Robust control of 8 DOF active vehicle suspension system

By

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To my mother and my father, who have been a source of encouragement and inspiration to me throughout my life.

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Abstract

Vehicle suspension system plays an important role in maintaining the standard of ride quality and road handling. Poor ride quality and road handling deteriorates the ride comfort and vehicle's health. Active suspension system has an active actuator to add and dissipate energy from the system to give better ride quality and road handling.

It is usually required to maximize the ride quality for driver. To achieve this, driver seat dynamics are very important to be considered. Hence an 8 DOF active suspension full car model is proposed this thesis. Based on this model, many control schemes for effective disturbance attenuation on seat can be derived.

Controller design for polytopic system involves complex mathematics. To reduce the complexity, COG representation of polytopic systems is introduced in this thesis.

Mass of the vehicle is dependent on the passengers and hence it is an uncertain quantity. Suspension system is required to be robust against such uncertainty. This thesis deals with the design of a Robust $\mathcal{H}_\infty$ controller controller which maintains ride quality in case of road disturbance and uncertain vehicle's mass.

Control input in case of active suspension system is delayed due to electro-mechanical nature of the actuator and microprocessors processing time. This delayed input can deteriorate the performance if controller is not robust against delay. This thesis deals with the design of such a Robust $\mathcal{H}_\infty$ controller that is robust against the input delay.

Suspension system health can deteriorate with the passage of time due to wear

and tear of the suspension parts. Active suspension actuator should be robust to deal with such condition. This thesis deals with the design of a actuator fault tolerant Robust $\mathcal{H}_\infty$ controller for full vehicle model.

Robust $\mathcal{H}_\infty$ controllers for disturbance rejection, parametric uncertainty effect minimization, delayed input effect minimization and fault tolerance have been designed in this thesis for active suspension system. Simulation results are provided to evaluate the effectiveness of design.

Nomenclature

Symbol	Nomenclature
$He\{A\}$	$A + A^T$
$Ha\{A\}$	AA^T
COG	Center of gravity
DOF	Degree of freedom
LMI	Linear matrix inequality
BMI	Bilinear matrix inequality

Chapter 1

Introduction

Tires are the vehicle's contact point with the road. They manage the input of forces and disturbances from the road and they are the final link in the driver's chain of output commands. Tire's characteristics are therefore a key factor in the effect the road has on the vehicle and in the effectiveness of the output forces that control vehicle's stability and cornering characteristics.

The tire's basic characteristics are managed by the system of springs, dampers and linkages that control the way in which tires move and react to disturbances and control inputs and connect tires to the body of vehicle. This system of springs, dampers and linkages is called suspension system. It plays an important role in maintaining the vehicle's performance, vehicle's health and passengers health. Suspension system's performance is evaluated in terms of ride quality and road handling provided by it.

Ride quality is the degree of weeding out the effects of road disturbances on the vehicle. Poor ride quality deteriorates the vehicle's health. Poor ride quality also results in unhealthy effects on passengers due to road disturbances. Isolation from road disturbance is achieved by elastic and dissipative elements.

Road handling can be described as the behavior of vehicle during cornering and sheering. Over steer and under steer are described as cornering dynamics. Over steer occurs when the under steer gradient is negative i.e. the car turns more than the input from the steering and under steer is opposite to it. Bump steer is the amount a wheel will toe in or out when the wheel hits a bump.

Ride quality and road handling have always been a compromise. High levels of comfort are difficult to reconcile with a low center of gravity, body roll resistance. This gives rise to the problem of designing a suspension system that gives an optimal compromise between these.

Suspension system can be classified as passive, semi-active and active.

Passive suspension system consists of conventional spring and shock absorbers with constant characteristics. Passive suspension system does not involve any sensors and electronics. The passive suspension system gives a compromise between ride quality and road handling. For example a heavily damped suspension system will give good road handling at the cost of ride quality. Opposite is the case of lightly damped suspension system. An optimally tuned passive suspension system will give an optimal compromise between ride quality and road handling.

Semi-active suspension system involves an active damper. Active damper allows controlled dissipation of energy. Active damper is used in parallel with conventional spring. Such suspension systems can be classified into variable and on-off semi-active suspension systems.

Active suspension system, in addition to springs and shock absorbers, includes force actuators. Force actuator's operation depends on the output from the sensors that monitor the conditions of vehicle for example heave acceleration of body. Active suspension involves electronics and control.

1.1 Motivation

Vehicle model is an uncertain model due to the parametric uncertainty associated with the vehicle's mass. Input delay, model uncertainty and road disturbance effect the ride quality and road handling offered by the suspension system. The design of robust controller to maximize road comfort for the driver is an open challenge.

Uncertain vehicle model can be modeled as polytopic system .Output feedback controller design for such system is an open challenge.
These challenges are addressed in this thesis and robust controllers have been proposed.

1.2 Contribution

This thesis focuses on the improvement of ride quality for vehicle driver. Following are the main contributions presented in this thesis.

- Mathematical model of 8 DOF full car model is proposed.
- COG representation of polytopic systems is proposed which reduces complexity associated with controller synthesis problem.
- Robust controller for 8 DOF full car is proposed which maintains performance against disturbance and parametric uncertainty. Both state feedback and output feedback controllers are proposed.
- Delay dependent robust controller is proposed to make suspension system robust to the delayed control input.
- A reliable fault tolerant controller has been proposed for vehicle suspension system.

1.3 Thesis layout

In chapter 2, overview of the papers studied to deal with the challenges addressed in this thesis has been presented.
In chapter 3, mathematical model of the system is proposed. It is an 8 DOF full car model which considers driver seat dynamics so that ride comfort for driver can be improved.
In chapter 4, COG representation of polytopic systems has been proposed. This

representation of polytopic systems is very helpful for controller synthesis problem.
In chapter 5, state and output feedback robust controller which maintain desired performance level in case of disturbances have been proposed.
In chapter 6, state and output feedback robust controller which maintains desired performance level in case of disturbances and parametric uncertainty are proposed.
In chapter 7, delay dependent state feedback robust controller which maintains desired performance level in case of disturbances, parametric uncertainty and input delay has been proposed.
In chapter 8, output feedback fault tolerant controller which maintains desired performance level in case of disturbances and actuator fault has been proposed.

Chapter 2

Literature overview

2.1 Mathematical modeling of suspension system

Research work for practical implementation of suspension system [1–3] have been started in middle of 80's. Surveys on active suspension control systems have been presented in [2, 4].

The suspension system has been studied and modeled by many researchers. In [5], quarter car model with active vehicle suspension system was used. Tire was being modeled as spring. In [6], mathematical model for half car active vehicle suspension system was derived. In [7], mathematical model for full car active vehicle suspension system was derived using four quarter car models. In [8], mathematical model for full car active vehicle suspension system with seven degrees of freedom was derived.

In most of the work done on active suspension system, dynamics of the driver seat were not taken into consideration. Tire was modeled as a linear spring ignoring the damping of tire. In absence of tire damping, at wheel hop frequency motion of sprung masses and unsprung masses are uncoupled and the vertical acceleration of sprung mass is unaffected which leads to misleading conclusions [9, 10]. In [11], it was concluded that performance of an active suspension system designed by ignoring tire damping may seriously deteriorate due to tire damping. Force balance analysis was used for the mathematical modeling. Force balance analysis is based on vector and force concepts which are difficult to apply in case of complex systems.

2.2 Types of Suspension system

Ride comfort, road handling and suspension deflection are mainly used to evaluate the suspension performances. Ride quality is related to vehicle acceleration sensed by passengers, road handling is depends on the contact of tires and road surface, and suspension deflection is the displacement between the sprung mass and unsprung mass [12, 13]. These performance criteria are conflicting in nature [13, 14]. Three types of suspension system including passive [15], semi-active [16], [17] and active suspension systems [18, 19] have been researched. In [20], the performance of the passive suspensions are optimized to improve the compromise between the conflicting demands and compared with the performances of active and semi-active suspensions for quarter car model. The tuning of passive, on/off semi active, continuously variable semi active, and fully active suspensions is discussed in [21].

2.3 Uncertain vehicle model

Vehicle suspended mass is dependent on passengers and is uncertain. It can vary within a certain limit. In [22], polytopic model of the vehicle is discussed. In this paper, vehicle's mass is considered an uncertain quantity and two vertex polytope model is used for the quarter vehicle model.

2.4 State feedback control

Based on the assumption that all states are available for feedback, the state-feedback control method was discussed in [23]. In [23] $\mathcal{H}_\infty$ control for active suspensions has been designed. Full car model has been considered. This paper also gives a comparison of passive, semi-active and active suspension system.

2.5 Output feedback control

It is not possible practically that all states of system are available for measurement so the state feedback $\mathcal{H}_\infty$ controller is not feasible for real time implementation. In [24], ride performance is characterized by $\mathcal{H}_2$ norm while road handling and constraints are specified by the generalized $\mathcal{H}_2$ norm to design an $\mathcal{H}_\infty$ output feedback controller for half vehicle design .In [25], the structurally constrained decentralized and robust dynamic output feedback control design have been discussed and a numerical cross decomposition algorithm is proposed.

2.6 Robust control of suspension system

Different control techniques has been utilized to address the trade-off between conflicting performance parameters of active suspension system such as fuzzy logic and neural network control [26], gain scheduling control [27] [28], linear optimal control [29, 30], adaptive control [22, 31, 32], back-stepping technique [33] and $\mathcal{H}_\infty$ control [34].

The development of $\mathcal{H}_\infty$ control theory [35] and linear matrix inequality (LMI) Toolbox [36] resulted in extensive research on LMI based $\mathcal{H}_\infty$ control methods [37] [38]. Robust and disturbance attenuation $\mathcal{H}_\infty$ control strategies for vehicle suspensions were discussed thoroughly in [39, 40]. These researches focus on finding an objective function incorporating all constraints which should be minimized to get an optimal controller.

In [41], it was observed that choosing appropriate and frequency dependent weights to manage the trade-off between conflicting requirements in single objective approaches is very difficult.

2.7 Control Design for Actuator Imperfect Information

2.7.1 Actuator delay

Electromagnetic actuators characteristics, sampling and processing time of microprocessors cause delay in control input. Actuator delay can cause degradation of the control performances or even instability of the closed loop system so input delay problem should be addressed while designing a controller. In [42, 43],constant actuator delay was considered and Moon's inequality method was utilized to derive existence conditions of $\mathcal{H}_\infty$ controller for quarter car and half-car active suspension systems. Parameter dependent robust $\mathcal{H}_\infty$ controller considering both vehicle parameter uncertainty and actuator time delay problem has been investigated in [42] .

2.7.2 Actuator Fault tolerant control

It is important to design a controller such that closed loop stability and performance can be guaranteed in the presences of faults. To avoid complete system failure during partial faults, fault tolerant controllers are designed. In [44] fault-tolerant $\mathcal{H}_\infty$ controller design problem for linear systems was designed for partial component failure. $\mathcal{H}_\infty$ control problem of seat suspension systems with actuator faults is discussed in [45].

Chapter 3

Mathematical modeling of full vehicle model

There has been a lot of research on accurate modeling of the vehicle model.An 8 degrees of freedom full car model has been derived in this chapter. To get an appropriate model of vehicle for control purpose, the dynamics of driver seat should be included in the mathematical model of the full car. The seat's damping has an important influence on the ride comfort [46]. So the dynamics of seat have been considered.Mathematical model has been obtained by euler lagrange modeling approach.

3.1 Euler Lagrange

Mechanical system can easily be by modeled by Euler Lagrange [47]. The Lagrangian energy function is defined as the difference of the kinetic energy and the potential energy.
Let

- T Kinetic energy of the entire system
- D Dissipation function of the system
- V Potential energy of the entire system

q_j Generalized j^{th}coordinate

$\dot{q}_j$ Generalized velocity of j^{th}coordinate

Q_j Generalized applied force at j^{th}coordinate

Lagrangian Function is $L = T - V$. Lagrange equation is given as

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_j} - \frac{\partial L}{\partial q_j} + \frac{\partial D}{\partial \dot{q}_j} = Q_j$$

where j=1,2,3..... are the independent coordinates or degrees of freedom which exist in the system.

3.2 Description of system model

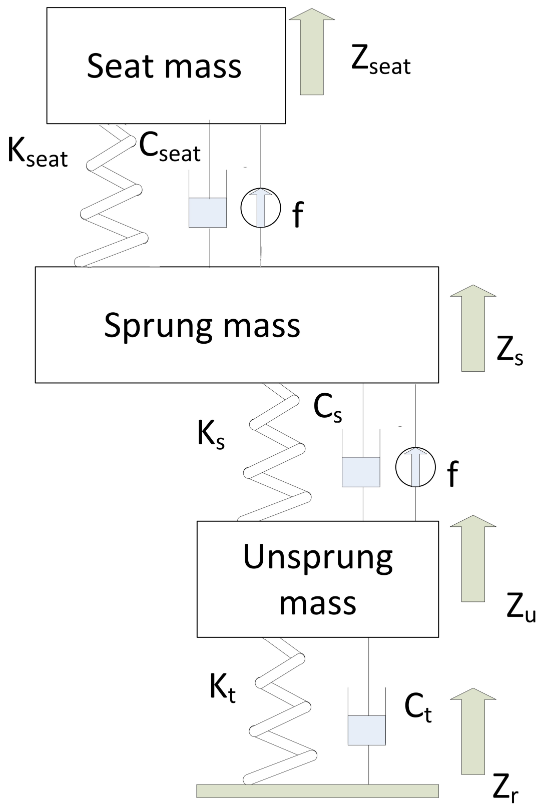

Figure 3.1: Model of one wheel with suspension, sprung mass and seat

Fig. 3.1 shows sprung mass is attached to unsprung mass by suspension system. Suspension system is modeled as combination of spring, damper and active force actuator. Unsprung masses can move vertically. The tires are modeled as linear spring and damper [48].

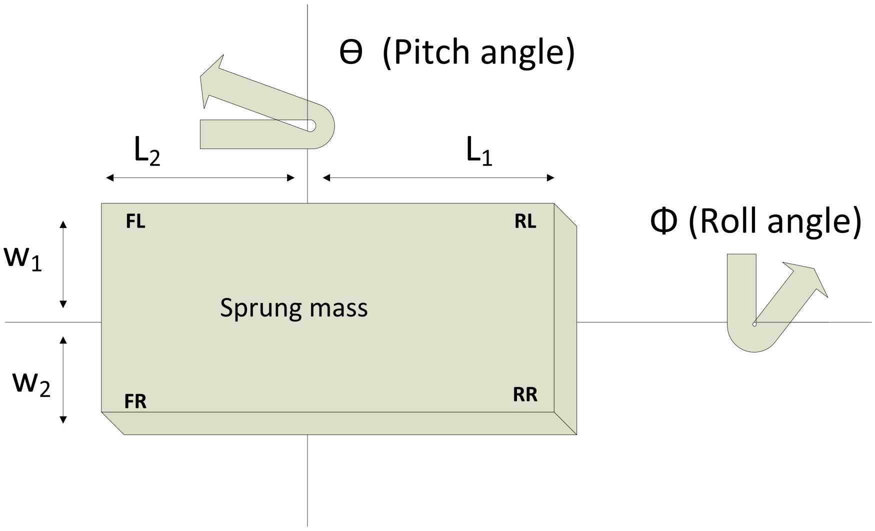

Figure 3.2: Sprung mass heave, pitch and roll motion

Fig. 3.2 shows sprung mass can heave, pitch and roll. Driver's seat can move vertically.

