Defuzzification and fuzzy control systems

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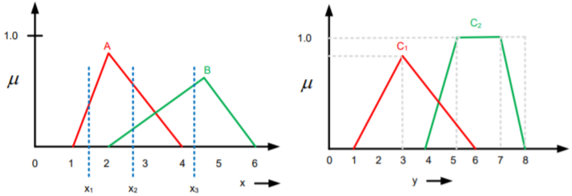
- Here the output fuzzy set $C = C_1 \cup C_2$



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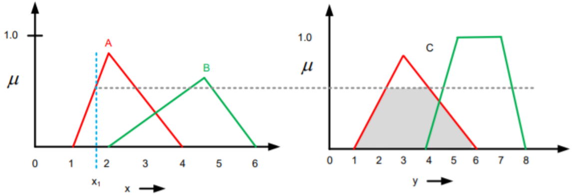

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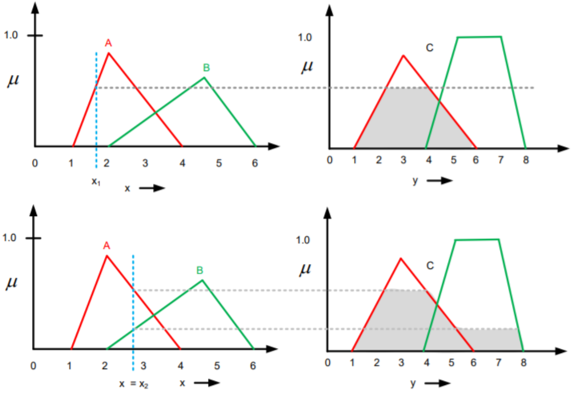

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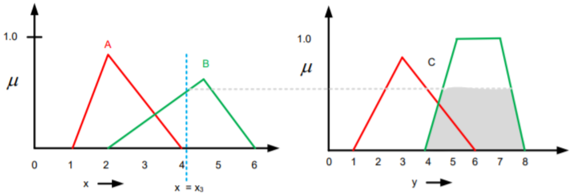

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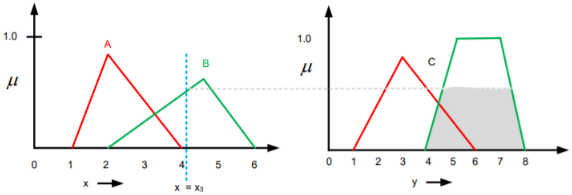
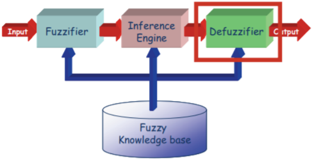


What is the **crisp value** that can be inferred from the above rules given an input x'?

Defuzzification



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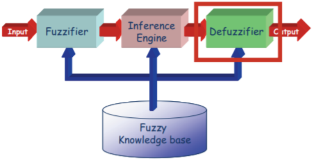

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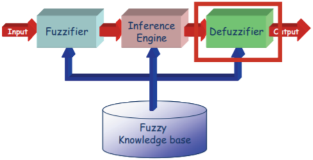

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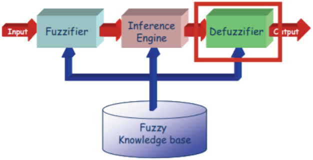

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- fuzzy results cannot be used to drive a control mechanism
- the defuzzified value represents the action to be taken in controlling the process
- there are many defuzzification techniques, each with its own advantages and drawbacks

Centroid method: CoG

Center of gravity method (CoG)

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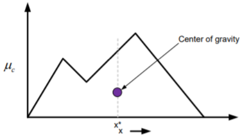

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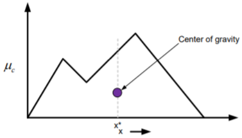

- Mathematically the CoG can be expressed as follows:

$$x* = \frac{\int x.\mu_C(x)dx}{\int \mu_C(x)}$$

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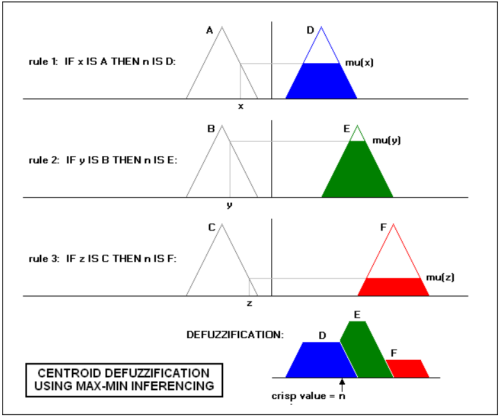

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  1. Max membership method
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  3. Last of Maxima method (LOM)
  4. Mean of Maxima method (MOM)

