SOLID MECHANICS
(CEN 102)

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CABLES

Cables are basically subjected to tension

Cable Stayed Bridge

Cable suspension Bridge

Power Transmission Lines

BOOKS

1. Structural Analysis: R. C. Hibbeler (Pearson)
2. Basic Structural Analysis: C. S. Reddy (McGraw Hill)
3. Intermediate Structural Analysis: C. K. Wang (McGraw Hill)
4. Theory of Structures: S. Ramamrutham and R. Narayan (Dhanpat Rai Publishing Company)

CABLE STRUCTURES

: Cables are basically subjected to tension

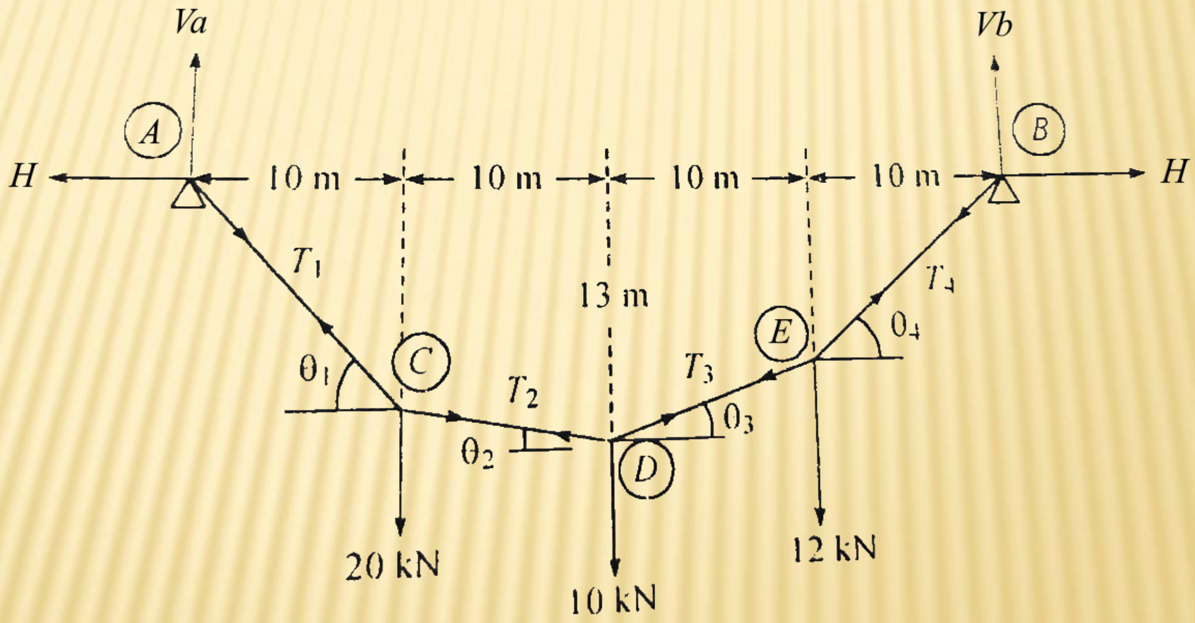
Cables may be subjected to concentrated load or uniformly distributed load. Accordingly cables may take the form of several straight line segments each of which is subjected to constant tensile force or they may take parabolic or any other shape.

In the analysis of cable the following assumptions are made:

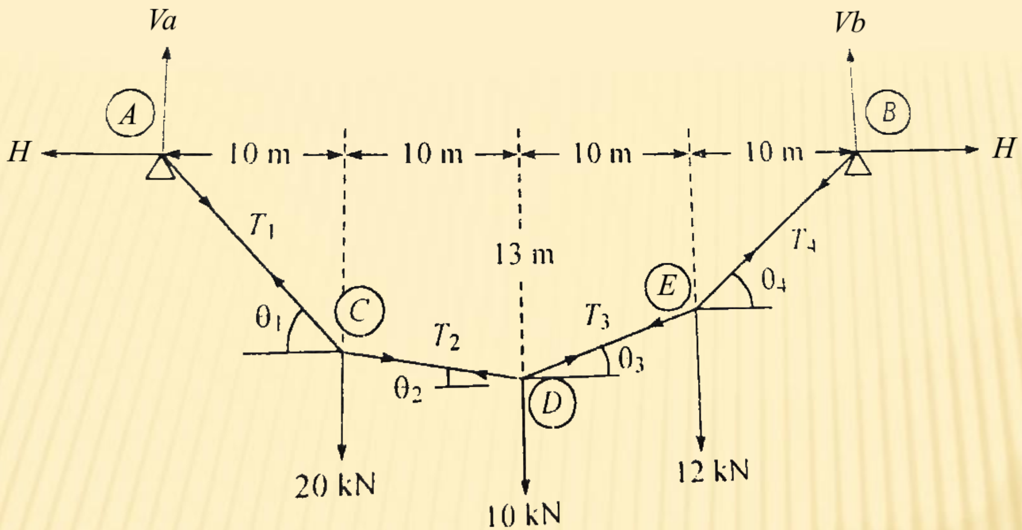
1. Cables are perfectly flexible (i.e., bending moment at each point is zero) and inextensible.
2. Applied loadings are coplanar with the cable.