The parameters related to system are

$z_{r(p)}$	Terrain disturbance height at "p" tire
$z_{u(p)}$	Height of "p" wheel unsprung mass
z_s	Height of sprung mass
z_{seat}	Height of driver seat
θ	Sprung mass roll angle
ϕ	Sprung mass pitch angle
$m_{r(p)}$	Mass of "p" wheel
$m_{u(p)}$	"p" wheel unsprung mass
m_s	Sprung mass
m_{seat}	Mass of driver seat
I_x	Moment of inertia about x axis
I_y	Moment of inertia about x axis

$k_{t(p)}$ Spring coefficient of "p" wheel

$k_{s(p)}$ Spring coefficient of "p" wheel suspension

k_{seat} Spring coefficient of driver seat

$c_{t(p)}$ Damping coefficient of "p" wheel

$c_{s(p)}$ Damping coefficient of "p" wheel suspension

c_{seat} Damping coefficient of driver seat

L_1 Distance of rear left wheel from axis of pitch

L_2 Distance of front left wheel from axis of pitch

w_1 Distance of front left wheel from axis of roll

w_2 Distance of front right wheel from axis of roll

s_1 Distance of driver's seat from axis of roll

s_2 Distance of driver's seat from axis of pitch

here p $\in$ Rear right,Rear Left,Front Right,Front left

3.3 Derivation of mathematical model

For simplicity of derivation, suppose

$$z_1 = z - z_{u(RL)} + w_1\phi - L_1\theta$$

$$z_2 = z - z_{u(RR)} - w_2\phi - L_1\theta$$

$$z_3 = z - z_{u(FL)} + w_1\phi + L_2\theta$$

$$z_4 = z - z_{u(FR)} + L_2\theta - w_2\phi$$

$$z_5 = z_{u(RL)} - z_{r(RL)}$$

$$z_6 = z_{u(RR)} - z_{r(RR)}$$

$$z_7 = z_{u(FL)} - z_{r(FL)}$$

$$z_8 = z_{u(FR)} - z_{r(FR)}$$

$$z_9 = z_{seat} - z - s_2\theta + s_1\phi$$

3.3.1 Kinetic energy of system

Driver's seat and unsprung masses can have vertical motion so their kinetic energy is translational. Sprung mass can heave, pitch and roll so it's kinetic energy will be the sum of translational and rotational kinetic energy. The total Kinetic energy (T) of the system is the sum of kinetic energies of suspended, unsuspended and seat masses i.e.

$$\frac{1}{2}m\dot{z_s}^2 + \frac{1}{2}I_x\dot{\phi}^2 + \frac{1}{2}I_y\dot{\theta}^2 + \frac{1}{2}m_{u(RR)}\dot{z}^2_{u(RR)} + \frac{1}{2}m_{seat}\dot{z}^2_{seat} + \frac{1}{2}m_{u(FR)}\dot{z}^2_{u(FR)} + \frac{1}{2}m_{u(FL)}\dot{z}^2_{u(FL)} + \frac{1}{2}m_{u(RL)}\dot{z}^2_{u(RL)}$$

3.3.2 Potential energy of system

Tires exhibit potential energy due to their stiffness represented by spring in their models. Drivers seat mass, suspended mass and unsuspended masses exhibit potential energy due to their height. Driver's seat and suspension springs also exhibit potential energy. Total potential energy (V) of the system is the sum of potential energies associated with suspended mass, driver's seat and tires. It is given as

$$\frac{1}{2}k_{s(RL)}z_1^2 + \frac{1}{2}k_{t(FR)}z_8^2 + \frac{1}{2}k_{s(RR)}z_2^2 + \frac{1}{2}k_{t(RR)}z_6^2 + \frac{1}{2}k_{s(FL)}z_3^2 + \frac{1}{2}k_{t(RL)}z_5^2 + \frac{1}{2}k_{s(FR)}z_4^2 + \frac{1}{2}k_{t(FL)}z_7^2 - m_{u(RL)}gz_{u(RL)} - m_{u(RR)}gz_{u(RR)} - m_{u(FL)}gz_{u(FL)} - m_{seat}z_{seat}g - m_{u(FR)}gz_{u(FR)} - m_szg + \frac{1}{2}k_{seat}z_9^2$$

3.3.3 Energy dissipation of system

Energy dissipating elements in vehicle are dampers associated with driver's seat, wheel and suspension system models. Total Energy dissipation of the system is the sum of energy dissipations associated with driver's seat, wheel and suspension system models. It is given as

$$\frac{1}{2}c_{s(RL)}\dot{z}_1^2 + \frac{1}{2}c_{t(RL)}\dot{z}_5^2 + \frac{1}{2}c_{s(RR)}\dot{z}_2^2 + \frac{1}{2}c_{t(RR)}\dot{z}_6^2 +$$
$$\frac{1}{2}c_{s(FL)}\dot{z}_3^2 + \frac{1}{2}c_{t(FL)}\dot{z}_7^2 + \frac{1}{2}c_{s(FR)}\dot{z}_4^2 + \frac{1}{2}c_{t(FR)}\dot{z}_8^2 +$$
$$\frac{1}{2}c_{seat}\dot{z_9}^2$$

3.3.4 Lagrangian function

Lagrangian Function is $L = T - V$ given as

$$\frac{1}{2}m\dot{z_s}^2 + \frac{1}{2}I_x\dot{\phi}^2 + \frac{1}{2}I_y\dot{\theta}^2 + \frac{1}{2}m_{u(RR)}\dot{z}_{u(RR)}^2 + \frac{1}{2}m_{seat}\dot{z}_{seat}^2$$
$$+ \frac{1}{2}m_{u(FR)}\dot{z}_{u(FR)}^2 + \frac{1}{2}m_{u(FL)}\dot{z}_{u(FL)}^2 + \frac{1}{2}m_{u(RL)}\dot{z}_{u(RL)}^2 -$$
$$\frac{1}{2}k_{s(RL)}z_1^2 - \frac{1}{2}k_{t(FR)}z_8^2 - \frac{1}{2}k_{s(RR)}z_2^2 - \frac{1}{2}k_{t(RR)}z_6^2 -$$
$$\frac{1}{2}k_{s(FL)}z_3^2 - \frac{1}{2}k_{t(RL)}z_5^2 - \frac{1}{2}k_{s(FR)}z_4^2 - \frac{1}{2}k_{t(FL)}z_7^2 +$$
$$m_{u(RL)}gz_{u(RL)} + m_{u(RR)}gz_{u(RR)} + m_{u(FL)}gz_{u(FL)} +$$
$$m_{seat}z_{seat}g + m_{u(FR)}gz_{u(FR)} - m_s zg$$

3.4 Equations of motion

Eight degrees of freedom has been considered so eight equations of motion will be obtained. Equation of motion for generalized coordinated $z_{u(RR)}$ is

$$m_{u(RR)}\ddot{z}_{u(RR)} + k_{t(RR)}z_6 - k_{s(RR)}z_2 + m_{u(RR)}g +$$
$$c_{t(RR)}\dot{z}_6 - c_{s(RR)}\dot{z}_2 = -f_{(RR)}$$

Equation of motion for generalized coordinated $z_{u(RL)}$ is

$$m_{u(RL)}\ddot{z}_{u(RL)} + k_{t(RL)}z_5 - k_{s(RL)}z_1 + m_{u(RL)}g +$$
$$c_{t(RL)}\dot{z}_5 - c_{s(RL)}\dot{z}_1 = -f_{(RL)}$$

Equation of motion for generalized coordinated $z_{u(FR)}$ is

$$m_{u(FR)}\ddot{z}_{u(FR)} + k_{t(FR)}z_8 - k_{s(FR)}z_4 + m_{u(FR)}g +$$
$$c_{t(FR)}\dot{z}_8 - c_{s(FR)}\dot{z}_4 = -f_{(FR)}$$

Equation of motion for generalized coordinated $z_{u(FL)}$ is

$$m_{u(FL)}\ddot{z}_{u(FL)} + k_{t(FL)}z_7 - k_{s(FL)}z_3 + m_{u(FL)}g +$$
$$c_{t(FL)}\dot{z}_7 - c_{s(FL)}\dot{z}_3 = -f_{(FL)}$$

Equation of motion for generalized coordinated z_{seat} is

$$m_{seat}\ddot{z}_{seat} + k_{seat}z_9 + m_{seat}g + c_{seat}\dot{z}_9 = -f_{seat}$$

Equation of motion for generalized coordinated z_s is

$$m_s\ddot{z} + k_{s(RL)}z_1 + k_{s(RR)}z_2 + k_{s(FR)}z_4 + k_{s(FL)}z_3 -$$
$$k_{seat}z_9 + m_sg + c_{s(RL)}\dot{z}_1 + c_{s(RR)}\dot{z}_2 + c_{s(FR)}\dot{z}_4 +$$
$$c_{s(FL)}\dot{z}_3 - c_{seat}\dot{z}_9 = f_{(RR)} + f_{(RL)} + f_{(FR)} + f_{(FL)} + f_{seat}$$

Equation of motion for generalized coordinated ϕ is

$$I_x\ddot{\phi} - w_1k_{s(RL)}z_1 - w_2k_{s(RR)}z_2 - w_2k_{s(FR)}z_4 +$$
$$w_1k_{s(FL)}z_3 + s_1k_{(seat)}z_9 - w_1c_{u(RL)}\dot{z}_1 - w_2c_{u(RR)}\dot{z}_2 -$$
$$w_2c_{u(FR)}\dot{z}_4 + w_1c_{u(FL)}\dot{z}_3 + s_1c_{seat}\dot{z}_9 = 0$$

Equation of motion for generalized coordinated θ is

$$
\begin{aligned}
& I_y\ddot{\theta} - L_1 k_{s(RL)} z_1 - L_1 k_{s(RR)} z_2 + L_2 k_{s(FR)} z_4 + \\
& L_2 k_{s(FL)} z_3 - s_2 k_{(seat)} z_9 - L_1 c_{u(RL)} \dot{z}_1 - L_1 c_{u(RR)} \dot{z}_2 + \\
& L_2 c_{u(FR)} \dot{z}_4 + L_2 c_{u(FL)} \dot{z}_3 - s_2 c_{seat} \dot{z}_9 = 0
\end{aligned}
$$

3.5 State space representation

3.5.1 Nominal state space model

The state space model of the active suspension full car is

$$
\begin{aligned}
\dot{x} =& Ax(t) + B_u u(t) + B_z w(t) \\
y =& C_y x(t) \\
z =& C_Z x(t)
\end{aligned}
\tag{3.1}
$$

Here $A \in \mathcal{R}_{n\times n}$ is the state matrix, $B_u \in \mathcal{R}_{n\times q}$ is the control input matrix, $B_z \in \mathcal{R}_{n\times z}$ is the disturbances input matrix, $C_y \in \mathcal{R}_{r\times n}$ is the measured outputs matrix and $C_z \in \mathcal{R}_{rz\times n}$ is the outputs to be minimized matrix.

3.5.2 Uncertain state space model

Mass of the vehicle m_s is uncertain quantity i.e $m_1 < m_s < m_2$ then matrices A, B_u, C_y are uncertain and can be represented as

$$
\begin{aligned}
A =& \alpha_1 A_1 + \alpha_1 A_1 \\
B_u =& \alpha_1 B_{u1} + \alpha_2 B_{u2} \\
C_y =& \alpha_1 C_{y1} + \alpha_2 C_{y2}
\end{aligned}
\tag{3.2}
$$

Here $\alpha_1 = \frac{\frac{1}{m_s} - \frac{1}{m_2}}{\frac{1}{m_1} - \frac{1}{m_2}}$ and $\alpha_2 = \frac{\frac{1}{m_1} - \frac{1}{m_s}}{\frac{1}{m_1} - \frac{1}{m_2}}$
The numerical values of parameters are shown in table 3.1

m_s	$m_{u(p)}$	k_{sf}	k_{tf}
1500 kg	40 kg	23500 N/m	190000 N/m
c_{sf}	L_2	W_2	I_y
1000 Ns/m	0.96 m	0.71 m	2100 kg/m^2
m_{ur}	k_{sr}	k_{tr}	c_{sr}
40 kg	25500 N/m	190000N/m	1100 Ns/m
L_1	W_1	I_x	c_{seat}
1.44 m	1.44 m	550 kg/m^2	500 Ns/m
S_2	S_1	m_{seat}	k_{seat}
0.25 m	0.25 m	190000N/m	1100 Ns/m

Table 3.1: Numerical values of vehicle model parameters

3.6 summary

An 8 DOF mathematical model has been obtained for full car.Driver seat dynamics have been included.This model will be used further for controller synthesis.

Chapter 4

Center of gravity representation of polytopic system

4.1 Polytopic system

Parametric uncertainty can be represented in the form of a polytope. Interval and linear parameter uncertain systems can be modeled as polytopes which are the convex hull of the parameters of a set of models [49].

The barycentric coordinate system is used for interpolation in a polytope. In this coordinates system; any point within polytope is specified as the barycenter of masses placed at its vertices. In order to locate a point within the polytope, barycentric coordinate should be positive and their sum should be less than 1.

As compared to polytopic forms, systems are also represented in norm bounded form. Polytopic representation has an advantage over norm bounded systems that it is less conservative and accurate representation of a system as compared norm bounded systems.

4.2 Control problem for polytopic systems

Polytopic systems are difficult to handle in control problems as their control problems extra mathematical calculations. In [50], load dependent controller has been obtained. In [**?**, 51] output feedback controller in terms of local bmi has been obtained.

4.3 COG representation for polytopic systems

In order to reduce the complexity attached with the polytopic systems, another representation of polytopic systems is proposed here. This representation will involve interpolation not only along the vertices of polytope but also along the center of gravity. This represntation is a calculation friendly as many existing lemma's can easily be applied to the control related problems of polytopic systems. Let us consider a function $f(x_1, x_2, x_3, ..., x_n)$. This function is dependent on n variables. A polytope of 2^n vertices will represent such a function.
Let f_k is the function's value at vertex k where $k = 1, 2, 3, ..., 2^n$. If α_k represent barycentric coordinate for k^{th} vertex then value of function at any point within the polytope is given as

$$f = \sum_{i=1}^{k} \alpha_i f_i$$

Systems represented in polytopic forms are called polytopic systems. Consider the value of function $f(x_1, x_2, x_3, ..., x_n)$ at center of gravity of the polytope is f_{cog}. At center of gravity the barycentric coordinates are $\alpha_{k(cog)}$ then

$$f_{cog} = \sum_{i=1}^{k} \alpha_{i(cog)} f_i$$

$$f_{cog} - \sum_{i=1}^{k} \alpha_{i(cog)} f_i = 0$$

Now

$$f = f_{cog} + (\sum_{i=1}^{k} (\alpha_i - \alpha_{i(cog)}) f_i)$$

It is obvious that $(\alpha_i - \alpha_{i(cog)}) < 1$.

4.4 COG polytopic model of active suspension system

Full car model is an uncertain model of the vehicle as the sprung mass of vehicle can vary. Uncertain model of car can be represented as either norm bounded or polytopic system. As stated earlier that polyopic representation is less conservative and more accurate so this representation will be adopted in this thesis.

