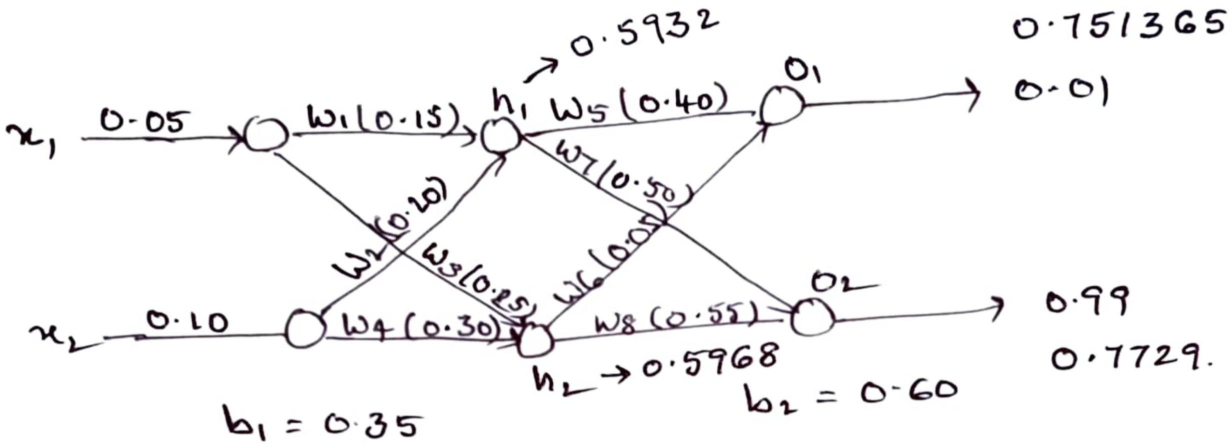
Back Propagation Algorithm:



part - I: Calculate forward Propagation Error

1) Calculate h_1 (in and out)

$$h_1(in) = w_1 x_1 + w_2 x_2 + b_1$$

$$= 0.15 \times 0.05 + 0.2 \times 0.1 + 0.35$$

$$= 0.377$$

$$h_1(out) = \frac{1}{1+e^{-h_1(in)}}$$

$$= \frac{1}{1+e^{-0.377}}$$

$$h_1(out) = 0.5932$$

2) Calculate h_2 (in and out)

$$h_2(in) = 0.05 \times 0.25 + 0.10 \times 0.3 + 0.35$$

$$= 0.3725$$

$$h_2(out) = \frac{1}{1+e^{-h_2(in)}} = \frac{1}{1+e^{-0.3725}} = 0.5968$$

$h_2(out) = 0.5968.$

3) Calculate O_1 (in and out):

$O_1(in) = h_1(out) \times w_5 + h_2(out) \times w_6$

$= 0.593 \times 0.4 + 0.596 \times 0.45 + 0.6$

$= 1.105$

$O_1(out) = \frac{1}{1+e^{-O_1(out)}}$

$= \frac{1}{1+e^{-1.105}}$

$O_1(out) = 0.751365$

4) Calculate O_2 (in and out):

$O_2(in) = h_1(out) \times w_7 + h_2(out) \times w_8 + b_2$

$= 0.5932 \times 0.50 + 0.5968 \times 0.55 + 0.6$

$= 1.22484$

$O_2(out) = \frac{1}{1+e^{-O_2(in)}}$

$= \frac{1}{1+e^{-1.22484}}$

$= 0.7729$

$= \frac{1}{2}(0.01 - 0.7513)^2 +$
$\frac{1}{2}(0.99 - 0.7739)^2$

$= 0.274 + 0.0235$

$= 0.29839.$

5) Calculate E_{Total} (Total Error):

$E_{Total} = \frac{1}{2} \sum (target - O/p)^2$

$= E_{O1} + E_{O2}$

part – II : calculate Back Propagation Error:

(out put Layer → hidden layer)

update weights W_5, W_6, W_7 and W_8

first let us adjust W_5

$$\boxed{\eta = 0.6}$$

$$W_5^* = W_5 - \eta \frac{\partial E_{total}}{\partial W_5}$$

Consider $$\frac{\partial E_{total}}{\partial W_5} = \frac{\partial E_{total}}{\partial out\,O_1} \times \frac{\partial out\,O_1}{\partial net\,O_1} \times \frac{\partial net\,O_1}{\partial W_5}$$

$$\therefore \frac{\partial E_{total}}{\partial out\,O_1} = out\,O_1 - target\,O_1$$

$$= 0.751365 - 0.01$$

$$= 0.741365$$

$$\frac{\partial out\,O_1}{\partial net\,O_1} = out\,O_1(1 - out\,O_1)$$

$$= 0.751365(1 - 0.751365)$$

$$= 0.18681560.$$

$$\frac{\partial net\,O_1}{W_5} = out\,h_1 = 0.5932$$

$$\frac{\partial E_{total}}{\partial W_5} = 0.741365 \times 0.18681560 \times 0.5932$$

$$= 0.08216704.$$

$$W_5^* = W_5 - \eta \frac{\partial E_{total}}{\partial W_5}$$

$$= 0.4 - 0.6 \times 0.08216704$$

$$W_5^* = 0.350699776.$$

Part 3: calculating backward propagation of Error

(Hidden Layer – input Layer)

