HW #1
SOLUTIONS

1.3 (b) $i(t) = (2t+5)$

$$q(t) = \int_0^t (2t+5)\,dt$$

$$= \frac{2t^2}{2} + 5t + C$$

Now, $q(0) = C = 0$

1 $\therefore\ q(t) = t^2 + 5t.$

(c) $i(t) = 20\cos\left(10t + \frac{\pi}{6}\right)$; $q(0) = 2\,\mu C$

$$q(t) = 20\int \cos\left(10t + \frac{\pi}{6}\right) dt$$

1

$$= \frac{20}{10}\sin\left(10t + \frac{\pi}{6}\right) + C$$

But, $q(0) = 2\sin\left(\frac{\pi}{6}\right) + C$

$$= 2\,\mu C$$

$\therefore\ C = 1\,\mu C.$

$\therefore\ q(t) = 2\sin\left(10t + \frac{\pi}{6}\right) + 1\ \mu C.$

1.7.

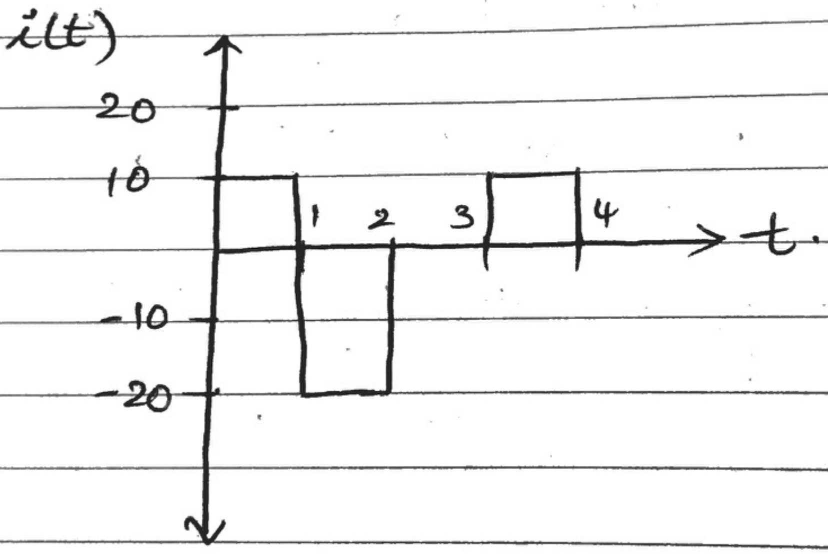
$$i(t) = \frac{dq(t)}{dt}$$

$$q(t) = \begin{cases} 10t & 0 \leq t \leq 1 \\ -20(t - \frac{3}{2}) & 1 \leq t \leq 2 \\ -10 & 2 \leq t \leq 3 \\ 10(t-4) & 3 \leq t \leq 4. \end{cases}$$

1

$$i(t) = \frac{dq(t)}{dt} = \begin{cases} 10 & 0 \leq t \leq 1 \\ -20 & 1 \leq t \leq 2 \\ 0 & 2 \leq t \leq 3 \\ 10 & 3 \leq t \leq 4. \end{cases}$$

1



1.9

$$q(t) = \int i(t)\,.\,dt$$

(a) $t = 1s.$

$$q = \int_0^1 i(t)\,.\,dt$$

$$= \int_0^1 10\,.\,dt$$

1 $$= 10t\Big|_0^1 = \underline{\underline{10C}}$$

(b) $t = 3s$

$$q = \int_0^3 i(t)\,.\,dt$$

$$= \int_0^1 10\,dt + (-5)\int_1^2 (t-3)\,dt + \int_2^3 5\,dt$$

1 $$= 10 + -5\left[\frac{t^2}{2} - 3t\right]_1^2 + 5$$

$$= 10 + \frac{15}{2} + 5$$

$$= \underline{\underline{22.5\ C}}$$

(c) $\cancel{q}$ $t = 5s.$

$$q(5) = q(3) + \int_3^5 i(t)\,.\,dt$$

1 $$= 22.5 + 5\int_3^4 dt + (-5)\int_4^5 (t-5)\,.\,dt$$

$$= 22.5 + 1\times5 - 5\left[\frac{t^2}{2} - 5t\right]_4^5 = \underline{\underline{30}}$$