Consider the system 3.2. At center of gravity, system matrices are given as

$$\begin{aligned} A_{cog} =& \alpha_{1\_cog} A_1 + \alpha_{2\_cog} A_2 \\ B_u cog =& \alpha_{1\_cog} B_{u1} + \alpha_{2\_cog} B_{u2} \\ C_y cog =& \alpha_{1\_cog} C_{y1} + \alpha_{2\_cog} C_{y2} \end{aligned} \tag{4.1}$$

Equation 4.1 is equal to

$$\begin{aligned} & A_{cog} - \alpha_{1\_cog} A_1 - \alpha_{2\_cog} A_2 = 0 \\ & B_u cog - \alpha_{1\_cog} B_{u1} - \alpha_{2\_cog} B_{u2} = 0 \\ & C_y cog - \alpha_{1\_cog} C_{y1} - \alpha_{2\_cog} C_{y2} = 0 \end{aligned} \tag{4.2}$$

where

$$\begin{aligned} \alpha_{1\_cog} =& \frac{\frac{1}{m_{s\_cog}} - \frac{1}{m_2}}{\frac{1}{m_1} - \frac{1}{m_2}} \\ \alpha_{2\_cog} =& \frac{\frac{1}{m_1} - \frac{1}{m_{s\_cog}}}{\frac{1}{m_1} - \frac{1}{m_2}} \end{aligned}$$

Adding 3.2 and 4.2 will give

$$\begin{aligned} A =& A_{cog} + \alpha_1 A_1 - \alpha_{1\_cog} A_1 + \alpha_2 A_2 - \alpha_{2\_cog} A_2 \\ B_u =& B_u cog + \alpha_1 B_u 1 - \alpha_{1\_cog} B_{u1} + \alpha_2 B_u 2 - \alpha_{2\_cog} B_{u2} \\ C_y =& C_y cog + \alpha_1 C_y 1 - \alpha_{1\_cog} C_{y1} + \alpha_2 C_y 2 - \alpha_{2\_cog} C_{y2} \end{aligned}$$

Let us suppose

$$\theta_1 = \alpha_1 - \alpha_{1\_cog}$$
$$\theta_2 = \alpha_2 - \alpha_{2\_cog}$$

then the vehicle model is

$$\begin{aligned} A &= A_{cog} + \theta_1 A_1 + \theta_2 A_2 \\ B_u &= B_{u\_cog} + \theta_1 B_{u1} + \theta_2 B_{u2} \\ C_y &= C_{y\_cog} + \theta_1 C_{y1} + \theta_2 C_{y2} \end{aligned} \tag{4.3}$$

and

$$\theta_1 = \frac{\frac{1}{m_s} - \frac{1}{m_{s\_cog}}}{\frac{1}{m_1} - \frac{1}{m_2}}$$
$$\theta_2 = \frac{\frac{1}{m_{s\_cog}} - \frac{1}{m_s}}{\frac{1}{m_1} - \frac{1}{m_2}}$$

such that $\theta_1 < 1$ and $\theta_2 < 1$.

The uncertain mathematical model of the full car can be written as

$$\begin{aligned} \dot{x}(t) &= (A + \Delta A)x(t) + (B_u + \Delta B_u)u(t) + (B_z + \Delta B_z)w(t) \\ y &= (C_y + \Delta C_y)x(t) \\ z &= C_z x(t) \end{aligned} \tag{4.4}$$

where

$$\Delta A = \theta_1 A_1 + \theta_2 A_2$$
$$\Delta B_u = \theta_1 B_{u\_1} + \theta_2 B_{u\_2}$$
$$\Delta C_y = \theta_1 C_{y\_1} + \theta_2 C_{y\_2}$$

4.5 Comparison of Nominal, Barycentric polytopic and COG polytopic systems

Variation of the mass of vehicle is $1200 < m_s < 1800$. Graphical comparison of the nominal system 3.1, Barycentric coordinate polytopic system 3.2 and COG representation of polytopic systems 4.4 for $m_s = 1700\ kg$ is shown.

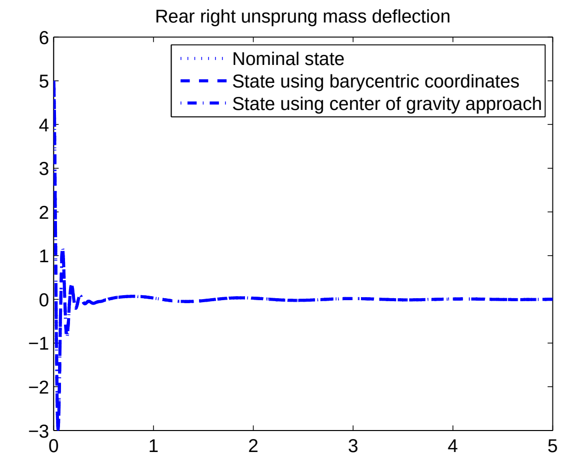

Figure 4.1: Rear right unsprung mass deflection

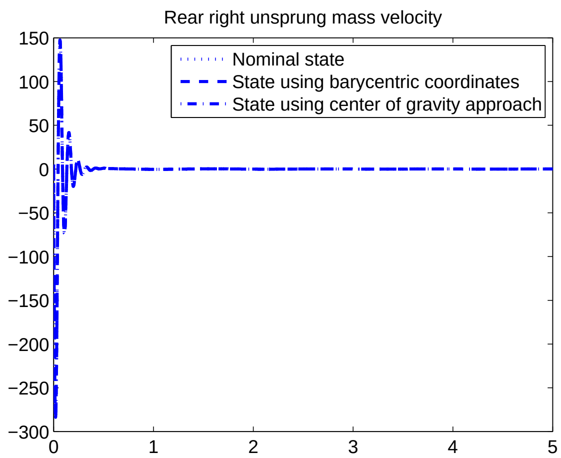

Figure 4.2: Rear right unsprung mass velocity

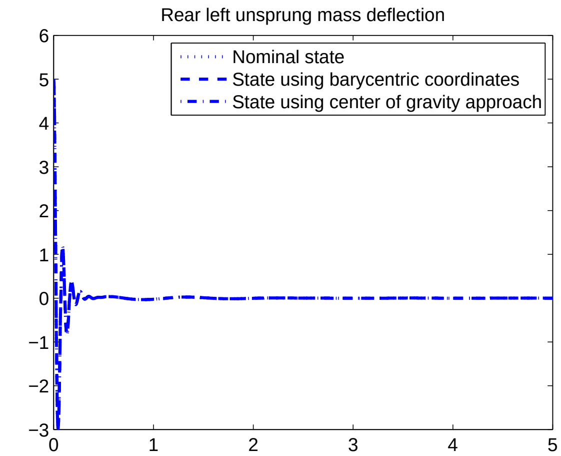

Figure 4.3: Rear left unsprung mass deflection

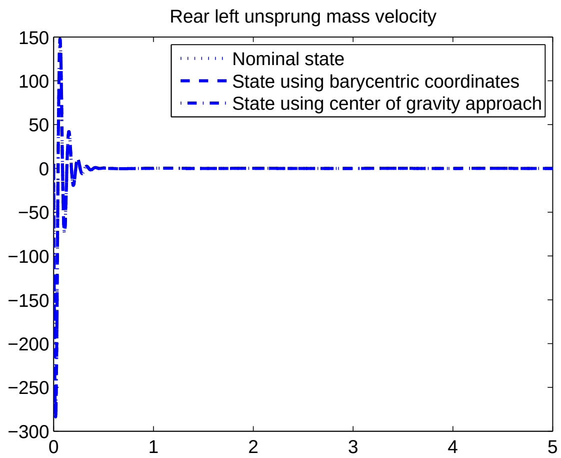

Figure 4.4: Rear left unsprung mass velocity

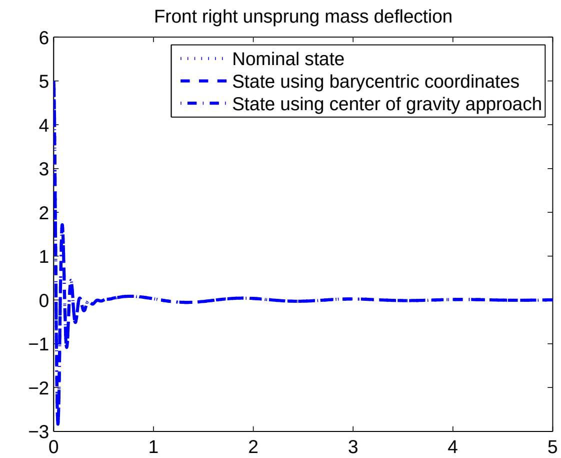

Figure 4.5: Front right unsprung mass deflection

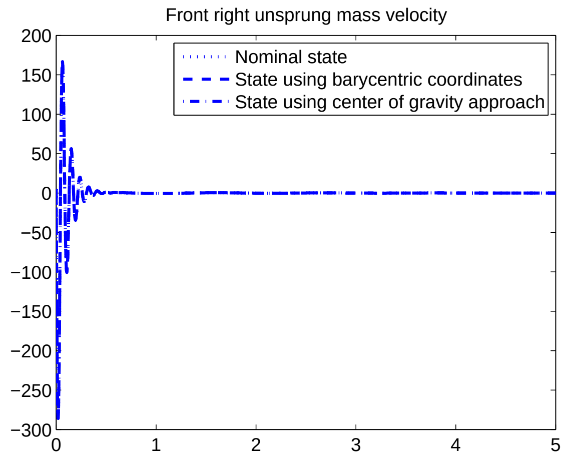

Figure 4.6: Front right unsprung mass velocity

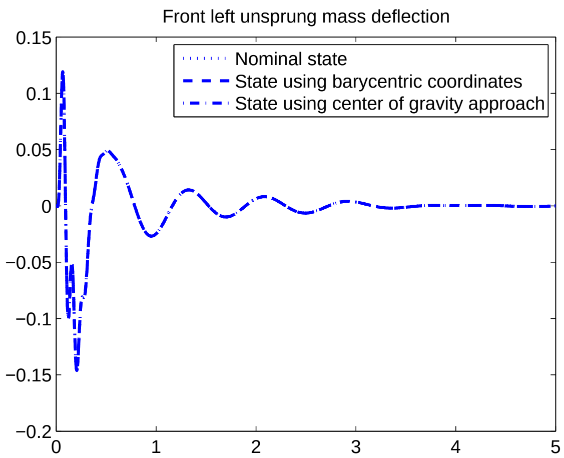

Figure 4.7: Front left unsprung mass deflection

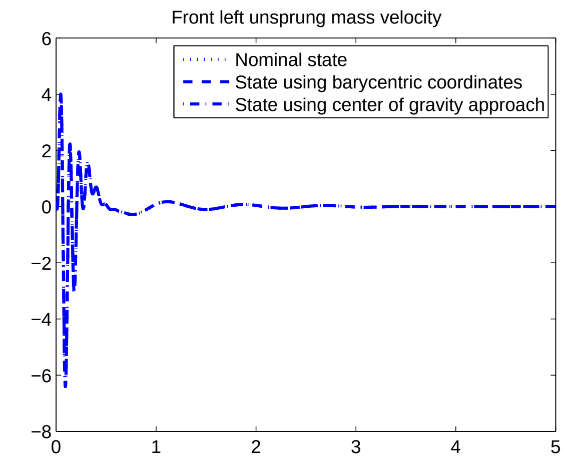

Figure 4.8: Front left unsprung mass velocity

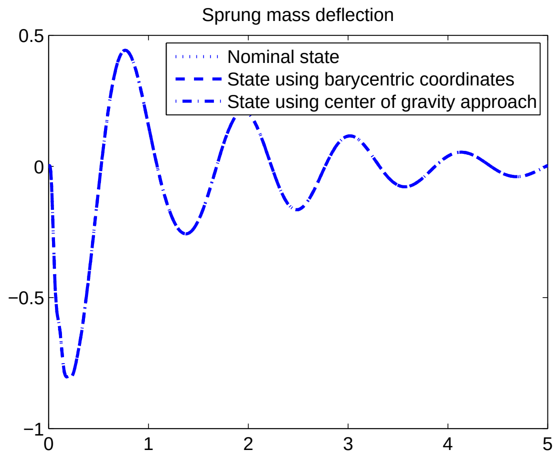

Figure 4.9: Sprung mass deflection

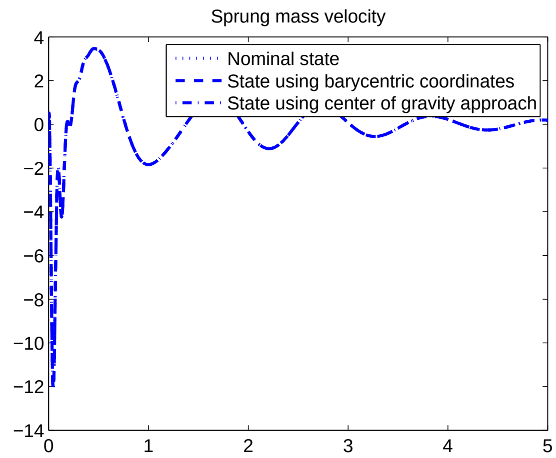

Figure 4.10: Sprung mass velocity

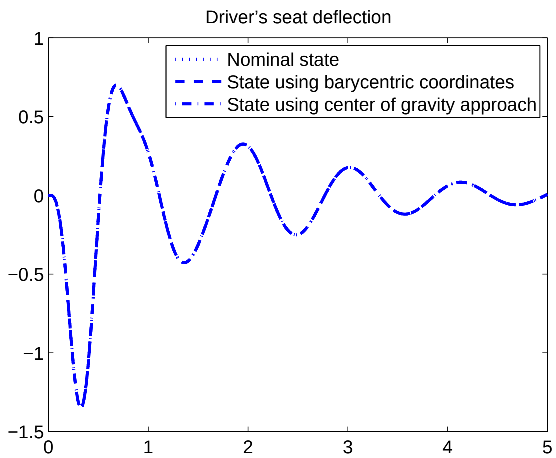

Figure 4.11: Driver's seat deflection

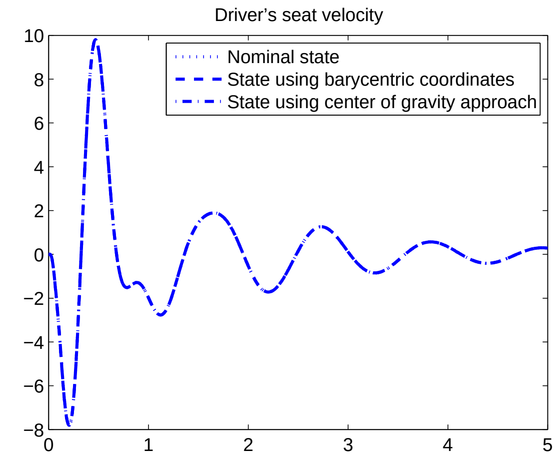

Figure 4.12: Driver's seat velocity

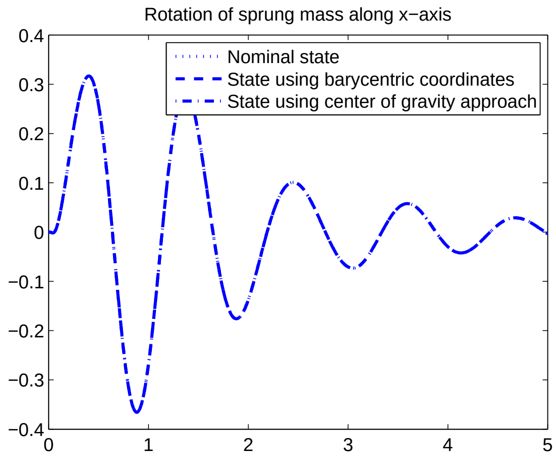

Figure 4.13: Rotation of sprung mass along x-axis

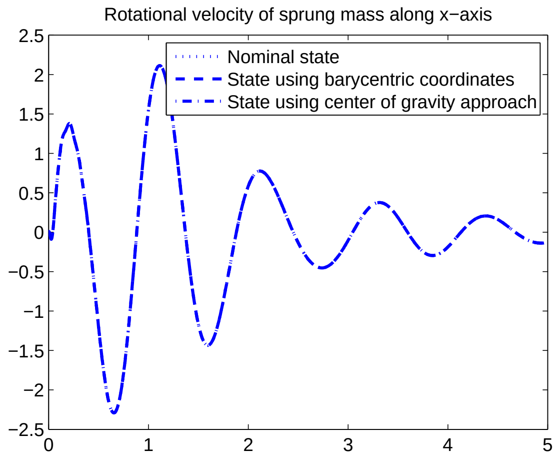

Figure 4.14: Rotational velocity of sprung mass along x-axis

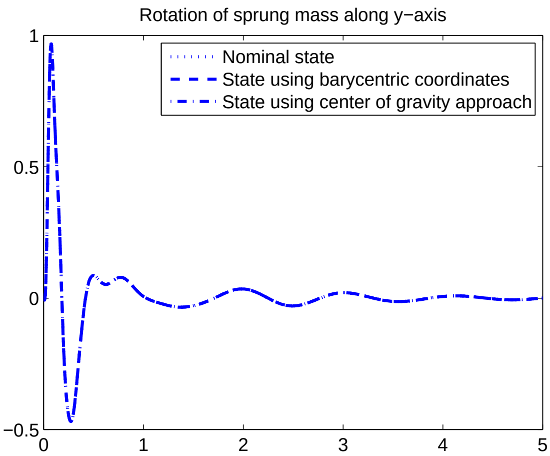

Figure 4.15: Rotation of sprung mass along y-axis

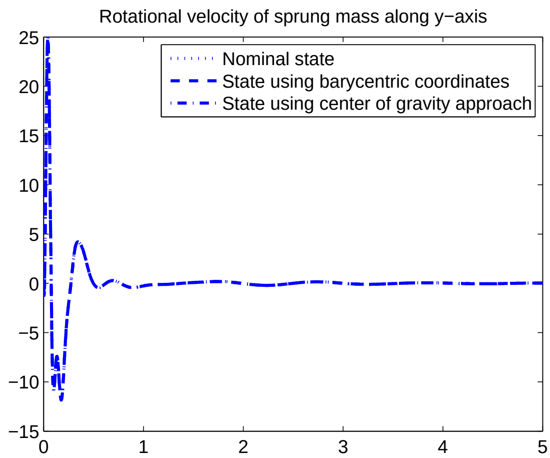

Figure 4.16: Rotational velocity of sprung mass along y-axis

These simulation results show that nominal system 3.1, Barycentric coordinate polytopic system 3.2 and COG representation of polytopic systems 4.4 are same for a fixed mass.