Example: 1 Cable Subjected to concentrated load

A cable (cord) is supported at its ends which are 40 m apart. It carries loads of 20 kN, 10 kN, 12 kN at distances 10 m, 20 m and 30 m from the left end. If the point on the cord where the 10 kN load is supported is 13 metres below the level of the end supports, determine (i) the reactions at the supports , (ii) tensions in the different parts of the cord and (iii) total length of the cord.



Solution: Let V_a and V_b be the vertical reactions at the supports *A* and *B*. Let *H* be the horizontal reaction at each support



Taking moment about the left support at *A*, we have:

V_b x 40 = 20x10+10x20+12x30

V_b = 19 kN, Therefore, V_a = (20+10+12) -19 = 23 kN.

Since the position of point *D* is known and B.M. at *D* is zero (considering the left side of *D*),

$$Hx13 + 20x10 = 23x20, H = 20\ kN$$

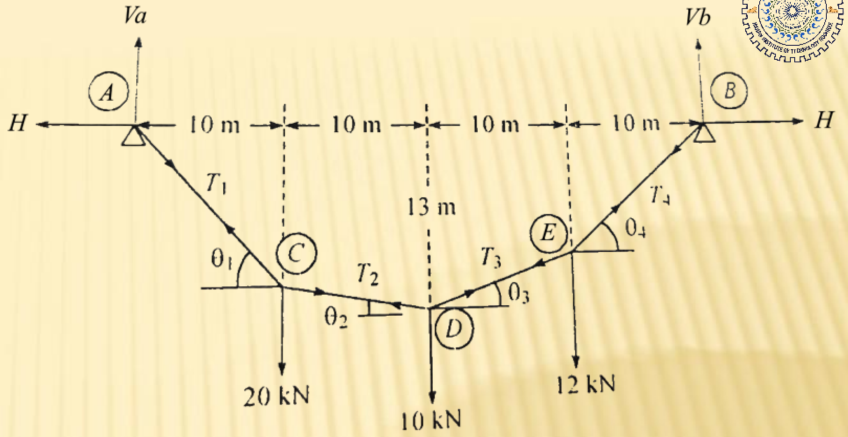
Tensions T_1, T_2, T_3 and T_4 in parts *AC*, *CD*, *DE* and *EB* can be easily calculated as,

$$T_1 = \sqrt{23^2 + 20^2} = 30.48\ kN$$

$T_2 = \sqrt{(23-20)^2+20^2}$ = 20.22 kN

$T_4 = \sqrt{19^2+20^2}$ = 27.59 kN

$T_3 = \sqrt{(19-12)^2+20^2}$ = 21.19 kN



Angles,

$\theta_1 = \tan^{-1}\frac{23}{20} = 49^\circ$, $\theta_2 = \tan^{-1}\frac{3}{20} = 8^\circ 32'$

$\theta_3 = \tan^{-1}\frac{7}{20} = 19^\circ 18'$, $\theta_4 = \tan^{-1}\frac{19}{20} = 43^\circ 32'$

