2. Linear Regression

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INDIAN INSTITUTE OF TECHNOLOGY PALAKKAD

Recap

- Artificial intelligence and machine learning

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- Datasets for machine learning

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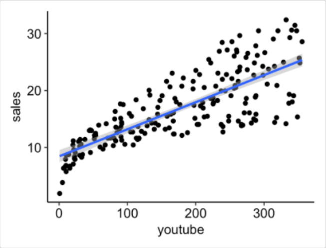
- Designing a machine learning system

Regression: Motivation

The following scatter plot shows the impact of YouTube advertising of a company on its sales (thousands of dollars). The advertising experiment has been repeated 200 times with different budgets and the observed sales have been recorded.



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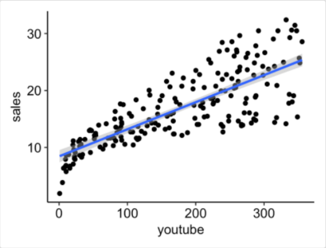

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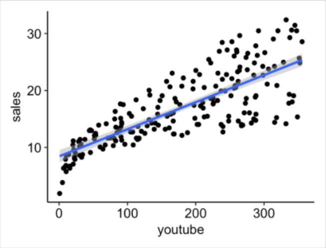

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Other applications: predicting the price of a house, predicting sales, weather forecasting, etc.

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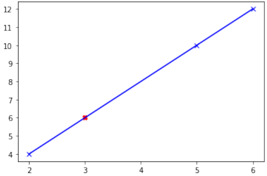

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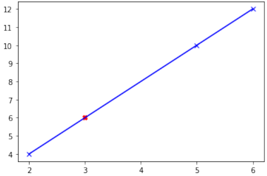

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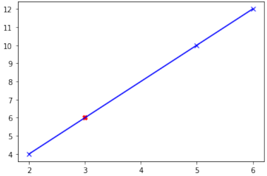

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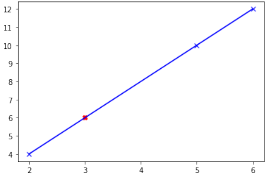

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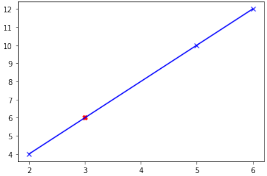

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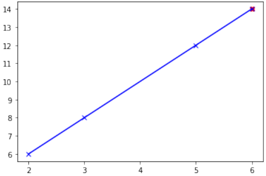
$y = 2x + 2$



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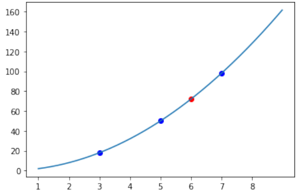

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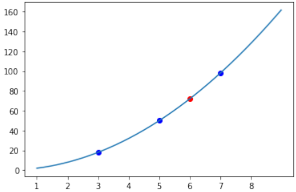

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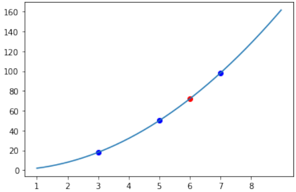

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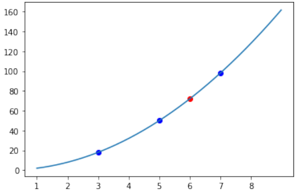

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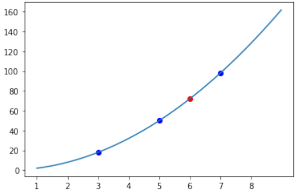

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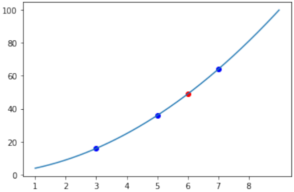
$$y = x^2 + 2x + 1$$



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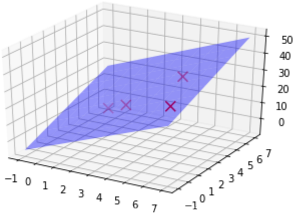

