1 Relativistic System Explained

In the relativistic regime, we are faced with a problem. Namely, the time variable changes by a factor of γ. As such, momentum changes.

$$p = m_0 dr/dt_0$$

but:

$$dt = \gamma dt_0$$

therefore:

$$p = m_0 \gamma v$$

Now, we can use this to define our Hamiltonian:

$$H = c\sqrt{m_0^2 c^2 + p^2} + q\phi$$

If we set $\phi = 0$ then $H^2 = E^2$ s.t.:

$$E^2 = c^2(m_0^2 c^2 + p^2)$$

which is the typical form of the relativistic energy of a free particle.

Secondly, we need to confirm that $dH/dp = v = qdot$, so:

$$dH/dp = cp(m_0^2 c^2 + p^2)^{-1/2}$$

Now, we know $p = m_0 \gamma v$ therefore if we assume $dH/dp = v$ we get:

$$\frac{\gamma m_0 v c}{\sqrt{m_0^2 c^2 + \gamma^2 m_0^2 v^2}} = v$$

$$\gamma^2 c^2 m_0^2 = m_0^2 c^2 + \gamma^2 m_0^2 v^2$$

simplifying this further:

$$m_0^2 c^2 (\gamma^2 - 1) = \gamma^2 m_0^2 v^2 / c^2$$

yields:

$$\frac{v^2/c^2}{1 - v^2/c^2} = \frac{v^2/c^2}{1 - v^2/c^2}$$