Length of different line segments ,

$AC = 10\sec 49^\circ$= 15.24 m, $CD = 10\sec 8^\circ 32'$= 10.11 m,

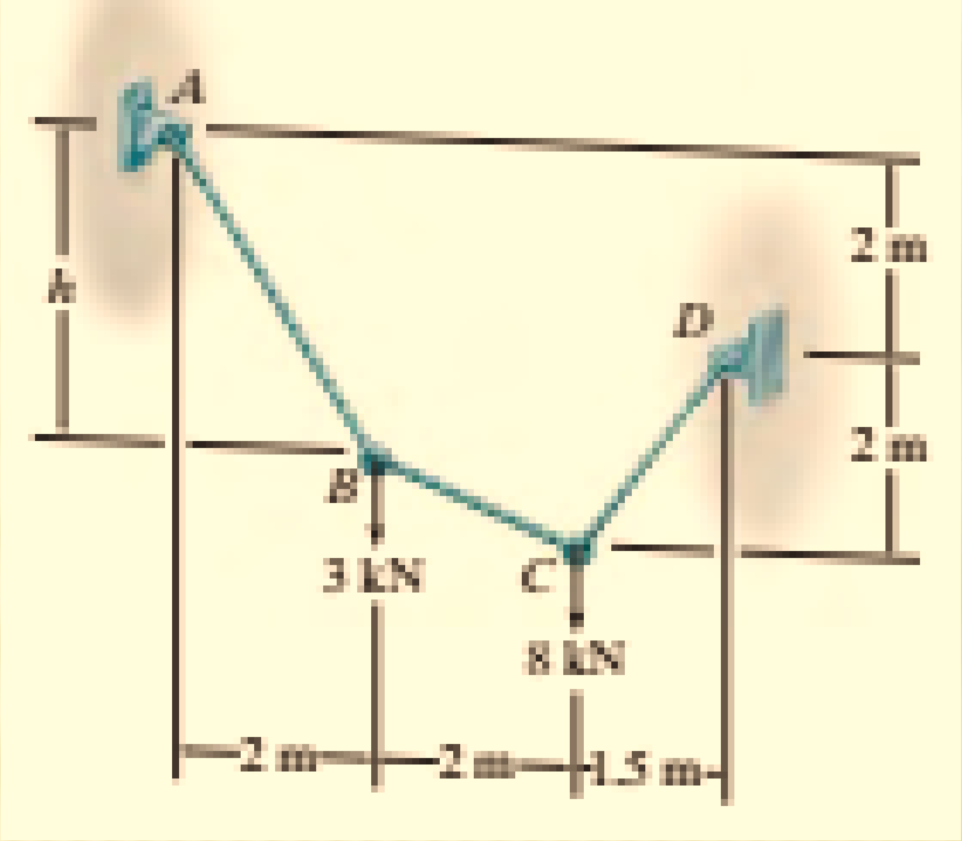
$DE = 10\sec 19^\circ 18'$ = 10.60 m, $EF = 10\sec 43^\circ 32'$= 13.79 m

Therefore,

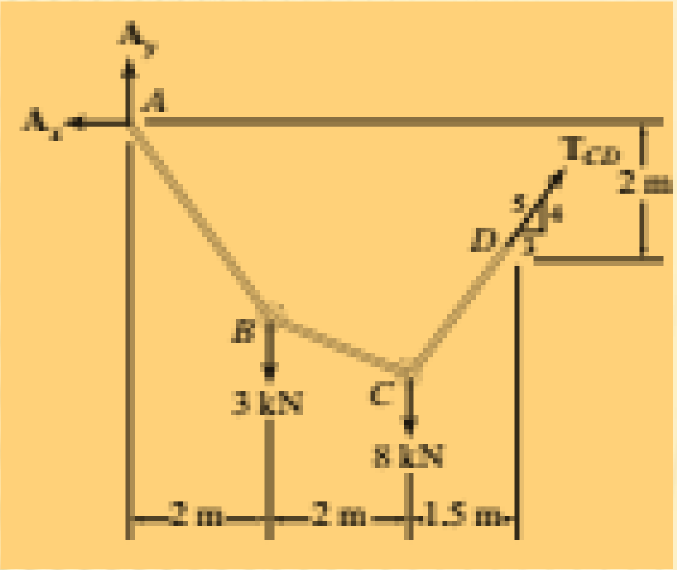
total length of the cord = 15.24 + 10.11 + 10.60 + 13.79 m = 49.74 m

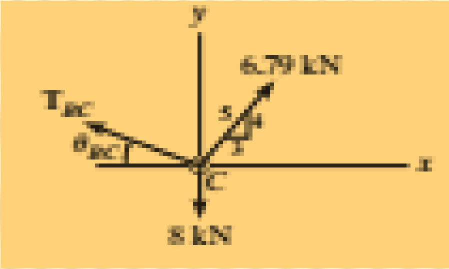
Example: 2

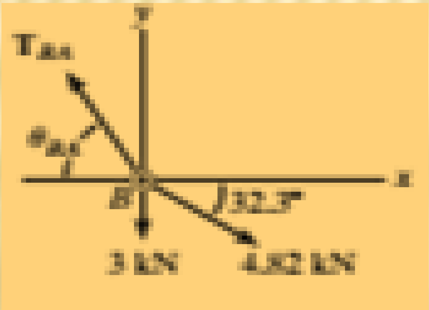
Determine the tension in each segment of the cable shown in the figure below. Also, determine the dimension *h*?.