Updated → W_1, W_2, W_3 and W_4

first Let's Adjust W_1

$$W_1^* = W_1 - \eta \frac{\partial E_{total}}{\partial W_1}$$

$$\therefore \frac{\partial E_{total}}{\partial W_1} = \frac{\partial E_{total}}{\partial out\, h_1} \times \frac{\partial out(h_1)}{\partial net\, h_1} \times \frac{\partial net\, h_1}{\partial W_1}$$

Consider

$$\frac{\partial E_{total}}{\partial out\, h_1} = \frac{\partial E_{o1}}{\partial out\, h_1} + \frac{\partial E_{o2}}{\partial out\, h_1}$$

$$\frac{\partial E_{o1}}{\partial out\, h_1} \rightarrow \frac{\partial E_{o1}}{\partial net\, o_1} \times \frac{\partial net\, o_1}{\partial out\, h_1}$$

$$\frac{\partial E_{o1}}{\partial net\, o_1} \rightarrow \frac{\partial E_{o1}}{\partial out\, o_1} \times \frac{\partial out\, o_1}{\partial \cancel{out\, h_1}\, net\, o_1}$$

$$\frac{\partial E_{o2}}{\partial out\, h_1} \rightarrow \frac{\partial E_{o2}}{\partial net\, O_2} \times \frac{\partial net\, O_2}{\partial out\, h_1}$$

$$\frac{\partial E_{o2}}{\partial net\, O_2} \rightarrow \frac{\partial E_{o2}}{\partial out\, O_2} \times \frac{\partial out\, O_2}{\partial \cancel{out\, h_1}\, net\, O_2}$$

$$\therefore \frac{\partial E_{o2}}{\partial out\, O_2} = (out\, O_2 - target\, O_2)$$

$$= 0.77292847 - 0.99$$

$$= -0.217071535$$

$$\frac{\partial out\ o_2}{\partial net\ o_2} = out\ o_2(1 - out\ o_2)$$
$$= 0.7729(1 - 0.7729)$$
$$= 0.175510052$$

$$\frac{\partial net\ o_1}{\partial out\ h_1} = \text{on } o_1 \text{ from } h_1 \Rightarrow w_5 = 0.40.$$

$$\frac{\partial net\ o_2}{\partial out\ h_1} = \text{on } o_2 \text{ from } h_1 \Rightarrow w_7 = 0.50$$

$$\frac{\partial E_{total}}{\partial out h_1} = \frac{\partial E_{o1}}{\partial out h_1} + \frac{\partial E_{o2}}{\partial out h_2} = 0.13849856 \times 0.4 + (-0.03809 \times 0.5)$$
$$= 0.05539 + (-0.019049119)$$
$$= 0.036350306.$$

$$\frac{\partial out h_1}{\partial net h_1} = out h_1(1 - out h_1)$$
$$= 0.5932(1 - 0.5932)$$
$$= 0.241300709$$

$$\frac{\partial net h_1}{\partial w_1} = \frac{\partial}{\partial w_1}(w_1 x_1 + \underset{0}{\underline{\underline{w_2 x_2}}} + \underset{0}{\underline{\underline{b_1}}})$$
$$= x_1$$
$$= 0.05$$

$$\therefore \frac{\partial E_{total}}{\partial w_1} = \frac{\partial E_{total}}{\partial out h_1} \times \frac{\partial out h_1}{\partial net h_1} \times \frac{\partial net h_1}{\partial w_1}$$
$$= 0.036350306 \times 0.241300709 \times 0.05$$
$$= 0.00438568.$$

$$w_1^* = w_1 - \eta \frac{\partial E_{total}}{\partial w_1}$$

$$= 0.15 - 0.6 \times 0.00438568$$

$$= 0.149736859 2$$

In the similar way we can update $w_2, w_3, w_4, w_6, w_7, w_8$

$$\frac{\partial E_{o1}}{\partial out_{o1}} = (out_{o1} - target_{o1})$$

$$= 0.751365 - 0.01$$

$$= 0.741365$$

$$\frac{\partial out_{o1}}{\partial net_{o1}} = out_{o1}(1 - out_{o1})$$

$$= 0.7729(1 - 0.7729)$$

$$= 0.17552559.$$