Your name: Solution Key

EECS 215.
Midterm Exam #1
February 9, 2016

This text consists of 8 problems with points as indicated to total 100 points.

Read through the entire exam before beginning.
Show all work (on the pages provided in this booklet) to earn partial credit.
Do not unstaple the pages.
Briefly explain major steps, include units, and write your final answers in the areas provided.

No credit will be given if no work is shown.

- Exam policies
  - No food allowed during exam
  - No books allowed (closed book exam)
  - One, 8.5x11 inch notes page (ONE SIDED) allowed
  - Calculators allowed (But scientific calculator only for future exams)
  - College of Engineering Honor Code is followed: No communication of any kind is allowed. No use of cell phones, computers, or any devices besides scientific calculators. Violation of this will be treated as an honor code violation. No credit will be given for this exam without a signed honor pledge.

Write and sign the honor pledge:

"I have neither given nor received unauthorized aid on this examination, nor have I concealed any violations of the Honor Code."

Signed: ______________________________

Do not write in this space

Problem 1: [ ]/10	Problem 5: [ ]/15
Problem 2: [ ]/10	Problem 6: [ ]/15
Problem 3: [ ]/10	Problem 7: [ ]/15
Problem 4: [ ]/10	Problem 8: [ ]/15

Total score [ ]/100

1. (10 points). Given $i(t) = \begin{cases} 12\, e^{-2t}\, A, & t \geq 0 \\ 0, & t < 0 \end{cases}$, find the **total charge delivered**. Additional work space on next page.

① $i(t) = 12e^{-2t}\ A,\ t \geq 0$. Find total charge.

$$q(t) = \int_{-\infty}^{\infty} i(t)\,dt = \int_0^{\infty} 12e^{-2t}\,dt = \frac{12}{-2} e^{-2t}\Big|_0^{\infty}$$

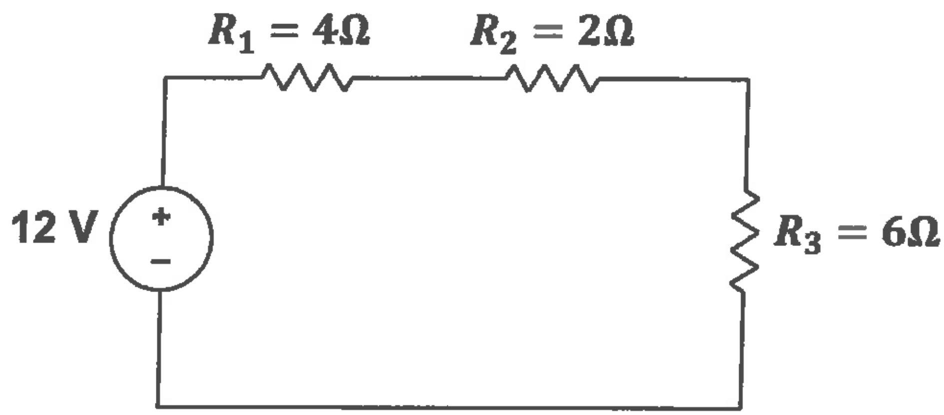
$$= -6(e^{-\infty} - e^{0}) = \boxed{6C}$$

Write your answer here:

Total charge delivered = 6C

Problem 1 score: [ ]/10

2. (10 points). Find the **power absorbed by each element** in the circuit below. Is power conserved in this circuit? Additional work space on next page.



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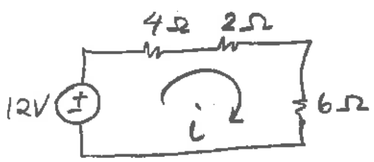


$i = \frac{12V}{12\Omega} = 1A$

$P_{abs-4\Omega} = i^2R = 1A(4\Omega) = \boxed{4W}$

$P_{abs-2\Omega} = \boxed{2W}$ $\quad P_{abs-6\Omega} = \boxed{6W}$

$P_{abs-12V} = -iV = (-1A)(12V) = \boxed{-12W}$

passive sign convention NOT followed

since $\sum P_{abs} = 0$, this confirms conservation of power

Write your answer here:

Power absorbed by 12V source = −12W

Power absorbed by R_1= 4W

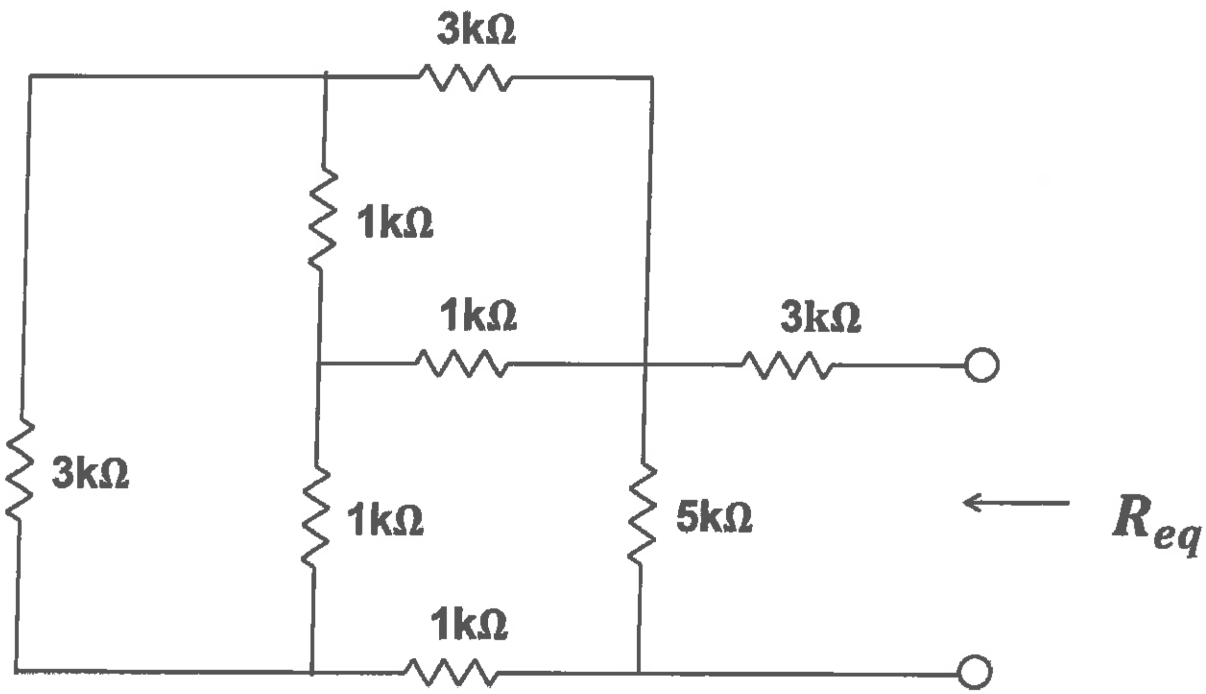
Power absorbed by R_2 = 2W

Power absorbed by R_3 = 6W

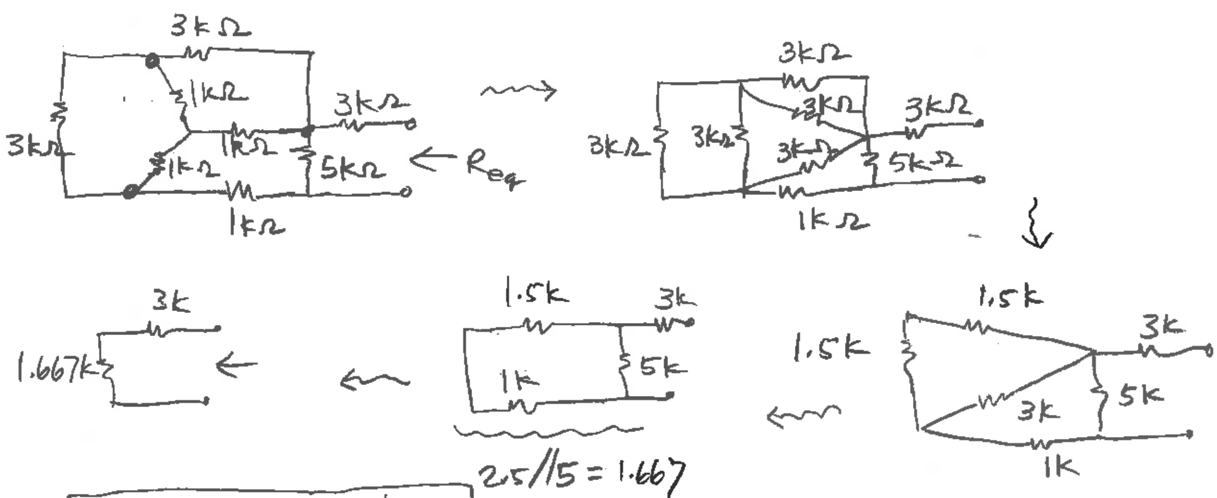
Is power conserved? yes − $\sum P = 0$

Problem 2 score: [ ]/10

3. (10 points). Find the **equivalent resistance** R_{eq}. Additional work space on next page.



