

# Modelling the provision of 5G-Wireless Internet in London



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## Executive Summary

SPEED Telecom is a Mobile Telecommunications Provider that would like to expand its internet service provision and upgrade to a 5<sup>th</sup> Generation Wireless Network. This report provides an analysis of potential designs for the 5G network for the London region, that is divided into 20 zones.

The 5G network comprises of numerous aerials spread out across the 20 zones and 5 high power antennas located in 5 zones: 1, 9 11, 17 and 18.

Models were developed using linear programming to analyze the total data loss and maximum capacity utilization of the antennas for three different scenarios.

For the first scenario, data loss that occurs during transmission wirelessly between an antenna and a mobile user was to be minimized and was found to be 6.83 Gbps. The maximum capacity utilization in this case was 400 Gbps.

For the second scenario, the maximum antenna capacity utilization was to be minimized to ensure that all customers received comparable services, and a maximum allocation capacity of 215 Gbps was achieved from a maximum transmission capacity of 400 Gbps. The data loss was found to be 11.96 Gbps.

For the third scenario, a combination of both scenarios was used to minimize the maximum antenna capacity utilization along with the added restriction that the data loss was to be no greater than 25% of the first scenario. The maximum capacity utilization was found to be 215 Gbps and the data loss was 8.53 Gbps.

The model was then changed to account for the more accurate data loss calculations. Using the new calculations for data loss, the maximum capacity utilization and data loss for each scenario are given below:

Scenario	Data Loss (Gbps)	Maximum Antenna Capacity Utilization (Gbs)
1	6.35	400
2	15.41	215
3	7.94	215

Table: Data Loss and Antenna Capacity per Scenario

Thus, from our analysis that scenario 1 provides the lowest data loss for both cases while scenario 3 provides the best antenna utilization along with minimal data loss. Scenario 3 provides a good balance between both data loss minimization and antenna capacity minimization.

# Management Report

## Introduction

SPEED Telecom would like to expand its services into the 5G network and is currently in its early planning stages. This report addresses the design requirements for SPEED Telecoms expansion into 5G Wireless network in the London region. The general problem involves deciding on an optimal network design that can serve the internet traffic demand across all zones subject to a number of other technical design specifications.

## Objectives

The key decisions this report aims to address are the transmission capacity of each of SPEED's main antennas and the allocating portions of the demand in each zone across its five main antennas.

## 5G Wireless Network Design

The London region comprises of 20 zones, and SPEED has numerous aerials spread across these zones. Five high power antennas are located in zones 1, 9, 11, 17 and 18, which handle the mobile internet traffic directed to it from the aerials from other zones. Although SPEED has a high degree of control over the location and transmission direction, the transmission capacity is determined through analysis carried out through his report.

## Design Constraints

The network design conforms to the following constraints:

- The transmission of internet traffic is limited to maximum distance of 25 km from the location of an antenna, for a given antenna.
- The demand for internet traffic in each zone is met by the antennas.
- The transmission capacity of each antenna was designed to cope with its demand requirements.
- The transmission capacity of each antenna was designed to be set between a maximum of 400 Gbs and 100 Gbs to account for available technology and reliability.
- The total wireless traffic transmitted to each antenna was below 40% of the total transmission capacity.

## Models

When data is transmitted wirelessly between a mobile user and an antenna, there is a loss that occurs that depends on the amount of data transmitted and the distance that the data is transmitted. The models are designed to minimize the data loss.

Also, to ensure that all customers within a region receive comparable service, a low capacity utilization has to be maintained over each of the antennas. The design also accounts for this.

Two different cases were explored in the analysis.

1. Case 1: For the first case, the model was designed based on the fact that the data loss due to wireless transmission was equal to one thousandth (1/1000) of the transmitted data is lost per second.
2. Case 2: In the second case, the model was upgraded to account for a change in the data loss calculation, which provided a more accurate analysis of data loss.

For each of these models, the objectives revolved around 3 scenarios:

- Scenario 1: Minimize the total amount of data lost per second.
- Scenario 2: Minimize maximum Antenna utilization.
- Scenario 3: Minimize maximum Antenna utilization with the added restriction that the data loss of this scenario is no more than 25% higher than the optimal data loss determined in scenario 1.

The findings for each of the models are discussed in detail below.

### Case 1: Model with Inaccurate Data Loss calculation

The findings for this case are detailed below:

Scenario	Data Loss (Gbps)	Maximum Antenna Capacity Utilization (Gbs)
1	6.83	400
2	11.94	215
3	8.53	215

Table 1a: Data loss and antenna utilization per scenario

Antenna	Antenna utilization (Gbs) for Scenario 1	Antenna utilization (Gbs) for Scenario 2	Antenna utilization (Gbs) for Scenario 3
1	400	215	215
9	400	215	215
11	400	215	215
17	400	215	135
18	400	100	180

Table 1b: Antenna utilization per antenna per scenario

Zones	Demand in Gbps	Antenna allocated in Scenario 1	Antenna allocated in Scenario 2	Antenna allocated in Scenario 3
1	50	1	18	1,18
2	50	11	17	17
3	50	1	18	18
4	35	18	9,17	18
5	35	17	9	17
6	70	11	1	1
7	70	11	11	11
8	25	11	1	1
9	20	9	9	9
10	20	1	9	1

11	15	11	11	11
12	50	18	17	18
13	50	9	17	9
14	60	9	9	9
15	60	11	11	11
16	60	9	9	9
17	50	17	17	17
18	50	18	1	1,19
19	70	9	1	1,9
20	70	11	11	11