4.6 Summary

A controller synthesis friendly polytopic representation of the system has been obtained. It will be further used for controller design.

Chapter 5

Robustness against road disturbance

There has been a lot of research on robust control of vehicle suspension system. In [52], a robust proportional-integral sliding mode control scheme has been proposed. In [53], robust H_∞ output-feedback controller is designed for a model decoupled 7 DOF vehicle active suspension system. The system is decoupled into heave-pitch and roll-warp subsystems and the uncontrollable mode in the roll-warp subsystem is removed. In [19], vibration isolation for quarter car model with limited actuator force, good track-holding capability and suspension travel has been achieved in the form of mixed H_2/H_∞ LMI.

The main goal of designing the robust controller is good ride quality and road handling. In this chapter, some existing robust control schemes have been designed for 8 DOF vehicle model. Actuator force limit and suspension travel limit constraints have also been considered.

This chapter includes

- State feedback controller
- Output feedback controller
- Antiwindup controller for actuator force limitation

5.1 Controller design

5.1.1 State feedback control

State feedback controller has been designed in this section. This controller is designed based on assumption that all states are available for feedback.

The state feedback controller is given as $u = Kx(t)$ where $K \in \mathcal{R}^{q\times n}$.

The closed loop system is then given as

$$\begin{aligned} \dot{x} =& Ax(t) + B_u K x(t) + B_z w(t) \\ z =& C_z x(t) \end{aligned} \tag{5.1}$$

Fig. 5.1 is a closed loop active suspension system with the state feedback active actuator.

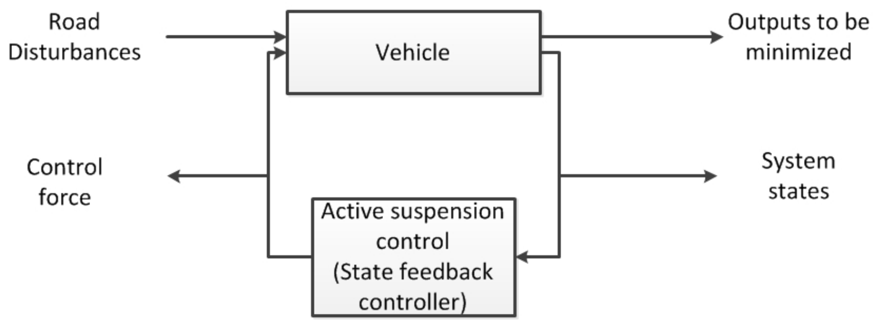

Figure 5.1: Closed loop system with state feedback

Proposition 5.1. For given scalar $\gamma > 0$, the closed-loop system 5.1 is robustly asymptotically stable with an $\mathcal{H}_\infty$ disturbance attenuation level less than γ, if there exist matrix $P > 0$ and controller matrix K with appropriate dimensions such that the following LMI's hold

$$\begin{bmatrix} He\{PA + PB_u K\} & PB_z & C_z^T \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} < 0 \tag{5.2}$$

Proof. Consider the Lyapunov function as follows $V(t) = x^T P x$

The derivative of lyapunov function is $\dot{V}(t) = \dot{x}^T P x + x^T P \dot{x}$

$\dot{V}(t) = (Ax(t) + B_uK + Bw(t))^T Px + x^T(PAx(t) + B_uK + PBw(t))$

Robust stability and objective minimization is guaranteed if $\dot{V}(t) < 0$.

The objective of the controller is to minimize effect of road disturbance $w(t)$ on desired output z. This is equivalent to $\|z\|_2 < \gamma\|w\|_2$ for all $w \in L_2[0, \infty]$. The objective function is $z^T z - \gamma^2 w^T w$. Then by S-procedure

$$\begin{aligned}\dot{V}(t) =&(Ax(t) + B_uK + Bw(t))^T Px + x^T(PAx(t) + B_uK + PBw(t))\\&+ x^T C_z^T C_z x - \gamma^2 w^T w\end{aligned}$$

It is obvious that $\dot{V}(t) < 0$ if

$$\begin{bmatrix} He\{PA + PB_uK\} + Ha\{C_z^T\} & PB \\ * & -\gamma^2 I \end{bmatrix} < 0 \tag{5.3}$$

Applying schur's complement to inequality 5.3, we get inequality 5.2 □

Theorem 5.1. For given scalars $\gamma > 0$,the closed-loop system 5.1 is robustly asymptotically stable with an $\mathcal{H}_\infty$ disturbance attenuation level less than γ, if there exist matrix $Q > 0$ and controller matrices K with appropriate dimensions such that the following LMI holds

$$\begin{bmatrix} He\{AQ + B_u\phi\} & B_z & QC_z^T \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} < 0 \tag{5.4}$$

Proof. Let $P^{-1} = Q$

Now multiplying diag $\{P^{-1}$, I , I $\}$and diag$\{P^{-1}$, I , I $\}$ on left and right of 5.2 gives 5.4.

As $P > 0$, it means $Q > 0$. □

5.1.1.1 Simulation Results

Inequality 5.4 has been solved for the active suspension system 5.1. It is assumed that all the states are available for feedback.

Fig. 5.2 shows bump response for the vehicle. Response for the rear part is

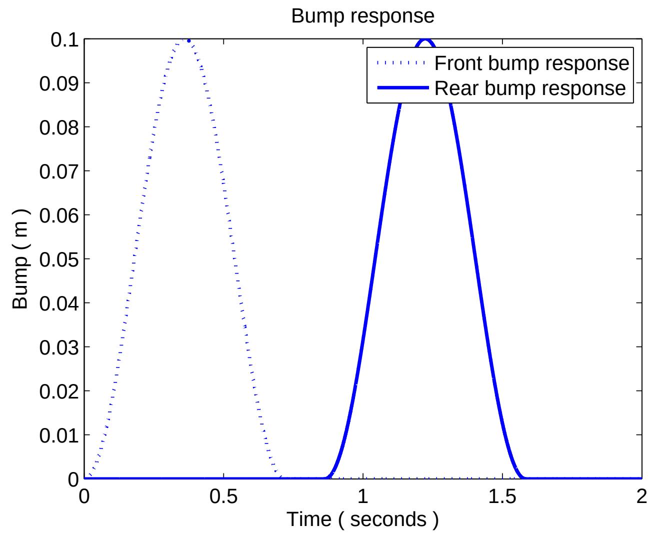

Figure 5.2: Bump input for vehicle

delayed due to the distance between front and real part.

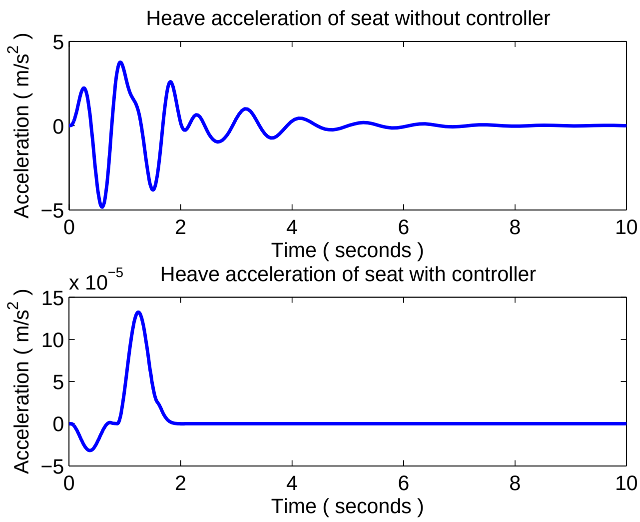

Figure 5.3: Heave acceleration of seat

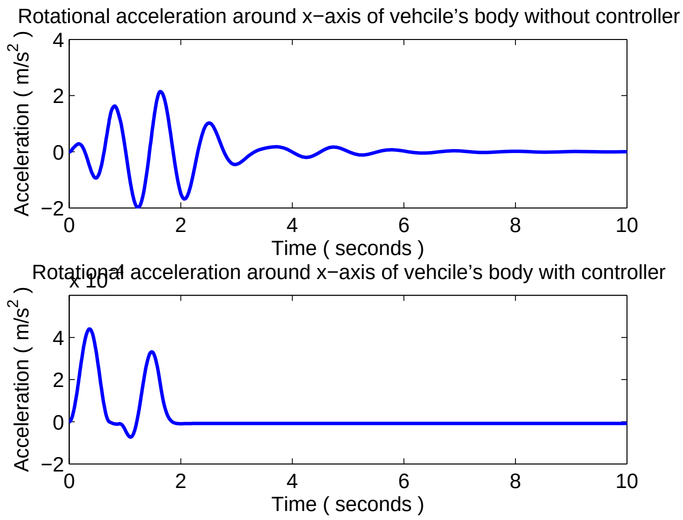

Figure 5.4: Rotational acceleration of car's body along x-axis

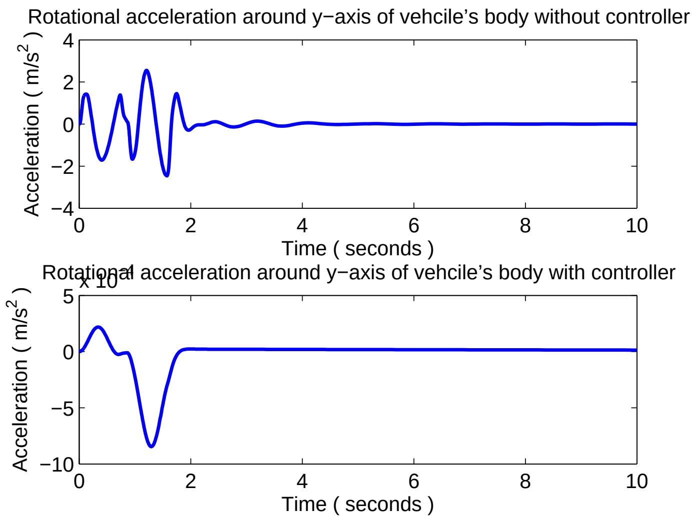

Figure 5.5: Rotational acceleration of car's body along y-axis

5.1.2 Dynamic output feedback control

The dynamic output feedback controller is given as

$$\begin{aligned} \dot{x}_c(t) &= A_c x_c(t) + B_c y(t) \\ u(t) &= C_c x_c(t) \end{aligned} \tag{5.5}$$

Fig. 5.6 shows active suspension system with output feedback active actuator.

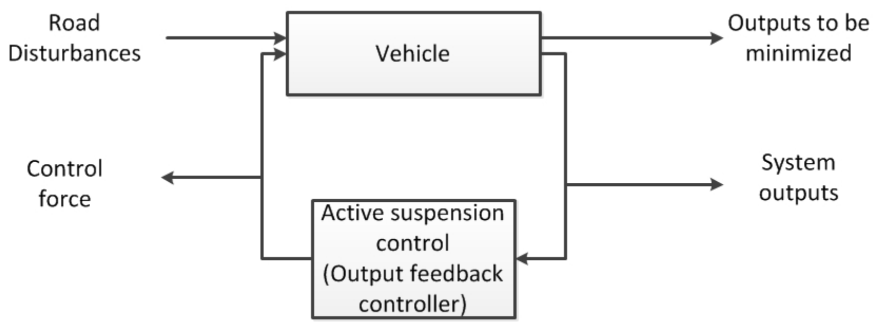

Figure 5.6: closed loop system with dynamic output feedback

Active suspension system 3.1 in closed loop with 5.9 is given as

$$\begin{aligned} \dot{x}_{cl}(t) &= A_{cl}x_{cl}(t) + B_{cl}w(t) \\ z(t) &= C_{z\_cl}x_{cl}(t) \end{aligned} \tag{5.6}$$

here $A_{cl} = \begin{bmatrix} A & B_uC_c \\ B_cC_y & A_c \end{bmatrix}$, $B_{cl} = \begin{bmatrix} B_z \\ 0 \end{bmatrix}$ and $C_{z\_cl} = \begin{bmatrix} C_z & 0 \end{bmatrix}$

Proposition 5.2. For given scalars $\gamma > 0$,the closed-loop system 5.6 is robustly asymptotically stable with an $\mathcal{H}_\infty$ disturbance attenuation level less than γ, if there exist matrix $P > 0$ and controller matrices A_c,B_c and C_c with appropriate dimensions such that the following LMI's hold

$$\begin{bmatrix} He\{PA_{cl}\} & PB_{cl} & C_{z\_cl}^T \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} < 0 \tag{5.7}$$

Proof. Consider the Lyapunov function as follows $V(t) = x_{cl}^T P x_{cl}$

The derivative of lyapunov function is $\dot{V}(t) = \dot{x}_{cl}^T P x_{cl} + x_{cl}^T P \dot{x}_{cl}$

$\dot{V}(t) = (A_{cl}x_{cl}(t) + B_{cl}w(t))^T P x_{cl} + x_{cl}^T P A_{cl} x_{cl}(t) + PB_{cl}w(t)$

Robust stability and objective minimization is guaranteed if $\dot{V}(t) < 0$.