③



$\boxed{R_{eq} = 4.667k\Omega = 14/3\,k\Omega}$

Write your answer here:

R_{eq} = 4.667kΩ = 14/3 kΩ

Problem 3 score: [ ]/10

4. (10 points). Use Cramer's rule to **solve the following system of equations** (show all work for finding Δ, Δ_1,and Δ_2; you will lose 5 points if you use a different approach). Check your solution by plugging your values for x and y into the original equations. Additional work space on next page.

$$2x + 3y = 5 \quad \text{and} \quad x - 2y = 4$$

4

$$\begin{aligned} 2x+3y&=5 \\ x-2y&=4 \end{aligned} \Rightarrow \begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

$$\Delta = \det \begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix} = -4-3=-7$$

$$\Delta_1 = \det \begin{bmatrix} 5 & 3 \\ 4 & -2 \end{bmatrix} = -10-12=-22$$

$$\Delta_2 = \det \begin{bmatrix} 2 & 5 \\ 1 & 4 \end{bmatrix} = 8-5=3$$

$$x = \frac{\Delta_1}{\Delta} = \boxed{\frac{22}{7}}$$

$$y = \boxed{\frac{-3}{7}}$$

check $2\left(\frac{22}{7}\right)+3\left(-\frac{3}{7}\right) = \frac{44-9}{7} = \frac{35}{7} = 5$ ✓

$\frac{22}{7} - 2\left(\frac{-3}{7}\right) = \frac{22+6}{7} = \frac{28}{7} = 4$ ✓

Write your answer here:

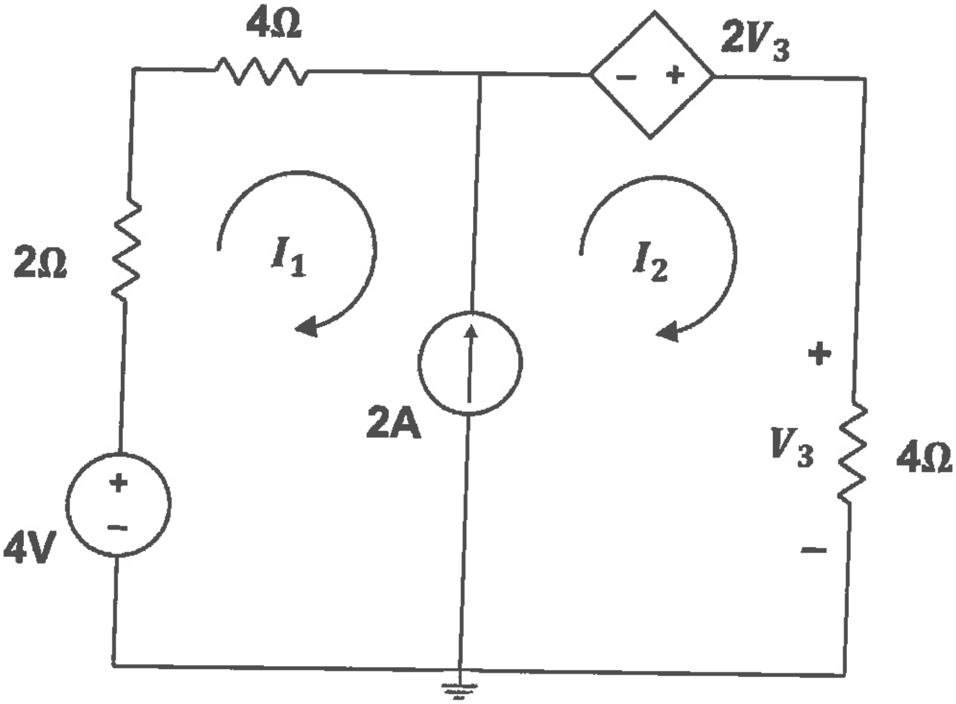
$x =$ 22/7

$y =$ −3/7

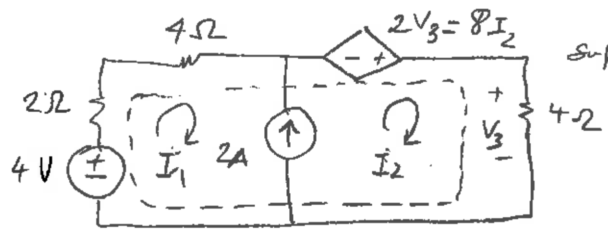
Does your solution work in the original set of equations? yes.