Table 1c: Demand and zone allocated per antenna

## Case 2: Model with accurate data loss calculation

The findings for this case are detailed below:

Scenario	Data Loss (Gbps)	Maximum Antenna Capacity Utilization (Gbs)
1	6.35	400
2	15.41	215
3	7.94	215

Table 2a: Data loss and antenna utilization per scenario

Antenna	Antenna utilization (Gbs) for Scenario 1	Antenna utilization (Gbs) for Scenario 2	Antenna utilization (Gbs) for Scenario 3
1	400	215	215
9	400	215	215
11	400	215	215
17	400	215	215
18	400	215	215

Table 2b: Antenna utilization per antenna per scenario

Zones	Demand in Gbps	Antenna allocated in Scenario 1	Antenna allocated in Scenario 2	Antenna allocated in Scenario 3
1	50	1	9	18
2	50	11	17	17
3	50	1	9	1,18
4	35	18	9	18
5	35	17	9	17
6	70	11	1,18	1
7	70	11	11	11
8	25	11	1	1
9	20	9	1	9
10	20	1	1	1

11	15	11	11	11
12	50	18	9	18
13	50	9	17	9
14	60	9	1	9
15	60	11	11	11
16	60	9	1	9
17	50	17	17	17
18	50	18	17	18
19	70	9	18,1	1
20	70	11	11	11

Table 2c: Demand and zone allocated per antenna

## Results

From our results, we can concur that the best case for minimum data loss is from Model 2 Scenario 1. As we know that model 2 has the more accurate data loss calculation, we can choose this model 1.

Focusing on model 2, given our three scenarios, the minimum data loss is obtained from our first scenario. But in this case the antennas are under maximum utilization at their highest of 400 Gbps. To avoid this, scenario 2 results in a lower antenna utilization but with the highest data loss. Thus, scenario 3 provides us with the best results, while achieving both objectives of minimum antenna utilization and minimizing data loss. It is also to be noted that, no significant dual values were observed in the scenario 3, which tells us that a change in its constraint values will not result in a significant change in the objective. We can also observe that the zone 3 demand was allocated between 2 antennas. This was allocated to 17.9 Gbps to antenna 1 and 7.1 Gbps to antenna 18, which meets the total zonal demand of 50 Gbps.

## Conclusion

The best design for the 5G network for London based on the given data is shown in the tables below. Using the following design for antenna utilization and demand allocation, we get the best case scenario for a reliable and fast 5G network that undergoes minimal data loss along with reliable connectivity.

Antenna	Antenna utilization (Gbs) for Scenario 3
1	215
9	215
11	215
17	215
18	215

Table 3a: Antenna capacity utilization design

Zones	Demand in Gbps	Antenna allocated in Scenario 3
1	50	18
2	50	17
3	50	1,18
4	35	18

5	35	17
6	70	1
7	70	11
8	25	1
9	20	9
10	20	1
11	15	11
12	50	18
13	50	9
14	60	9
15	60	11
16	60	9
17	50	17
18	50	18
19	70	1
20	70	11

Table 3b: Zonal demand allocation design



## Technical Appendices

### Model Entities

The model was made using different sets, variable and parameters as listed below:

1. Sets
  - a. ZONES: set of all the zones in London ranging from 1 to 20.
  - b. ANTENNAS: set of all the antennas listed. This is a subset of ZONES.
2. Parameters
  - a. Zonedemand: The demand in Gbps for each zone.
  - b. Zone\_dist: The average distance between zone in kilometers.
  - c. Max\_trm: The maximum transmission capacity per antenna.
  - d. Min\_trm: The minimum transmission capacity per antenna.
  - e. Max\_dist: The maximum distance that the data can be transmitted from an antenna.
  - f. Data\_loss: Data loss found in scenario 1 of the models
3. Variables
  - a. Trm\_cap: The transmission capacity of each antenna.
  - b. Demand\_alloc: The demand allocated to each antenna from each zone.
  - c. Dataloss: Data lost in the transmission from antenna to zone.
  - d. Max\_alloc: The maximum antenna capacity allocation.
  - e. Theta1, Theta2, Theta3: Variables used for piecewise approximation in model 2.
  - f. Delta1, Delta2: Binary variables used in piecewise approximation of model 2.

Zones	Demand in Gbps
1	50
2	50
3	50
4	35
5	35
6	70
7	70
8	25
9	20
10	20
11	15
12	50
13	50
14	60