The objective of the controller is to minimize effect of road disturbance $w(t)$ on desired output z. This is equivalent to $\|z\|_2 < \gamma\|w\|_2$ for all $w \in L_2[0, \infty]$. The objective function is $z^T z - \gamma^2 w^T w$. Then by S-procedure

$$\dot{V}(t) = (A_{cl}x_{cl}(t) + B_{cl}w(t))^T P x_{cl} + x_{cl}^T P A_{cl} x_{cl}(t) + P B_{cl} w(t) + x_{cl}^T C_{z\_cl}^T C_{z\_cl} x_{cl} - \gamma^2 w^T w$$

It is obvious that $\dot{V}(t) < 0$ if

$$\begin{bmatrix} He\{PA_{cl}\} + Ha\{C_{z\_cl}^T\} & PB_{cl} \\ * & -\gamma^2 I \end{bmatrix} < 0 \tag{5.8}$$

Applying schur's complement to inequality 5.8 and we get inequality 5.7 □

Theorem 5.2. For given scalar $\gamma > 0$, the closed-loop system 5.6 is robustly asymptotically stable with an $\mathcal{H}_\infty$ disturbance attenuation level less than γ, if the following linear matrix inequalities hold

$$\begin{bmatrix} \phi_{11} & \phi_{12} & B_z & \phi_{13} \\ * & \phi_{21} & P_1 B_z & C_{zl}^T \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -I \end{bmatrix} < 0 \tag{5.9}$$

$$\begin{bmatrix} P_{-1} & I \\ I & P_1 \end{bmatrix} \tag{5.10}$$

$$\begin{aligned} \phi_{11} &= He\{AP_{-1} + B_u \phi\} \\ \phi_{12} &= A + M^T \\ \phi_{13} &= P_{-1} C_z^T \\ M &= P_1 A P_{-1} + P_1 B_u \phi + \lambda C_y P_{-1} + P_2 A_c P_{-2}^T \end{aligned}$$

The controller matrices are then given as

$$
\begin{aligned}
A_c =& P_2^{-1}(M - P_1AP_{-1} - P_1B_u\phi - \lambda C_yP_{-1})P_{-2}^{-T} \\
B_c =& P_2^{-1}\lambda \\
C_c =& \phi P_{-2}^{-T}
\end{aligned}
$$

Proof. Let P= $\begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix}$ and $P^{-1} = \begin{bmatrix} P_{-1} & P_{-2} \\ * & P_{-3} \end{bmatrix}$

Consider $\pi = \begin{bmatrix} P_{-1} & I \\ P_{-2}^T & 0 \end{bmatrix}$ such that $\pi^T P = \begin{bmatrix} I & 0 \\ P_1 & P_2 \end{bmatrix}$

Now multiplying diag $\{\pi^T$, I , I $\}$and diag $\{\pi$, I , I $\}$ on left and right of 5.7 gives 5.9.

As $P > 0$, it means $\pi^T P\pi > 0$ which gives 5.10 □

Remark 5.1. As $PP^{-1} = I$, it gives $P_1P_{-1} + P_2P_{-2}^T = I$. After solving inequality 5.9 P_2 and P_{-2} can be obtain by singular value decomposition of $I - P_1P_{-1}$

5.1.3 Simulation results

Inequality 5.9 has been solved for the active suspension system 5.6. Dynamic output feedback controller has been obtained.

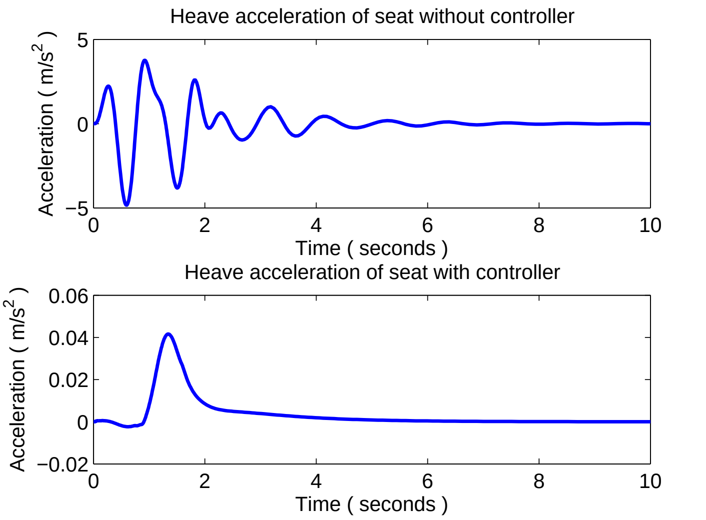

Figure 5.7: Heave acceleration of seat

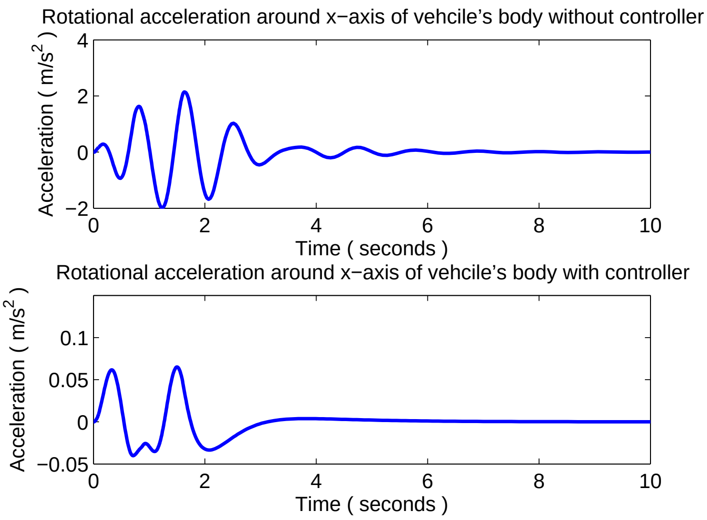

Figure 5.8: Rotational acceleration of car's body along x-axis

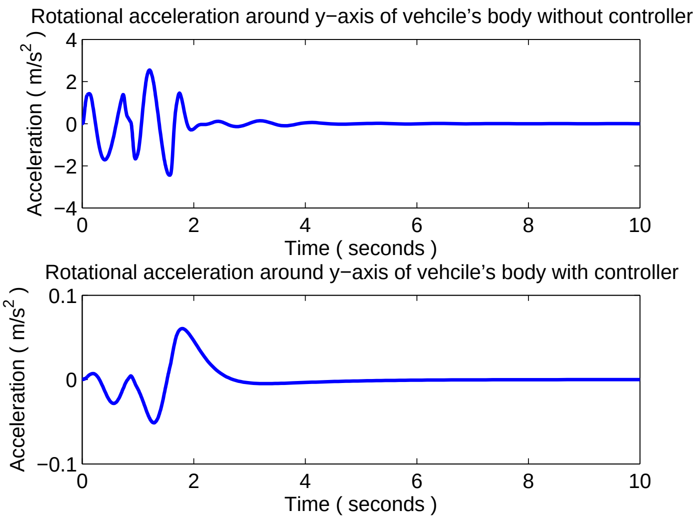

Figure 5.9: Rotational acceleration of car's body along y-axis

5.1.4 Antiwindup control

Actuator saturation in a control system can cause instability and degradation of performance. Antiwindup controller are designed to avoid this. A survey on the antiwindup techniques is given in [54].Recent advancements on antiwindup control are [55], [56], [57]. The state space representation of antiwindup controller [58] is

$$
\begin{aligned}
\dot{\eta} =&(A + B_u F)\eta + B_u\tilde{u} \\
u_d =&F\eta \\
y_d =&C_y\eta
\end{aligned}
\tag{5.11}
$$

Fig. 5.10 shows decoupled antiwindup controller for active suspension system.

Proposition 5.3. For given scalars $\gamma > 0$ and $\tau > 0$, the anti-windup controller 5.11 is asymptotically stable with an $\mathcal{H}_\infty$ disturbance attenuation level less than γ, if there exist matrix $P > 0$ and controller matrix F_1 with appropriate dimensions

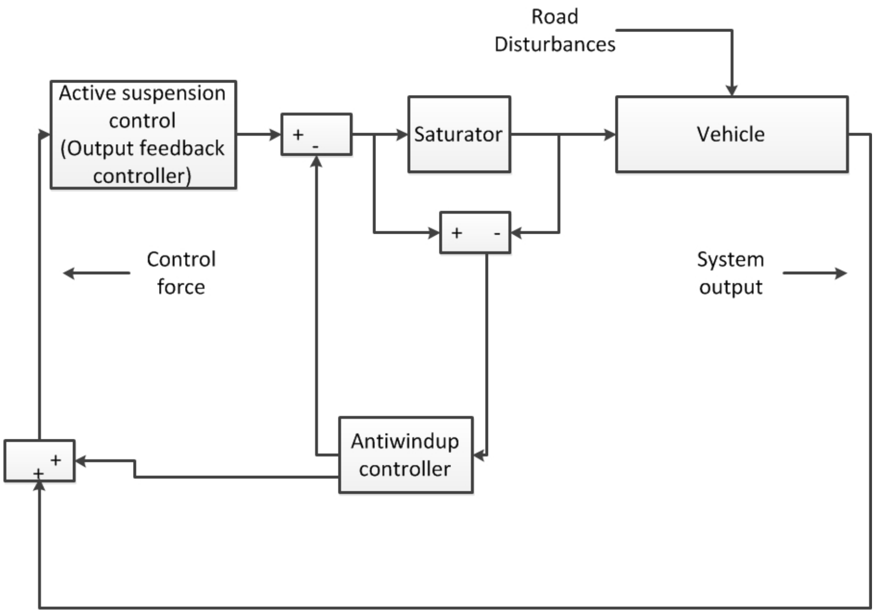

Figure 5.10: Antiwindup controller

such that the following LMI's hold

$$\begin{bmatrix} He\{PA + PB_uF\} & PB_uV^{-1} - F_1^T & 0 & C_y^T \\ * & -2V^{-1} & I & 0 \\ * & * & -\gamma I & 0 \\ * & * & * & -I \end{bmatrix} < 0 \tag{5.12}$$

Proof. Consider the Lyapunov function as follows $V(t) = \eta(t)^T P \eta(t)$

The derivative of lyapunov function is $\dot{V}(t) = \dot{\eta}(t)^T P \eta(t) + \eta(t)^T P \dot{\eta}(t)$

$\dot{V}(t) = (A_{aw}\eta(t) + B_u\tilde{u})^T P \eta(t) + \eta(t)^T (A_{aw}\eta(t) + B_u\tilde{u})$

here $A_{aw} = A + B_uF$ and $C_{aw} = C_y$

Robust stability and objective minimization is guaranteed if $\dot{V}(t) < 0$.

The objective of the controller is equivalent to $\|y_d\|_2 < \gamma \|u_{lin}\|_2$. The objective function is $y_d^T y_d - \gamma^2 u_{lin}^T u_{lin}$ where u_{lin} is the output of controller. Then by S-procedure

$$\dot{V}(t) = (A_{aw}\eta(t) + B_u\tilde{u})^T P \eta(t) + \eta(t)^T (A_{aw}\eta(t) + B_u\tilde{u}) + \eta(t)^T C_{aw}^T C_{aw} \eta(t) - \gamma^2 u_{lin}^T u_{lin}$$

It is obvious that $\dot{V}(t) < 0$ if

$$\begin{bmatrix} He\{PA_{aw}\} + Ha\{C_{aw}^T\} & PB & 0 \\ * & 0 & 0 \\ * & * & -\gamma^2 I \end{bmatrix} < 0 \tag{5.13}$$

The sector boundedness is $2\tilde{u}^T W(u_{lin} - F\eta - \tilde{u}) > 0$ .Then by S-procedure inequality becomes

$$\begin{bmatrix} He\{PA + PB_uF\} + Ha\{C_y^T\} & PB_u - F_1^T W\tau & 0 \\ * & -2W\tau & I \\ * & * & -\gamma I \end{bmatrix} < 0 \tag{5.14}$$

Multiplying by diag{ I , W^{-1} , I } on left and right side of 5.14 and then applying schur's complement we get inequality 5.12 □

Theorem 5.3. For given scalars $\gamma > 0$ and $\tau > 0$, the antiwindup controller asymptotically stable with an $\mathcal{H}_\infty$ disturbance attenuation level less than γ, if there exist matrix $P > 0$ and controller matrix F_1 with appropriate dimensions such that the following LMI's hold

$$\begin{bmatrix} He\{AQ + PB_u\phi\} & B_uV^{-1} - \phi_1^T & 0 & QC_y^T \\ * & -2V^{-1} & I & 0 \\ * & * & -\gamma I & 0 \\ * & * & * & -I \end{bmatrix} < 0 \tag{5.15}$$

The controller matrix is $F_1 = Q^{-1}\phi$.

Proof. Let $P^{-1} = Q$

Now multiplying diag $\{P^{-1}$, I , I , I $\}$ on left and right of 5.12 gives 5.15. As $P > 0$, it means $Q > 0$. □

5.2 summary

A state feedback controller has been designed to minimize the effect of road disturbance on ride comfort. An output feedback controller has also been designed. An anti-windup controller has been proposed for actuator saturation constraint.

Chapter 6

Robustness against parametric uncertainty

Polytopic system stability and control problem has been investigated by [49,59–61]. In [62], a homogeneous polynomial parameter dependent lyapunov function has been proposed for the stability of polytopic uncertain systems. In [63], stability of polytopic systems is discussed in terms of parameter dependent lyapunov function and state feedback control problem is discussed. In [53], load-dependent controller for multi-objective control of quarter vehicle active suspension systems is proposed. Parameter dependent lyapunov function has been used to ensure less conservative results.

6.1 Controller design

State feedback controller and output feedback controller which are uncertain against polytopic uncertainty have been proposed for the active suspension system.

6.1.1 State feedback control

State feedback controller has been designed in this section. This controller is designed based on assumption that all states are available for feedback.

The state feedback controller is given as $u = Kx(t)$ where $K \in \mathcal{R}^{q\times n}$.

This state feedback controller in closed loop with system 4.4 is then given as

$$\begin{aligned} \dot{x}(t) &= (A+\Delta A)x(t) + (B_u+\Delta B_u)Kx(t) + (B_z+\Delta B_z)w(t) \\ z &= C_z x(t) \end{aligned} \tag{6.1}$$

lemma 1. [64] For any matrices $D \in \mathcal{R}^{n\times n}$, $D \in \mathcal{R}^{n\times n}$ and $D \in \mathcal{R}^{n\times n}$ with $F^T F \leq I$ and scalar $\epsilon > 0$, we have

$$DFE + E^T F^T D^T \leq \epsilon^{-1} DD^T + \epsilon E^T E$$

Proposition 6.1. For given scalar $\gamma > 0$, the closed-loop system 6.1 is robustly asymptotically stable with an $\mathcal{H}_\infty$ disturbance attenuation level less than γ, if there exist matrix $P > 0$ and controller matrix K with appropriate dimensions such that the following LMI holds

$$\begin{bmatrix} He\{P(A+B_uK)\} & PB_{cl} & C_{z1\_cl}^T & \epsilon PA_1 & \epsilon P & \epsilon PA_2 & \epsilon P & I & K^T B_{u\_1}^T & I & K^T B_{u\_2}^T \\ * & -\gamma^2 I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\epsilon & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\epsilon & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\epsilon & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\epsilon & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\epsilon & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -\epsilon & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -\epsilon & 0 \\ * & * & * & * & * & * & * & * & * & * & -\epsilon \end{bmatrix} < 0 \tag{6.2}$$

Proof. Consider the Lyapunov function as follows $V(t) = x_{cl}^T P x_{cl}$
The derivative of lyapunov function is $\dot{V}(t) = \dot{x}_{cl}^T P x_{cl} + x_{cl}^T P \dot{x}_{cl}$

$$\begin{aligned}\dot{V}(t) =& ((A + \Delta A + B_u K + \Delta B_u K)x(t) + (B_z + \Delta B_z)w(t))^T Px \\ & + x^T P(A + \Delta A + B_u K + \Delta B_u K)x(t) + P(B_z + \Delta B_z)w(t)\end{aligned}$$

Robust stability and objective minimization is guaranteed if $\dot{V}(t) < 0$.
The objective of this controller is to minimize effect of parametric uncertainties and road disturbance $w(t)$ on desired output z. This is equivalent to $\|z\|_2 < \gamma\|w\|_2$ for all $w \in L_2[0, \infty]$. The objective function is $z^T z - \gamma^2 w^T w$. Then by S-procedure

$$\begin{aligned}\dot{V}(t) =& ((A + \Delta A + B_u K + \Delta B_u K)x(t) + (B_z + \Delta B_z)w(t))^T Px \\ & + x^T P(A + \Delta A + B_u K + \Delta B_u K)x(t) + P(B_z + \Delta B_z)w(t) + x^T C_z^T C_z x - \gamma^2 w^T w\end{aligned}$$

