3. a) $F_Y(y) = P(Y \le y) = P(\sigma X + \mu \le y) = P(X \le \frac{y-\mu}{\sigma})$

$$\boxed{= F_X\left(\frac{y-\mu}{\sigma}\right)}$$

b) $Q_Y(p) = F_Y^{-1}(p) \rightarrow F_Y(Q_Y(p)) = p$

$$\Rightarrow F_Y(Q_Y(p)) = F_X\left(\frac{Q_Y(p)-\mu}{\sigma}\right) = p$$

$$\Rightarrow \frac{Q_Y(p)-\mu}{\sigma} = F^{-1}(p)$$

$$Q_Y(p) = \sigma F_X^{-1}(p) + \mu \boxed{= \sigma Q_X(p) + \mu}$$

4. Kernel Density Estimator $\hat{f}(x) = \frac{1}{nb} \sum_i K\left(\frac{y_i - x}{b}\right)$

a) $b, n > 0 \quad \therefore \quad \frac{1}{nb} > 0$

$K(u) > 0$ for all u by definition.

$\therefore \sum_i K\left(\frac{y_i - x}{b}\right) \ge 0 \qquad \therefore \hat{f}(x) \ge 0$

b) $\int \hat{f}(x) = \int_{-\infty}^{\infty} \frac{1}{nb} \sum_i K\left(\frac{y_i - x}{b}\right) dx$. Let $u = \frac{y_i - x}{b}$

$$= \frac{1}{nb} \int_{-\infty}^{\infty} \sum_i K(u)\, dx \qquad \frac{du}{dx} = -\frac{1}{b}$$

$$= \frac{1}{nb} \int_{-\infty}^{\infty} \sum_i K(u) \cdot -b\, du \qquad \therefore -du \cdot b = dx$$

$$= \frac{1}{n} \sum_i 1 = 1$$

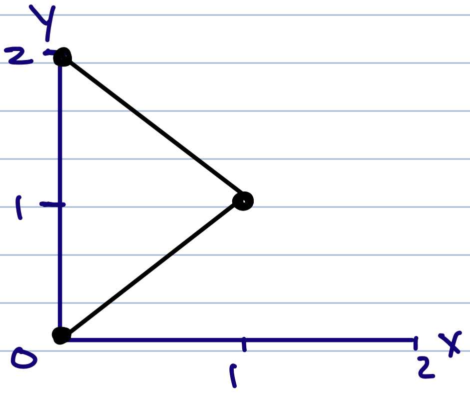
5.

a) Using the iris dataset on Sepal Length:
density(data, bw="nrd0") => bw = 0.2736
density(data, bw = "nrd") => bw = 0.3222

b) Using the formula provided in class: bw = 0.2735831 for nrd0
bw = 0.3222201 for nrd

=> they yield the same result.

6.



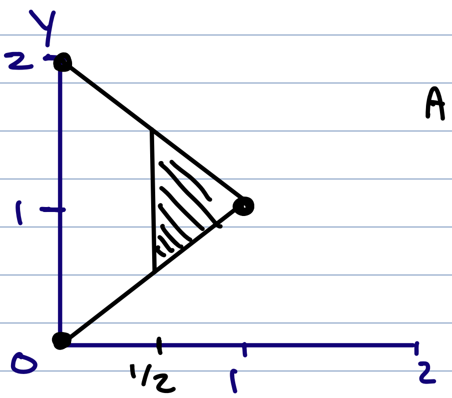
$A = \frac{1}{2}bh = \frac{1}{2} \cdot 1 \cdot 2 = 1$

a) $x \in [0, 1]$ and is uniform, so area under curve $= 1$:

$\therefore X \sim \text{Uniform}(0,1)$

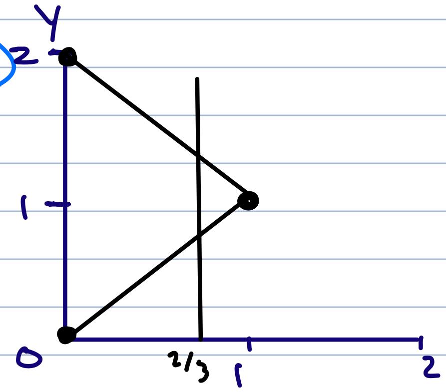
b) $y \in [0, 2]$ $\therefore y \sim \text{Uniform}(0, 2)$

c)



$A = \frac{1}{2}bh = \frac{1}{2}(1/2)(1)$ $\boxed{= 1/4}$

d)



When $x = 2/3$, y is at least $y = x = 2/3$

$\therefore P(Y > 1/3 \mid X = 2/3) \boxed{= 1}$