15	60
16	60
17	50
18	50
19	70
20	70

Table 4a: Demand Data

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	2	25.76	11.64	7.81	26.65	21.7	30.58	23.06	21.15	8.1	30.14	8.68	20.33	20.94	30.35	18.15	28.25	5.44	21.87	34.14
2	25.76	2	37.16	20.7	19.07	16.09	18.04	29.96	36.88	31.92	16.59	20.55	33.63	39.07	13.47	40.66	17.74	23.32	45.92	11.02
3	11.64	37.16	2	17.21	34.44	32.67	41.61	30.42	18.87	6.41	41.31	17.74	20.23	16.69	41.81	10.12	36.57	14.3	11.25	45.78
4	7.81	20.7	17.21	2	18.84	21.89	30.04	27.61	18.6	11.37	29.25	2.93	16.47	19.76	28.48	20.04	20.48	2.93	25.21	30.53
5	26.65	19.07	34.44	18.84	2	31.65	36.42	42.68	24.58	28.02	35.08	17.98	20.78	27.87	32.35	32.84	2.73	21.44	38.95	29.52
6	21.7	16.09	32.67	21.89	31.65	2	8.94	14.27	40.48	29.75	8.73	22.43	38.31	41.34	10.24	39.79	31.33	22.93	43.49	16.99
7	30.58	18.04	41.61	30.04	36.42	8.94	2	19.41	48.62	38.59	1.49	30.46	46.17	49.75	5.26	48.59	35.49	31.42	52.41	12.94
8	23.06	29.96	30.42	27.61	42.68	14.27	19.41	2	44.21	30.39	20.21	28.46	43.17	43.88	23.24	39.79	43	27.08	41.6	30.79
9	21.15	36.88	18.87	18.6	24.58	40.48	48.62	44.21	2	14.76	47.78	18.15	3.8	3.64	46.73	11.11	27.29	17.76	16.73	47.65
10	8.1	31.92	6.41	11.37	28.02	29.75	38.59	30.39	14.76	2	38.09	11.76	15.16	13.66	38.07	10.04	30.18	8.61	14.2	41.22
11	30.14	16.59	41.31	29.25	35.08	8.73	1.49	20.21	47.78	38.09	2	29.63	45.25	49	3.81	48.06	34.11	30.73	52.01	11.58
12	8.68	20.55	17.74	2.93	17.98	22.43	30.46	28.46	18.15	11.76	29.63	2	15.88	19.47	28.74	20.11	19.66	3.58	25.41	30.56
13	20.33	33.63	20.23	16.47	20.78	38.31	46.17	43.17	3.8	15.16	45.25	15.88	2	7.26	43.99	13.92	23.49	16.18	19.87	44.5
14	20.94	39.07	16.69	19.76	27.87	41.34	49.75	43.88	3.64	13.66	49	19.47	7.26	2	48.21	7.87	30.55	18.41	13.19	49.65
15	30.35	13.47	41.81	28.48	32.35	10.24	5.26	23.24	46.73	38.07	3.81	28.74	43.99	48.21	2	47.89	31.19	30.27	52.19	7.86
16	18.15	40.66	10.12	20.04	32.84	39.79	48.59	39.79	11.11	10.04	48.06	20.11	13.92	7.87	47.89	2	35.37	17.75	6.13	50.54
17	28.25	17.74	36.57	20.48	2.73	31.33	35.49	43	27.29	30.18	34.11	19.66	23.49	30.55	31.19	35.37	2	23.2	41.46	27.8
18	5.44	23.32	14.3	2.93	21.44	22.93	31.42	27.08	17.76	8.61	30.73	3.58	16.18	18.41	30.27	17.75	23.2	2	22.65	32.83
19	21.87	45.92	11.25	25.21	38.95	43.49	52.41	41.6	16.73	14.2	52.01	25.41	19.87	13.19	52.19	6.13	41.46	22.65	2	55.42
20	34.14	11.02	45.78	30.53	29.52	16.99	12.94	30.79	47.65	41.22	11.58	30.56	44.5	49.65	7.86	50.54	27.8	32.83	55.42	2

Table 4b: Data of average distance between zones

## Algebraic Formation of Models

1. Sets
  - a. ZONES: Z
  - b. ANTENNAS: A belongs to Z
2. Parameters
  - a. Zonedemand: ZD
  - b. Zone\_dist: D
  - c. Max\_trm: 400 Gbps
  - d. Min\_trm: 100 Gbps
  - e. Max\_dist: 25 km
  - f. Data\_loss: Data loss found in scenario 1 of the models
3. Variables
  - a. Trm\_cap: TC
  - b. Demand\_alloc: DA
  - c. Dataloss: DL
  - d. Max\_alloc: MA
  - e. Theta1, Theta2, Theta3
  - f. Delta1, Delta2

## Case 1:

**Scenario 1: Minimize the total amount of data lost per second.**

objective : to minimise total transmission loss

**minimize** total\_transmission\_loss :  $0.001 * \sum$  where  $a \in \text{ANTENNAS}, z \in \text{ZONES}$   
demand\_alloc[a,z]\*zone\_dist[a,z];

**subject to**

maximum distance transmitted

min\_distance{ $a \in \text{ANTENNAS}, z \in \text{ZONES}$ : zone\_dist[a,z] > max\_dist : demand\_alloc[a,z] = 0;

total demand met by antennas

tot\_demand where  $z \in \text{ZONES}$  :  $\sum \{a \in \text{ANTENNAS} \text{ demand\_alloc}[a,z] \geq \text{zonedemand}[z]$ ;

transmission capacity enough to meet demand

capacity\_reqr where  $a \in \text{ANTENNAS}$  :  $\text{trm\_cap}[a] \geq \sum$  where  $z \in \text{ZONES}$  demand\_alloc[a,z];

total traffic transmitted to an Antenna below 40% of the total network transmission capacity

max\_antenna\_cap where  $a \in \text{ANTENNAS}$  :  $\sum$  where  $z \in \text{ZONES}$  demand\_alloc[a,z] <= 0.4\* $\sum$  where  $i \in \text{ANTENNAS}$  trm\_cap[i];