It is obvious that $\dot{V}(t) < 0$ if

$$\begin{bmatrix} HeP(A + \Delta A + B_u K + \Delta B_u K) + Ha\{C_z^T\} & PB_z \\ * & -\gamma^2 I \end{bmatrix} < 0 \tag{6.3}$$

Applying schur's complement to inequality 6.3 we get

$$\begin{aligned}&\begin{bmatrix} He\{P(A + B_u K)\} & PB_{cl} & C_{z\_cl}^T \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} + \begin{bmatrix} He\{P\theta_1 A_1 + P\theta_1 B_{u\_1} K\} & 0 & 0 \\ * & 0 & 0 \\ * & * & 0 \end{bmatrix} \\ &+ \begin{bmatrix} HeP\theta_2 A_2 + P\theta_2 B_{u\_2} K & 0 & 0 \\ * & 0 & 0 \\ * & * & 0 \end{bmatrix} < 0\end{aligned}$$

$$\begin{bmatrix} He\{P(A+B_uK)\} & PB_{cl} & C_{z\_cl}^T \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} + \begin{bmatrix} PA_1 & P & PA_2 & P \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_1 & 0 \\ * & \theta_2 \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ B_{u\_1}K & 0 & 0 \\ I & 0 & 0 \\ B_{u\_2}K & 0 & 0 \end{bmatrix} < 0 \tag{6.4}$$

It is obvious that $\begin{bmatrix} \theta_1 & 0 \\ 0 & \theta_2 \end{bmatrix}^T \begin{bmatrix} \theta_1 & 0 \\ 0 & \theta_2 \end{bmatrix} <= I$ then by Lemma. 1, we have

$$\begin{bmatrix} PA_1 & P & PA_2 & P \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_1 & 0 \\ * & \theta_2 \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ B_{u\_1}K & 0 & 0 \\ I & 0 & 0 \\ B_{u\_2}K & 0 & 0 \end{bmatrix} = \epsilon * Ha \begin{bmatrix} PA_1 & P & PA_2 & P \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \end{bmatrix}$$

$$+ \frac{1}{\epsilon} * Ha \begin{bmatrix} I & 0 & 0 \\ B_{u\_1}K & 0 & 0 \\ I & 0 & 0 \\ B_{u\_2}K & 0 & 0 \end{bmatrix}^T = \begin{bmatrix} \epsilon PA_1 & \epsilon P & \epsilon PA_2 & \epsilon P & I & K^T B_{u\_1}^T & I & K^T B_{u\_2}^T \\ * & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{\epsilon} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\epsilon} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\epsilon} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\epsilon} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\epsilon} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\epsilon} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\epsilon} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\epsilon} \end{bmatrix} \begin{bmatrix} \epsilon PA_1 & \epsilon P & \epsilon PA_2 & \epsilon P & I & K^T B_{u\_1}^T & I & K^T B_{u\_2}^T \\ * & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

Putting above result in inequality 6.4 and applying schur's complement leads to inequality 6.2 □

Theorem 6.1. For given scalars $\gamma > 0$,the closed-loop system 6.1 is robustly asymptotically stable with an $\mathcal{H}_\infty$ disturbance attenuation level less than γ, if there exist matrix $Q > 0$ and controller matrix K with appropriate dimensions such that the following LMI's hold

$$\begin{bmatrix} He\{AQ + B_u\phi\} & B_{cl} & QC_{z\_cl}^T & \epsilon A_1 & \epsilon & \epsilon A_2 & \epsilon & Q & \phi^T B_{u\_1}^T & Q & \phi^T B_{u\_2}^T \\ * & -\gamma^2 I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\epsilon & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\epsilon & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\epsilon & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\epsilon & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\epsilon & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -\epsilon & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -\epsilon & 0 \\ * & * & * & * & * & * & * & * & * & * & -\epsilon \end{bmatrix} < 0 \tag{6.5}$$

The controller matrix is given as $K = \phi Q^{-1}$

Proof. Let $P^{-1} = Q$

Now multiplying diag $\{P^{-1}$, I , I , I , I , I , I , I , I , I , I $\}$ and diag$\{P^{-1}$, I , I , I , I , I , I , I , I , I , I $\}$ on left and right of 6.2 gives 6.5.

As $P > 0$, it means $Q > 0$. □

6.1.2 Dynamic output feedback control

The dynamic output feedback controller is given as

$$\begin{aligned} \dot{x}_c(t) &= A_c x_c(t) + B_c y(t) \\ u(t) &= C_c x_c(t) \end{aligned} \tag{6.6}$$

Active suspension system 4.4 in closed loop with 6.6 is given as

$$\begin{aligned} \dot{x}_{cl}(t) &= (A_{cl} + \Delta A_{cl}) x_{cl}(t) + B_{cl} w(t) \\ z_{cl}(t) &= C_{z\_cl} x_{cl}(t) \end{aligned} \tag{6.7}$$

here $A_{cl} = \begin{bmatrix} A_{cen} & B_{u\_cen} C_c \\ B_c C_{y\_cen} & A_c \end{bmatrix}$, $\Delta A_{cl} = H_1 \theta_1 G_1 + H_2 \theta_1 G_2$,

, $B_{cl} = \begin{bmatrix} B_z \\ 0 \end{bmatrix}$ and $C_{z\_cl} = \begin{bmatrix} C_z & 0 \end{bmatrix}$

$H_1 = \begin{bmatrix} A_1 & I \\ B_c C_{y\_1} & 0 \end{bmatrix}$ $G_1 = \begin{bmatrix} I & 0 \\ 0 & B_{u\_1} Cc \end{bmatrix}$ $H_2 = \begin{bmatrix} A_2 & I \\ B_c C_{y\_2} & 0 \end{bmatrix}$ $G_2 = \begin{bmatrix} I & 0 \\ 0 & B_{u\_2} Cc \end{bmatrix}$

Proposition 6.2. For given scalars $\gamma > 0$ and $\epsilon > 0$,the closed-loop system 6.7 is robustly asymptotically stable with an $\mathcal{H}_\infty$ disturbance attenuation level less than γ, if there exist matrix $P > 0$ and controller matrices A_c, B_c and C_c with appropriate dimensions such that the following LMI's hold

$$\begin{bmatrix} A_{cl}^T P + P A_{cl} & P B_{cl} & C_{z\_cl}^T & \epsilon P H_1 & \epsilon P H_2 & G_1^T & G_2^T \\ * & -\gamma^2 I & 0 & 0 & 0 & 0 & 0 \\ * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & -\epsilon & 0 & 0 & 0 \\ * & * & * & * & -\epsilon & 0 & 0 \\ * & * & * & * & * & -\epsilon & 0 \\ * & * & * & * & * & * & -\epsilon \end{bmatrix} < 0 \tag{6.8}$$

Proof. Consider the Lyapunov function as follows $V(t) = x_{cl}^T P x_{cl}$
The derivative of lyapunov function is $\dot{V}(t) = \dot{x}_{cl}^T P x_{cl} + x_{cl}^T P \dot{x}_{cl}$

$$\begin{aligned}\dot{V}(t) =& ((A_{cl} + \Delta A_{cl})x_{cl}(t) + (B_{cl} + \Delta B_{cl})w(t))^T P x_{cl} + x_{cl}^T P(A_{cl} + \Delta A_{cl})x_{cl}(t) \\ &+ P(B_{cl} + \Delta B_{cl})w(t)\end{aligned}$$

Robust stability and objective minimization is guaranteed if $\dot{V}(t) < 0$.
The objective of the controller is to minimize effect of road disturbance $w(t)$ on desired output z. This is equivalent to $\|z\|_2 < \gamma\|w\|_2$ for all $w \in L_2[0, \infty]$. The objective function is $z^T z - \gamma^2 w^T w$. Then by S-procedure

$$\begin{aligned}\dot{V}(t) =& ((A_{cl} + \Delta A_{cl})x_{cl}(t) + (B_{cl} + \Delta B_{cl})w(t))^T P x_{cl} + x_{cl}^T P(A_{cl} + \Delta A_{cl})x_{cl}(t) \\ &+ P(B_{cl} + \Delta B_{cl})w(t) + x_{cl}^T C_{z\_cl}^T C_{z\_cl} x_{cl} - \gamma^2 w^T w\end{aligned}$$

It is obvious that $\dot{V}(t) < 0$ if

$$\begin{bmatrix} (A_{cl} + \Delta A_{cl})^T P + P(A_{cl} + \Delta A_{cl}) + C_{z\_cl}^T C_{z\_cl} & PB_{cl} \\ * & -\gamma^2 I \end{bmatrix} < 0 \tag{6.9}$$

Applying schur's complement to inequality 6.9 and putting we get

$$\begin{aligned}&\begin{bmatrix} A_{cl}^T P + PA_{cl} & PB_{cl} & C_{z\_cl}^T \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} + \begin{bmatrix} PH_1\theta_1 G_1 + G_1^T \theta_1 H_1^T P + & 0 & 0 \\ * & 0 & 0 \\ * & * & 0 \end{bmatrix} \\ &+ \begin{bmatrix} PH_2\theta_2 G_2 + G_2^T \theta_2 H_2^T P & 0 & 0 \\ * & 0 & 0 \\ * & * & 0 \end{bmatrix} < 0\end{aligned}$$

$$\begin{bmatrix} A_{cl}^T P + P A_{cl} & P B_{cl} & C_{z\_cl}^T \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} + \begin{bmatrix} PH_1 & PH_2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_1 & 0 \\ 0 & \theta_2 \end{bmatrix} \begin{bmatrix} G_1 & 0 & 0 \\ G_2 & 0 & 0 \end{bmatrix} < 0 \quad (6.10)$$

It is obvious that $\begin{bmatrix} \theta_1 & 0 \\ 0 & \theta_2 \end{bmatrix}^T \begin{bmatrix} \theta_1 & 0 \\ 0 & \theta_2 \end{bmatrix} <= I$ then by Lemma. 1, we have

$$\begin{bmatrix} PH_1 & PH_2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_1 & 0 \\ 0 & \theta_2 \end{bmatrix} \begin{bmatrix} G_1 & 0 & 0 \\ G_2 & 0 & 0 \end{bmatrix} = \epsilon * Ha \begin{bmatrix} PH_1 & PH_2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} + \frac{1}{\epsilon} * Ha \begin{bmatrix} G_1 & 0 & 0 \\ G_2 & 0 & 0 \end{bmatrix}^T$$

$$= \begin{bmatrix} \epsilon PH_1 & \epsilon PH_2 & G_1^T & G_2^T \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\epsilon} & 0 & 0 & 0 \\ 0 & \frac{1}{\epsilon} & 0 & 0 \\ 0 & 0 & \frac{1}{\epsilon} & 0 \\ 0 & 0 & 0 & \frac{1}{\epsilon} \end{bmatrix} \begin{bmatrix} \epsilon PH_1 & \epsilon PH_2 & G_1^T & G_2^T \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}^T$$

Putting above result in inequality 6.10 and applying schur's complement leads to inequality 6.8 □

Theorem 6.2. For given scalars $\gamma > 0$ and $\epsilon > 0$, the closed-loop system 6.7 is robustly asymptotically stable with an $\mathcal{H}_\infty$ disturbance attenuation level less than

γ, if the following linear matrix inequalities hold

$$\begin{bmatrix} \phi_{11} & \phi_{12} & B_z & \phi_{13} & \epsilon A_1 & \epsilon I & \epsilon A_2 & \epsilon I & P_{-1} & \phi_{14} & P_{-1} & \phi_{15} \\ * & \phi_{21} & P_1 B_z & C_z^T & \phi_{22} & \epsilon P_1 & \phi_{23} & \epsilon P_1 & I & 0 & I & 0 \\ * & * & -\gamma^2 I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\epsilon & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\epsilon & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\epsilon & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\epsilon & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -\epsilon & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -\epsilon & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & -\epsilon & 0 \\ * & * & * & * & * & * & * & * & * & * & * & -\epsilon \end{bmatrix} < 0 \quad (6.11)$$

$$\begin{bmatrix} P_{-1} & I \\ I & P_1 \end{bmatrix} > 0 \quad (6.12)$$

$$\phi_{11} = He\{AP_{-1} + B_u \phi\}$$

$$\phi_{12} = A + M^T$$

$$\phi_{13} = P_{-1} C_z^T$$

$$\phi_{14} = \phi^T b_{u\_1}$$

$$\phi_{15} = \phi^T b_{u\_2}$$

$$\phi_{21} = He\{P_1 A + \lambda C_y\}$$

$$\phi_{22} = \epsilon P_1 A_1 + \epsilon \lambda C_{y\_1}$$

$$\phi_{23} = \epsilon P_1 A_2 + \epsilon \lambda C_{y\_2}$$

$$M = P_1 A P_{-1} + P_1 B_u \phi + \lambda C_y P_{-1} + P_2 A_c P_{-2}^T$$

The controller matrices are then given as

$$
\begin{aligned}
A_c =& P_2^{-1}(M - P_1AP_{-1} - P_1B_u\phi - \lambda C_yP_{-1})P_{-2}^{-T} \\
B_c =& P_2^{-1}\lambda \\
C_c =& \phi P_{-2}^{-T}
\end{aligned}
$$

Proof. Let P= $\begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix}$ and $P^{-1} = \begin{bmatrix} P_{-1} & P_{-2} \\ * & P_{-3} \end{bmatrix}$

Consider $\pi = \begin{bmatrix} P_{-1} & I \\ P_{-2}^T & 0 \end{bmatrix}$ such that $\pi^T P = \begin{bmatrix} I & 0 \\ P_1 & P_2 \end{bmatrix}$

Now multiplying diag $\{\pi^T$, I , I , I , I , I , I $\}$and diag$\{\pi$, I , I , I , I , I , I $\}$ on left and right of 6.8 gives 6.11.

As $P > 0$, it means $\pi^T P\pi > 0$ which gives 6.12 □

6.2 Simulation

Inequality 6.11 is solved for vehicle's mass $m_1 = 1200\ kg$ and $m_2 = 1800\ kg$. Simulation results for $m_s = 1650\ kg$ show that controller's performance is robust to disturbance and uncertainties.

