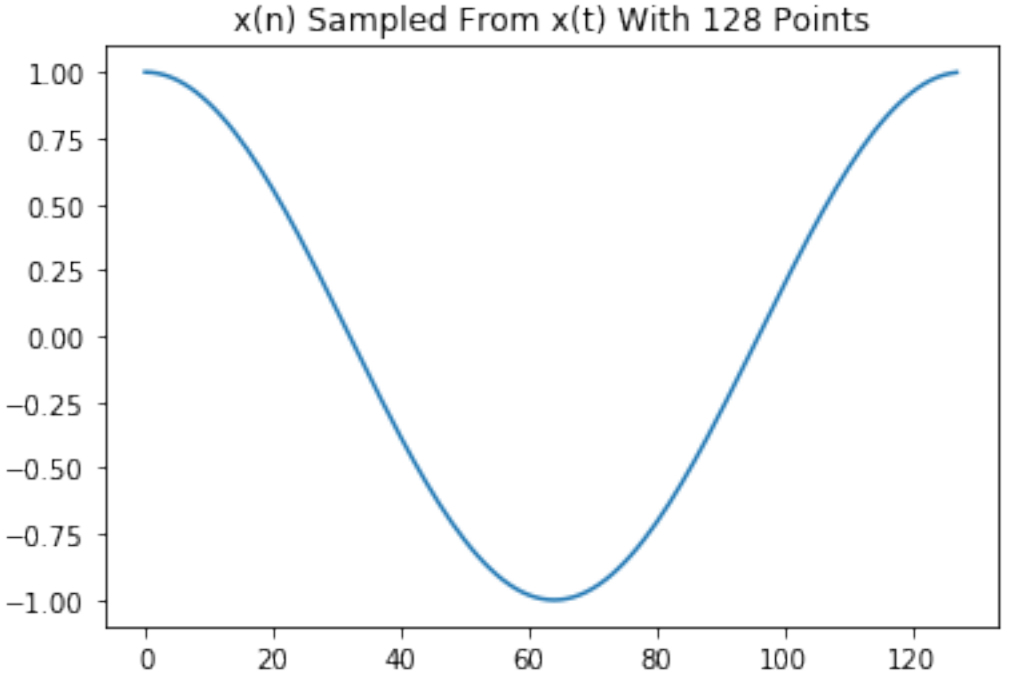
```
('The frequency (Hz) of the saxophone is ', 439)
```

3. Implement the FFT

Part A

In [9]:

```
t = np.arange(0, 1, 1./128)
x_n = np.cos(2 * math.pi * t)
plt.plot(x_n)
plt.title('x(n) Sampled From x(t) With 128 Points')
plt.show()
```



This sampling rate is above the Nyquist rate. We can see that the original continuous signal x(t) can be reconstructed from x(n). Additionally, the sampling rate (128 Hz) is greater than the Nyquist rate (2 * 1 Hz).

Part B

In [10]:

```
def N_DFT(x_n, N):
    X_k = np.zeros(N, dtype=complex)

    for k in range(N):
        for n in range(N):
            c = np.exp(-1j * 2 * np.pi * k * n / N)
            X_k[k] += x_n[n] * c
    return X_k
```

Part C

I.

In [11]:

```
def separate(x_n, N):
    n = np.arange(0, N/2)
    x_1 = np.zeros(N/2)
    x_2 = np.zeros(N/2)
    for i in range(len(x_1)):
        x_1[i] = x_n[2 * i]
        x_2[i] = x_n[2 * i + 1]

    return x_1, x_2
```

II.

