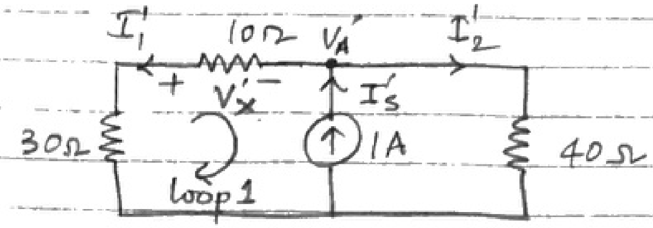
HW #4

SOLUTIONS

Problems :

* 4.7
* 4.16
* 4.27
* 4.48
* 4.57
* 4.72
* 4.88

4.7



* Initial assumption: $V_x' = 1V$

Current through 10Ω resistor:

$$I_1' = -\frac{1V}{10\Omega} = -0.1A$$

Applying KVL in loop 1:

½

$$-30I_1' - 10I_1' + V_A' = 0$$

$$\therefore V_A = 40I_1' = 40(-0.1) = -4V$$

Applying KCL at node A (V_A):

½

$$I_1' + \frac{V_A'}{40} = I_s'$$

$$\Rightarrow I_s' = -0.2\ A$$

But, actual value of $I_s = 1A = -5 \times I_s'$

½

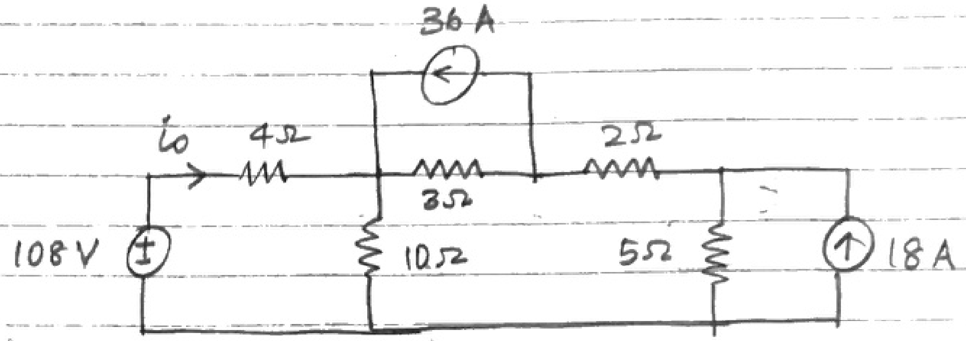
$\therefore$ Multiplication factor $= -5$

$$\therefore I_1 = 5 \times I_1' = -0.5A$$

½

$$\therefore V_x = -5 \times V_x' = -5 \times (-I_1' \times 10) = \underline{\underline{5V}}$$

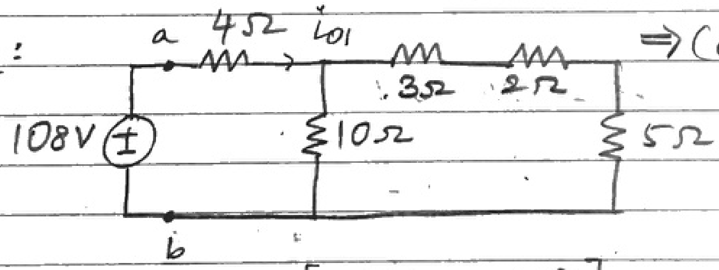
4.16



$$i_o = i_{o1} + i_{o2} + i_{o3} \quad — (1)$$

i_{o1} (from 108V source), i_{o2} (from 36A source), i_{o3} (from 18A source)

i_{o1} :



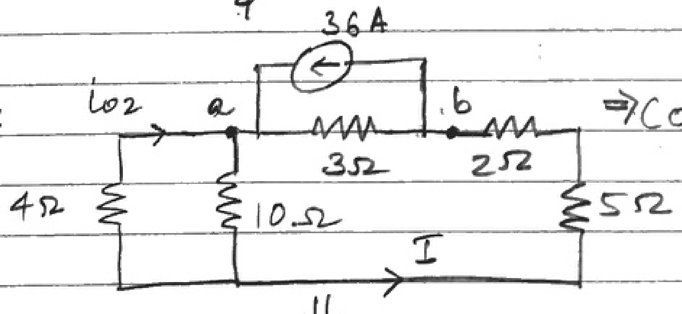
⇒ (Open ckt for 18A and 36A sources)

$$R_{ab} = 4 + [10 \| (3+2+5)]$$

$$= 4 + 5 = 9\Omega.$$

$$\therefore i_{o1} = \frac{108}{9} = 12A \quad — (2)$$

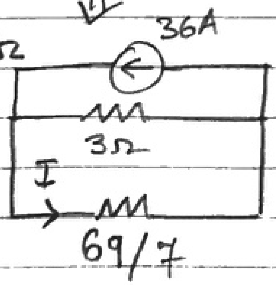
i_{o2} :