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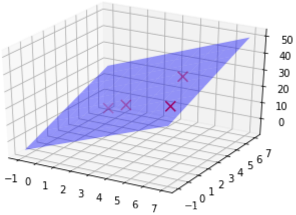

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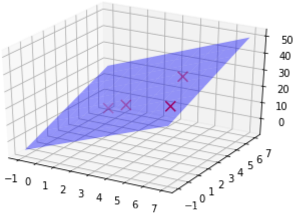

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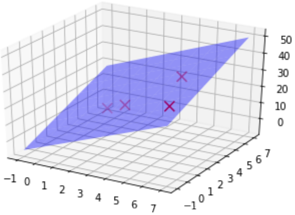

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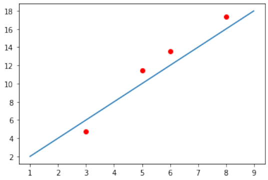

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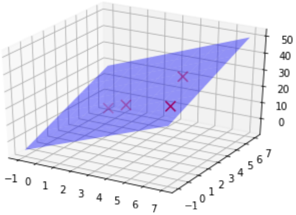

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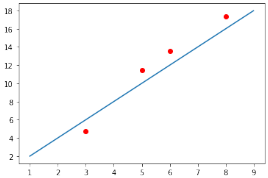

Not an easy task!!

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  - the data points are considered to be representative of the population at large

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   - sum of absolute differences between the target and predicted variables

$$MAE = \frac{\sum_{i=1}^{n}|y_i - \hat{y}_i|}{n}$$

Regression techniques estimate the weights of the model such that the value of the chosen loss function is minimized

Different regression models

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Logistic regression: used when the dependent variable is binary (yes/no), (0/1), etc.

A small mathematics refresher

Scalar:

24

Vector:

[2, -6, 9]

row

or column

$$\begin{bmatrix} 2, \\ 6, \\ 9 \end{bmatrix}$$

Matrix:

$$\begin{bmatrix} 2, & -6, & 9 \\ 4, & 5, & -7 \end{bmatrix}$$

row(s) x column(s)

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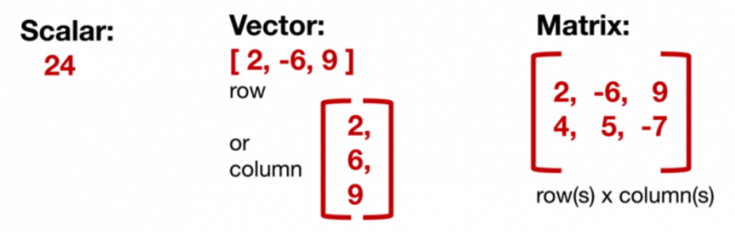

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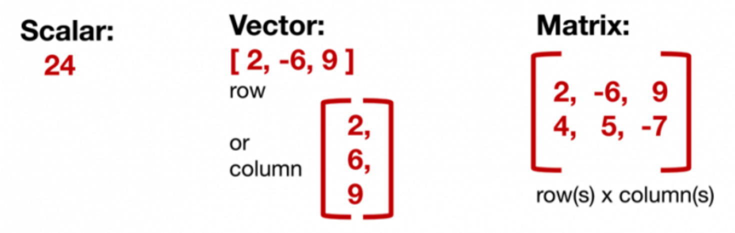

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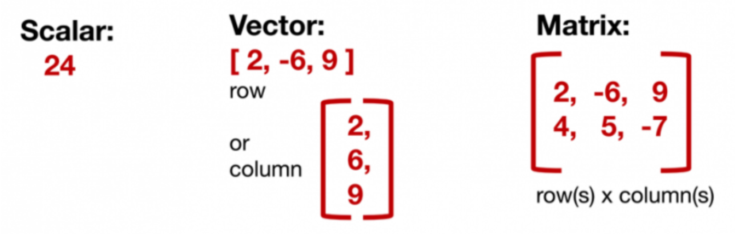

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- the model can then be used for predicting the output y for new x values

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Given: Training data set comprising N observations $(x_n, y_n)_{n=1}^{N}$, where $x_n = [x_{n1}, x_{n2}, ..., x_{nD}]$ is the input and y_n is the corresponding output