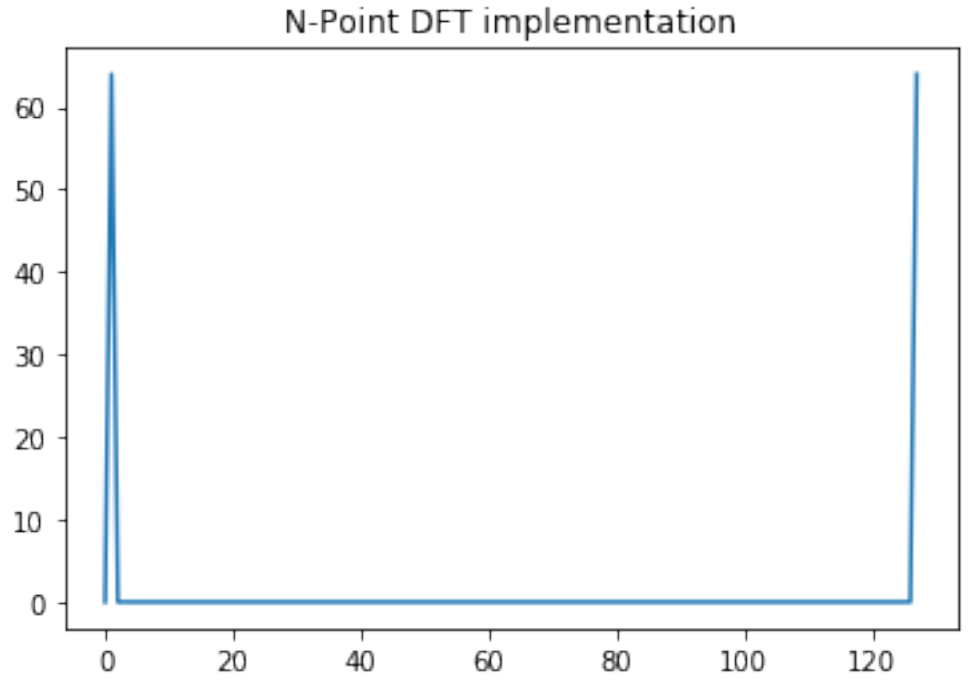
In [12]:

```
def My_FFT(x_n, N):
    x_n = x_n.astype(complex)

    # base case
    if (N < 2):
        return x_n

    # recursive case
    else:
        x_1, x_2 = separate(x_n, N)
        x1_fft = My_FFT(x_1, N/2)
        x2_fft = My_FFT(x_2, N/2)

        for k in range(N/2):
            W = np.exp(-1j * 2 * np.pi * k / N)
            temp = W * x2_fft[k]
            x_n[int(k)] = temp + x1_fft[k]
            x_n[int(k + N/2)] = x1_fft[k] - temp

        # return the final result
        return x_n
```