⇒ (open ckt for 18A source and short ckt for 108 V source)

⇓

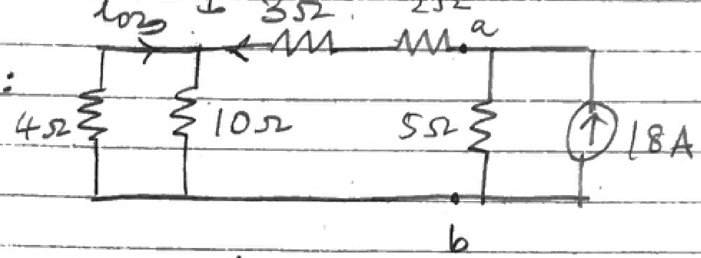
$$R_{ab} = ((4\|10) + 2 + 5)\Omega = \frac{69}{7}\Omega$$



$$\Rightarrow I = \left(\frac{3}{\frac{69}{7} + 3}\right) \times 36 = 8.4 \text{ A}$$

$$i_{o2} = \frac{-10}{14}(I) = -6A. \quad - (3)$$

i_{o3}:



⇒ (short ckt 108 V source and open ckt 36A source)

$$R_{ab} = (4 \| 10) + 5 = \frac{55}{7}\ \Omega$$

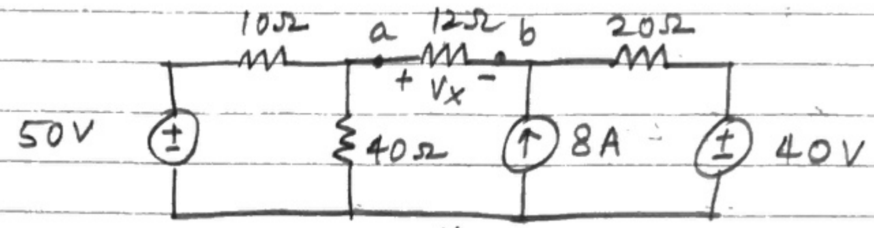
$$I = \frac{5}{5+\frac{55}{7}} \times (18) = 7A$$

$$i_{o3} = -\frac{10}{14} \times 7 = -5A. \quad - (4)$$

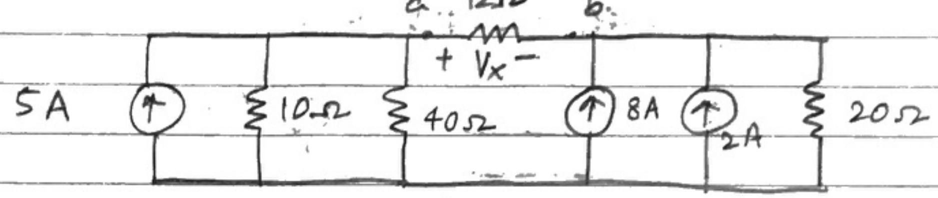
From (1), (2), (3) and (4)

$$i_o = 12 - 6 - 5 = \underline{\underline{1A}}$$

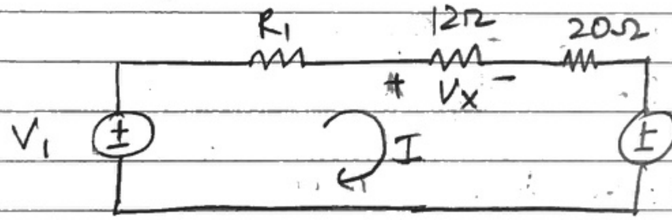
4.27



Applying source transformation to 50V & 40V sources:



Applying current to vltg source transformation:



$V_2 = 20 \times (8+2) = 200V$

$V_1 = 5 \times (10 \| 40) = 40V$

$R_1 = 10 \| 40 = 8 \Omega$

Applying KVL around the loop:

