CS 4442 Assignment 3 Report

Problem 1 – Proof of separable convolution for the Gaussian kernel

$$y=[m,n]=x[m,n]*h[m,n]=\sum_{j=-\infty}^{\infty}\sum_{i=-\infty}^{\infty}x[i,j]\cdot h[m-i,n-j]$$ // By 2D definition of Convolution

$$y=[m,n]=h[m,n]*x[m,n]=\sum_{j=-\infty}^{\infty}\sum_{i=-\infty}^{\infty}h[i,j]\cdot x[m-i,n-j]$$ // Commutative law since convolution is Commutative

If $h[m,n]$ is separable to $(M\times 1)$ and $(1\times N)$, then $h[m,n]=h_1[m]\cdot h_2[n]$.
In the case of the Gaussian Kernel, assuming the matrix size is a $(2k+1)\times(2k+1)$, it can be separated into a $M\times 1$ column vector and a $1\times N$ row vector (1D).

I.e.

$$\underbrace{\begin{bmatrix}1&2&1\\2&4&2\\1&2&1\end{bmatrix}}_{\text{Gaussian Matrix}}=\underbrace{\begin{bmatrix}1\\2\\1\end{bmatrix}}_{\substack{M\times 1\\ \text{Column Vector}}}\times\underbrace{\begin{bmatrix}1&2&1\end{bmatrix}}_{\substack{1\times N\\ \text{Row Vector}}}$$

$$G_{\sigma}(x,y)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(x^2+y^2)}{2\sigma^2}\right)$$

Thus,

$$y[m,n]=h[m,n]*x[m,n]=\sum_{j=-\infty}^{\infty}\sum_{i=-\infty}^{\infty}h[i,j]\cdot x[m-i,n-j]$$
$$=\sum_{j=-\infty}^{\infty}\sum_{i=-\infty}^{\infty}h_1[i]\cdot h_2[j]\cdot x[m-i,n-j]$$
$$=\sum_{j=-\infty}^{\infty}h_2[j]\left(\sum_{i=-\infty}^{\infty}h_1[i]\cdot x[m-i,n-j]\right)$$

Since the definition of 1D convolution is:

$$y[n]=x[n]*h[n]=\sum_{k=-\infty}^{\infty}x[k]\cdot h[n-k]$$

It convolves with input and h_1, then convolves once again with the result of the previous convolution and h_2.
Therefore, separable 2D convolution performs twice the 1D convolution in horizontal and vertical direction.
Thus, since the Gaussian Kernel can be separated into $(M\times 1)$ and $(1\times N)$ vectors, it is a spatially separable convolution.

Problem 1 – Is the Sobel kernel spatially separable?

Sobel Kernel

$$\underbrace{\begin{bmatrix}-1&0&1\\-2&0&2\\-1&0&1\end{bmatrix}}_{S_x}=\begin{bmatrix}1\\2\\1\end{bmatrix}\times\begin{bmatrix}-1&0&1\end{bmatrix}\qquad \underbrace{\begin{bmatrix}1&2&1\\0&0&0\\-1&-2&-1\end{bmatrix}}_{S_y}=\begin{bmatrix}1\\0\\-1\end{bmatrix}\times\begin{bmatrix}1&2&1\end{bmatrix}$$

The Sobel Kernel is spatially separable since it can be decomposed into a $(M\times 1)$ and $(1\times N)$ vectors. ∴ Spatially separable convolution can be performed.

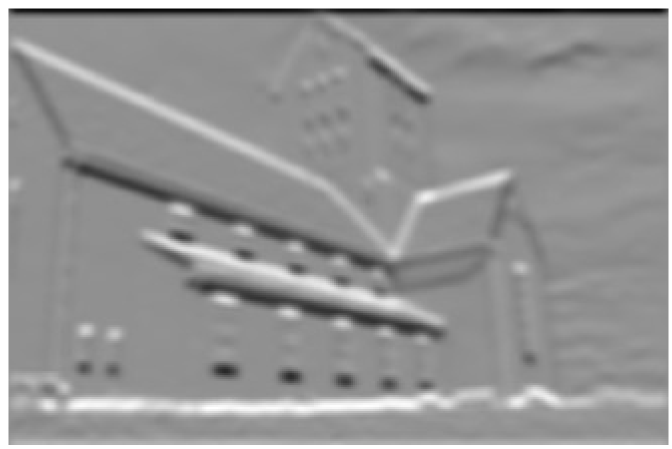
Problem 2 – Edge Detection
*See Jupyter Notebook for solution code

Original Image

Gaussian Smoothing sigma=1

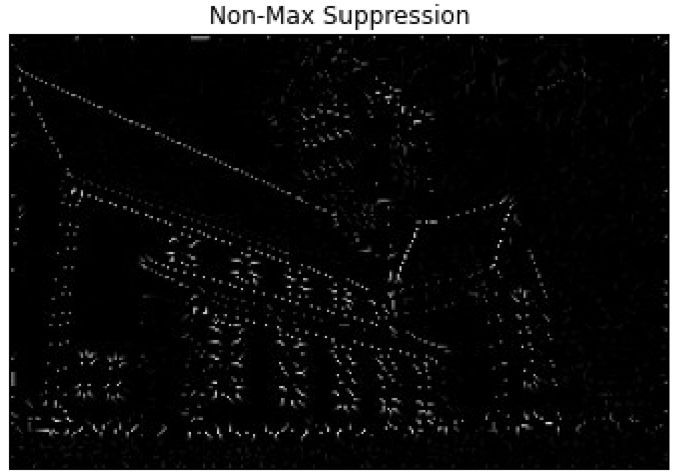
Derivative wrt Y

Derivative wrt X

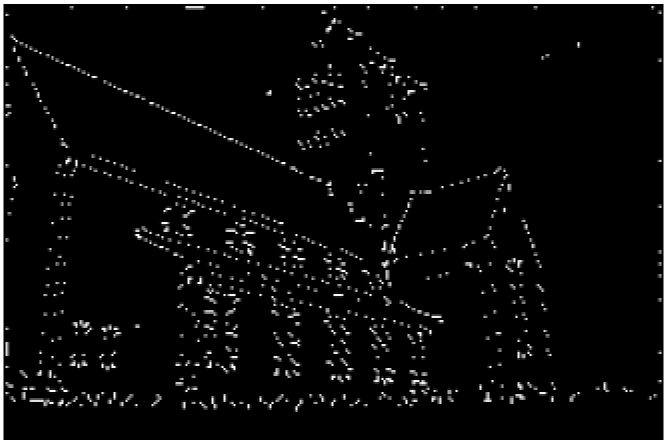


Gradient Intensity

Non-Max Suppression



Binary Threshold Hystersis:low=40 high=50



Problem 3 – Corner Detection
*See Jupyter Notebook for solution code

Corners Found

Corners (threshold=0.1)