Solution:







As the slope of portion *CD* is given, it is required to take moment about *A* as *h* is unknown.

$\sum M_A = 0;$

$T_{CD}(3/5)(2) + T_{CD}(4/5)(5.5) – 3x2-8x4 = 0$, $T_{CD} = 6.79$ kN

Now we consider the equilibrium of point *C* and *B* in sequence.

Point *C*:

$\sum F_x = 0$, $6.79(3/5) - T_{BC}\cos\theta_{BC} = 0$

$\sum F_y = 0$, $6.79(4/5) - 8 + T_{BC}\sin\theta_{BC} = 0$

$\theta_{BC} = 32.3°$ and $T_{BC} = 4.82$ kN

Point *B*:

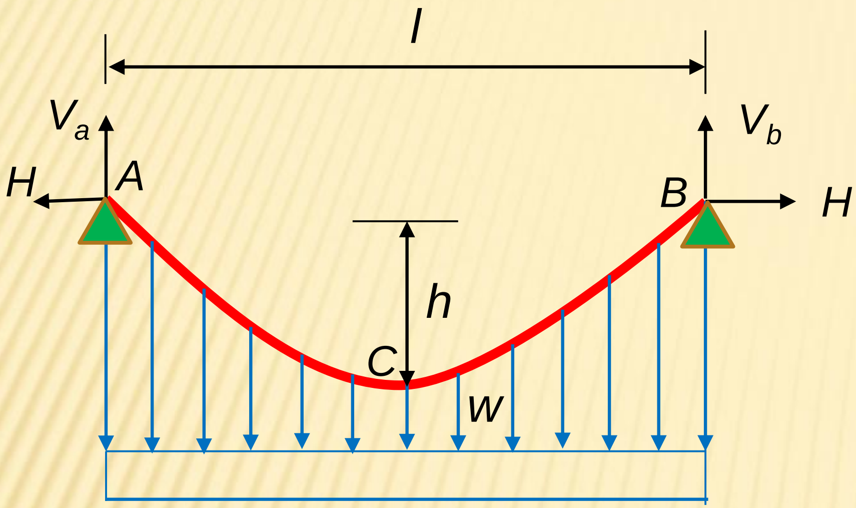
$\sum F_x = 0$, $- T_{BA}\cos\theta_{BA} + 4.82\cos 32.3° = 0$

$\sum F_y = 0$, $- T_{BA}\sin\theta_{BA} - 4.82\sin 32.3° - 3 = 0$

$\theta_{BA} = 53.8°$ and $T_{BA} = 6.90$ kN

From figure, $h = 2\tan 53.8° = 2.74$ m

Example: 3 Cable Subjected to Uniformly distributed load



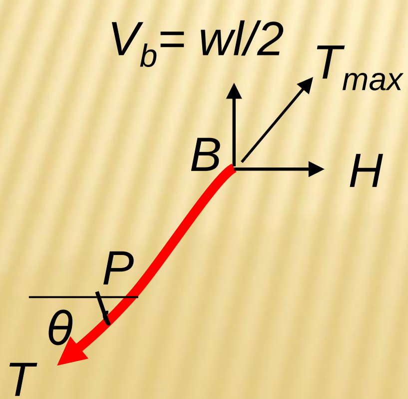
From symmetry, we have:

$V_a = V_b = wl/2$

Taking moment about *C* of the forces on the right hand side of it, we have:

$H.h + w.l/2.l/4 = w.l/2.l/2$, i.e. $H = \frac{wl^2}{8h}$

Maximum tension at the supports, $T_{max} = \sqrt{\left(\frac{wl}{2}\right)^2 + \left(\frac{wl^2}{8h}\right)^2} = \frac{wl}{2}\sqrt{1 + \frac{l^2}{16h^2}}$