Problem 4 score: [ ]/10

5. (15 points). Use **mesh analysis to find V_3**, using mesh currents defined in the circuit below (you will lose 5 points if you use a different approach). Additional work space on next page.



⑤ mesh analysis



Supermesh:

KVL: $-4 + 6I_1 - 8I_2 + 4I_2 = 0$

$3I_1 - 2I_2 = 2$

aux. eqn: $[-I_1 + I_2 = 2] * 3$

$I_2 = 8\ A$

$I_1 = 6\ A$

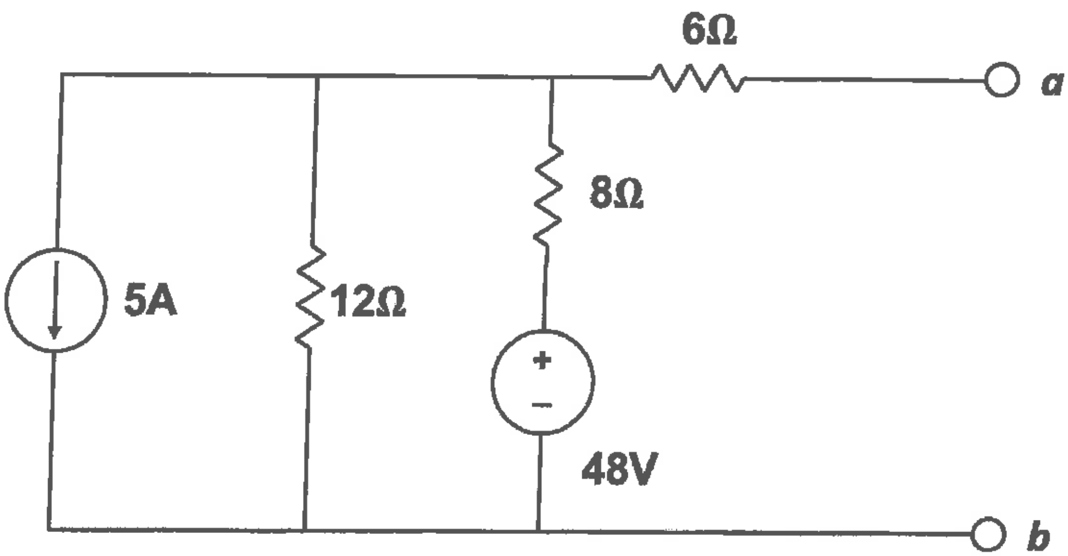
$V_3 = 4I_2 = \boxed{32V}$

Write your answer here:

V_3= 32V

Problem 5 score: [ ]/15

6. (15 points) Find the Thevenin equivalent circuit at terminals(a, b). Additional work space on next page.



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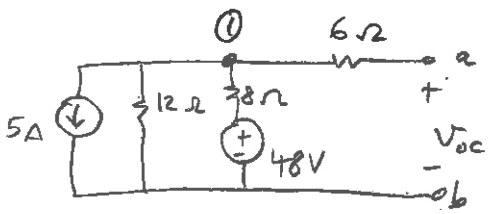
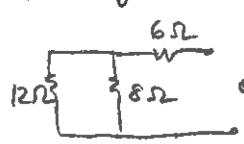


finding V_{oc}: no current in 6Ω resistor: $V_1 = V_{oc}$

node analysis: $5 + \frac{V_{oc}}{12} + \frac{V_{oc}-48}{8} = 0$

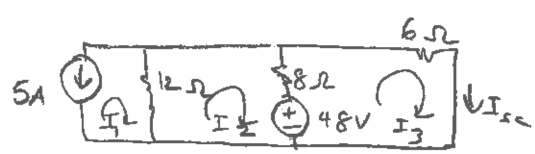
find R_{TH} by deactivating source

$5V_{oc} = 24 \Rightarrow V_{oc} = \boxed{4.8\,V}$



$\leftarrow R_{TH} = \boxed{10.8\,\Omega}$

Could also find I_{sc}



$I_1 = -5$;

