Your name: ______________________________

EECS 215
Midterm Exam #2
November 16, 2016

This exam consists of 6 problems with points as indicated to total 60 points.

Read through the entire exam before beginning.
Show all work (on the pages provided in this booklet) to earn partial credit.
Briefly explain major steps, include units, and write your final answers in the areas provided.
Do not unstaple the pages.

No credit will be given if no work is shown.

- **Exam policies**
  - No food allowed during exam.
  - No books allowed (closed book exam).
  - One, 8.5x11 inch notes page (ONE SIDED) allowed
  - Calculators allowed (But **you may not use the following functions: graphs, integrals, derivatives**).
  - Full credit will not be awarded if you do not show your work.
  - No communication of any kind is allowed. No use of cell phones, computers, or any devices besides calculators. Violation of this will be treated as an honor code violation.
  - No credit will be given for this exam without a signed honor pledge.

In which section are you enrolled? ☐**EECS 215-001 (Finelli)** ☐**EECS 215-002 (Zhang)**

Write and sign the honor pledge:

Signed: Answer Key

Do not write in this space

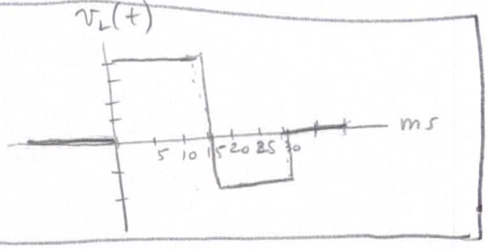
Problem 1: [ ]/10	Problem 4: [ ]/10
Problem 2: [ ]/10	Problem 5: [ ]/10
Problem 3: [ ]/10	Problem 6: [ ]/10
	Total score [ ]/60

1. (10 points) A 6H inductor has a voltage and current defined according to the passive sign convention. The initial current on the inductor is $\boxed{i_L(0) = 5mA}$. The voltage across the inductor is defined as follows:

$$v_L(t) = \begin{cases} 0, & t < 0 \\ 4V, & 0 \le t < 15\ ms \\ -2V, & 15 \le t < 30\ ms \\ 0, & t \ge 30\ ms \end{cases}$$

a. (2 points) Plot the voltage versus time for $-5 < t < 35\ ms$.
b. (5 points) Find a piecewise expression for the current through the inductor $i_L(t)$.
c. (3 points) Plot the current versus time for $-5 < t < 35\ ms$.

a)



b) $i_L(t) = \frac{1}{6}\int_{t_0}^{t} v_L(\tau)\,d\tau + i_L(t_0)$

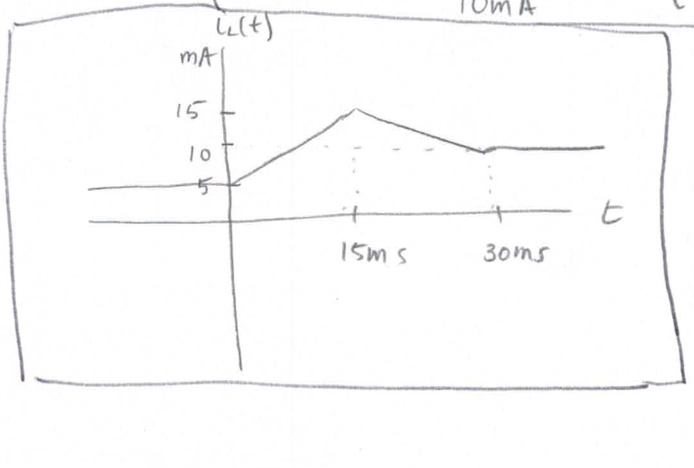
$0<t<15ms$: $i_L(t) = \frac{1}{6}\int_0^t 4\,d\tau + 5mA = \frac{1}{6}4t + 5 = \left(\frac{2}{3}t + .005\right)A$