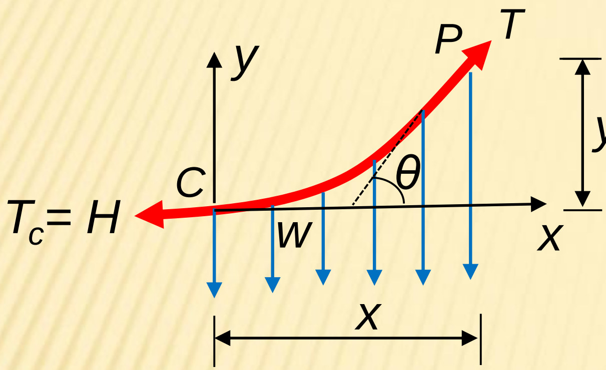


Let, *T* be the tension at any point *P* which is having an inclination of θ with the horizontal line,

Then at point *P*, $T \cos\theta = H$

At $\theta = 0$, i.e., at Point *C*, $T_c = H$

Shape of loaded cord



*Co-ordinates defined with respect to point C

$$T\sin\theta = wx$$

$$T\cos\theta = T_C = H, \therefore \tan\theta = \frac{wx}{H} = \frac{dy}{dx}$$

$$\therefore dy = \frac{wx}{H}\,dx$$

Integrating, $y = \frac{wx^2}{2H} + k$

At C, x =0 and y =0, k = 0, $\therefore y = \frac{wx^2}{2H}$

Replacing $H = \frac{wl^2}{8h}$ in the above, $y = \frac{4h}{l^2}x^2$

If the origin is taken at any end (A or B), then, $y = \frac{4hx}{l^2}(l - x)$

Length of cable

$$y = \frac{4h}{l^2}x^2, \frac{dy}{dx} = \frac{8hx}{l^2}$$

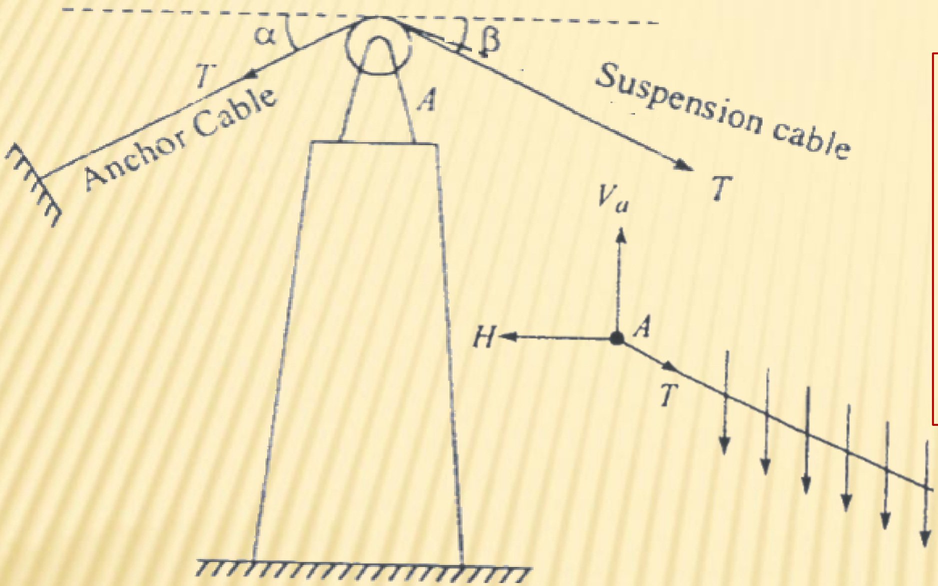
$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx = \sqrt{1 + \left(\frac{8hx}{l^2}\right)^2}\,dx$$

Neglecting higher terms, $ds = \left(1 + \frac{32h^2x^2}{l^4}\right)dx$

$\therefore$ **Total length of cable, L** $= 2\int_0^{l/2}\left(1 + \frac{32h^2x^2}{l^4}\right)dx = \boldsymbol{l + \frac{8}{3}\frac{h^2}{l}}$

