Homework 2 Solution.

[1.]

a. $b \cdot a = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 7 & 6 & 3 & 10 & 2 & 11 & 1 & 5 & 4 & 9 & 8 \end{pmatrix}$

$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 9 & 11 & 7 & 8 & 10 & 2 & 1 & 5 & 3 & 4 & 6 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 4 & 8 & 1 & 5 & 9 & 6 & 7 & 2 & 3 & 10 & 11 \end{pmatrix}$

b. $b \cdot a = (1, 4, 5, 9, 3)(2, 8)$.

c. $b^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 7 & 5 & 3 & 9 & 8 & 2 & 1 & 11 & 10 & 4 & 6 \end{pmatrix}$

d. $b^{-1} = (7, 1)(5, 8, 11, 6, 2)(9, 10, 4)$.

ADDITION TABLE

2.1

+	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	0
2	2	3	4	5	6	7	8	9	10	11	12	13	14	0	1
3	3	4	5	6	7	8	9	10	11	12	13	14	0	1	2
4	4	5	6	7	8	9	10	11	12	13	14	0	1	2	3
5	5	6	7	8	9	10	11	12	13	14	0	1	2	3	4
6	6	7	8	9	10	11	12	13	14	0	1	2	3	4	5
7	7	8	9	10	11	12	13	14	0	1	2	3	4	5	6
8	8	9	10	11	12	13	14	0	1	2	3	4	5	6	7
9	9	10	11	12	13	14	0	1	2	3	4	5	6	7	8
10	10	11	12	13	14	0	1	2	3	4	5	6	7	8	9
11	11	12	13	14	0	1	2	3	4	5	6	7	8	9	10
12	12	13	14	0	1	2	3	4	5	6	7	8	9	10	11
13	13	14	0	1	2	3	4	5	6	7	8	9	10	11	12
14	14	0	1	2	3	4	5	6	7	8	9	10	11	12	13

MULTIPLICATION TABLE

*	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
2	0	2	4	6	8	10	12	14	1	3	5	7	9	11	13
3	0	3	6	9	12	0	3	6	9	12	0	3	6	9	12
4	0	4	8	12	1	5	9	13	2	6	10	14	3	7	11
5	0	5	10	0	5	10	0	5	10	0	5	10	0	5	10
6	0	6	12	3	9	0	6	12	3	9	0	6	12	3	9
7	0	7	14	6	3	5	12	4	11	3	10	2	9	1	8
8	0	8	1	9	2	10	3	11	4	12	5	13	6	14	7
9	0	9	3	12	6	0	9	3	12	6	0	9	3	12	6
10	0	10	5	0	10	5	0	10	5	0	10	5	0	10	5
11	0	11	7	3	14	10	6	2	13	9	5	1	12	8	4
12	0	12	9	6	3	0	12	9	6	3	0	12	9	6	3
13	0	13	11	9	12	5	3	1	14	12	10	8	6	4	2
14	0	14	13	12	11	10	9	8	7	6	5	4	3	2	1

Non Trivial divisors

3 5 6 9 10 12

Units

1 2 4 7 8 11 13 14

3 ADDITION TABLE

+	0	1	x	$x+1$	x^2	x^2+1	x^2+x	x^2+x+1
0	0	1	x	$x+1$	x^2	x^2+1	x^2+x	x^2+x+1
1	1	0	$x+1$	x	x^2+1	x^2	x^2+x+1	x^2+x
x	x	$x+1$	0	1	x^2+x	x^2+x+1	x^2	x^2+1
$x+1$	$x+1$	x	1	0	x^2+x+1	x^2+x	x^2+1	x^2
x^2	x^2	x^2+1	x^2+x	x^2+x+1	0	1	x	$x+1$
x^2+1	x^2+1	x^2	x^2+x+1	x^2+x	1	0	$x+1$	x
x^2+x	x^2+x	x^2+x+1	x^2	x^2+1	x	$x+1$	0	1
x^2+x+1	x^2+x+1	x^2+x	x^2+1	x^2	$x+1$	x	1	0

MULTIPLICATION TABLE

×	0	1	x	$x+1$	x^2	x^2+1	x^2+x	x^2+x+1
0	0	0	0	0	0	0	0	0
1	0	1	x	$x+1$	x^2	x^2+1	x^2+x	x^2+x+1
x	0	x	x^2	x^2+x	1	$x+1$	x^2+1	x^2+x+1
$x+1$	0	$x+1$	x^2+x	x^2+1	x^2+1	x^2+x	$x+1$	0
x^2	0	x^2	1	x^2+1	x	x^2+x	$x+1$	x^2+x+1
x^2+1	0	x^2+1	$x+1$	x^2+x	x^2+x	$x+1$	x^2+1	0
x^2+x	0	x^2+x	x^2+1	$x+1$	$x+1$	x^2+1	x^2+x	0
x^2+x+1	0	x^2+x+1	x^2+x+1	0	x^2+x+1	0	0	x^2+x+1

Units - 1, x, x^2

Non trivial divisors - $x+1$, x^2+1, x^2+x, x^2+x+1

4.