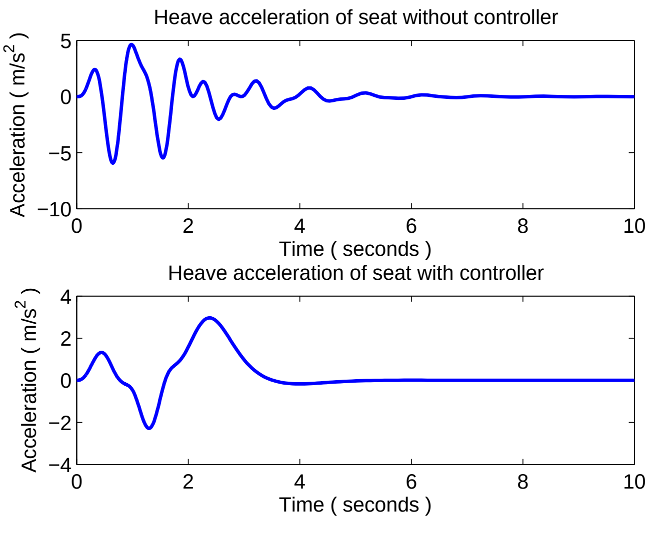

Figure 6.1: Heave acceleration of seat

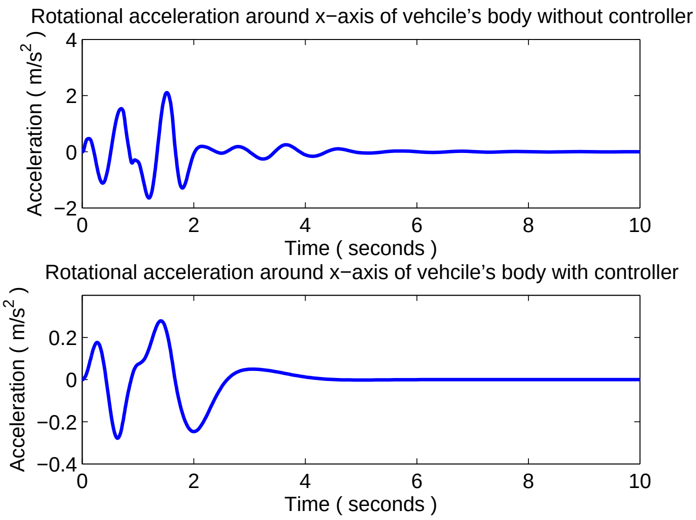

Figure 6.2: Rotational acceleration of car's body along x-axis

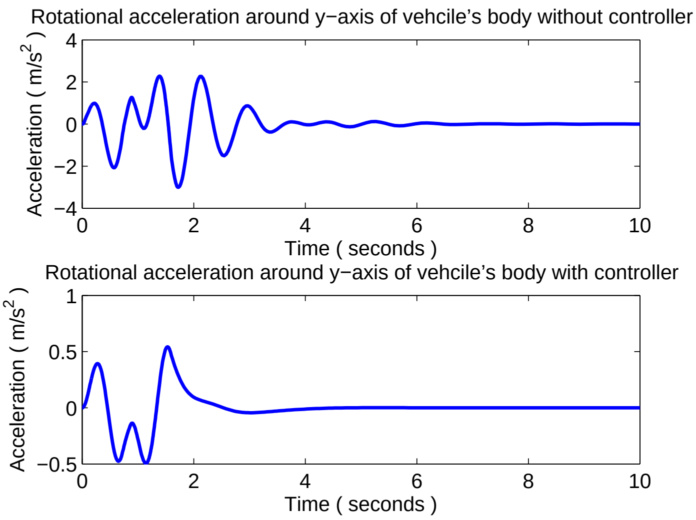

Figure 6.3: Rotational acceleration of car's body along y-axis

Inequality 6.11 is solved for vehicle's mass $m_s = 1450kg$. Simulation results show that controller's performance is robust to disturbance and uncertainties.

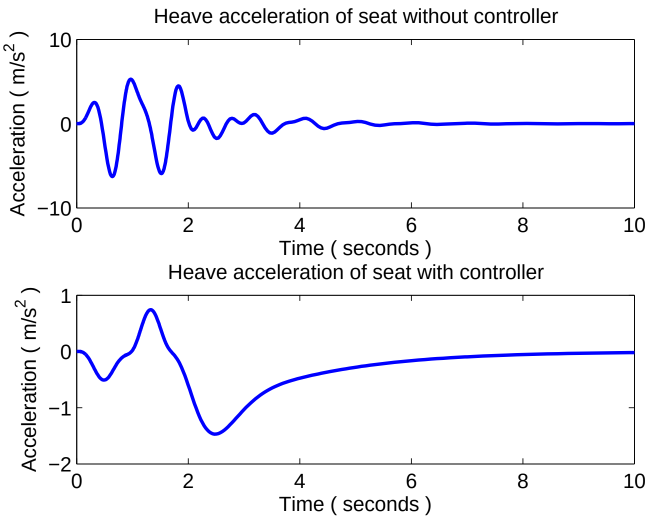

Figure 6.4: Heave acceleration of seat

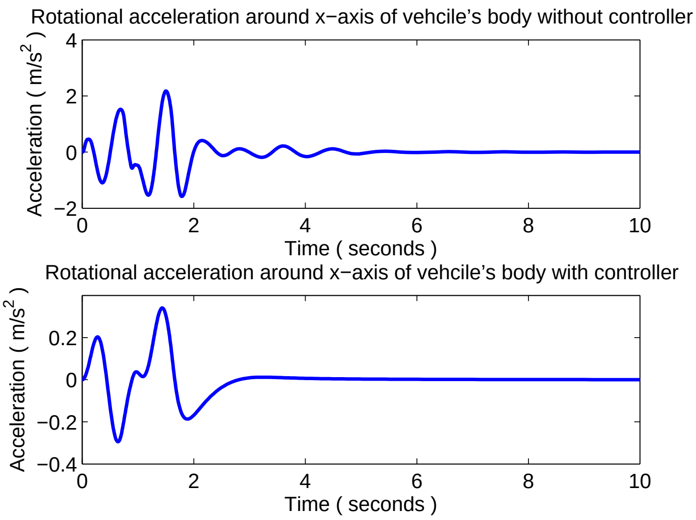

Figure 6.5: Rotational acceleration of car's body along x-axis

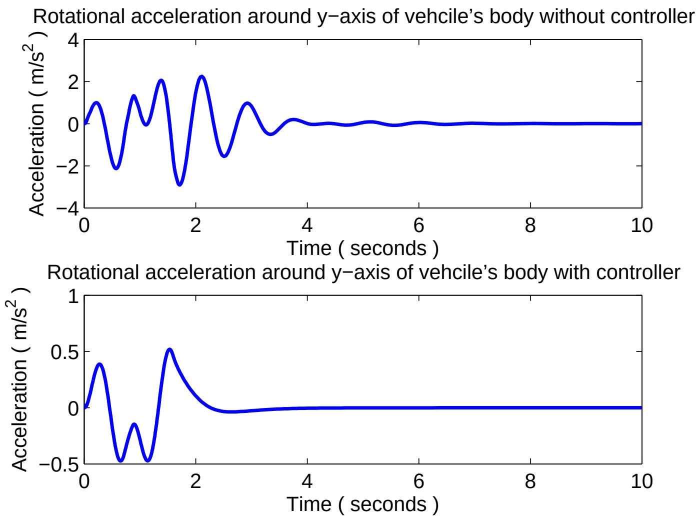

Figure 6.6: Rotational acceleration of car's body along y-axis

6.3 summary

Using COG representation for the polytopic system has reduced the complexity associated with controller design for polytopic systems. A state feedback controller and an output feedback controller have been developed for the active suspension system.

Chapter 7

Robustness against input delay

Active suspension system requires input from microprocessors. Microprocessors take data from sensors, sample and process it. This sampling and processing requires certain time. This causes delayed input.

Delayed input can cause system instability and degradation of closed loop performance. To avoid this, controller are designed to be robust against the time delay. Stability criteria for the system with delay can be delay dependent and delay independent. Delay dependent criteria is less conservative while delay independent criteria is more conservative.

Delay independent stability and robust control is discussed in [65, 66]. Delay dependent stability and robust control is discussed in [67–70, 70, 71]. In [72], delay independent controller has been designed for norm bounded uncertain system. In [73], delay dependent controller has been designed. Time delay system has been represented in descriptor form and vector cross product bound lemma has been utilized to derive controller.

7.1 Controller design

The state space model of the active suspension full car with delayed control input is

$$\begin{aligned}\dot{x} =& Ax(t) + B_u u(t-\tau) + B_z w(t)\\ z =& C_Z x(t)\end{aligned} \tag{7.1}$$

The state feedback control law is $u(t) = Kx(t)$.Then the closed loop system is given as

$$\begin{aligned}\dot{x} =& Ax(t) + B_u Kx(t-\tau) + B_z w(t)\\ z =& C_Z x(t)\end{aligned} \tag{7.2}$$

lemma 2. [74] Assume that $a(.) \in \mathcal{R}^{n_a}$, $b(.) \in \mathcal{R}^{n_b}$ and $\mathcal{N} \in \mathcal{R}^{n_a \times n_a}$ are defined on the interval Ω. Then for any matrices $\mathcal{X} \in \mathcal{R}^{n_a \times n_a}$, $\mathcal{Y} \in \mathcal{R}^{n_a \times n_b}$ and $\mathcal{Z} \in \mathcal{R}^{n_b \times n_b}$, the following holds

$$-2\int_{\Omega} a^T(\alpha)\mathcal{N}b(\alpha)d\alpha \leq \begin{bmatrix} a(\alpha) \\ b(\alpha) \end{bmatrix}^T \begin{bmatrix} X & Y - \mathcal{N} \\ Y^T - \mathcal{N}^T & Z \end{bmatrix} \begin{bmatrix} a(\alpha) \\ b(\alpha) \end{bmatrix} d\alpha$$

where $\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} > 0$

Proposition 7.1. For given scalar $\gamma > 0$,the closed-loop system 7.2 is robustly asymptotically stable with an $\mathcal{H}_\infty$ disturbance attenuation level less than γ, if there exist matrix $L = L^T > 0$,$R = R^T > 0$,$W = W^T > 0$ and controller matrix

K with appropriate dimensions such that the following LMI's hold

$$\begin{bmatrix} LA^T + AL + \tau M + N + N^T + W & B_uV - N & B_z & \tau LA^T & LC_z^T & & \\ * & -W & 0\tau V^T B_u^T & 0 & & & \\ * & * & -\gamma I & \tau B_z^T & 0 & & \\ * & * & * & -\tau L & 0 & & \\ * & * & * & * & * & * & -I \end{bmatrix} < 0 \tag{7.3}$$

Proof. Consider the lyapunov function as
$V(t) = x^T Px + \int_{-\tau}^{0} \int_{t+\beta}^{t} \dot{x}^T(s)Z\dot{x}(s)dsd\beta + \int_{t-\tau}^{t} x^T(s)Qx(s)ds$
The derivative $\dot{V}(t)$ is given as

Applying newton Leibniz theorem to equation. 7.2, we get

$$\begin{aligned} \dot{x} =& (A + B_uK)x(t) - B_uK \int_{t-\tau}^{t} \dot{x}(s) + B_z w(t) \\ z =& C_Z x(t) \end{aligned} \tag{7.4}$$

Now applying Lemma. 2 to ,we get The objective of the controller is to minimize effect of road disturbance $w(t)$ on desired output z. This is equivalent to $\|z\|_2 < \gamma\|w\|_2$ for all $w \in L_2[0, \infty]$. The objective function is $z^T z - \gamma^2 w^T w$. Then by S-procedure

$$\begin{aligned} \dot{V}(t) =& (Ax(t) + B_uK + Bw(t))^T Px + x^T(PAx(t) + B_uK + PBw(t)) \\ &+ x^T C_z^T C_z x - \gamma^2 w^T w \end{aligned}$$

$\square$

7.2 Simulation

A robust controller has been designed which gives required performance in case of delayed input. Maximum delay can be 50 μs. Simulation results show that this controller is robust to input delay.

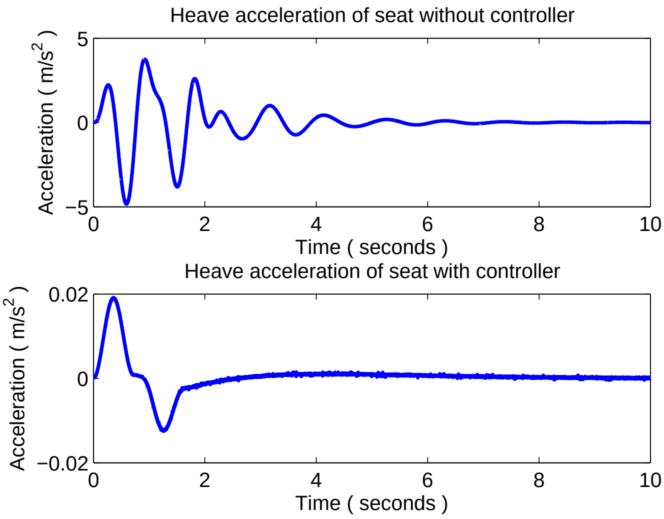

Figure 7.1: Heave acceleration of seat

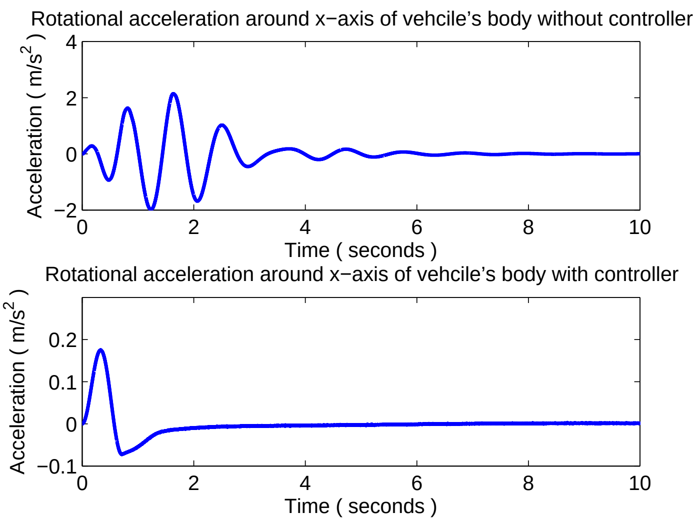

Figure 7.2: Rotational acceleration of car's body along x-axis

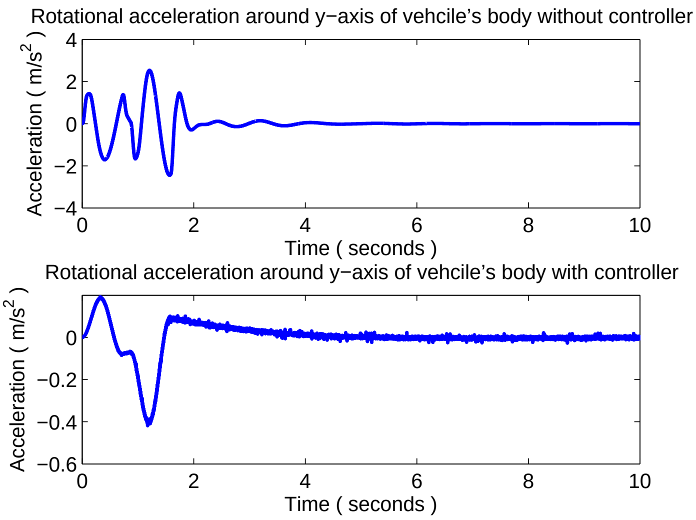

Figure 7.3: Rotational acceleration of car's body along y-axis

7.3 Summary

A delayed input robust controller has been designed. This controller is delay dependent and hence less conservative. This technique can be extended to output feedback control problem.

Chapter 8

Fault tolerant control

Fault tolerant controllers are designed to have desirable robust performance and stability properties even if the fault occurs. They are usually referred to be "reliable controllers". A lot of research has been done in this field of control [75–79]. Fault tolerant control can be classified into passive and active control.

Passive fault tolerant controllers are designed for a predefined set of faults and their performance is guaranteed only if the fault lies within the predefined set [80]. They do not require any fault detection scheme and are not re-configurable [81]. In [82–85], passive fault tolerant control schemes have been developed and discussed.

Active fault tolerant control requires fault detection scheme. Reconfigureable controllers are designed to preserve the performance. In [86], youla parameterization has been used to develop a decoupled active fault-tolerant controller. Robustness and performance controllers are designed separately. In [87], an active Fault Tolerant Control is developed for polytopic systems. Fault detection scheme include a polytopic unknown input observer. Gain scheduling technique is then employed for fault-free and faulty cases in order to preserve the system performances over a wide operating range.

8.1 Design of fault tolerant controller

In this thesis, a new fault tolerant controller is presented. Fault detection scheme involves an observer that estimates the states of system. Residual generator basically compares the states of the observer and the system and generates residual in case of fault. The residual is fed to the robust output feedback controller then the controller maintains the performance in case of fault.