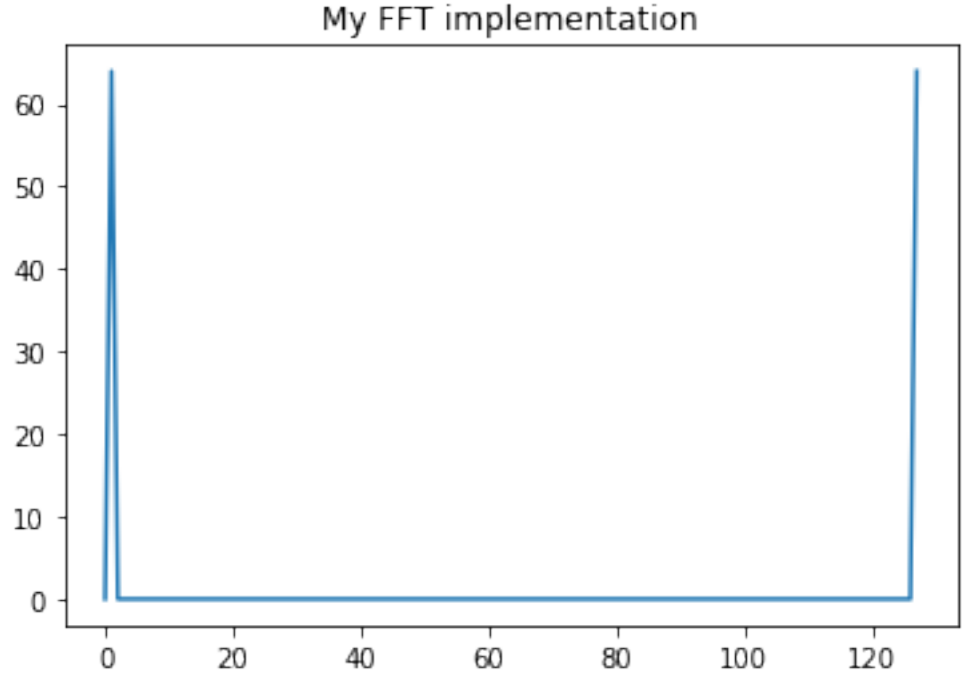
Part D

In [13]:

```
t = np.arange(0, 1, 1./128)
x_n = np.cos(2 * math.pi * t)

# part (b) DFT
plt.plot(N_DFT(x_n, len(x_n)))
plt.title("N-Point DFT implementation")
plt.show()

# part (c) FFT
plt.plot(My_FFT(x_n, len(x_n)))
plt.title("My FFT implementation")
plt.show()
```




```
/usr/local/lib/python2.7/site-packages/ipykernel_lau
ncher.py:6: ComplexWarning: Casting complex values t
o real discards the imaginary part

/usr/local/lib/python2.7/site-packages/ipykernel_lau
ncher.py:7: ComplexWarning: Casting complex values t
o real discards the imaginary part
  import sys
```



Part E

In [14]:

```
clip_rate, clip_data = wavfile.read('./HW1-Files/clip.wav')
clip_data = np.array([i[1] for i in clip_data]).astype(complex)
signal = clip_data[0:8192]
```

```
/usr/local/lib/python2.7/site-packages/scipy/io/wavf
ile.py:273: WavFileWarning: Chunk (non-data) not und
erstood, skipping it.
  WavFileWarning)
```

In [15]:

```
dft_startTime = time.time()
dft_result = N_DFT(signal, len(signal))
dft_time = time.time() - dft_startTime

print(dft_time)
```

```
118.154667139
```

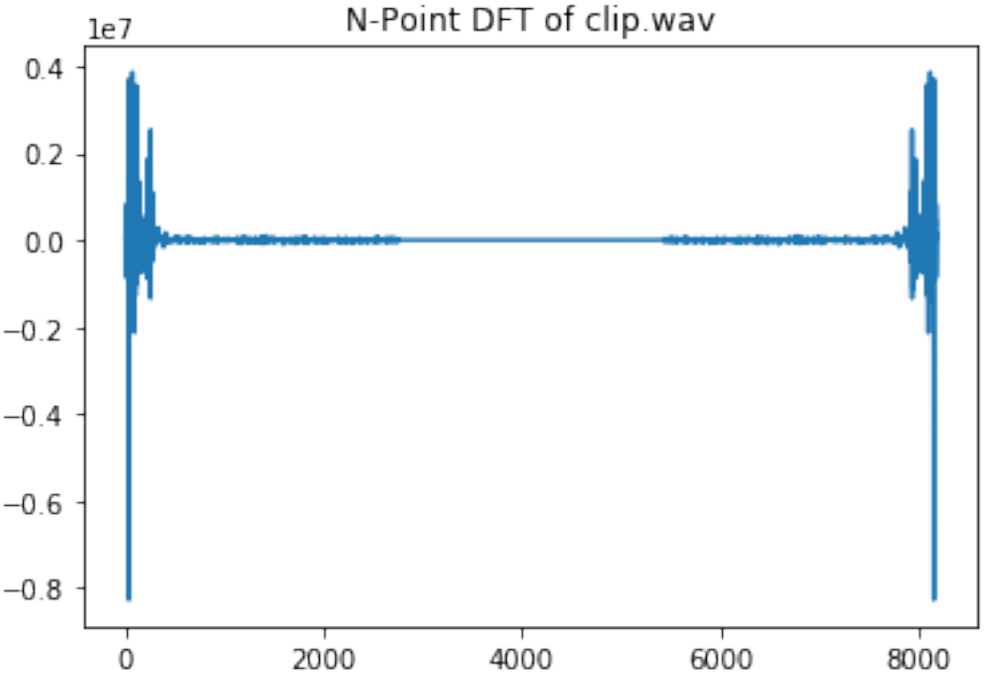
In [16]:

```
fft_startTime = time.time()
fft_result = My_FFT(signal, len(signal))
fft_time = time.time() - fft_startTime

print(fft_time)
```