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Minimize: `Mean-squared error`: $l(w) = \frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2$
where $\hat{y}_i = f(w, x_i)$

Simple linear regression

Independent Variable

Years of Experience	Salary in 1000$
2	15
3	28
5	42
13	64
8	50
16	90
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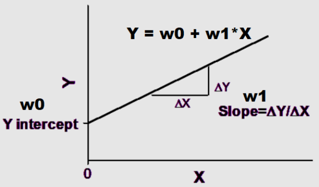
9	54

Dependent Variable



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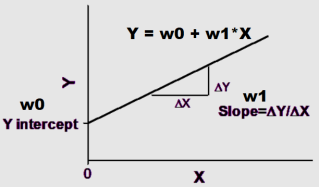

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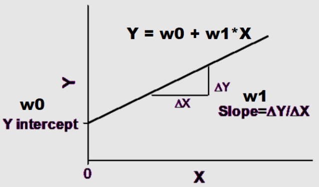

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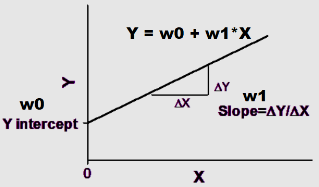

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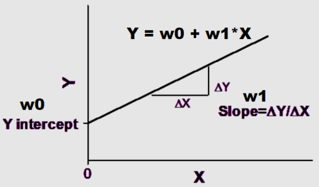

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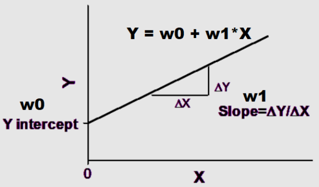

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Ordinary least squares (OLS)

- `residual` $\hat{\epsilon}_i$: difference between the actual and predicted values of the dependent variable y_i

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- can be obtained using methods of calculus

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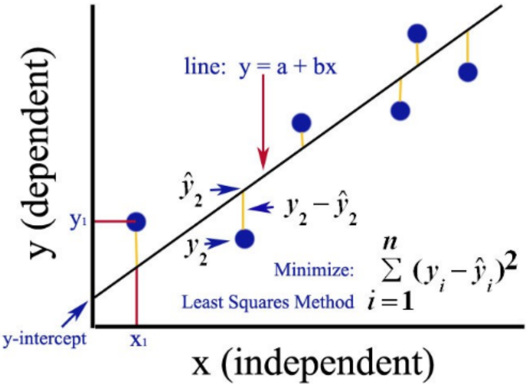

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$$w_1 = \frac{\sum_{i=1}^{N} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{N} (x_i - \bar{x})^2} = \frac{cov(x, y)}{var(x)}$$

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Applications of linear regression

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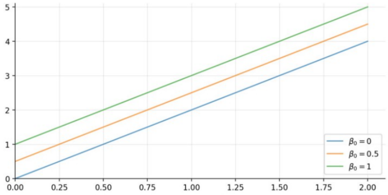

(a) Regression lines on varying the intercept value β_0 for the model: $y = \beta_0 + 2x$

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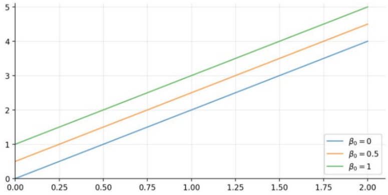

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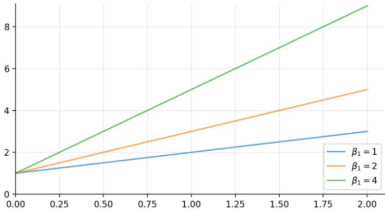

(b) Regression lines on varying the coefficient value β_1 for the model: $y = 1 + \beta_1 x$

References

1. https://towardsdatascience.com/a-beginners-guide-to-regression-analysis-in-machine-learning-8a828
2. https://en.wikipedia.org/wiki/Regression_analysis
3. https://towardsdatascience.com/linear-regression-in-real-life-4a78d7159f16

Thanks Google for the pictures!