note: $i_L(15ms) = \frac{2}{3}(.015) + .005 = 15mA$

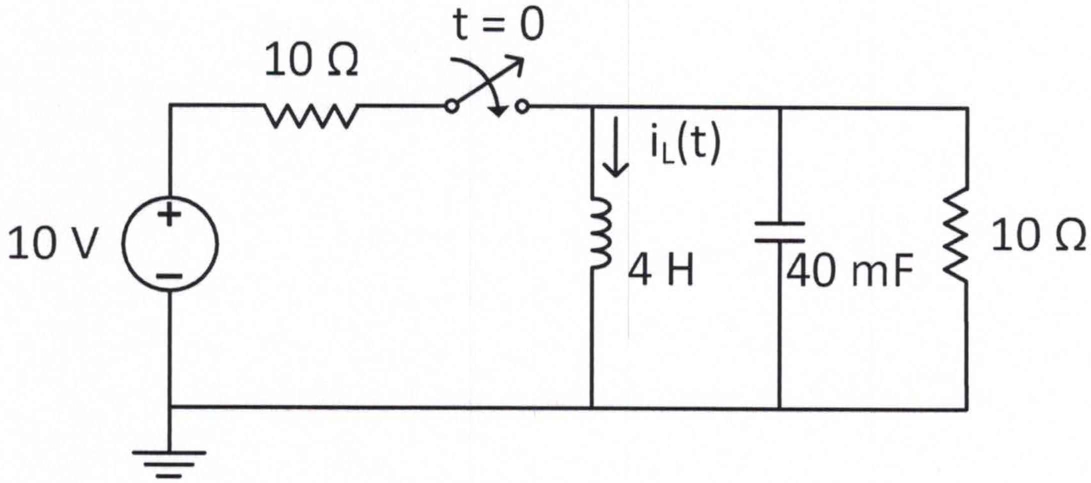
$15ms<t<30ms$: $i_L(t) = \frac{1}{6}\int_{15ms}^t -2\,d\tau + 15mA = -\frac{1}{3}(t-.015) + .015 = -\frac{t}{3} + .020$

$i_L(30ms) = 10mA$

$$i_L(t) = \begin{cases} 5mA & t<0 \\ \frac{2}{3}t + .005 & 0 \le t < 15ms \\ -\frac{t}{3} + .020 & 15ms \le t \le 30ms \\ 10mA & t \ge 30ms \end{cases}$$



2. (10 points) The switch in the circuit below has been open for a long time and the circuit has reached steady state. Then, the switch is closed at $t = 0$. Solve completely for the current through the inductor, $i_L(t)$ for $t > 0$.

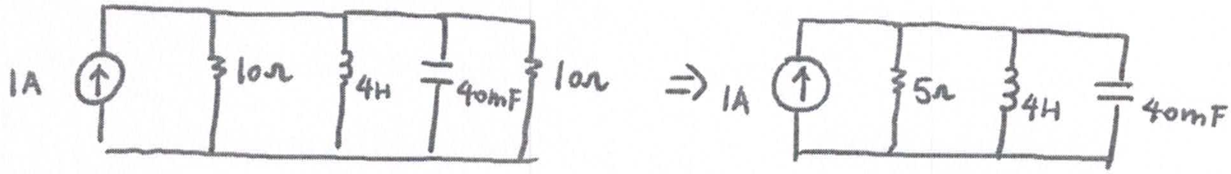


$i_L(0^-) = 0$ $\quad$ $i_L(0) = i_L(0^-) = 0$ $\quad$ $i_L(\infty) = 1A$

$V_c(0^-) = 0$ $\quad$ $V_c(0) = V_c(0^-) = 0$

$V_L(0) = V_C(0) = 0$ $\quad$ $i_L'(0) = \dfrac{V_L(0)}{L} = 0$

$t > 0$ Equivalent circuit :



Parallel RLC

$$\alpha = \frac{1}{2RC} = \frac{1}{2\times5\times40\times10^{-3}} = 2.5 \quad (1/s)$$

$$W_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4\times40\times10^{-3}}} = 2.5 \quad (\text{rad/s})$$