### Scenario 2: Minimize maximum Antenna utilization.

minimise  $z$

subject to  $z \geq t^1 x$

$z \geq t^2 x$

$\vdots$

$z \geq t^k x$

$Ax \leq b.$

objective : to minimise maximum Antenna utilization

**minimize** maximum\_transmission\_allocation : max\_alloc ;

**subject to**

maximum distance transmitted

min\_distance where  $a \in \text{ANTENNAS}, z \in \text{ZONES}$ : zone\_dist[a,z] > max\_dist : demand\_alloc[a,z] = 0;

total demand met by antennas

tot\_demand where  $z \in \text{ZONES}$  :  $\sum$  where  $a \in \text{ANTENNAS}$  demand\_alloc[a,z] >= zonedemand[z];

transmission capacity enough to meet demand

capacity\_reqr where  $a \in \text{ANTENNAS}$  :  $\text{trm\_cap}[a] \geq \sum$  where  $z \in \text{ZONES}$  demand\_alloc[a,z];

total traffic transmitted to an Antenna below 40% of the total network transmission capacity  
 $\max\_antenna\_cap$  where  $a \in \text{ANTENNAS}$  :  $\sum$  where  $z \in \text{ZONES}$   $\text{demand\_alloc}[a,z] \leq 0.4 * \sum$  where  $i \in \text{ANTENNAS}$   $\text{trm\_cap}[i]$ ;

maximum antenna allocation requirement  
 $\max\_antenna\_alloc$  where  $a \in \text{ANTENNAS}$  :  $\max\_alloc \geq \text{trm\_cap}[a]$ ;

data loss value  
 $\max\_data\_loss : 0.001 * \sum$  where  $a \in \text{ANTENNAS}, z \in \text{ZONES}$   
 $\text{demand\_alloc}[a,z] * \text{zone\_dist}[a,z] = \text{data\_loss}$ ;

**Scenario 3: Minimize maximum Antenna utilization with the added restriction that the data loss of this scenario is no more than 25% higher than the optimal data loss determined in scenario 1.**

objective : to minimise maximum Antenna utilization  
**minimize**  $\max\_transmission\_allocation$  :  $\max\_alloc$  ;

**subject to**

maximum distance transmitted  
 $\min\_distance$  where  $a \in \text{ANTENNAS}, z \in \text{ZONES}$  :  $\text{zone\_dist}[a,z] > \max\_dist$  :  $\text{demand\_alloc}[a,z] = 0$ ;

total demand met by antennas  
 $\text{tot\_demand}$  where  $z \in \text{ZONES}$  :  $\sum$  where  $a \in \text{ANTENNAS}$   $\text{demand\_alloc}[a,z] \geq \text{zonedemand}[z]$ ;

transmission capacity enough to meet demand  
 $\text{capacity\_reqr}$  where  $a \in \text{ANTENNAS}$  :  $\text{trm\_cap}[a] \geq \sum$  where  $z \in \text{ZONES}$   $\text{demand\_alloc}[a,z]$ ;

total traffic transmitted to an Antenna below 40% of the total network transmission capacity  
 $\max\_antenna\_cap$  where  $a \in \text{ANTENNAS}$  :  $\sum$  where  $z \in \text{ZONES}$   $\text{demand\_alloc}[a,z] \leq 0.4 * \sum$  where  $i \in \text{ANTENNAS}$   $\text{trm\_cap}[i]$ ;

maximum antenna allocation requirement  
 $\max\_antenna\_alloc$  where  $a \in \text{ANTENNAS}$  :  $\max\_alloc \geq \text{trm\_cap}[a]$ ;

data loss constraint  
 $\max\_data\_loss : 0.001 * \sum$  where  $a \in \text{ANTENNAS}, z \in \text{ZONES}$   
 $\text{demand\_alloc}[a,z] * \text{zone\_dist}[a,z] \leq \text{data\_loss}$ ;

**Case 2:**

$$\begin{aligned}
& \text{minimise } \sum_{i=1}^k \theta_i f(a_i) \\
& \text{subject to } \theta_1 \leq \delta_1 \\
& \quad \theta_i \leq \delta_{i-1} + \delta_i \text{ for } i = 2, 3, \dots, k-1 \\
& \quad \theta_k \leq \delta_{k-1} \\
& \quad x = \sum_{i=1}^k \theta_i a_i \\
& \quad \sum_{i=1}^k \theta_i = 1 \\
& \quad \sum_{i=1}^{k-1} \delta_i = 1 \\
& \quad \theta_i \geq 0 \text{ for } i = 1, \dots, k \\
& \quad \delta_i \in \{0, 1\} \text{ for } i = 1, \dots, k-1
\end{aligned}$$

**Scenario 1:**

objective : to minimise total transmission loss

**minimize** total\_transmission\_loss : **sum** where a **in** ANTENNAS, z **in** ZONES zone\_dist[a,z]\*(theta1[a,z]\*0 + theta2[a,z]\*0.05 + theta3[a,z]\*0.075);

**subject to**

maximum distance transmitted

min\_distance where a **in** ANTENNAS, z **in** ZONES: zone\_dist[a,z] > max\_dist : demand\_alloc[a,z] = 0;

total demand met by antennas

tot\_demand where z **in** ZONES : **sum** where a **in** ANTENNAS demand\_alloc[a,z] >= zonedemand[z];

transmission capacity enough to meet demand

capacity\_reqr where a **in** ANTENNAS : trm\_cap[a] >= **sum** where z **in** ZONES demand\_alloc[a,z];

total traffic transmitted to an Antenna below 40% of the total network transmission capacity

max\_antenna\_cap where a **in** ANTENNAS : **sum** where z **in** ZONES demand\_alloc[a,z] <= 0.4 \* **sum** where i **in** ANTENNAS trm\_cap[i];

sum of thetas is one

piecewise where a **in** ANTENNAS, z **in** ZONES : theta1[a,z]+theta2[a,z]+theta3[a,z]=1;

theta and demand constraint

demand\_theta where a **in** ANTENNAS, z **in** ZONES :