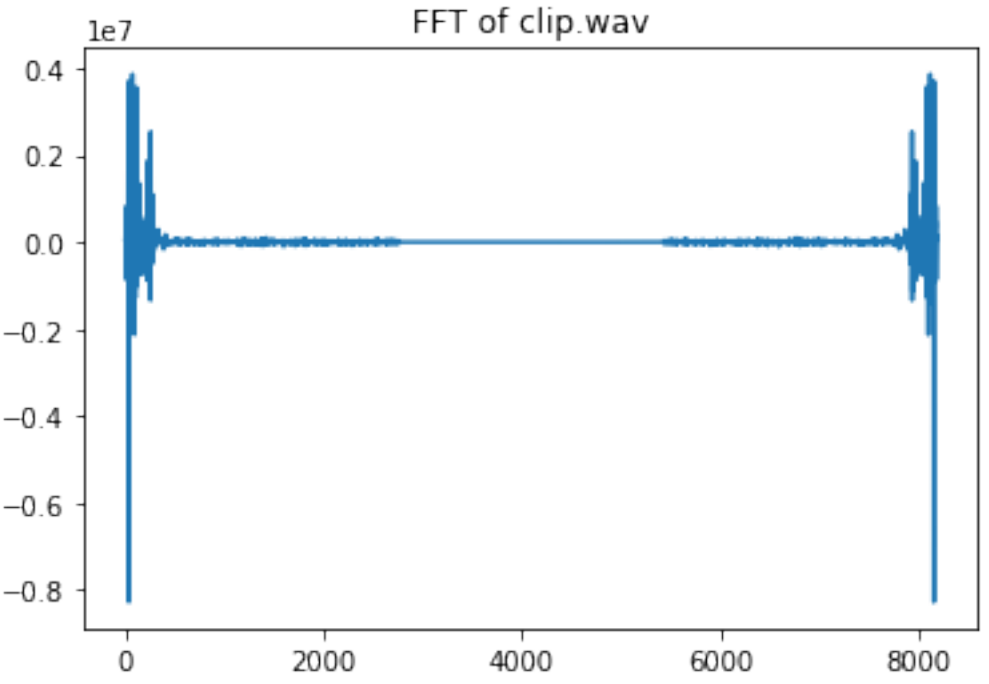
```
/usr/local/lib/python2.7/site-packages/ipykernel_lau
ncher.py:6: ComplexWarning: Casting complex values t
o real discards the imaginary part

/usr/local/lib/python2.7/site-packages/ipykernel_lau
ncher.py:7: ComplexWarning: Casting complex values t
o real discards the imaginary part
  import sys

0.356539964676
```

In [31]:

```
plt.plot(dft_result)
plt.title('N-Point DFT of clip.wav')
plt.show()

print('N-Point DFT computing time: ', dft_time)

plt.plot(fft_result)
plt.title('FFT of clip.wav')
plt.show()

print('FFT computing time: ', fft_time)
```




```
('N-Point DFT computing time: ', 121.77304196357727)
```




```
('FFT computing time: ', 0.29848599433898926)
```

In [ ]: