Homework - 4 Solutions.

1)
$$\begin{bmatrix} 0 & 0 & 2 & 2 & 0 & 2 \\ 2 & 2 & 0 & 2 & 1 & 2 \\ 1 & 1 & 2 & 0 & 2 & 2 \\ 1 & 1 & 0 & 1 & 2 & 1 \end{bmatrix}$$

$R_1 \rightarrow R_1 + R_3$

$$\begin{bmatrix} 1 & 1 & 1 & 2 & 2 & 1 \\ 2 & 2 & 0 & 2 & 1 & 2 \\ 1 & 1 & 2 & 0 & 2 & 2 \\ 1 & 1 & 0 & 1 & 2 & 1 \end{bmatrix}$$

$R_2 \rightarrow R_2 + R_1$, $R_3 \rightarrow 2R_1 + R_3$, $R_4 \rightarrow R_4 + 2R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 2 & 2 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 2 & 2 & 0 & 0 \end{bmatrix}$$

$R_3 \rightarrow R_3 + R_2$, $R_4 \rightarrow R_4 + R_2$

$$\begin{bmatrix} 1 & 1 & 1 & 2 & 2 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 \rightarrow R_1 + 2R_2$,

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_1 \to R_1 + 2R_3$

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

⟨2⟩ $\begin{bmatrix} 0 & 0 & 2 & 2 & 0 & 2 \\ 2 & 2 & 6 & 8 & 4 & 8 \\ 1 & 1 & 5 & 6 & 2 & 5 \\ 1 & 1 & 3 & 4 & 2 & 7 \end{bmatrix}$

$R_1 \to R_1 + R_3$

$$\begin{bmatrix} 1 & 1 & 7 & 8 & 2 & 7 \\ 2 & 2 & 6 & 8 & 4 & 8 \\ 1 & 1 & 5 & 6 & 2 & 5 \\ 1 & 1 & 3 & 4 & 2 & 7 \end{bmatrix}$$

$R_2 \to R_2 + 9R_1$, $R_3 \to R_3 + 10R_1$, $R_4 \to R_4 + 10R_1$

$$\begin{matrix} 1 & 1 & 7 & 8 & 2 & 7 \\ 0 & 0 & 3 & 3 & 0 & 5 \\ 0 & 0 & 9 & 9 & 0 & 9 \\ 0 & 0 & 7 & 7 & 0 & 0 \end{matrix}$$

$R_3 \to R_3 + 2R_4$

$$\begin{bmatrix} 1 & 1 & 7 & 8 & 2 & 7 \\ 0 & 0 & 3 & 3 & 0 & 5 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

$R_2 \to R_2 + 8R_4$, $R_3 \to R_3 + 10R_4$

$$\begin{bmatrix} 1 & 1 & 7 & 8 & 2 & 7 \\ 0 & 0 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

$R_2 \rightarrow R_2 + 6R_3$

$$\begin{bmatrix} 1 & 1 & 7 & 8 & 2 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

$R_2 \rightarrow R_4$

$$\begin{bmatrix} 1 & 1 & 7 & 8 & 2 & 7 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 \rightarrow R_1 + 4R_2 + 4R_3$

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

3) $\begin{bmatrix} 1 & 2 & 1 \\ 3 & -2 & 4 \\ 0 & 1 & 0 \end{bmatrix}$

Using Gauss-Jordan elimination to find matrix inverse

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 3 & 4 & 4 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \to R_2 + 3R_1$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 4 & 1 & 3 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to R_2 + 2R_3$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 & 2 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_1 \to R_1 + 5R_2 + 4R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 4 & 5 & 2 \\ 0 & 0 & 1 & 3 & 1 & 2 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_2 \Longleftrightarrow R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 4 & 5 & 2 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 3 & 1 & 2 \end{array}\right]$$

$$\qquad I \qquad\qquad\qquad A^{-1}$$

Inverse

$$= \begin{bmatrix} 4 & 5 & 2 \\ 0 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

(4) $G = \begin{bmatrix} 0 & 2 & 1 & 2 & 0 \\ 2 & 1 & 1 & 0 & 2 \\ 2 & 2 & 0 & 1 & 1 \end{bmatrix}$

a) Length of V, $n = 5$

b) $\begin{bmatrix} 0 & 2 & 1 & 2 & 0 \\ 2 & 1 & 1 & 0 & 2 \\ 2 & 2 & 0 & 1 & 1 \end{bmatrix}$

$R_1 \to R_1 + 2R_2$

$\begin{matrix} 1 & 1 & 0 & 2 & 1 \\ 2 & 1 & 1 & 0 & 2 \\ 2 & 2 & 0 & 1 & 1 \end{matrix}$

$R_2 \to R_2 + R_1$, $R_3 \to R_3 + R_1$

$\begin{bmatrix} 1 & 1 & 0 & 2 & 1 \\ 0 & 2 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$

$R_2 \to 2R_2$, $R_3 \to 2R_3$

$\begin{matrix} 1 & 1 & 0 & 2 & 1 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{matrix}$

$R_1 \to R_1 + 2R_2 + 2R_3$

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Dimension $k = 3$

(c) Parity Check matrix H for the linear code V.

$$G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Interchanging C_3 and C_5

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$[I \mid P]$$

$$H = [-P^T \mid I_{n-k}]$$

$$= \begin{bmatrix} -1 & -1 & 0 & 1 & 0 \\ -1 & -2 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Interchanging C_3 and C_5

$$H = \begin{bmatrix} 2 & 2 & 0 & 1 & 0 \\ 2 & 1 & 1 & 0 & 0 \end{bmatrix}$$

(5)

a) $$G = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \times G = \begin{bmatrix} 000 \\ 001 \\ 010 \\ 011 \\ 100 \\ 101 \\ 110 \\ 111 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

List of code Vectors. $$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$
\begin{array}{c|c}
d & \\
\begin{array}{c}0\\3\\4\\3\\4\\3\\4\\3\end{array} &
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 1 & 0 \\
0 & 1 & 1 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 & 1 & 1 \\
1 & 0 & 1 & 1 & 0 & 0 \\
1 & 1 & 0 & 1 & 0 & 1 \\
1 & 1 & 0 & 0 & 1 & 0
\end{bmatrix}
\end{array}
$$

$\therefore$ Minimum distance of linear codes is the min. weight of code vectors $\underline{=} 3$

<c> In linear codes, the linear combination of code words is also a codeword.

Therefore minimum distance is also the minimum weight among all the code words.

This might not be case for non-linear codes.

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(d) $G = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$

Interchange columns C_3 and C_4

$$G' = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$R_2 \rightarrow R_2 + R_3$

$$G' = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

$$G' = [I \mid P]$$

$$H' = [-P^T \mid I]$$

$$H' = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Interchange columns C_3 and C_4

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

(6)

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$R_2 \to R_2 + R_3$,

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$R_3 \to R_3 + R_1$

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$R_1 \to R_1 + R_3$.

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$$H = [I \mid P]$$

$$G = [-P^T \mid I]$$

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$