(theta1[a,z]\*0+theta2[a,z]\*50+theta3[a,z]\*100)=demand\_alloc[a,z];

binary constraints

sum\_of\_binary where a **in** ANTENNAS, z **in** ZONES :  $\delta_1[a,z] + \delta_2[a,z] = 1$ ;

binary1 where a **in** ANTENNAS, z **in** ZONES :  $\theta_1[a,z] \leq \delta_1[a,z]$ ;

binary2 where a **in** ANTENNAS, z **in** ZONES :  $\theta_2[a,z] \leq \delta_1[a,z] + \delta_2[a,z]$ ;

binary3 where a **in** ANTENNAS, z **in** ZONES :  $\theta_3[a,z] \leq \delta_2[a,z]$ ;

Scenario 2:

objective : to minimise maximum Antenna utilization

**minimize** maximum\_transmission\_allocation : max\_alloc ;

**subject to**

maximum distance transmitted

min\_distance where a **in** ANTENNAS, z **in** ZONES :  $\text{zone\_dist}[a,z] > \text{max\_dist} : \text{demand\_alloc}[a,z] = 0$ ;

total demand met by antennas

tot\_demand where z **in** ZONES : **sum** where a **in** ANTENNAS  $\text{demand\_alloc}[a,z] \geq \text{zonedemand}[z]$ ;

transmission capacity enough to meet demand

capacity\_reqr where a **in** ANTENNAS :  $\text{trm\_cap}[a] \geq \text{sum where z in ZONES demand\_alloc}[a,z]$ ;

total traffic transmitted to an Antenna below 40% of the total network transmission capacity

max\_antenna\_cap where a **in** ANTENNAS : **sum** where z **in** ZONES  $\text{demand\_alloc}[a,z] \leq 0.4 * \text{sum where i in ANTENNAS trm\_cap}[i]$ ;

maximum antenna allocation requirement

max\_antenna\_alloc where a **in** ANTENNAS :  $\text{max\_alloc} \geq \text{trm\_cap}[a]$ ;

total transmission loss

total\_transmission\_loss : **sum** where a **in** ANTENNAS, z **in** ZONES  $\text{zone\_dist}[a,z] * (\theta_1[a,z] * 0 + \theta_2[a,z] * 0.05 + \theta_3[a,z] * 0.075) = \text{dataloss}$ ;

sum of thetas is one

piecewise where a **in** ANTENNAS, z **in** ZONES :  $\theta_1[a,z] + \theta_2[a,z] + \theta_3[a,z] = 1$ ;

theta and demand constraint

demand\_theta where a **in** ANTENNAS, z **in** ZONES :

$(\theta_1[a,z] * 0 + \theta_2[a,z] * 50 + \theta_3[a,z] * 100) = \text{demand\_alloc}[a,z]$ ;

binary constraints

sum\_of\_binary where a **in** ANTENNAS, z **in** ZONES :  $\delta_1[a,z] + \delta_2[a,z] = 1$ ;

binary1 where a **in** ANTENNAS, z **in** ZONES :  $\theta_1[a,z] \leq \delta_1[a,z]$ ;

binary2 where a **in** ANTENNAS, z **in** ZONES :  $\theta_2[a,z] \leq \delta_1[a,z] + \delta_2[a,z]$ ;

binary3 where a **in** ANTENNAS, z **in** ZONES :  $\theta_3[a,z] \leq \delta_2[a,z]$ ;

Scenario 3:

objective : to minimise maximum Antenna utilization

**minimize** maximum\_transmission\_allocation : max\_alloc ;

**subject to**

maximum distance transmitted

min\_distance where a **in** ANTENNAS, z **in** ZONES: zone\_dist[a,z] > max\_dist : demand\_alloc[a,z] = 0;

total demand met by antennas

tot\_demand where z **in** ZONES : **sum** where a **in** ANTENNAS demand\_alloc[a,z] >= zonedemand[z];

transmission capacity enough to meet demand

capacity\_reqr where a **in** ANTENNAS : trm\_cap[a] >= **sum** where z **in** ZONES demand\_alloc[a,z];

total traffic transmitted to an Antenna below 40% of the total network transmission capacity

max\_antenna\_cap where a **in** ANTENNAS : **sum** where z **in** ZONES demand\_alloc[a,z] <= 0.4 \* **sum** where i **in** ANTENNAS trm\_cap[i];

maximum antenna allocation requirement

max\_antenna\_alloc where a **in** ANTENNAS : max\_alloc >= trm\_cap[a];

data loss constraint

max\_data\_loss : **sum** where a **in** ANTENNAS, z **in** ZONES zone\_dist[a,z] \* (theta1[a,z] \* 0 + theta2[a,z] \* 0.05 + theta3[a,z] \* 0.075) <= data\_loss;

sum of thetas is one

piecewise where a **in** ANTENNAS, z **in** ZONES : theta1[a,z] + theta2[a,z] + theta3[a,z] = 1;

theta and demand constraint

demand\_theta where a **in** ANTENNAS, z **in** ZONES :

(theta1[a,z] \* 0 + theta2[a,z] \* 50 + theta3[a,z] \* 100) = demand\_alloc[a,z];

binary constraints

sum\_of\_binary where a **in** ANTENNAS, z **in** ZONES : delta1[a,z] + delta2[a,z] = 1;

binary1 where a **in** ANTENNAS, z **in** ZONES : theta1[a,z] <= delta1[a,z];

binary2 where a **in** ANTENNAS, z **in** ZONES : theta2[a,z] <= delta1[a,z] + delta2[a,z];

binary3 where a **in** ANTENNAS, z **in** ZONES : theta3[a,z] <= delta2[a,z];