DIFFERENT TYPES OF CABLE SUPPORTS

Case: I Cable passed over guide pulley at the supports (Frictionless pulley)



$$T = \sqrt{V_a^2 + H^2} \text{ and } \tan\beta = \frac{V_a}{H}$$

The vertical force transmitted to the cable,

$= T\sin\alpha + T\sin\beta = T(\sin\alpha + \sin\beta)$

Net horizontal force transmitted at the top of the pier,

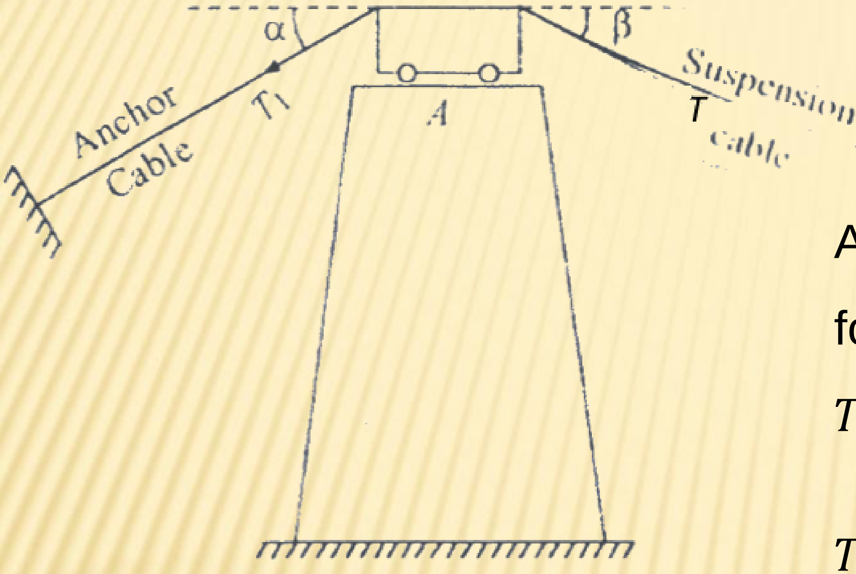
$= T\cos\alpha \sim T\cos\beta$

$= T(\cos\alpha \sim \cos\beta)$

Maximum bending moment in the pier

= Net Horizontal force x Height of the pier

Case: II Cable clamped to saddle carried on smooth rollers on the top of a pier



$$T = \sqrt{V_a^2 + H^2} \text{ and } \tan\beta = \frac{V_a}{H}$$

As the rollers cannot resist any horizontal force, for the equilibrium of the saddle,