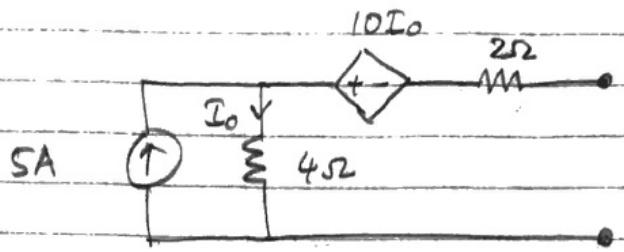
$-V_1 + R_1 I + 12I + 20I + 200 = 0$

$\Rightarrow -40 + 8I + 32I + 200 = 0$

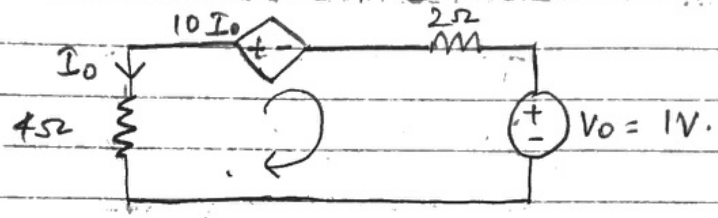
$\Rightarrow I = -4 \ A$

Now, $V_x = 12I = -48 \ V$

4.48



Step I : Find R_N : Open ckt for independent current source.



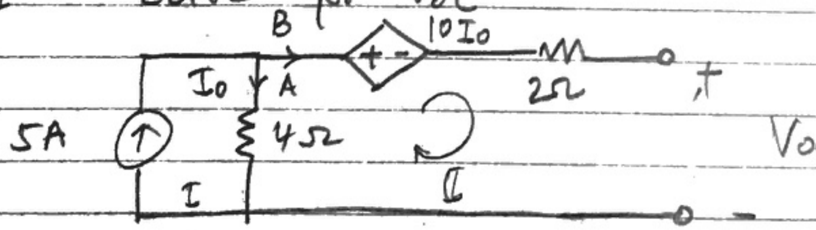
Applying KVL around the loop:

$$-4I_o + 10I_o - 2I_o + 1V = 0$$

$$\Rightarrow \quad I_o = +\tfrac{1}{4} \text{ A}$$

$$\therefore R_N = \frac{V_o}{I_o} = 4\ \Omega$$

Step II : Solve for V_{oc}



$\because$ loop II is open circuit, no current flows through branch B.

$\therefore I_o = 5A$.

Applying KVL around loop II :

1

$$-4I_0 + 10I_0 = V_{oc}$$

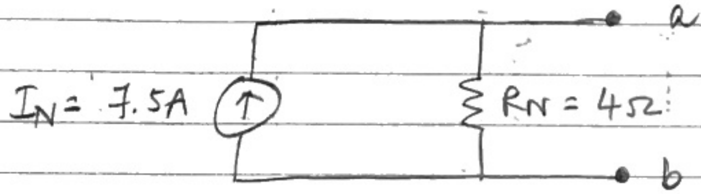
$$\Rightarrow V_{oc} = 30\,V$$

Step III : Find I_N :

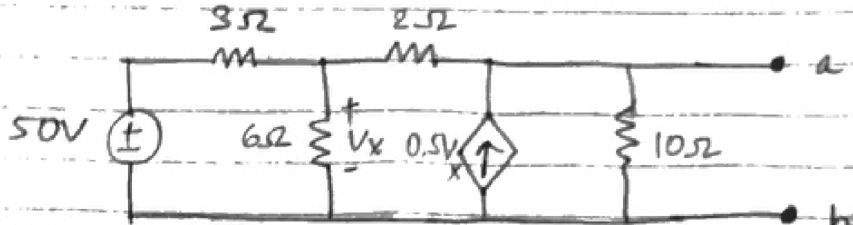
1/2

$$I_N = \frac{V_{oc}}{R_N} = \frac{30}{4} = 7.5A$$

$\therefore$ Norton equivalent ckt :

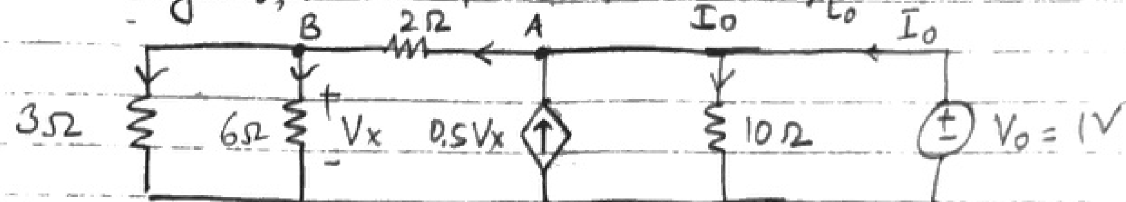


4.57



Step I: Find R_N or R_{th}: Since there is a dependent current source, we apply a vltg source, $V_o = 1V$, between a & b.