## AMPLDev Code

### Data file

```
set ZONES:= 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20;
```

```
set ANTENNAS:= 1 9 11 17 18;
```

```
param max_trm:=400;
```

```
param min_trm:=100;
```

```
param max_dist=25;
```

```
param zonedemand := 1 50
```

```
2 50
3 50
4 35
5 35
6 70
7 70
8 25
9 20
10 20
11 15
12 50
13 50
14 60
15 60
16 60
17 50
18 50
19 70
20 70;
```

```
param zone_dist :
      1      2      3      4      5
6      7      8      9      10     11
12     13     14     15     16     17
18     19     20 :=
1      2      25.76  11.64  7.81  26.65  21.7
30.58  23.06  21.15  8.1    30.14  8.68  20.33  20.94  30.35  18.15  28.25
5.44   21.87  34.14
2      25.76  2      37.16  20.7  19.07  16.09
18.04  29.96  36.88  31.92  16.59  20.55  33.63  39.07  13.47  40.66  17.74  23.32
45.92  11.02
3      11.64  37.16  2      17.21  34.44  32.67
41.61  30.42  18.87  6.41  41.31  17.74  20.23  16.69  41.81  10.12  36.57  14.3
11.25  45.78
4      7.81  20.7  17.21  2      18.84  21.89
30.04  27.61  18.6  11.37  29.25  2.93  16.47  19.76  28.48  20.04  20.48  2.93
25.21  30.53
5      26.65  19.07  34.44  18.84  2      31.65
36.42  42.68  24.58  28.02  35.08  17.98  20.78  27.87  32.35  32.84  2.73  21.44
38.95  29.52
```





### Case 1: Scenario 1

#Minimise the total amount of data lost per second

#sets

**set** ZONES; #set of all zones

**set** ANTENNAS **within** ZONES; #set of all antennas

#parameters

**param** zonedemand{ZONES};#demand per zone

**param** zone\_dist{ZONES,ZONES};#avg distance between zones

**param** max\_trm;#max transmission capacity per antenna

**param** min\_trm;#min transmission capacity per antenna

**param** max\_dist;# max distance that can be transmitted

#variables

**var** trm\_cap{ANTENNAS}<=max\_trm, >=min\_trm;#transmission capacity of each Antenna

**var** demand\_alloc{ANTENNAS,ZONES}>=0;#demand allocated to each zone per antenna

#objective : to minimise total transmission loss

**minimize** total\_transmission\_loss : 0.001\***sum**{a **in** ANTENNAS,z **in** ZONES} demand\_alloc[a,z]\*zone\_dist[a,z];

**subject to**

#maximum distance transmitted

min\_distance{a **in** ANTENNAS, z **in** ZONES: zone\_dist[a,z] > max\_dist}: demand\_alloc[a,z] = 0;

#total demand met by antennas

tot\_demand{z **in** ZONES}: **sum**{a **in** ANTENNAS}demand\_alloc[a,z]>= zonedemand[z];

#transmission capacity enough to meet demand

capacity\_reqr{a **in** ANTENNAS}: trm\_cap[a]>= **sum**{z **in** ZONES}demand\_alloc[a,z];

#total traffic transmitted to an Antenna below 40% of the total network transmission capacity

max\_antenna\_cap{a **in** ANTENNAS}: **sum**{z **in** ZONES}demand\_alloc[a,z]<= 0.4\***sum**{i **in** ANTENNAS}trm\_cap[i];

**option** solver **cp**lex;

### Scenario 2:

#Minimise maximum Antenna utilization

#sets

**set** ZONES; #set of all zones

**set** ANTENNAS **within** ZONES; #set of all antennas

#parameters

**param** zonedemand{ZONES};#demand per zone

**param** zone\_dist{ZONES,ZONES};#avg distance between zones

**param** max\_trm;#max transmission capacity per antenna

**param** min\_trm;#min transmission capacity per antenna

**param** max\_dist;# max distance that can be transmitted

```

#variables
var trm_cap{ANTENNAS}<=max_trm, >=min_trm;#transmission capacity of each Antenna
var demand_alloc{ANTENNAS,ZONES}>=0;#demand allocated to each zone per antenna
var max_alloc>=0;#maximum antenna allocation
var data_loss;

#objective : to minimise maximum Antenna utilization
minimize maximum_transmission_allocation : max_alloc ;

subject to
#maximum distance transmitted
min_distance{a in ANTENNAS, z in ZONES: zone_dist[a,z] > max_dist}: demand_alloc[a,z] = 0;

#total demand met by antennas
tot_demand{z in ZONES}: sum{a in ANTENNAS}demand_alloc[a,z]>= zonedemand[z];

#transmission capacity enough to meet demand
capacity_reqr{a in ANTENNAS}: trm_cap[a]>= sum{z in ZONES}demand_alloc[a,z];

#total traffic transmitted to an Antenna below 40% of the total network transmission capacity
max_antenna_cap{a in ANTENNAS}: sum{z in ZONES}demand_alloc[a,z]<= 0.4*sum{i in ANTENNAS}trm_cap[i];

#maximum antenna allocation requirement
max_antenna_alloc{a in ANTENNAS}:max_alloc >= trm_cap[a];

#data loss value
max_data_loss :0.001*sum{a in ANTENNAS,z in ZONES} demand_alloc[a,z]*zone_dist[a,z]=data_loss;

option solver cplex;