$$T_1 \cos\alpha = T\cos\beta$$

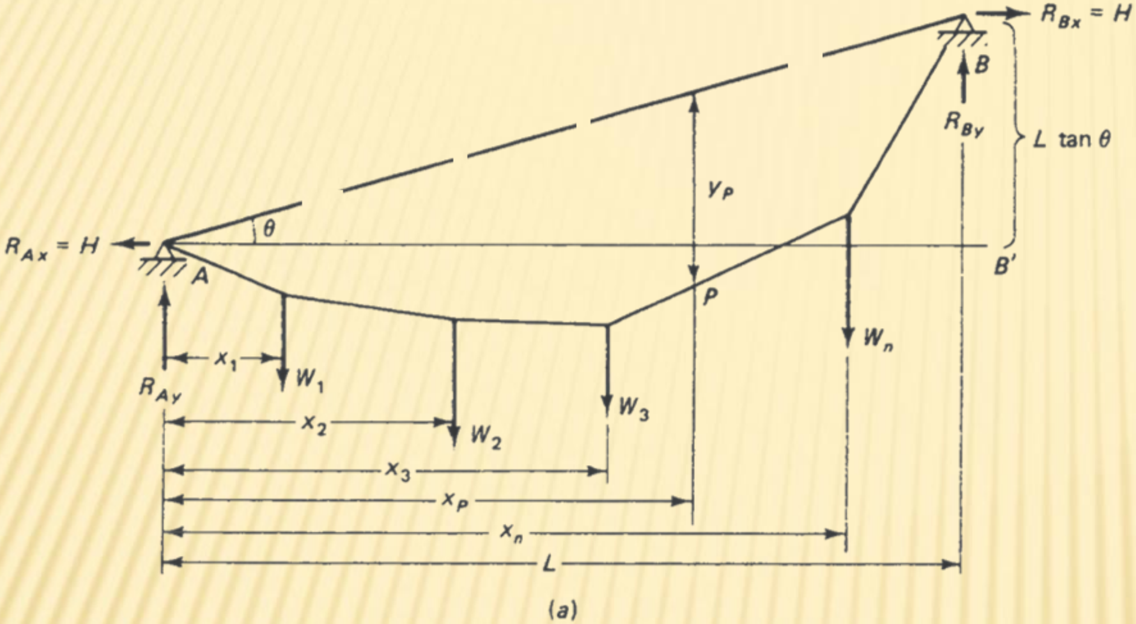
$$T_1 = T\frac{\cos\beta}{\cos\alpha}$$

Total vertical load transmitted to the pier $= T_1 \sin\alpha + T\sin\beta$

$$= T\frac{\cos\beta}{\cos\alpha}\sin\alpha + T\sin\beta$$

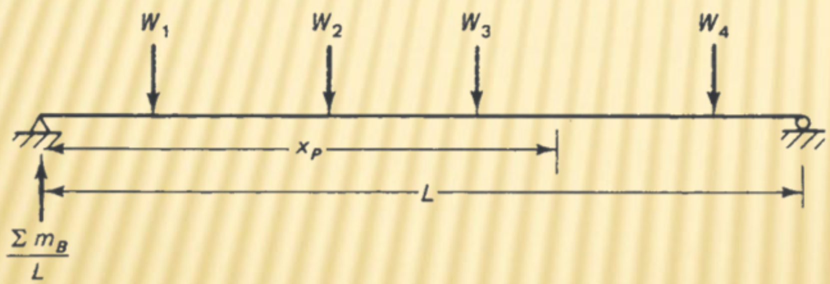
$$= T(\tan\alpha\cos\beta + \sin\beta)$$

CABLE: DERIVATION FOR GENERAL CASE



$\sum m_B =$ Summation of moments about *B* of all external loads W_1, W_2, …W_n

$\sum m_P =$ Summation of moments about *P* of those loads W_1, W_2, …W_n acting on cable to the left of point *P*

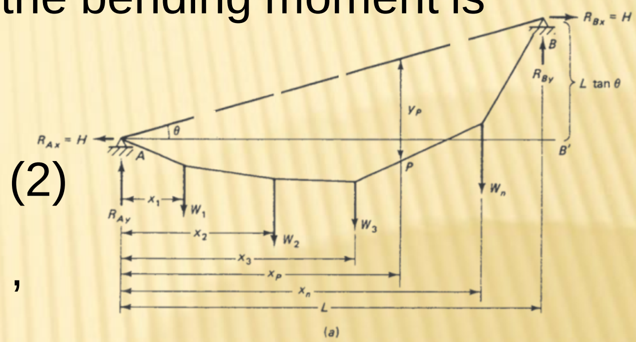


For moment equilibrium , moment of all forces acting on the structure about point, *B* =0. Assuming clockwise rotation to be positive,

$$R_{AY}.L + H.L.\tan\theta - \sum m_B = 0,$$

$$R_{AY} = \frac{\sum m_B}{L} - H\tan\theta \qquad (1)$$

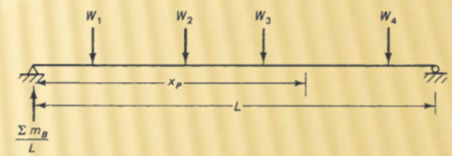
Since the cable is assumed to be perfectly flexible, the bending moment is zero at all points. Applying that condition to point *P*,



$$R_{AY}(X_p) - H(Y_p - X_p \tan\theta) - \sum m_P = 0 \quad (2)$$

Substituting for R_{AY} from (1) into (2) and simplifying ,

$$HY_p = \frac{X_p}{L}\sum m_B - \sum m_P$$



The right hand side of the above equation is nothing but bending moment at *P* of the simply supported beam subjected to the same external load as that on the cable.