$\alpha = W_0$ $\quad\therefore$ Critically damped case

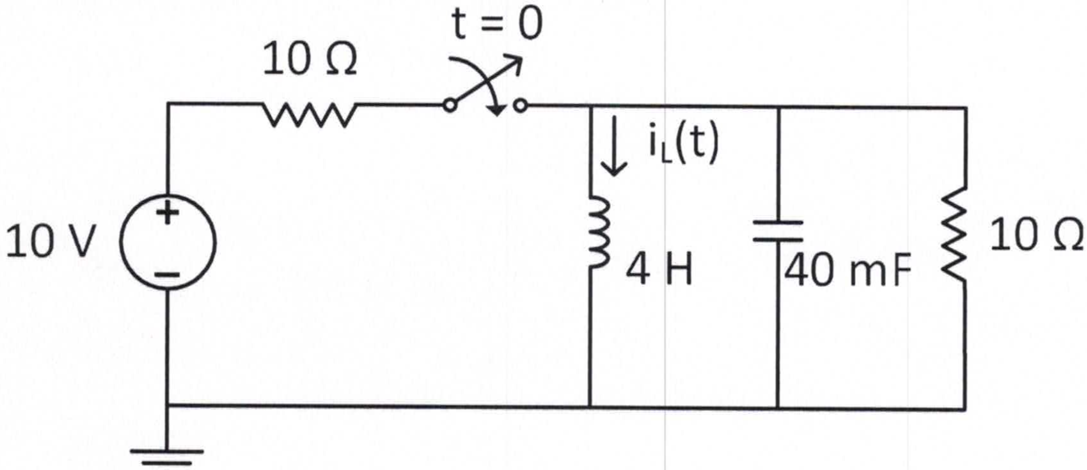
$$i_L(t) = B_1 e^{-2.5t} + B_2 t e^{-2.5t} + 1$$

$$\boxed{i_L(t) = -e^{-2.5t} - 2.5te^{-2.5t} + 1 \quad t \geq 0}$$

$$B_1 = i_L(0) - i_L(\infty) = 0 - 1 = -1$$

$$B_2 = \frac{1}{L} V_L(0) + \alpha\,(i_L(0) - i_L(\infty)) = 0 + 2.5\,(0-1) = -2.5$$

3. (10 points) The switch in the circuit below has been closed for a long time, and the circuit has reached steady state. Then, the switch is opened at $t = 1$ second. Solve completely for the current through the inductor, $i_L(t)$ for $t \geq 1$.



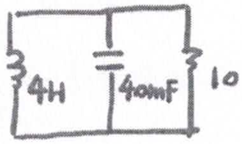
$i_L(1^-) = 1A$ $\quad$ $i_L(1) = i_L(1^-) = 1A$ $\quad$ $i_L(\infty) = 0$

$V_C(1^-) = 0$ $\quad$ $V_C(1) = V_C(1^-) = 0$

$V_L(1) = V_C(1) = 0$ $\quad$ $i_L'(1) = \frac{V_L(1)}{L} = 0$

$t>1$

Equivalent Circuit



parallel RLC circuit

$$\alpha = \frac{1}{2RC} = \frac{1}{2\times 10\times 40\times 10^{-3}} = 1.25 \ (\tfrac{1}{s})$$

$$w_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4\times 40\times 10^{-3}}} = 2.5 \ (\text{rad/s})$$

$\alpha < w_0$, under damped case

$$w_d = \sqrt{w_0^2 - \alpha^2} = \sqrt{2.5^2 - 1.25^2} = 2.165 \ (\text{rad/s})$$

$$i_L(t) = D_1 e^{-1.25(t-1)} \cos 2.165(t-1) + D_2 e^{-1.25(t-1)} \sin 2.165(t-1)$$

