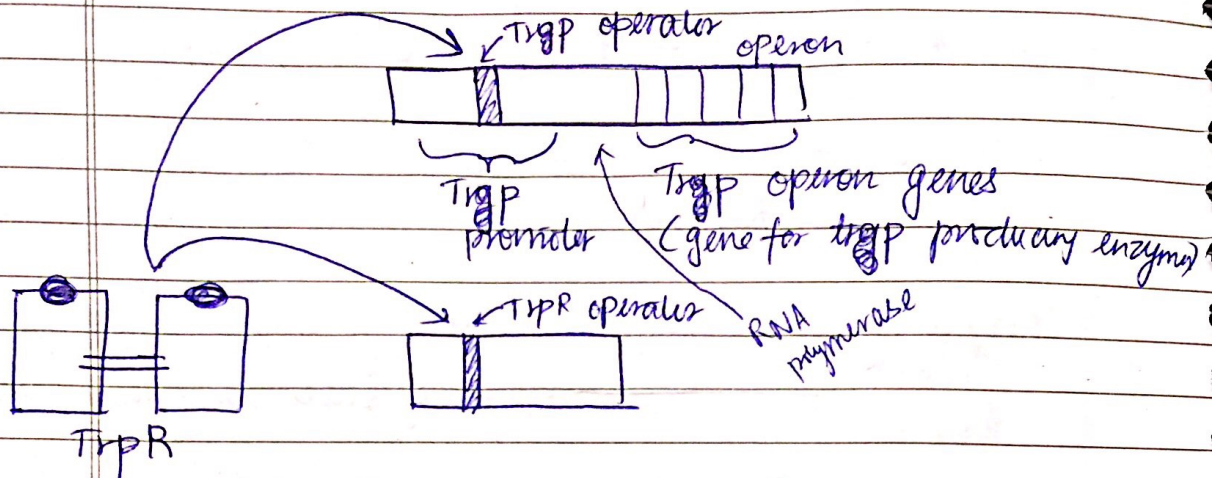


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### Homework # 5



Now, ODE of gene regulation model for tryptophan producing enzymes is as follows:

$$\frac{d(t_r)}{dt} = K - K_d(t_r) - K'(t_r)$$

$K$ ,  $K_d$ ,  $K'$  are the constants

$K'$  which is the repressing constant.

Now, we see that in case of no repressor, our ODE comes out to be

$$\frac{d(t_r)}{dt} = K - K_d(t_r)$$

At steady state,  $\frac{d(t_r)}{dt} = 0$

$$\Rightarrow K - K_d(t_r) = 0 \Rightarrow t_r = \frac{K}{K_d} \text{ --- (1)}$$

When repressor is present,  $K'$  is not 0  
ODE is

$$\frac{d(t_r)}{dt} = K - K_d(t_r) - K'(t_r)$$

At steady state,  $\frac{d(t_r)}{dt} = 0$

$$\Rightarrow K - K_d(t_r) - K'(t_r) = 0$$

$$\Rightarrow t_r = \frac{K}{K_d + K'} \quad - (2)$$

Comparing  $t_r$  level at steady state from eq (1) & (2)

\_\_\_\_\_  $\leftarrow$  steady state when no repressor

$\downarrow K'$

\_\_\_\_\_  $\leftarrow$  steady state when repressor

The steady state level drops

Now, suppose we have an additional regulation (tryptophan bound  $t_p R$  binding with operator region of  $t_p R$  promoter).

In such a case we can see that our ~~repressor~~ ~~constant~~ repressing factor changes (decreases).

Say, our new repressor is  $(K' - c R_A)$   
Then our new ODE is

$$\frac{d(t_r)}{dt} = K - K_d(t_r) - (K' - c R_A) t_r$$

$R \rightarrow$  Total Repressor Concentration

$R_A \rightarrow$  Active repressor Concentration

At steady state  $\frac{d(t_r)}{dt} = 0$



$$R_A = \frac{t_r(t) \cdot R}{t_r(t) + K_t}$$

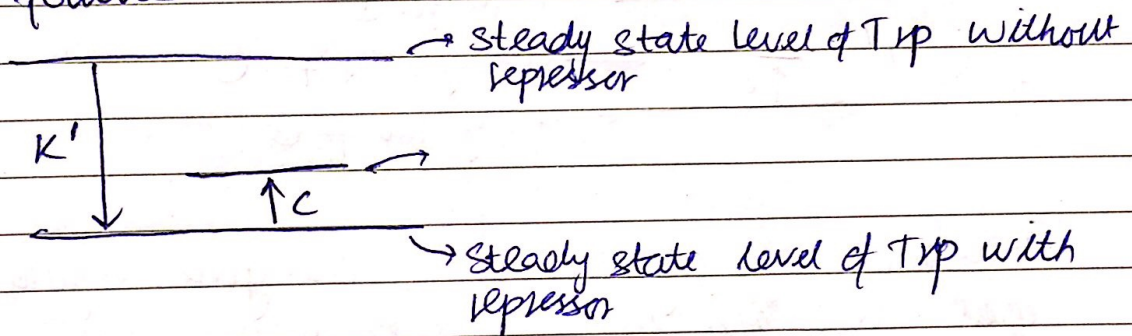
$R \rightarrow$  Total repressor concentration

$R_A \rightarrow$  Active repressor concentration

Modified  
ODE

$$\frac{d(t_r)}{dt} = k - k_d(t_r) - (k' - c R_A) t_r \quad - (3)$$

The variation in steady state levels of  $T_{ip}$  is as follows.



Biological ~~sig~~ significance of the gene regulatory circuit that modulates the level of tryptophan inside a cell.  
(Adapt to fluctuating levels of tryptophan in the environment)

Trp R produces the repressor & they dimerize. This protein is in inactive form & does not recognise the operator region. When repressor is not bound to the operator, the operon is active and Trp is synthesised.

When Trp level raises and no more Trp is required, Trp binds to dimeric repressor in one to one ratio. Binding of Trp to repressor makes it active and the protein now binds to the operator & tries to block RNA transcription.

This mechanism is called Attenuator mechanism in which cells are involved in.

When concentration of Tip becomes too high, the dimer ( $\text{Tip} + \text{repressor}$ ) now also gets attached to the ~~top~~ operator of Tip R, hence reducing the repressor concentration (& its effect) & slightly increasing the amount of Tip at steady state.

Note: ~~The~~ Diagram for the same given on ~~front~~ first page. & modified ODE given by eqn. (3).