ASSIGNMENT 3.

1.
$$p(x) = x^{12} + x^9 + x^8 + x^6 + x^4 + x + 1$$
$$q(x) = x^{11} + x^{10} + x^6 + x^5 + x^4 + x^3 + 1$$
for $GF(2)[x]$

$$
\begin{array}{r}
x+1 \\
x^{11}+x^{10}+x^6+x^5+x^4+x^3+1 \overline{) x^{12}+x^9+x^8+x^6+x^4+x+1} \\
x^{12}+x^{11}+x^7+x^6+x^5+x^4+x \\
\hline
x^{11}+x^9+x^8+x^7+x^5+1 \\
x^{11}+x^{10}+x^6+x^5+x^4+x^3+1 \\
\hline
x^{10}+x^9+x^8+x^7+x^6+x^4+x^3
\end{array}
$$

$$
\begin{array}{r}
x \\
x^{10}+x^9+x^8+x^7+x^6+x^4+x^3 \overline{) x^{11}+x^{10}+x^6+x^5+x^4+x^3+1} \\
x^{11}+x^{10}+x^9+x^8+x^7+x^5+x^4 \\
\hline
x^9+x^8+x^7+x^6+x^3+1
\end{array}
$$

$$
\begin{array}{r}
x \\
x^9+x^8+x^7+x^6+x^3+1 \overline{) x^{10}+x^9+x^8+x^7+x^6+x^4+x^3} \\
x^{10}+x^9+x^8+x^7+x^4+x \\
\hline
x^6+x^3+x.
\end{array}
$$

$$
\begin{array}{r}
x^3 + x^2 + x
\end{array}
$$

$x^6 + x^3 + x \,\overline{\smash{\big)}\, x^9 + x^8 + x^7 + x^6 + x^3 + 1}$

$ x^9 + x^6 + x^4$

$ \overline{}$

$ x^8 + x^7 + x^4 + x^3 + 1$

$ x^8 + x^5 + x^3$

$ \overline{}$

$ x^7 + x^5 + x^4 + 1$

$ x^7 + x^4 + x^2$

$ \overline{}$

$ x^5 + x^2 + 1.$

$$
\begin{array}{r}
x
\end{array}
$$

$x^5 + x^2 + 1 \,\overline{\smash{\big)}\, x^6 + x^3 + x}$

$ x^6 + x^3 + x$

$ \overline{}$

$ \textcircled{0}$

$\therefore \; GCD(p(x), q(x))$
$$= $$
$$x^5 + x^2 + 1.$$

(2) Irreducible polynomial
$P(x) = x^5 + x^2 + 1$ for $GF(2^5)$

$2^5 = 32$ elements.

$P(x) = x^5 + x^2 + 1$

$y = x \mod x^5 + x^2 + 1 = 0$
$\qquad y^5 + y^2 + 1 = 0$
$\qquad y^5 = y^2 + 1$

$y^{-\infty} = 0$

$y^1 = y$

$y^2 = y^2$

$y^3 = y^3$

$y^4 = y^4$

$y^5 = y^2 + 1$

$y^6 = y^3 + y.$

$$y^7 = y^4 + y^2$$

$$y^8 = y^5 + y^3 = y^3 + y^2 + 1$$

$$y^9 = y^4 + y^3 + y$$

$$y^{10} = y^5 + y^4 + y^2 = y^2 + 1 + y^4 + y^2$$
$$= y^4 + 1$$

$$y^{11} = y^5 + y = y^2 + y + 1$$

$$y^{12} = y^3 + y^2 + y.$$

$$y^{13} = y^4 + y^3 + y^2$$

$$y^{14} = y^5 + y^4 + y^3 = y^4 + y^3 + y^2 + 1$$

$$y^{15} = y^5 + y^4 + y^3 + y = y^2 + 1 + y^4 + y^3 + y$$
$$= y^4 + y^3 + y^2 + y + 1$$

$$y^{16} = y^5 + y^4 + y^3 + y^2 + y$$
$$= y^2 + 1 + y^4 + y^3 + y^2 + y$$
$$= y^4 + y^3 + y + 1$$

$$y^{17} = y^5 + y^4 + y^2 + y$$
$$= y^2 + 1 + y^4 + y^2 + y$$
$$= y^4 + y + 1$$

$$y^{18} = y^5 + y^2 + y = y + 1$$

$$y^{19} = y^2 + y$$

$$y^{20} = y^3 + y^2$$

$$y^{21} = y^4 + y^3$$

$$y^{22} = y^5 + y^4 = y^4 + y^2 + 1$$

$$y^{23} = y^5 + y^3 + y = y^3 + y^2 + y + 1$$

$$y^{24} = y^4 + y^3 + y^2 + y$$

$$y^{25} = y^5 + y^4 + y^3 + y^2$$
$$= y^4 + y^3 + y^2 + y^2 + 1$$
$$= y^4 + y^3 + 1$$

$$y^{26} = y^5 + y^4 + y = y^4 + y^2 + y + 1$$

$$y^{27} = y^5 + y^3 + y^2 + y = y^3 + y^2 + y^2 + y + 1$$
$$= y^3 + y + 1$$

$$y^{28} = y^4 + y^2 + y$$

$$y^{29} = y^5 + y^3 + y^2 = y^3 + 1$$

$$y^{30} = y^4 + y$$

$$y^{31} = y^5 + y^2 = 1$$

LOG	ANTILOG $a_0\ a_1\ a_2\ a_3\ a_4$
$-\infty$	00000
0	10000
1	01000
2	00100
3	00010
4	00001
5	10100
6	01010
7	00101
8	10110
9	01011
10	10001
11	11100

12	01110
13	00111
14	10111
15	11111
16	11011
17	11001
18	11000
19	01100
20	00110
21	00011
22	10101
23	11110
24	01111
25	10011
26	11101
27	11010
28	01101
29	10010
30	01001

(3)

$$p(x) = x^2 + x + 2 \text{ for } GF(3^2)$$

$$\beta = x \bmod x^2 + x + 2$$

$$\beta^2 + \beta + 2 = 0$$

$$\beta^2 = 2\beta + 1$$

CALCULATIONS

$$\beta^{-\infty} = 0$$

$$\beta^0 = 1$$

$$\beta^1 = \beta$$

$$\beta^2 = 2\beta + 1$$

$$\beta^3 = 2\beta^2 + \beta = 2(2\beta + 1) + \beta = 4\beta + 2 + \beta$$
$$= 2\beta + 2$$

$$\beta^4 = 2\beta^2 + 2\beta = 2(2\beta + 1) + 2\beta = 4\beta + 2 + 2\beta$$
$$= 2$$

$$\beta^5 = 2\beta$$

$$\beta^6 = 2\beta^2 = 2(2\beta + 1) = \beta + 2$$

$$\beta^7 = \beta^2 + 2\beta = 2\beta + 1 + 2\beta = \beta + 1$$
$$\beta^8 = \beta^2 + \beta = 2\beta + \beta + 1 = 1^{\beta+1}$$

LOG	ANTILOG $a_0 a_1$
$-\infty$	00
0	10
1	01
2	12
3	22
4	20
5	02
6	21
7	11
8	01