```

### Scenario 3:

#Minimise maximum Antenna utilization with data loss constraint

```

#sets
set ZONES; #set of all zones
set ANTENNAS within ZONES; #set of all antennas

#parameters
param zonedemand{ZONES};#demand per zone
param zone_dist{ZONES,ZONES};#avg distance between zones
param max_trm;#max transmission capacity per antenna
param min_trm;#min transmission capacity per antenna
param max_dist;# max distance that can be transmitted
param data_loss= 6.82655*1.25;

#variables
var trm_cap{ANTENNAS}<=max_trm, >=min_trm;#transmission capacity of each Antenna
var demand_alloc{ANTENNAS,ZONES}>=0;#demand allocated to each zone per antenna
var max_alloc>=0;#maximum antenna allocation

```

```

#objective : to minimise maximum Antenna utilization
minimize maximum_transmission_allocation : max_alloc ;

subject to
#maximum distance transmitted
min_distance{a in ANTENNAS, z in ZONES: zone_dist[a,z] > max_dist}: demand_alloc[a,z] = 0;

#total demand met by antennas
tot_demand{z in ZONES}: sum{a in ANTENNAS}demand_alloc[a,z]>= zonedemand[z];

#transmission capacity enough to meet demand
capacity_reqr{a in ANTENNAS}: trm_cap[a]>= sum{z in ZONES}demand_alloc[a,z];

#total traffic transmitted to an Antenna below 40% of the total network transmission capacity
max_antenna_cap{a in ANTENNAS}: sum{z in ZONES}demand_alloc[a,z]<= 0.4*sum{i in ANTENNAS}trm_cap[i];

#maximum antenna allocation requirement
max_antenna_alloc{a in ANTENNAS}:max_alloc >= trm_cap[a];

#data loss constraint
max_data_loss :0.001*sum{a in ANTENNAS,z in ZONES} demand_alloc[a,z]*zone_dist[a,z]<=data_loss;

option solver cplex;

```

## Case 2:

### Scenario 1

#Minimise the total amount of data lost per second

```

#sets
set ZONES; #set of all zones
set ANTENNAS within ZONES; #set of all antennas

#parameters
param zonedemand{ZONES};#demand per zone
param zone_dist{ZONES,ZONES};#avg distance between zones
param max_trm;#max transmission capacity per antenna
param min_trm;#min transmission capacity per antenna
param max_dist;# max distance that can be transmitted

#variables
var trm_cap{ANTENNAS}<=max_trm, >=min_trm;#transmission capacity of each Antenna
var demand_alloc{ANTENNAS,ZONES}>=0;#demand allocated to each zone per antenna
var theta1{ANTENNAS,ZONES}>=0;
var theta2{ANTENNAS,ZONES}>=0;
var theta3{ANTENNAS,ZONES}>=0;
var delta1{ANTENNAS,ZONES} binary;
var delta2{ANTENNAS,ZONES} binary;

```

```

#objective : to minimise total transmission loss
minimize total_transmission_loss : sum{a in ANTENNAS,z in ZONES} zone_dist[a,z]*(theta1[a,z]*0 +
theta2[a,z]*0.05 + theta3[a,z]*0.075);

subject to
#maximum distance transmitted
min_distance{a in ANTENNAS, z in ZONES: zone_dist[a,z] > max_dist}: demand_alloc[a,z] = 0;

#total demand met by antennas
tot_demand{z in ZONES}: sum{a in ANTENNAS}demand_alloc[a,z]>= zonedemand[z];

#transmission capacity enough to meet demand
capacity_reqr{a in ANTENNAS}: trm_cap[a]>= sum{z in ZONES}demand_alloc[a,z];

#total traffic transmitted to an Antenna below 40% of the total network transmission capacity
max_antenna_cap{a in ANTENNAS}: sum{z in ZONES}demand_alloc[a,z]<= 0.4*sum{i in ANTENNAS}trm_cap[i];

#sum of thetas is one
piecewise{a in ANTENNAS, z in ZONES}: theta1[a,z]+theta2[a,z]+theta3[a,z]=1;

#theta and demand constraint
demand_theta{a in ANTENNAS,z in ZONES}: (theta1[a,z]*0+theta2[a,z]*50+theta3[a,z]*100)=demand_alloc[a,z];

#binary constraints
sum_of_binary{a in ANTENNAS, z in ZONES}:delta1[a,z]+delta2[a,z]=1;
binary1{a in ANTENNAS, z in ZONES}: theta1[a,z]<=delta1[a,z];
binary2 {a in ANTENNAS, z in ZONES}: theta2[a,z]<= delta1[a,z]+delta2[a,z];
binary3 {a in ANTENNAS, z in ZONES}: theta3[a,z]<= delta2[a,z];

option solver cplex;

```

## Scenario 2

#Minimise maximum Antenna utilization

```

#sets
set ZONES; #set of all zones
set ANTENNAS within ZONES; #set of all antennas

#parameters
param zonedemand{ZONES};#demand per zone
param zone_dist{ZONES,ZONES};#avg distance between zones
param max_trm;#max transmission capacity per antenna
param min_trm;#min transmission capacity per antenna
param max_dist;# max distance that can be transmitted

#variables
var trm_cap{ANTENNAS}<=max_trm, >=min_trm;#transmission capacity of each Antenna
var demand_alloc{ANTENNAS,ZONES}>=0;#demand allocated to each zone per antenna
var max_alloc>=0;#maximum antenna allocation
var theta1{ANTENNAS,ZONES}>=0;
var theta2{ANTENNAS,ZONES}>=0;
var theta3{ANTENNAS,ZONES}>=0;
var delta1{ANTENNAS,ZONES} binary;