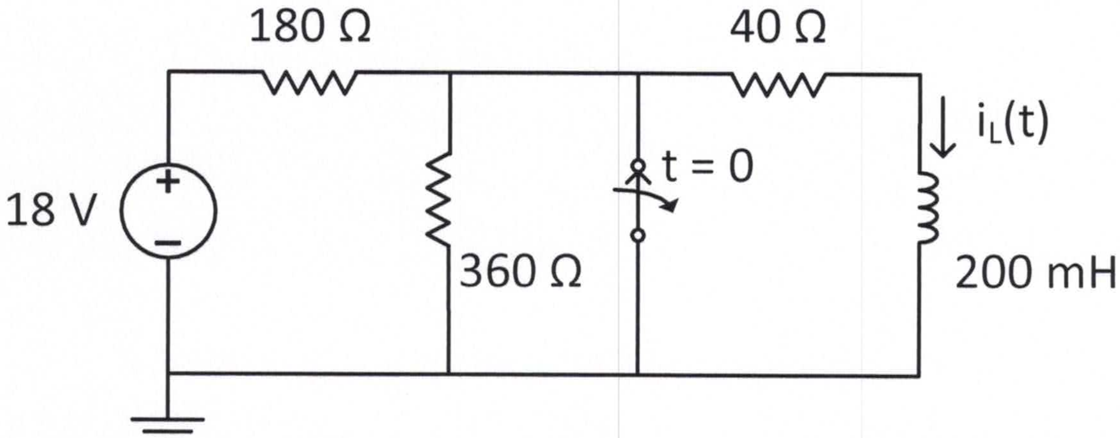
$D_1 = i_L(1) - i_L(\infty)$
$= 1 - 0 = 1$

$$D_2 = \frac{\frac{1}{L} V_L(1) + \alpha\,(i_L(1) - i_L(\infty))}{w_d} = \frac{0 + 1.25\,(1-0)}{2.165} = 0.577$$

$$\boxed{i_L(t) = e^{-1.25(t-1)} \cos 2.165(t-1) + 0.58 e^{-1.25(t-1)} \sin 2.165(t-1)}$$

$t \geq 1$

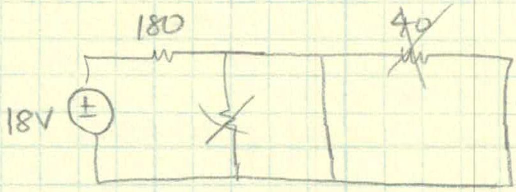
4. (10 points) The switch in the following circuit has been closed for a long time. At $t = 0$ the switch opens.



a. (2 points) Find the initial current through the inductor, $i_L(0)$.

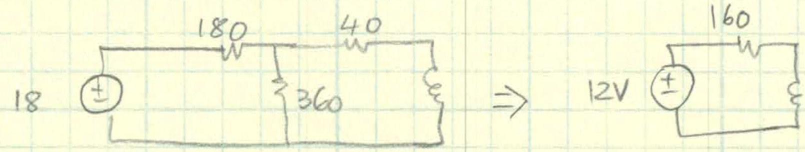
b. (8 points) Find an expression for the current through the inductor, $i_L(t), t \geq 0$.