+	$S0$ 0	$S1$ 1	$S2$ x	$S3$ $x+1$	$S4$ x^2	$S5$ x^2+1	$S6$ x^2+x	$S7$ x^2+x+1
$S0$	0	$S0$	$S1$	$S2$	$S3$	$S4$	$S5$	$S6$	$S7$
$S1$	1	$S1$	$S0$	$S3$	$S2$	$S5$	$S4$	$S7$	$S6$
$S2$	x	$S2$	$S3$	$S0$	$S1$	$S6$	$S7$	$S4$	$S5$
$S3$	$x+1$	$S3$	$S2$	$S1$	$S0$	$S7$	$S6$	$S5$	$S4$
$S4$	x^2	$S4$	$S5$	$S6$	$S7$	$S0$	$S1$	$S2$	$S3$
$S5$	x^2+1	$S5$	$S4$	$S7$	$S6$	$S1$	$S0$	$S3$	$S2$
$S6$	x^2+x	$S6$	$S7$	$S4$	$S5$	$S2$	$S3$	$S0$	$S1$
$S7$	x^2+x+1	$S7$	$S6$	$S5$	$S4$	$S3$	$S2$	$S1$	$S0$
$S8$	x^3	$S8$	$S9$	$S10$	$S11$	$S12$	$S13$	$S14$	$S15$
$S9$	x^3+1	$S9$	$S8$	$S11$	$S10$	$S13$	$S12$	$S15$	$S14$
$S10$	x^3+x	$S10$	$S11$	$S8$	$S9$	$S14$	$S15$	$S12$	$S13$
$S11$	x^3+x+1	$S11$	$S10$	$S9$	$S8$	$S15$	$S14$	$S13$	$S12$
$S12$	x^3+x^2	$S12$	$S13$	$S14$	$S15$	$S8$	$S9$	$S10$	$S11$
$S13$	x^3+x^2+1	$S13$	$S12$	$S15$	$S14$	$S9$	$S8$	$S11$	$S10$
$S14$	x^3+x^2+x	$S14$	$S15$	$S12$	$S13$	$S10$	$S11$	$S8$	$S9$
$S15$	x^3+x^2+x+1	$S15$	$S14$	$S13$	$S12$	$S11$	$S10$	$S9$	$S8$

$S8$	$S9$	$S10$	$S11$	$S12$	$S13$	$S14$	$S15$
x^3	x^3+1	x^3+x	x^3+x+1	x^3+x^2	x^3+x^2+1	x^3+x^2+x	x^3+x^2+x+1
$S8$	$S9$	$S10$	$S11$	$S12$	$S13$	$S14$	$S15$
$S9$	$S8$	$S11$	$S10$	$S13$	$S12$	$S15$	$S14$
$S10$	$S11$	$S8$	$S9$	$S14$	$S15$	$S12$	$S13$
$S11$	$S10$	$S9$	$S8$	$S15$	$S14$	$S13$	$S12$
$S12$	$S13$	$S14$	$S15$	$S8$	$S9$	$S10$	$S11$
$S13$	$S12$	$S15$	$S14$	$S9$	$S8$	$S11$	$S10$
$S14$	$S15$	$S12$	$S13$	$S10$	$S11$	$S8$	$S9$
$S15$	$S14$	$S13$	$S12$	$S11$	$S10$	$S9$	$S8$
$S0$	$S1$	$S2$	$S3$	$S4$	$S5$	$S6$	$S7$
$S1$	$S0$	$S3$	$S2$	$S5$	$S4$	$S7$	$S6$
$S2$	$S3$	$S0$	$S1$	$S6$	$S7$	$S4$	$S5$
$S3$	$S2$	$S1$	$S0$	$S7$	$S6$	$S5$	$S4$
$S4$	$S5$	$S6$	$S7$	$S0$	$S1$	$S2$	$S3$
$S5$	$S4$	$S7$	$S6$	$S1$	$S0$	$S3$	$S2$
$S6$	$S7$	$S4$	$S5$	$S2$	$S3$	$S0$	$S1$
$S7$	$S6$	$S5$	$S4$	$S3$	$S2$	$S1$	$S0$