```

```

var delta2{ANTENNAS,ZONES} binary;
var dataloss;

#objective : to minimise maximum Antenna utilization
minimize maximum_transmission_allocation : max_alloc ;

subject to
#maximum distance transmitted
min_distance{a in ANTENNAS, z in ZONES: zone_dist[a,z] > max_dist}: demand_alloc[a,z] = 0;

#total demand met by antennas
tot_demand{z in ZONES}: sum{a in ANTENNAS}demand_alloc[a,z]>= zonedemand[z];

#transmission capacity enough to meet demand
capacity_reqr{a in ANTENNAS}: trm_cap[a]>= sum{z in ZONES}demand_alloc[a,z];

#total traffic transmitted to an Antenna below 40% of the total network transmission capacity
max_antenna_cap{a in ANTENNAS}: sum{z in ZONES}demand_alloc[a,z]<= 0.4*sum{i in ANTENNAS}trm_cap[i];

#maximum antenna allocation requirement
max_antenna_alloc{a in ANTENNAS}:max_alloc >= trm_cap[a];

#total transmission loss
total_transmission_loss : sum{a in ANTENNAS,z in ZONES} zone_dist[a,z]*(theta1[a,z]*0 + theta2[a,z]*0.05 +
theta3[a,z]*0.075)=dataloss;

#sum of thetas is one
piecewise{a in ANTENNAS, z in ZONES}: theta1[a,z]+theta2[a,z]+theta3[a,z]=1;

#theta and demand constraint
demand_theta{a in ANTENNAS,z in ZONES}: (theta1[a,z]*0+theta2[a,z]*50+theta3[a,z]*100)=demand_alloc[a,z];

#binary constraints
sum_of_binary{a in ANTENNAS, z in ZONES}:delta1[a,z]+delta2[a,z]=1;
binary1{a in ANTENNAS, z in ZONES}: theta1[a,z]<=delta1[a,z];
binary2 {a in ANTENNAS, z in ZONES}: theta2[a,z]<= delta1[a,z]+delta2[a,z];
binary3 {a in ANTENNAS, z in ZONES}: theta3[a,z]<= delta2[a,z];

option solver cplex;

```

### Scenario 3

#Minimise maximum Antenna utilization with data loss constraint

#sets

set ZONES; #set of all zones

set ANTENNAS within ZONES; #set of all antennas

#parameters

param zonedemand{ZONES};#demand per zone

param zone\_dist{ZONES,ZONES};#avg distance between zones

param max\_trm;#max transmission capacity per antenna

```

param min_trm;#min transmission capacity per antenna
param max_dist;# max distance that can be transmitted
param data_loss= 6.34845*1.25;

#variables
var trm_cap{ANTENNAS}<=max_trm, >=min_trm;#transmission capacity of each Antenna
var demand_alloc{ANTENNAS,ZONES}>=0;#demand allocated to each zone per antenna
var max_alloc>=0;#maximum antenna allocation
var theta1{ANTENNAS,ZONES}>=0;
var theta2{ANTENNAS,ZONES}>=0;
var theta3{ANTENNAS,ZONES}>=0;
var delta1{ANTENNAS,ZONES} binary;
var delta2{ANTENNAS,ZONES} binary;

#objective : to minimise maximum Antenna utilization
minimize maximum_transmission_allocation : max_alloc ;

subject to
#maximum distance transmitted
min_distance{a in ANTENNAS, z in ZONES: zone_dist[a,z] > max_dist}: demand_alloc[a,z] = 0;

#total demand met by antennas
tot_demand{z in ZONES}: sum{a in ANTENNAS}demand_alloc[a,z]>= zonedemand[z];

#transmission capacity enough to meet demand
capacity_reqr{a in ANTENNAS}: trm_cap[a]>= sum{z in ZONES}demand_alloc[a,z];

#total traffic transmitted to an Antenna below 40% of the total network transmission capacity
max_antenna_cap{a in ANTENNAS}: sum{z in ZONES}demand_alloc[a,z]<= 0.4*sum{i in ANTENNAS}trm_cap[i];

#maximum antenna allocation requirement
max_antenna_alloc{a in ANTENNAS}:max_alloc >= trm_cap[a];

#data loss constraint
max_data_loss :sum{a in ANTENNAS,z in ZONES} zone_dist[a,z]*(theta1[a,z]*0 + theta2[a,z]*0.05 +
theta3[a,z]*0.075)<=data_loss;

#sum of thetas is one
piecewise{a in ANTENNAS, z in ZONES}: theta1[a,z]+theta2[a,z]+theta3[a,z]=1;

#theta and demand constraint
demand_theta{a in ANTENNAS,z in ZONES}: (theta1[a,z]*0+theta2[a,z]*50+theta3[a,z]*100)=demand_alloc[a,z];

#binary constraints
sum_of_binary{a in ANTENNAS, z in ZONES}:delta1[a,z]+delta2[a,z]=1;
binary1 {a in ANTENNAS, z in ZONES}: theta1[a,z]<=delta1[a,z];
binary2 {a in ANTENNAS, z in ZONES}: theta2[a,z]<= delta1[a,z]+delta2[a,z];
binary3 {a in ANTENNAS, z in ZONES}: theta3[a,z]<= delta2[a,z];

option solver cplex;

```

### Collaboration Statement

I have collaborated with candidate 86466 on building my model for case 2: scenario 1 and for scenario 2 when I was implementing the piecewise approximation and the min-max function.