∴ By calculating I_o, we can find $R_{Th} = \frac{V_o}{I_o} = 1/I_o$



Now, $V_x = V_B$ and $V_A = 1V$

Applying KCL at node A:

1/2 $$\frac{1 - V_B}{2} + \frac{1}{10} = I_o + 0.5V_B$$

$$\Rightarrow 5 - 5V_B + 1 = 10 I_o + 5V_B$$

$$\Rightarrow 6 - 10I_o = 10V_B$$

$$\Rightarrow 6 = 10V_B + 10I_o \quad — (1)$$

Applying KCL at node B:

1/2 $$\frac{V_B}{6} + \frac{V_B}{3} = \frac{1 - V_B}{2}$$

$$\Rightarrow V_B + 2V_B = 3 - 3V_B$$

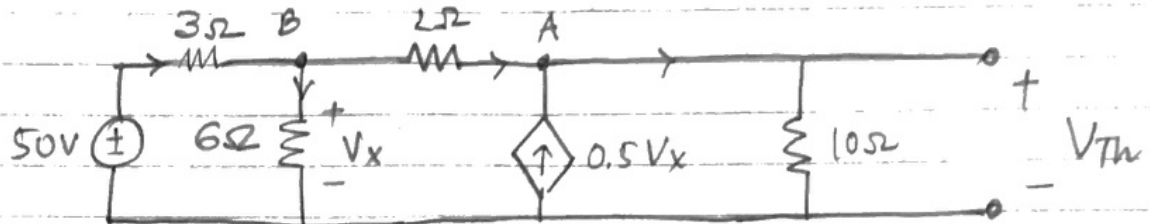
$$\Rightarrow V_B = 0.5V \quad — (2)$$

∴ From (1) & (2)

$$I_o = 0.1 A$$

1/2 ∴ $$R_N = R_{Th} = \frac{1}{0.1} = 10Ω$$

Step II : Find V_{Th}



Again $V_B = V_x$

Applying KCL at node A :

½ $$\frac{V_B - V_A}{2} + 0.5\,V_B = \frac{V_A}{10}$$

$\Rightarrow$ $5V_B + 5V_B - 5V_A - V_A = 0$

$\Rightarrow$ $10V_B - 6V_A = 0$ — (3)

Applying KCL at node B :

½ $$\frac{50 - V_B}{3} = \frac{V_B}{6} + \frac{V_B - V_A}{2}$$

$100 - 2V_B = V_B + 3V_B - 3V_A$

$\Rightarrow$ $6V_B - 3V_A = 100$ — (4)

Solving (3) & (4) :

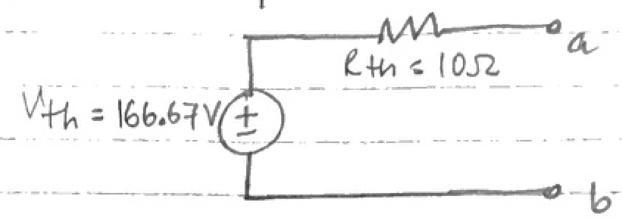
½ $V_A = V_{Th} = 166.67\ V$

Step III : Find I_N

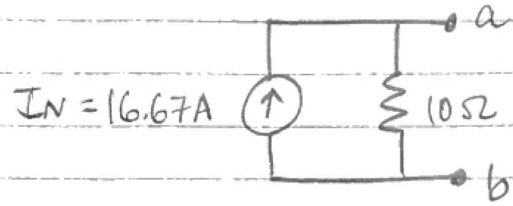
½ $I_N = \dfrac{V_{Th}}{R_{Th}} = 16.67\ A$

Resulting equivalent ckts :

