CMSC 442/653
HOMEWORK 1

1)

	1	ρ	ρ^2	σ	$\rho\sigma$	$\rho^2\sigma$
1	1	ρ	ρ^2	σ	$\rho\sigma$	$\rho^2\sigma$
ρ	ρ	ρ^2	1	$\rho\sigma$	$\rho^2\sigma$	σ
ρ^2	ρ^2	1	ρ	$\rho^2\sigma$	σ	$\rho\sigma$
σ	σ	$\rho^2\sigma$	$\rho\sigma$	1	ρ^2	ρ
$\rho\sigma$	$\rho\sigma$	σ	$\rho^2\sigma$	ρ	1	ρ^2
$\rho^2\sigma$	$\rho^2\sigma$	$\rho\sigma$	σ	ρ^2	ρ	1

Distinct group elements

$\{1, \rho, \rho^2, \sigma, \rho\sigma, \rho^2\sigma\}$

where $\rho^3 = 1$, $\sigma^2 = 1$, $\rho\sigma = \sigma\rho^2$

	1	ρ	ρ^2	ρ^3	σ	$\rho\sigma$	$\rho^2\sigma$	$\rho^3\sigma$
1	1	ρ	ρ^2	ρ^3	σ	$\rho\sigma$	$\rho^2\sigma$	$\rho^3\sigma$
ρ	ρ	ρ^2	ρ^3	1	$\rho\sigma$	$\rho^2\sigma$	$\rho^3\sigma$	σ
ρ^2	ρ^2	ρ^3	1	ρ	$\rho^2\sigma$	$\rho^3\sigma$	σ	$\rho\sigma$
ρ^3	ρ^3	1	ρ	ρ^2	$\rho^3\sigma$	σ	$\rho\sigma$	$\rho^2\sigma$
σ	σ	$\rho^3\sigma$	$\rho^2\sigma$	$\rho\sigma$	1	ρ^3	ρ^2	ρ
$\rho\sigma$	$\rho\sigma$	σ	$\rho^3\sigma$	$\rho^2\sigma$	ρ	1	ρ^3	ρ^2
$\rho^2\sigma$	$\rho^2\sigma$	$\rho\sigma$	σ	$\rho^3\sigma$	ρ^2	ρ	1	ρ^3
$\rho^3\sigma$	$\rho^3\sigma$	$\rho^2\sigma$	$\rho\sigma$	σ	ρ^3	ρ^2	ρ	1

Distinct group element
$\{ \rho^m \sigma^n \quad 0 \le m < 4, \; 0 \le n < 2 \}$

$\{ 1, \rho, \rho^2, \rho^3, \sigma, \rho\sigma, \rho^2\sigma, \rho^3\sigma \}$

where $\rho^4 = 1, \sigma^2 = 1$
$\rho\sigma = \sigma\rho^3$

<3> a>

Given :

$e_L \circ s = s \quad \forall s \in S \longrightarrow (1)$

$s \circ e_R = s \quad \forall s \in S \longrightarrow (2)$

To prove :

$e_L = e_R$

Proof :

Substitute $s = e_R$ in (1)

$e_L \circ e_R = e_R \longrightarrow (3)$

Substitute $s = e_L$ in (2)

$e_L \circ e_R = e_L. \longrightarrow (3)$

From (3) and (4)

$e_L = e_R$

Hence proved.

Scanned by CamScanner

(b) **Given :**

$e_L \circ s = s \quad \forall s \in S$

$s \circ e_R = s \quad \forall s \in S$

$e_L = e_R = e$ from (a).

To prove :

S can have atmost one 2 sided identity

Proof :

Proof by Contradiction :
Assume there exist two distinct 2 sided identity e and e'

$$e \neq e'$$

Since e is identity element

$e \circ s = s \longrightarrow (1)$
$s \circ e = s \longrightarrow (2)$

Since e' is also identity element

$e' \circ s = s \longrightarrow (3)$
$s \circ e' = s \longrightarrow (4)$

Substitute $s = e'$ in 1

$$e \circ e' = e' \longrightarrow (5)$$

Substitute $s = e$ in 4

$$e \circ e' = e \longrightarrow (6)$$

From 5 and 6

$$e = e'$$

Therefore our initial assumption that there are 2 distinct 2 sided identity element is wrong.

Hence proved.

⟨4⟩ Given :

$$\tilde{S}_L \circ s = e \quad s \in S \rightarrow (1)$$

$$s \circ \tilde{S}_R = e \quad s \in S \rightarrow (2)$$

To prove :-

$$S_L = S_R$$

Proof :

By property of identity element

$$s \circ e = s.$$

Let $s = \tilde{S}_L$

$\therefore \tilde{S}_L \circ e = S_L$

substitute $e = s \circ \tilde{S}_R$ from (2)

$$\tilde{S}_L \circ (s \circ \tilde{S}_R) = \tilde{S}_L$$

By property of Associativity

$$(\tilde{S}_L \circ s) \circ \tilde{S}_R = S_L$$

Since $\tilde{S}_L \circ s = e$ from (1)

$$e \circ \tilde{S}_R = \tilde{S}_L$$

By property of identity element

$$e \circ \tilde{S}_R = \tilde{S}_R = \tilde{S}_L$$

Hence proved

$$\tilde{S}_L = \tilde{S}_R$$