HOMEWORK - 5

(1) $$G = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Converting G into echelon form.

$$G = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Interchanging C_3 and C_4

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$R_2 \to R_2 + R_3$.

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

This is of form $[I_3\ P] = G'$

Then $H' = [-P^T\ I]$

$$= \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Interchanging C_3 & C_4.

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

(b). constructing syndrome table.

Syndrome	Coset Leader
000	000000
001	000001
010	000010
011	000100
100	001000
101	010000
110	100001
111	100000

(c) if $r = 111101$.

then Computing VH^T

$$= \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}_{1\times 6} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{6\times 3}$$

$= 1\ 0\ 0$

Since $r = c + e$

Codeword ↲ ↳ error.

$$rH^T = (c+e)H^T$$

$$= cH^T + eH^T$$

$$= 0 + eH^T$$

$\therefore$ if $eH^T = rH^T = 100$

Then for $e = 001\ 000$
$eH^T = 100$

Original code word sent

$$r = c + e$$

$$c = r + e$$

$$= 111101 + 001000$$

$$= 110101$$

<2>

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

(b) $r = 1000\ 1000\ 0000\ 001$

$rH^T = 1\ 1\ 0\ 1$

By creating the syndrome table,
it can be observed that
the 8th bit contains error

$\therefore$ Actual Codeword sent

$= 1000\ 1001\ 0000\ 001$

(3) Using the parity check matrix of 15, 11 to construct that of 16, 11 hamming code.

$$H = \left[\begin{array}{c|c} 1000,1001,1010111 & 0 \\ 0100,1101,0111100 & 0 \\ 0010,0110,1011110 & 0 \\ 0001,0011,0101111 & 0 \\ \hline 1111,1111,1111,111 & 1 \end{array}\right]$$

$\vec{r} = 10x1\ 1001\ y001\ 1001$

For this to be a valid codevector

$rH^T = 00000$

$rH^T = [y \quad 0 \quad x+y \quad 0 \quad 0]$

$= [0 \quad 0 \quad 0 \quad 0 \quad 0]$

$\therefore\ y = 0,\ x = 0.$

(4) $497^{-1} \pmod{899}$

$x = 497^{-1} \pmod{899}$

$497x = 1 \pmod{899}$.

$GCD(497, 899) = 1$

$899 = 497 \times 1 + 402$
$497 = 402 \times 1 + 95$
$402 = 95 \times 4 + 22$
$95 = 22 \times 4 + 7$
$22 = 7 \times 3 + 1$

Now $1 = 22 - (7 \times 3)$

$1 = 22 - [[95 - (22 \times 4)] \times 3]$

$= [4 \times 3 + 1] 22 - 95 \times 3$

$= 13 \times 22 - 95 \times 3$

$= 13 \times [402 - 95 \times 4] - 95 \times 3$

$= 13 \times 402 - 95 [13 \times 4 + 3]$

$= 13 \times 402 - 95 \times 55$

$= 13 \times 402 - (497 - 402) \times 55$

$= [13 + 55] \times 402 - 55 \times 497$

$= 68 \times 402 - 55 \times 497$

$= 68 \times (899 - 497) - 55 \times 497$

$= 68 \times 899 - (68 + 55) \times 497$

$= 68 \times 899 - 123 \times 497$

$497^{-1} \pmod{899} \equiv -123$

$= 776$

(5) $(x^5+x^2+1)^{-1} \pmod{x^{10}+x^3+1}$

Using similar approach as before

$$x^{10}+x^3+1 = (x^5+x^2+1)(x^5+x^2+1) + x^4+x^3$$

$$x^5+x^2+1 = (x^4+x^3)(x+1) + x^3+x^2+1$$

$$x^4+x^3 = (x^3+x^2+1)(x) + x$$

$$x^3+x^2+1 = (x^2+x)x + 1$$

$$\therefore \quad 1 = (x^3+x^2+1) - (x^2+x)x$$

$$1 = (x^3+x^2+1) - (x^2+x)((x^4+x^3) - x(x^3+x^2+1))$$

$$1 = (x^3+x^2+1)(x^3+x^2+1) - (x^2+x)(x^4+x^3)$$

$$1 = (x^3+x^2+1)((x^5+x^2+1) - (x+1)(x^4+x^3))$$

$$- (x^2+x)(x^4+x^3)$$

$$1 = (x^3+x^2+1)(x^5+x^2+1)$$
$$-(x^4+x^3)(x^2+x+(x+1)(x^3+x^2+1))$$
$$1 = (x^3+x^2+1)(x^5+x^2+1)$$
$$-(x^4+x^3)(\cancel{x^2}+x+x^4+\cancel{x^3}+\cancel{x}+\cancel{x^2}+\cancel{x^3}+1)$$
$$1 = (x^3+x^2+1)(x^5+x^2+1)$$
$$-(x^4+x^3)(x^4+1)$$
$$1 = (x^3+x^2+1)(x^5+x^2+1)$$
$$-(x^4+1)[(x^{10}+x^3+1)$$
$$-(x^5+x^2+1)(x^5+x^2+1)]$$
$$1 = [(x^3+x^2+1)+(x^4+1)(x^5+x^2+1)]$$
$$(x^5+x^2+1) - (x^4+1)(x^{10}+x^3+1)$$
$$= (x^9+x^6+x^5+x^4+x^3)$$
$$(x^5+x^2+1) - (x^4+1)(x^{10}+x^3+1)$$
$$\therefore (x^5+x^2+1)^{-1} (\text{mod } x^{10}+x^3+1)$$
$$= x^9+x^6+x^5+x^4+x^3$$