1.12

$0 \leq t < 6$

$$q(t) = \int_0^t i\,dt + q(0)$$

(Assuming $q(0)=0$)

$$= \frac{3t^2}{2}$$

$$\therefore q(6) = 54C$$

$6 \leq t < 10$

$$q(t) = \int_6^t i\,dt + q(6)$$

$$= 18t - 108 + 54$$

$$= 18t - 54$$

$$\therefore q(10) = 126C$$

2

$10 \leq t < 15$

$$q(t) = \int_{10}^{\cancel{15}} i(t)\,dt + i(10)$$

$$= -12t + 120 + 126$$

$$= -12t + 246$$

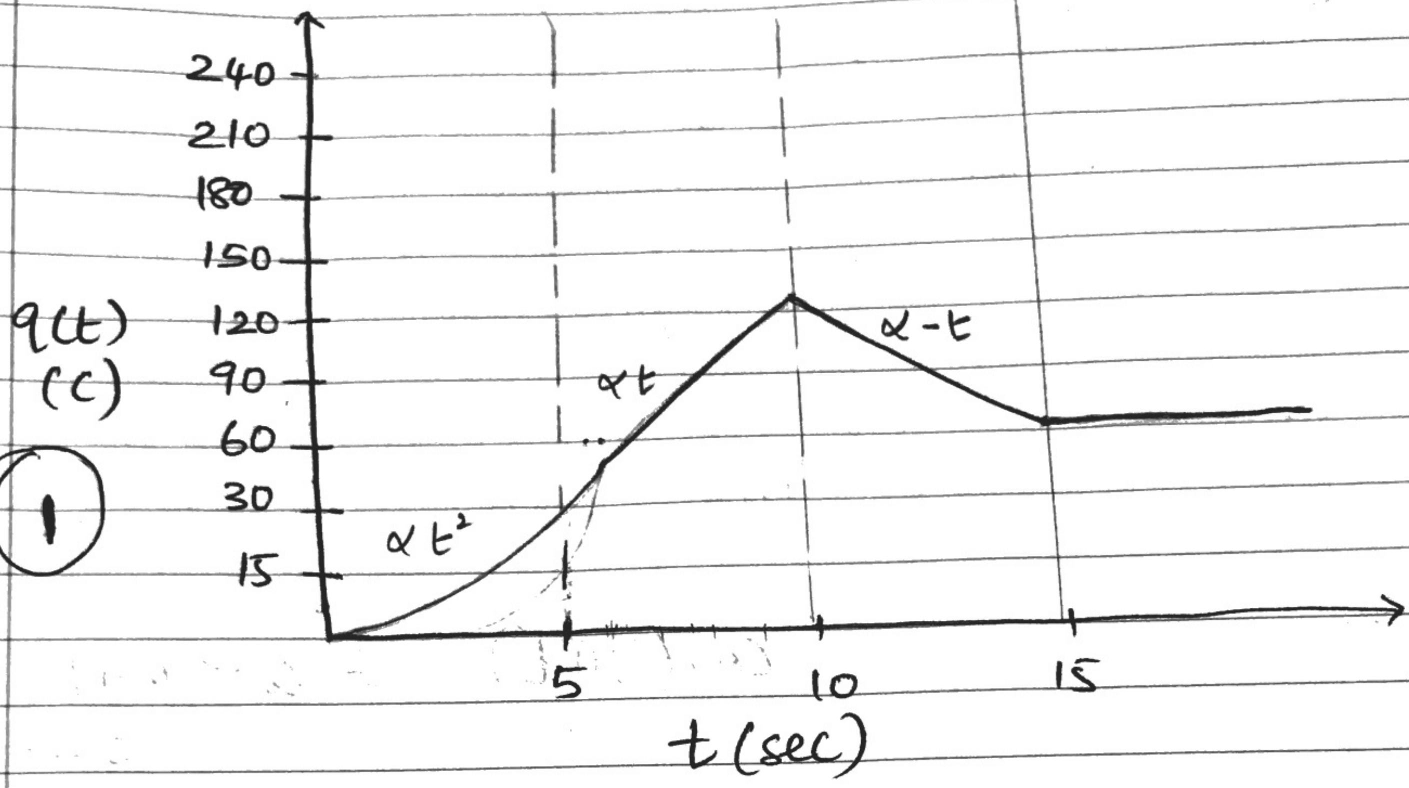
$$\therefore q(15) = 66C$$

$t \geq 15$

$$q(t) = \int_{15}^t 0\,dt + q(15)$$

$$= 66C$$

1



q(t)
(C)
240
210
180
150
120
90
60
30
15
αt^2
αt
$\alpha - t$
5
10
15
t (sec)

1.13 (a) $q(t) = 5 \sin 4\pi t$ mC

$\therefore i(t) = \frac{d}{dt} q(t)$

$= 20\pi \cos 4\pi t$ mA.

$v(t) = 3 \cos 4\pi t$ V.

1

$\therefore p(t) = 60\pi \cos^2 4\pi t$ mW