(4) $t=0^-$



$\boxed{i_L(0^-) = \boxed{i_L(0^+) = 0}}$

$t>0$



$\downarrow i_L(t)$

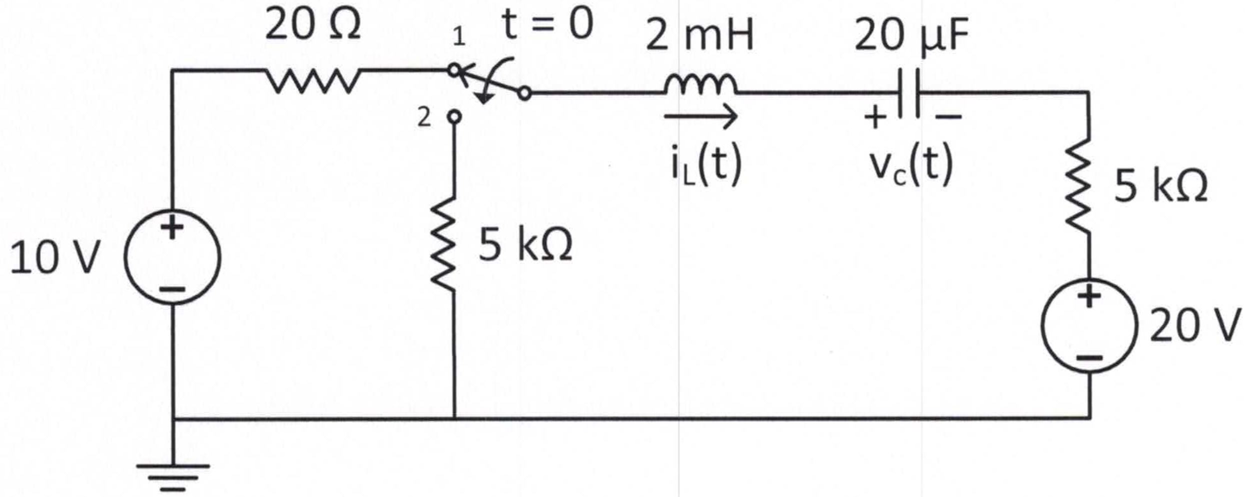
$i_L(\infty) = \frac{12}{160} = 75\,mA$

$\tau = L/R = \frac{200\times10^{-3}}{160} = 1.25\times10^{-3}$

$i_L(t) = i_L(\infty) + (i_L(0) - i_L(\infty))e^{-t/\tau}$

$\boxed{i_L(t) = 75(1 - e^{-800t})\ mA}$

5. (10 points) The switch in the circuit below has been in position 1 for a long time, and it moves to position 2 at $t = 0$.



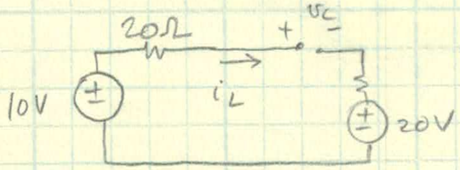
Determine the initial and final conditions as follows:

a. (3 points) $i_L(0^+), v_C(0^+)$

b. (4 points) $i_L'(0^+), v_C'(0^+)$

c. (3 points) $i_L(\infty), v_C(\infty)$

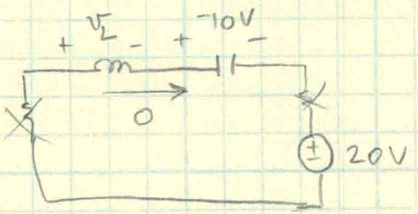
5 $t=0^-$



$i_L(0^-) = \boxed{i_L(0^+) = 0A}$

$v_c(0^-) = \boxed{v_c(0^+) = -10V}$

$t=0^+$

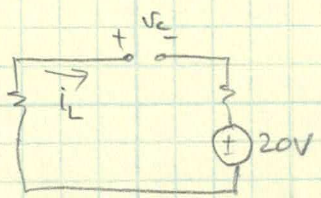


$i_c(0^+) = 0 \Rightarrow \boxed{v_c'(0^+) = 0 V/s}$

$v_L(0^+) = -10 \Rightarrow i_L'(0^+) = \frac{-10}{2\times10^{-3}}$

$\boxed{= -5000 A/s}$

$t \to \infty$



$\boxed{\begin{array}{l} i_L(\infty) = 0\ A \\ v_c(\infty) = -20V. \end{array}}$

6. (10 points) Given the function $v_1(t) = -5\sin(20t + 30°)$ V

   a. (2 points) Express the function in the standard cosine form (e.g., as a cosine function with a positive amplitude and an angle between $-180°$ and $180°$).

   b. (6 points) Determine the amplitude, frequency (f), period (T), and phase angle of the function.

   c. (2 points) For a second function $v_2(t) = 2\cos(20t - 30°)$, does $v_1(t)$ lead or lag $v_2(t)$ and by what angle?

a. $V_1(t) = -5(-\cos(20t + 30° + 90°))$ V

$\boxed{V_1(t) = 5\cos(20t + 120°) \quad V}$

b. amplitude : $\boxed{5\ V}$

frequency : $\frac{W}{2\pi} = \frac{20}{2\pi} = \boxed{\frac{10}{\pi} \quad Hz}$

period : $\frac{2\pi}{W} = \boxed{\frac{\pi}{10} \quad s}$

phase : $\boxed{120°}$

c. $\alpha_1 - \alpha_2 = 120° - (-30°) = 150°$

$\therefore$ $\boxed{v_1 \text{ leads } v_2 \text{ by } 150°.}$