KVL ②

$12(I_2 - I_1) + 8(I_2 - I_3) + 48 = 0$

$5I_2 - 2I_3 = -27$

KVL ③

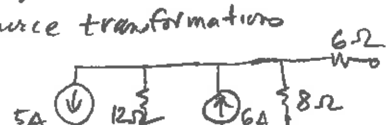
$-48 + 8(I_3 - I_2) + 6I_3 = 0$

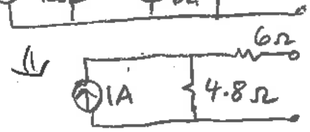
$-4I_2 + 7I_3 = 24$

solving ⇒ $I_2 = \frac{-47}{9}$, $I_3 = I_{sc} = \boxed{\frac{4}{9}\,A}$

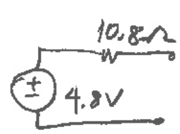
(OR)

source transformations





⇒



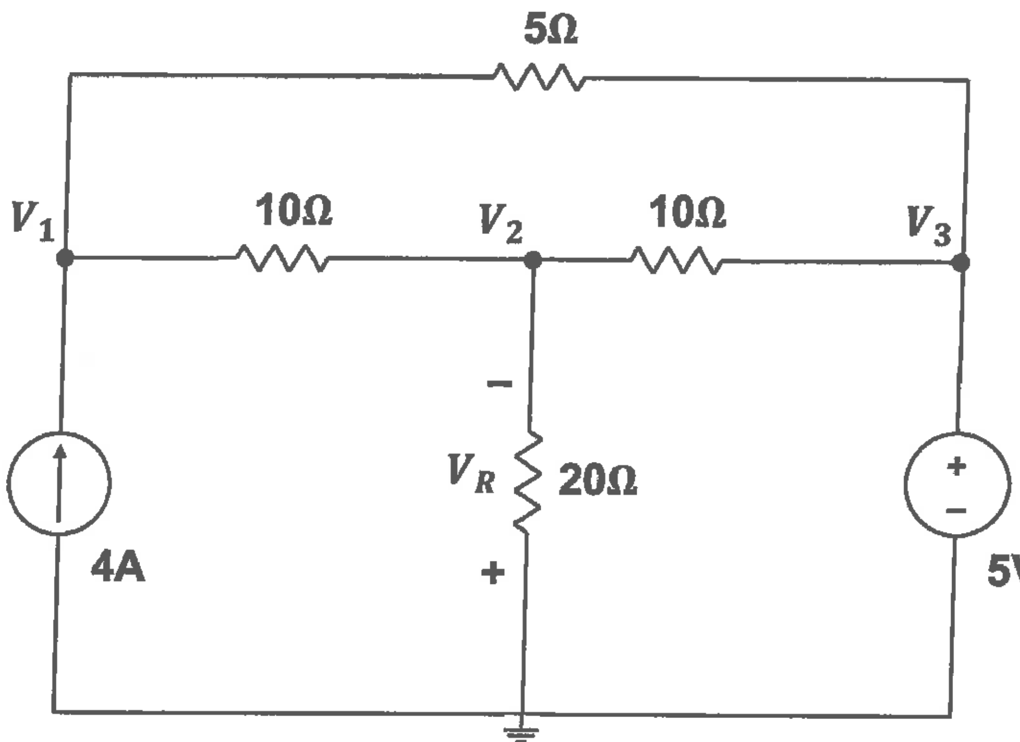
Write your answer here:

V_{TH} = 4.8V

R_{TH} = 10.8 Ohms

Problem 6 score: [ ]/15

7. (15 points). Using nodal analysis (with the node voltages as defined in the circuit), solve for the voltage V_R (you will lose 5 points if you use a different approach). Additional work space on next page.



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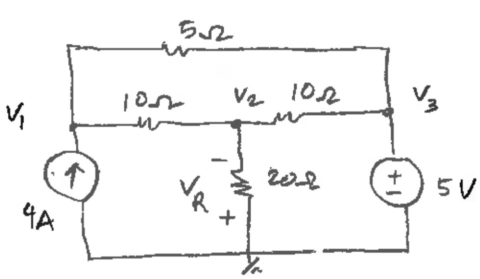