$p(0.3) = 123.37$ mW

(b)

$W = \int p \, dt$

$= 60\pi \int \cos^2(4\pi t) \, dt$

1

$= \frac{60\pi}{2} \int_0^{0.6} \cos(8\pi t) + 1 \, dt$

$= 30\pi \left[\frac{\sin(8\pi t)}{8\pi} + t \right]_0^{0.6}$

$= 30\pi \left[\frac{\sin(4.8\pi)}{8\pi} + 0.6 \right]$

$= 58.75$ mJ.

1.14

(a) $q = \int_0^1 10(1+e^{-2t}) \cdot dt$

$= 10 \left[t - \frac{e^{-2t}}{2} \right]_0^1$

$= 10 \left[1 - \frac{e^{-2}}{2} + \frac{e^{-0}}{2} \right]$

1

$= 10 \left[\frac{3}{2} - \frac{e^{-2}}{2} \right]$

$= 14.32 \text{ mC}.$

(b) $p = v(t)\, i(t)$ @ $t = 1$

$= 20 \sin(4) \times 10(1+e^{-2})$

1

$= -171.844 \text{ mW}.$

1.19 Approach I :

Element	Power.
10 A current source	-150 W.
15 V element	$15 \times 4 = 60$ W.
9 V element	$-9I$ W
6 V source	$-6I$ W

