CMSC 442/653
HOMEWORK 1

1)

	1	ρ	ρ^2	σ	$\rho\sigma$	$\rho^2\sigma$
1	1	ρ	ρ^2	σ	$\rho\sigma$	$\rho^2\sigma$
ρ	ρ	ρ^2	1	$\rho\sigma$	$\rho^2\sigma$	σ
ρ^2	ρ^2	1	ρ	$\rho^2\sigma$	σ	$\rho\sigma$
σ	σ	$\rho^2\sigma$	$\rho\sigma$	1	ρ^2	ρ
$\rho\sigma$.	$\rho\sigma$	σ	$\rho^2\sigma$	ρ	1	ρ^2
$\rho^2\sigma$	$\rho^2\sigma$	$\rho\sigma$	σ	ρ^2	ρ	1

Distinct group elements

$\{1, \rho, \rho^2, \sigma, \rho\sigma, \rho^2\sigma\}$

where $\rho^3 = 1$, $\sigma^2 = 1$, $\rho\sigma = \sigma\rho^2$

(2)

	1	ρ	ρ^2	ρ^3	σ	$\rho\sigma$	$\rho^2\sigma$	$\rho^3\sigma$
1	1	ρ	ρ^2	ρ^3	σ	$\rho\sigma$	$\rho^2\sigma$	$\rho^3\sigma$
ρ	ρ	ρ^2	ρ^3	1	$\rho\sigma$	$\rho^2\sigma$	$\rho^3\sigma$	σ
ρ^2	ρ^2	ρ^3	1	ρ	$\rho^2\sigma$	$\rho^3\sigma$	σ	$\rho\sigma$
ρ^3	ρ^3	1	ρ	ρ^2	$\rho^3\sigma$	σ	$\rho\sigma$	$\rho^2\sigma$
σ	σ	$\rho^3\sigma$	$\rho^2\sigma$	$\rho\sigma$	1	ρ^3	ρ^2	ρ
$\rho\sigma$	$\rho\sigma$	σ	$\rho^3\sigma$	$\rho^2\sigma$	ρ	1	ρ^3	ρ^2
$\rho^2\sigma$	$\rho^2\sigma$	$\rho\sigma$	σ	$\rho^3\sigma$	ρ^2	ρ	1	ρ^3
$\rho^3\sigma$	$\rho^3\sigma$	$\rho^2\sigma$	$\rho\sigma$	σ	ρ^3	ρ^2	ρ	1

Distinct group element

$\{\rho^m\sigma^n \quad 0 \leq m < 4,\ 0 \leq n < 2\}$

$\{1, \rho, \rho^2, \rho^3, \sigma, \rho\sigma, \rho^2\sigma, \rho^3\sigma\}$

where $\rho^4 = 1$, $\sigma^2 = 1$

$\rho\sigma = \sigma\rho^3$

2) a)

Given :

$e_L \circ s = s \quad \forall s \in S \longrightarrow (1)$

$s \circ e_R = s \quad \forall s \in S \longrightarrow (2)$

To prove :

$e_L = e_R$

Proof :

Substitute $s = e_R$ in (1)

$e_L \circ e_R = e_R \longrightarrow (3)$

Substitute $s = e_L$ in (2)

$e_L \circ e_R = e_L. \longrightarrow (3)$

From (3) and (4)

$e_L = e_R$

Hence proved.

<b> Given :

$e_L \circ s = s \quad \forall s \in S$

$s \circ e_R = s \quad \forall s \in S$

$e_L = e_R = e$ from (a)

To prove :

S can have atmost one 2 sided identity

Proof :

Proof by Contradiction :
Assume there exist two distinct 2 sided identity e and e'

$$e \neq e'$$

Since e is identity element

$e \circ s = s \longrightarrow (1)$
$s \circ e = s \longrightarrow (2)$

Since e' is also identity element

$e' \circ s = s \longrightarrow (3)$
$s \circ e' = s \longrightarrow (4)$

Substitute $c = e'$ in (1)

$$e \cdot e' = e' \quad \text{--- (5)}$$

Substitute $c = e$ in (4)

$$e \cdot e' = e \quad \text{--- (6)}$$

From (5) and (6)

$$e = e'$$

Therefore our initial assumption that there are 2 distinct 2 sided identity element is wrong.

Hence proved.

⟨4⟩ <u>Given</u> :

$$\tilde{S}_L \circ s = e \quad s \in S \rightarrow (1)$$

$$s \circ \tilde{S}_R = e \quad s \in S \rightarrow (2)$$

<u>To prove</u> :-

$$S_L = S_R$$

<u>Proof</u> :

By property of identity element

$$s \circ e = s.$$

Let $s = \tilde{S}_L$

$\therefore \tilde{S}_L \circ e = S_L$

Substitute $e = s \circ \tilde{S}_R$ from (2)

$$\tilde{S}_L \circ (s \circ \tilde{S}_R) = \tilde{S}_L$$

By property of Associativity

$$(\tilde{S}_L \circ s) \circ \tilde{S}_R = S_L$$

Since $\tilde{S}_L \circ s = e$ from (1)

$$e \circ \tilde{S}_R = \tilde{S}_L$$

By property of identity element

$$e \circ \tilde{S}_R = \tilde{S}_R = \tilde{S}_L$$

Hence proved

$$\tilde{S}_L = \tilde{S}_R$$

5 (a) $b \cdot a = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 7 & 6 & 3 & 10 & 2 & 11 & 1 & 5 & 4 & 9 & 8 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 9 & 11 & 7 & 8 & 10 & 2 & 1 & 5 & 3 & 4 & 6 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 4 & 8 & 1 & 5 & 9 & 6 & 7 & 2 & 3 & 10 & 11 \end{pmatrix}$

(b) $b \cdot a = (1, 4, 5, 9, 3)(2, 8)$

(c) $b^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 7 & 5 & 3 & 9 & 8 & 2 & 1 & 11 & 10 & 4 & 6 \end{pmatrix}$

(d) $b^{-1} = (7, 1)(5, 8, 11, 6, 2)(9, 10, 4)$