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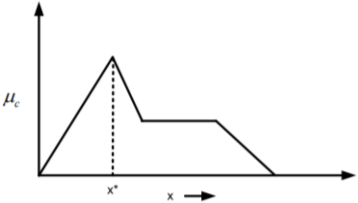

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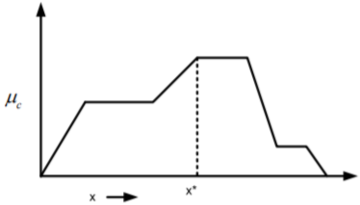

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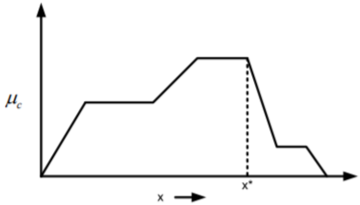

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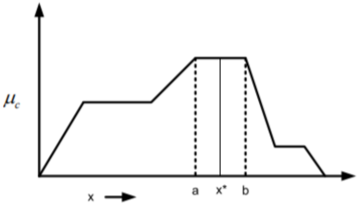

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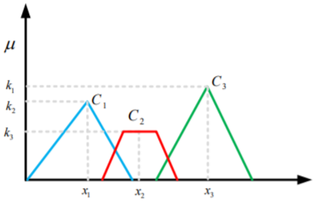

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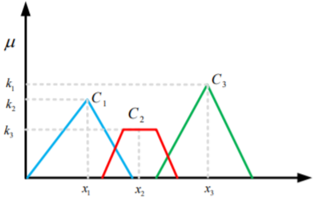

- Here $x^* = \frac{k_2.x_1 + k_3.x_2 + k_1 * x_3}{k_1 + k_2 + k_3}$

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- The output membership functions are only linear or constant for the Sugeno-type fuzzy inference

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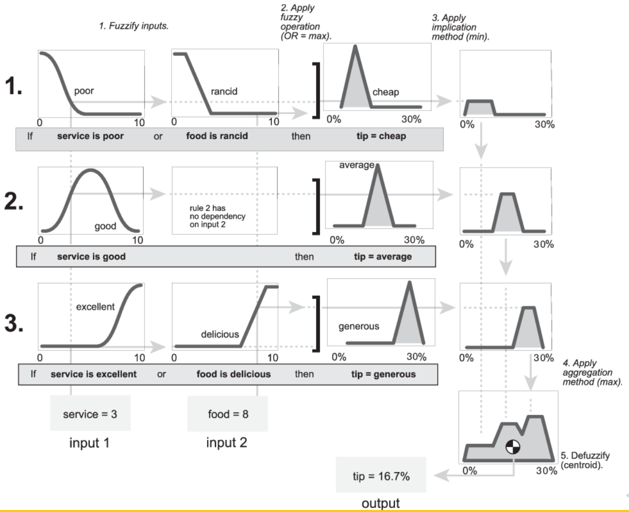

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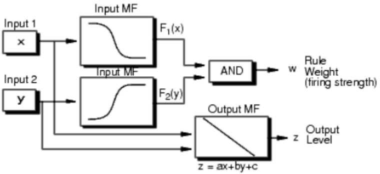

Each rule generates two values:

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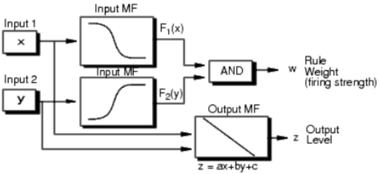

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  - x and y are values of input 1 and input 2 respectively
  - a_i, b_i and c_i are constant coefficients

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- The final output of the system is the weighted average over all rule outputs:

$$\text{final output} = \frac{\sum_{i=1}^{N} w_i . z_i}{\sum_{i=1}^{N} w_i}$$

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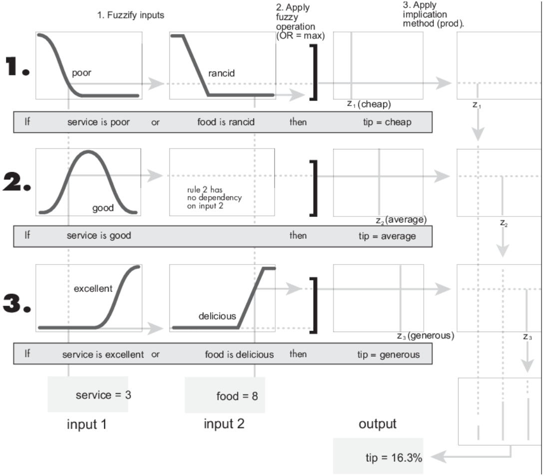

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References

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