$\frac{1}{2}$

$$\Sigma \text{ Power} = \underline{90 - 15I}$$

$\because \Sigma$ Power $= 0$,

$$-90 - 15I = 0$$

$$\therefore I = \frac{90}{15}$$

$$= -6A.$$

1

$\therefore$ Power absorbed by

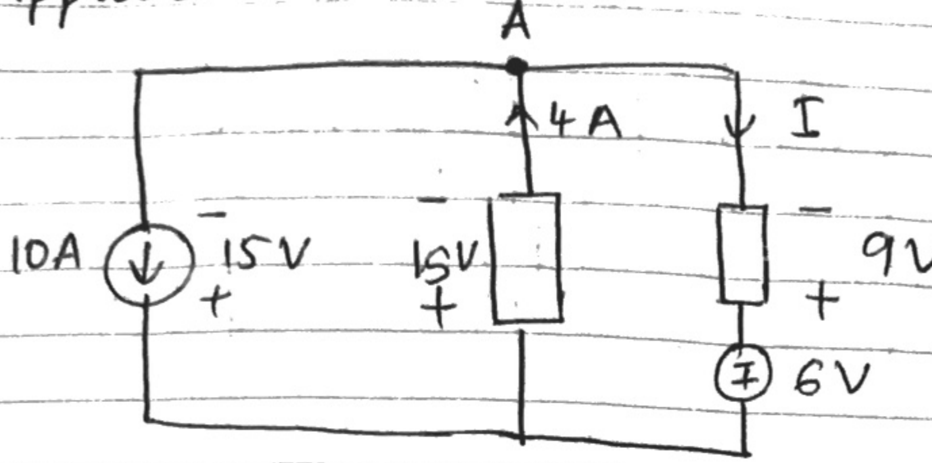
9V element $= +9 \times 6 = +54$ W

6V source $= 6 \times 6 = 36$ W.

$\frac{1}{2}$

Approach II:

1.19



The total current @ node A must $= \underline{\underline{0}}$

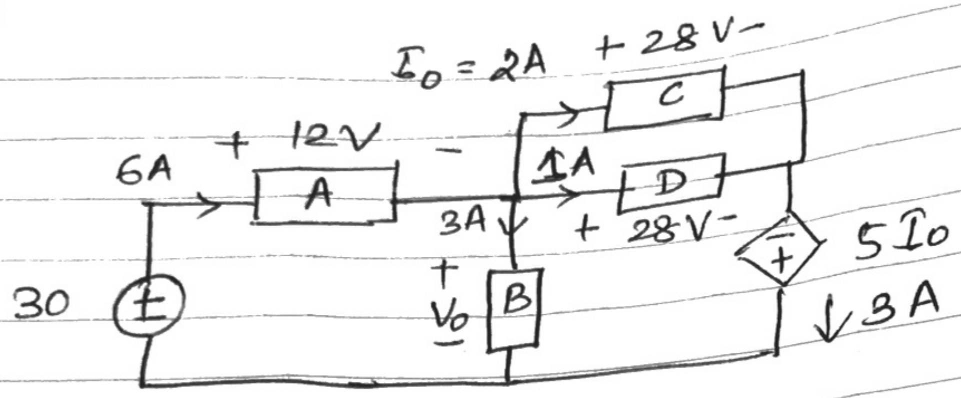
$$\underbrace{10 + I}_{\text{current flowing out of A}} = \underbrace{4}_{\text{current flowing into A}}$$

1 $I = -6A.$

10 A current source ⇒ $P = -10 \times 15 = -150W$

1 Power absorbed
- element with 15V ⇒ $P = 4 \times 15 = 60W.$
- element with 9V ⇒ $P = 9 \times 6 = 54W$
- 6V voltage source ⇒ $P = 6 \times 6 = 36W.$

1.20



Element	Power
30 V supply	$-30 \times 60 = -180\ W$
A	$12 \times 6 = 72\ W$.
B	$3 \times V_o = 3V_o\ W$
C	$2 \times 28 = 56\ W$
D	$1 \times 28 = 28\ W$.
$5I_o$ supply	$-10 \times 3 = -30\ W$
	Σ Power $= -54 + 3V_o$

(1)

(1) Now Σ Power $= 0\ W$.

$\therefore 3V_o = 54$

$\therefore V_o = 18\ V$.

1.25

Total ~~power~~ energy consumed (kW-h).

$$= 1.2 \times \frac{4}{60} \times 2 \times 14.$$

(1) $= 2.24$ kW-h. ($\frac{1}{2}$)

$\therefore$ Total cost $= 2.24 \times 9$

$= 20.16$ cents. ($\frac{1}{2}$)

1.28

(a) ½ current (I) $= \frac{P}{V} = \frac{150}{120} = 1.25\ A$

(b) ½ Total energy $= 150 \times (12 \times 365)$

$= 657 \times 10^3\ W.$

1 Cost $= 657 \times 9.5$ cents

$= 62415$ Cents.

$= 62.415\ \$$