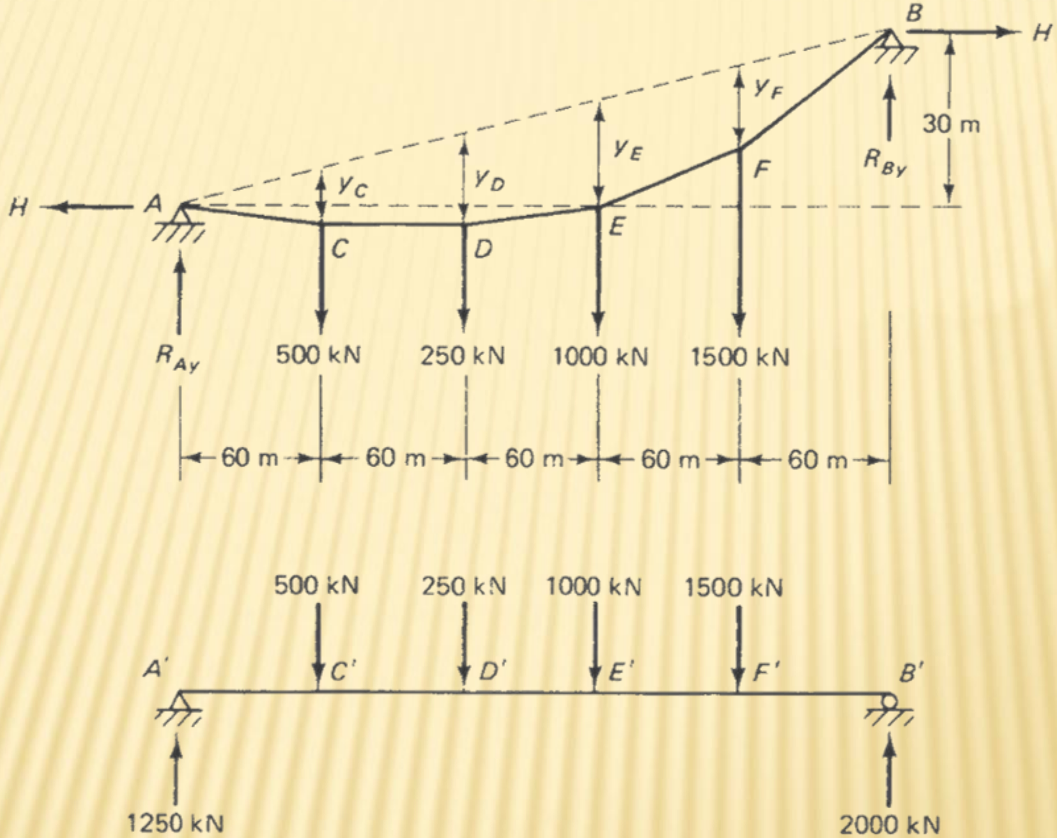
Theorem:

Product of horizontal component of cable tension and vertical distance between the cable chord and cable at any point

= Bending moment at that point on a horizontal simply supported beam subjected to the same external load as that on the cable

Example:

For the cable shown in the figure below, calculate the vertical distances of different points *C*, *D*, *F* and tension in different segments if the vertical distance at *E*, Y_E = 15 m.



Solution:

Bending moment at *C´* = 1250x60 = 75000 kN-m

Bending moment at *D´* = 1250x120 - 500x60 = 120000 kN-m

Bending moment at *E´* = 1250x180 – 500x120 – 250x60 = 150000 kN-m

Bending moment at *F´* = 2000x60 = 120000 kN-m

Since the maximum bending moment occurs at E', the maximum value of Y will also be at E', i.e., $Y_E = Y_{max.}$

∴ H. Y_{max} = H. Y_E = 150000, i.e., H = 10000 kN

Referring to the figure,

$$\sum F_Y = 0;\ R_{AY} + R_{BY} - 500 - 250 - 1000 - 1500 = 0$$

Also, $\sum M_B = 0;$

$$R_{AY}\mathrm{x}300 + H\mathrm{x}30 - 500\mathrm{x}240 - 250\mathrm{x}180 - 1000\mathrm{x}120 - 1500\mathrm{x}60 = 0$$

Giving, $R_{AY} = 250\ \mathrm{kN}, R_{BY} = 3000\ \mathrm{kN}$

Therefore, $$Y_c = \frac{M_c'}{H} = 7.5\ \mathrm{m}$$

$$Y_D = \frac{M_D'}{H} = 12\ \mathrm{m}$$

$$Y_E = \frac{M_E'}{H} = 15\ \mathrm{m}$$

$$Y_F = \frac{M_F'}{H} = 12\ \mathrm{m}$$

Table: Tension in different segments

Segment	Horizontal component, H (kN)	Vertical Component, V (kN)	Resultant Tension (kN) = $\sqrt{H^2 + V^2}$
AC	10000	250	10003
CD	10000	250-500 = -250	10003
DE	10000	250-500-250 = -500	10012
EF	10000	250-500-250-1000 = -1500	10112
FB	10000	250-500-250-1000 - 1500 = -3000	10440