MULTIPLICATION TABLE

	S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7
$*$	0	1	x	$x+1$	x^2	x^2+1	x^2+x	x^2+x+1
S_0	0	S_0	S_0	S_0	S_0	S_0	S_0	S_0
S_1	1	S_0	S_1	S_2	S_3	S_4	S_5	S_6
S_2	x	S_0	S_2	S_4	S_6	S_8	S_{10}	S_{12}
S_3	$x+1$	S_0	S_3	S_6	S_5	S_{12}	S_{15}	S_{10}
S_4	x^2	S_0	S_4	S_8	S_{12}	S_1	S_5	S_9
S_5	x^2+1	S_0	S_5	S_{10}	S_{15}	S_5	S_0	S_{15}
S_6	x^2+x	S_0	S_6	S_{12}	S_{10}	S_9	S_{15}	S_5
S_7	x^2+x+1	S_0	S_7	S_{14}	S_9	S_{13}	S_{10}	S_3
S_8	x^3	S_0	S_8	S_1	S_9	S_2	S_{10}	S_3
S_9	x^3+1	S_0	S_9	S_3	S_{10}	S_6	S_{15}	S_5
S_{10}	x^3+x	S_0	S_{10}	S_5	S_{15}	S_{10}	S_0	S_{15}
S_{11}	x^3+x+1	S_0	S_{11}	S_7	S_{12}	S_{14}	S_5	S_9
S_{12}	x^3+x^2	S_0	S_{12}	S_9	S_5	S_3	S_{15}	S_{10}
S_{13}	x^3+x^2+1	S_0	S_{13}	S_{11}	S_6	S_7	S_{10}	S_{12}
S_{14}	x^3+x^2+x	S_0	S_{14}	S_{13}	S_3	S_{11}	S_5	S_6
S_{15}	x^3+x^2+x+1	S_0	S_{15}	S_{15}	S_0	S_{15}	S_0	S_0

Units - S_1 S_2 S_4 S_7 S_8 S_{14} S_{11} S_{13}

Non trivial divisors

S_3 S_5 S_6 S_9 S_{10} S_{12} S_{15}

S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}
x^3	x^3+1	x^3+x	x^3+x+1	x^3+x^2	x^3+x^2+1	x^3+x^2+x	x^3+x^2+x+1
S_0	S_0	S_0	S_0	S_0	S_0	S_0	S_0
S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}
S_1	S_3	S_5	S_7	S_9	S_{11}	S_{13}	S_{15}
S_9	S_{10}	S_{15}	S_{12}	S_5	S_6	S_3	S_0
S_2	S_6	S_{10}	S_{14}	S_3	S_7	S_{11}	S_{15}
S_{10}	S_{15}	S_0	S_5	S_{15}	S_{10}	S_5	S_0
S_3	S_5	S_{15}	S_9	S_{10}	S_{12}	S_6	S_0
S_{11}	S_{12}	S_5	S_2	S_6	S_1	S_8	S_{15}
S_4	S_{12}	S_5	S_{13}	S_6	S_{14}	S_7	S_{15}
S_{12}	S_5	S_{15}	S_6	S_{10}	S_3	S_9	S_0
S_5	S_{15}	S_0	S_{10}	S_{15}	S_5	S_{10}	S_0
S_{13}	S_6	S_{10}	S_1	S_3	S_8	S_4	S_{15}
S_6	S_{10}	S_{15}	S_3	S_5	S_9	S_{12}	S_0
S_{14}	S_3	S_5	S_8	S_9	S_4	S_2	S_{15}
S_7	S_9	S_{10}	S_4	S_{12}	S_2	S_1	S_{15}
S_{15}	S_0	S_0	S_{15}	S_0	S_{15}	S_{15}	S_0

+	S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7
	0	1	x	$x+1$	x^2	x^2+1	x^2+x	x^2+x+1
0	S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7
1	S_1	S_0	S_3	S_2	S_5	S_4	S_7	S_6
x	S_2	S_3	S_0	S_1	S_6	S_7	S_4	S_5
$x+1$	S_3	S_2	S_1	S_0	S_7	S_6	S_5	S_4
x^2	S_4	S_5	S_6	S_7	S_0	S_1	S_2	S_3
x^2+1	S_5	S_4	S_7	S_6	S_1	S_0	S_3	S_2
x^2+x	S_6	S_7	S_4	S_5	S_2	S_3	S_0	S_1
x^2+x+1	S_7	S_6	S_5	S_4	S_3	S_2	S_1	S_0

×	S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7
	0	1	x	$x+1$	x^2	x^2+1	x^2+x	x^2+x+1
0	S_0	S_0	S_0	S_0	S_0	S_0	S_0	S_0
1	S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7
x	S_0	S_2	S_4	S_6	S_3	S_1	S_7	S_5
$x+1$	S_0	S_3	S_6	S_5	S_7	S_4	S_1	S_2
x^2	S_0	S_4	S_3	S_7	S_6	S_2	S_5	S_1
x^2+1	S_0	S_5	S_1	S_4	S_2	S_7	S_3	S_6
x^2+x	S_0	S_6	S_7	S_1	S_5	S_3	S_2	S_4
x^2+x+1	S_0	S_7	S_5	S_2	S_1	S_6	S_4	S_3

No non trivial divisors

Units - $S_1, S_2, S_3, S_4, S_5, S_6, S_7$