KCL ①. $\left(\frac{1}{5}+\frac{1}{10}\right)V_1 - \frac{1}{10}V_2 - \frac{1}{5}V_3 = 4$

$$3V_1 - V_2 - 2V_3 = 40$$

KCL ② $-\frac{1}{10}V_1 + \left(\frac{1}{10}+\frac{1}{10}+\frac{1}{20}\right)V_2 - \frac{1}{10}V_3 = 0$

$$-2V_1 + 5V_2 - 2V_3 = 0$$

KCL ③ $V_3 = 5V$

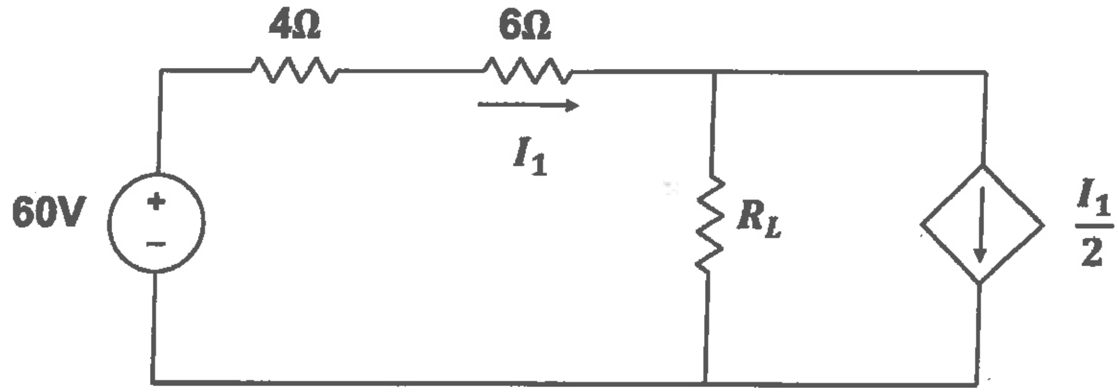
$$\left.\begin{aligned} 3V_1 - V_2 &= 50 \\ -2V_1 + 5V_2 &= 10 \end{aligned}\right\} \quad \begin{aligned} V_1 &= 20V \\ V_2 &= 10V = -V_R \\ V_R &= \boxed{-10V} \end{aligned}$$

Write your answer here:

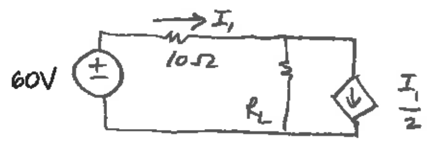
V_R = −10V

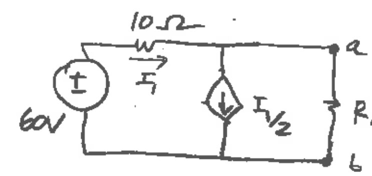
Problem 7 score: [ ]/15

8. (15 points) For the circuit below, derive the value for R_L that will result in maximum power being transferred to R_L? Calculate the maximum power that can be delivered to R_L. Additional work space on next page.



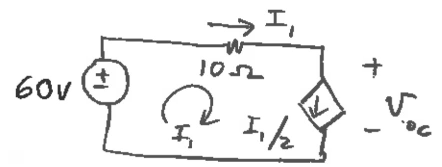
(8)





need thevenin equivalent for a, b.

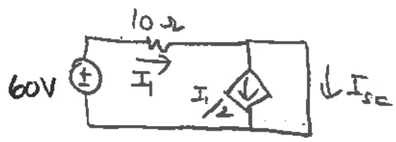
V_{oc}



$I_1 = I_1/2 \Rightarrow I_1 = 0$

$\Rightarrow V_{oc} = \underline{60V}$

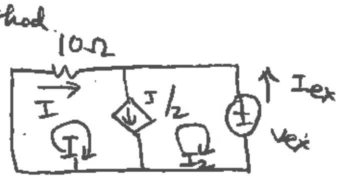
I_{sc}



KCL at top: $I_{sc} = I_1/2$

KVL outside: $-60 + 10I_1 = 0$, $I_1 = 6$, $I_{sc} = \underline{3A}$

check via test source method



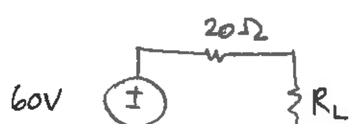
$I = I_1$

$I_2 = -I_{ex}$

supermesh: $10I_1 + V_{ex} = 0 \rightarrow V_{ex} = -10I_1$

aux: $\frac{I_1}{2} = I_1 - I_2 = I_1 + I_{ex}$

$-\frac{I_1}{2} = I_{ex}$



P_{max} when $R_L = 20\Omega$

$P_{max} = \frac{(V_{TH})^2}{4R_L}$

$= \frac{(60)^2}{4(20)} = \boxed{45W}$

$R_{TH} = \frac{V_{ex}}{I_{ex}} = \frac{-10I_1}{-I_1/2}$

$\boxed{R_{TH} = 20\Omega}$

Write your answer here:

R_L for maximum power transfer = 20Ω

In that case, power delivered to R_L = 45W

Problem 8 score [1/15