The state space model of the active suspension full car with actuator fault matrix is

$$\begin{aligned}\dot{x} =& Ax(t) + B_u u(t) + B_z w(t) + B_f f(t)\\ y =& C_y x(t)\\ z =& C_Z x(t)\end{aligned} \tag{8.1}$$

here $A \in \mathcal{R}^{n\times n}$ is the state matrix, $B_u \in \mathcal{R}^{n\times q}$ is the input matrix, $B_z \in \mathcal{R}^{n\times z}$ is the disturbance input matrix, $B_f \in \mathcal{R}^{n\times f}$ is the fault input matrix, $C_y \in \mathcal{R}^{r\times n}$ is the measured output matrix and $C_z \in \mathcal{R}^{rz\times n}$ is the output to be minimized matrix.

The dynamic output feedback fault tolerant controller is given as

$$\begin{aligned}\dot{x}_c(t) &= A_c x_c(t) + B_c y(t) + B_e e(t)\\ u(t) &= C_c x_c(t)\end{aligned} \tag{8.2}$$

The system states observer is given as

$$\begin{aligned}\dot{\tilde{x}}(t) &= A\tilde{x}(t) + B_u u(t)\\ \tilde{y} &= C_y \tilde{x}\end{aligned} \tag{8.3}$$

The state difference equation is given as

$$\dot{e}(t) = (A - LC_y)e(t) + B_f u(t) + B_x w(t) \tag{8.4}$$

Residual is the difference between output of the system and observer i.e $r(t) = y(t) - \tilde{y}(t)$. The closed loop system is then

$$\begin{bmatrix} \dot{x} \\ \dot{x}_c \end{bmatrix} = \begin{bmatrix} A & B_u C_c \\ B_c C_y & A_c \end{bmatrix} \begin{bmatrix} x \\ x_c \end{bmatrix} + \begin{bmatrix} B_z \\ 0 \end{bmatrix} w(t) + \begin{bmatrix} B_f \\ 0 \end{bmatrix} F(t) + \begin{bmatrix} 0 \\ B_e \end{bmatrix} e(t) \tag{8.5}$$

Fig. 8.1 shows a new fault tolerant control scheme in which observer feds the

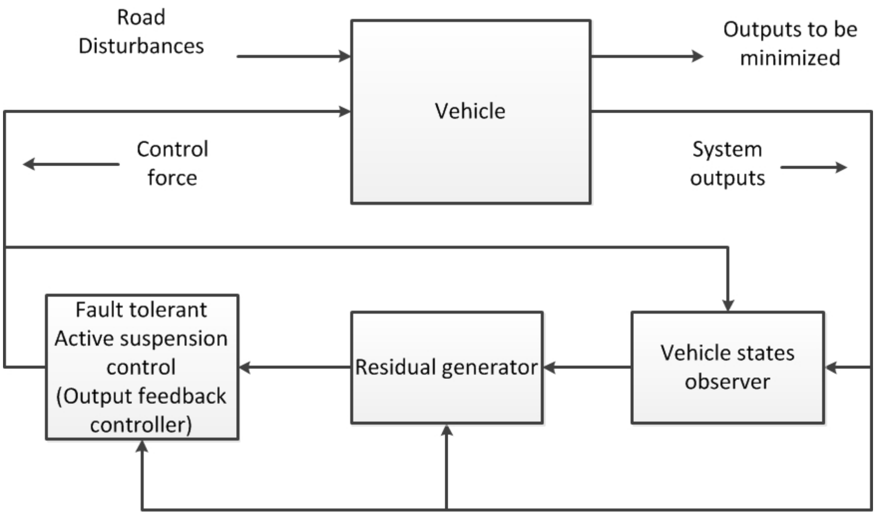

Figure 8.1: Fault tolerant control scheme

residual to the controller which in turn makes the performance robust to the fault.

Proposition 8.1. For given scalar $\gamma > 0$,the closed-loop system 8.5 is robustly asymptotically stable with an $\mathcal{H}_\infty$ disturbance attenuation level less than γ, if there exist matrix $P = P^T > 0$, observer gain matrix L and controller matrices A_c,B_c,B_e and C_c with appropriate dimensions such that the following LMI's hold

$$\begin{bmatrix} He\{PA_{cl}\} & PB_{cl(e)} & PB_{cl} & PB_{cl(f)} & C_{z1\_cl}^T & 0 \\ * & He\{R(A-LC)\} & RB_z & RB_f & 0 & C_y' \\ * & * & -\gamma^2 I & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0 \tag{8.6}$$

Proof. Consider the Lyapunov function as follows $V(t) = x_{cl}^T P x_{cl} + e^T P e$

The derivative of lyapunov function is $\dot{V}(t) = \dot{x}_{cl}^T P x_{cl} + x_{cl}^T P \dot{x}_{cl} + \dot{e}^T P e + e^T R \dot{e}$

$\dot{V}(t) = (A_{cl}x_{cl}(t) + B_{cl}w(t) + B_{cl(e)}e(t) + +B_{cl(f)}f(t))^T P x_{cl} + x_{cl}^T P(A_{cl}x_{cl}(t) + B_{cl}w(t) + B_{cl(e)}e(t) + +B_{cl(f)}f(t)) + e^T R((A - LC)e + B_f f(t) + B_z w) + ((A - LC)e + B_f f(t) + B_z w)^T Re$

Robust stability and objective minimization is guaranteed if $\dot{V}(t) < 0$.

The objective of this controller is to minimize effect of road disturbance $w(t)$ on desired output z and residual e. This is equivalent to $\|z\|_2 + \|e\|_2 < \gamma\|w\|_2$ for all $w \in L_2[0, \infty]$. The objective function is $z^T z + e^T e - \gamma^2 w^T w$. Then by S-procedure

$V(t) = (A_{cl}x_{cl}(t) + B_{cl}w(t) + B_{cl(e)}e(t) + +B_{cl(f)}f(t))^T P x_{cl} + x_{cl}^T P(A_{cl}x_{cl}(t) + B_{cl}w(t) + B_{cl(e)}e(t) + +B_{cl(f)}f(t)) + e^T R((A - LC)e + B_f f(t) + B_z w) + ((A - LC)e + B_f f(t) + B_z w)^T Re + x_{cl}^T C_{z\_cl}^T C_{z\_cl} x_{cl} - \gamma^2 w^T w$

It is obvious that $\dot{V}(t) < 0$ if

$$\begin{bmatrix} He\{PA_{cl}\} + Ha\{C_{z\_cl}^T\} & PB_{cl(e)} & PB_{cl} & PB_{cl(f)} \\ * & He\{R(A - LC)\} + Ha\{C_y'\} & RB_z & RB_f \\ * & * & -\gamma^2 I & 0 \\ * & * & * & 0 \end{bmatrix} < 0 \tag{8.7}$$

Applying schur's complement to inequality 8.7 and we get inequality 8.6 □

Theorem 8.1. For given scalars $\gamma > 0$, the closed-loop system 8.5 is robustly asymptotically stable with an $\mathcal{H}_\infty$ disturbance attenuation level less than γ, if the following linear matrix inequalities hold

$$\begin{bmatrix} \phi_{11} & \phi_{12} & 0 & B_z & B_f & \phi_{13} & 0 \\ * & \phi_{21} & EC_y & P_1B_z & P_1B_f & C_{zl}^T & 0 \\ * & * & He\{RA - \lambda C_y\} & Rb_z & Rb_f & 0 & C_y^T \\ * & * & * & -\gamma I & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} < 0 \tag{8.8}$$

$$\begin{bmatrix} P_{-1} & I \\ I & P_1 \end{bmatrix} > 0 \tag{8.9}$$

$$\phi_{11} = He\{AP_{-1} + B_u\phi\}$$

$$\phi_{12} = A + M^T$$

$$\phi_{13} = P_{-1}C_z^T$$

$$M = P_1AP_{-1} + P_1B_u\phi + \lambda C_yP_{-1} + P_2A_cP_{-2}^T$$

The controller matrices are then given as

$$\begin{aligned} A_c =& P_2^{-1}(M - P_1AP_{-1} - P_1B_u\phi - \lambda C_yP_{-1})P_{-2}^{-T} \\ B_c =& P_2^{-1}\lambda \\ B_e =& P_1^{-1}E \\ C_c =& \phi P_{-2}^{-T} \end{aligned}$$

The observer gain matrix is

$$L = R^{-1}\lambda$$

Proof. Let P= $\begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix}$ and $P^{-1} = \begin{bmatrix} P_{-1} & P_{-2} \\ * & P_{-3} \end{bmatrix}$

Consider $\pi = \begin{bmatrix} P_{-1} & I \\ P_{-2}^T & 0 \end{bmatrix}$ such that $\pi^T P = \begin{bmatrix} I & 0 \\ P_1 & P_2 \end{bmatrix}$

Now multiplying diag $\{\pi^T$, I , I , I $\}$and diag$\{\pi$, I , I , I $\}$ on left and right of 8.6 gives 8.8.

As $P > 0$, it means $\pi^T P \pi > 0$ which gives 8.9 □

8.2 Simulation

A fault tolerant controller has been designed. Simulation results show that the controller performance is not degraded by fault.

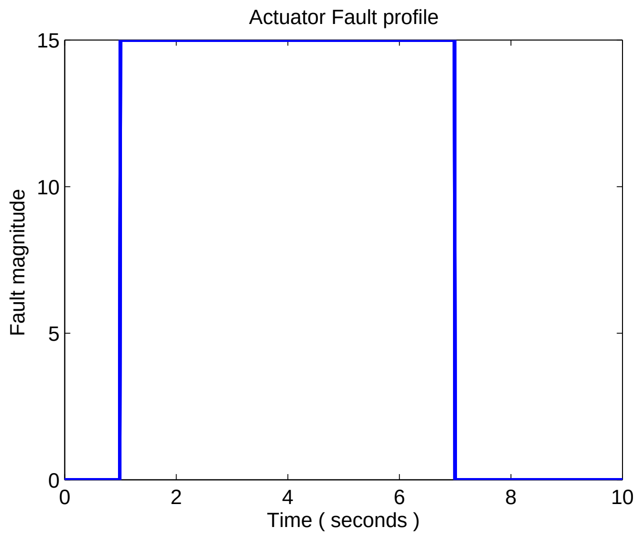

Figure 8.2: Actuator fault profile for vehicle suspension system

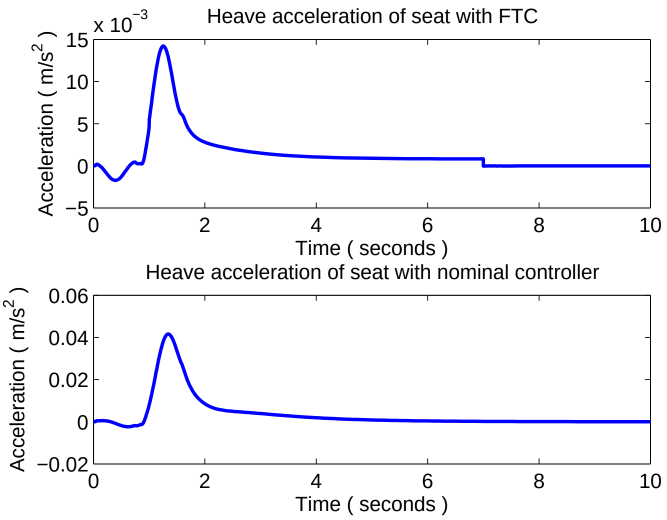

Figure 8.3: Heave acceleration of seat

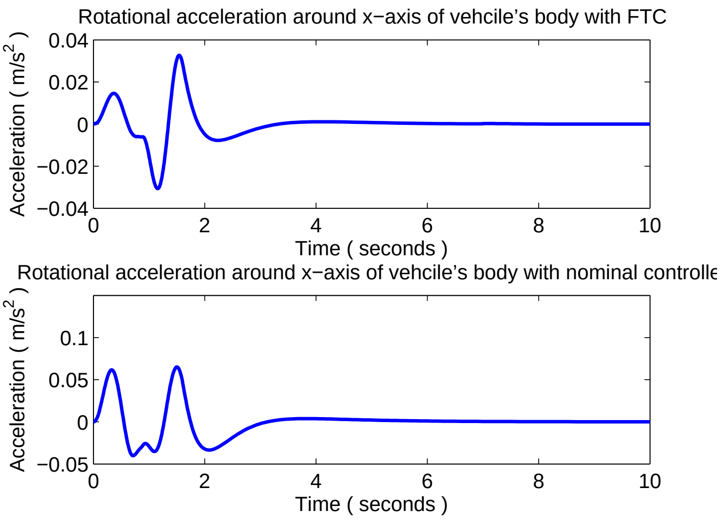

Figure 8.4: Rotational acceleration of car's body along x-axis

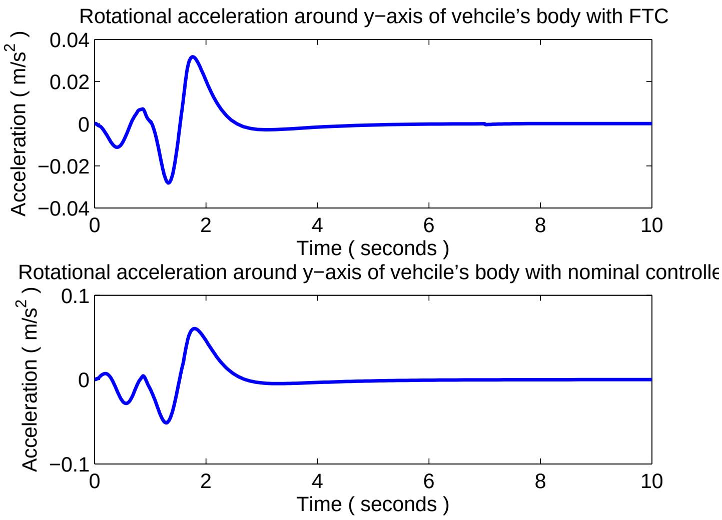

Figure 8.5: Rotational acceleration of car's body along y-axis

8.3 summary

A fault tolerant control scheme has been proposed. An observer has been designed to estimate the states of the system to detect the fault. The controller then takes both the outputs of the system and residual from the residual generator as inputs. This technique can easily be extended to polytopic systems by representing them in COG form.

Chapter 9

Conclusion and future recommendation

9.1 Conclusion

To maximize the ride quality for driver, driver's seat dynamics have been included in mathematical model to derive an 8 DOF active suspension full car model. Based on this model, many robust control schemes for effective disturbance attenuation on seat have been derived.

Mass of the vehicle is considered an uncertain quantity and a polytopic uncertain model of vehicle has been obtained. A new representation of polytopic systems i.e. COG representation has been obtained to reduce the complexity associated with the controller design.

A robust $\mathcal{H}_\infty$ controller has been designed which maintains the performance in presence of mass uncertainty.

Control input in case of active suspension system is delayed due to electro-mechanical nature of the actuator and microprocessors processing time. This delayed input can deteriorate the performance if controller is not robust against delay. In this thesis, a robust $\mathcal{H}_\infty$ controller has been designed that is robust against the input delay.

Suspension system health can deteriorate with the passage of time due to wear and tear of the suspension parts. Active suspension actuator should be robust to deal with such condition. In this thesis, a fault tolerant robust $\mathcal{H}_\infty$ controller for full vehicle model has been designed.

Robust $\mathcal{H}_\infty$ controllers for disturbance rejection, parametric uncertainty effect minimization, delayed input effect minimization and fault tolerance have been designed in this thesis for active suspension system. Ride quality has been the main focus of controller design.

9.2 Future recommendation

This thesis deals with the design of robust active suspension system. The main focus in thesis is on ride quality for the driver. There are still some open challenges like constrained control to address road handling and actuator force limitation. Output feedback controller for active suspension system with delayed input is still an open challenge. Fault tolerant control can be extended to polytopic systems by COG approach.Fault tolerant control can be extended to sensor faults.

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