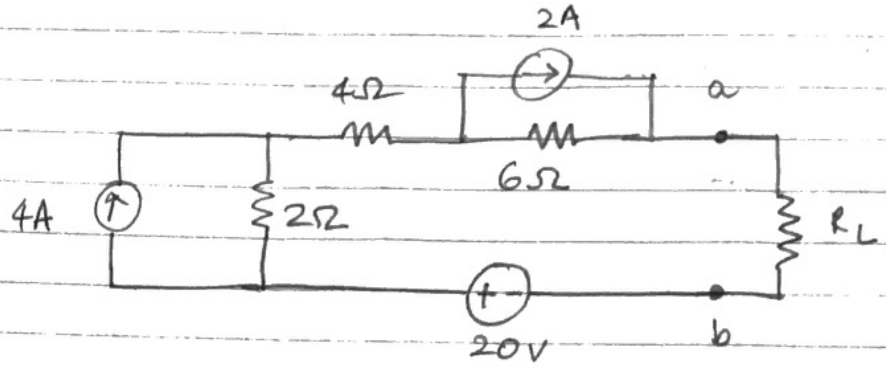
* Thevenin equivalent :



* Norton equivalent :



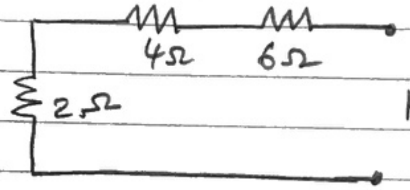
4.72



(a) Thevenin Equivalent:

1/2

R_{Th}:

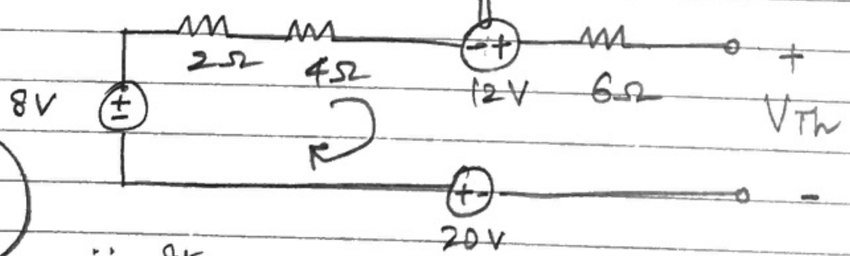


$R_L = R_{Th}$

$R_{Th} = 12\Omega$.

V_{Th}: Applying current to voltage source transformation:

1/2

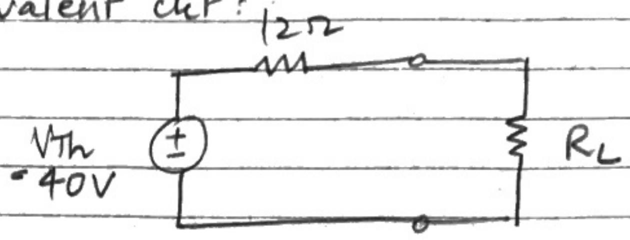


∵ Its an open ckt, current = 0
Applying KVL in the loop:

$$-8 - 12 + V_{Th} - 20 = 0$$

$$\Rightarrow V_{Th} = 40V$$

Equivalent ckt:



(b) ∴ If $R_L = 13$

1/2

$$i = \frac{V_{Th}}{R_{Th} + 13} = \frac{40}{25} = 1.6A$$

(c) For maximum power transfer:

1/2

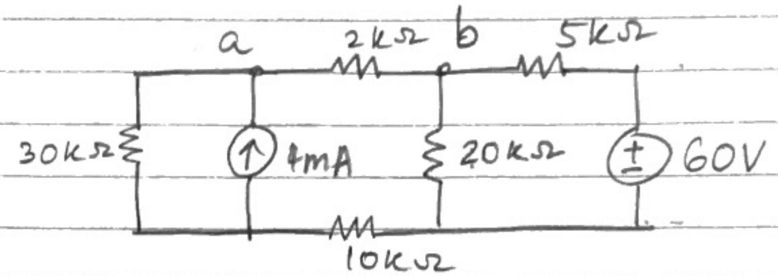
$$R_L = R_{Th} = 12\,\Omega$$

(d)

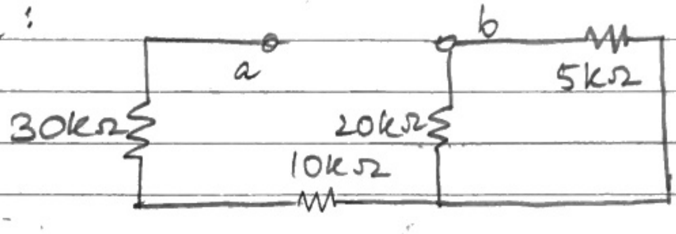
1/2

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = 33.33\ W$$

4.88



R_{Th} :

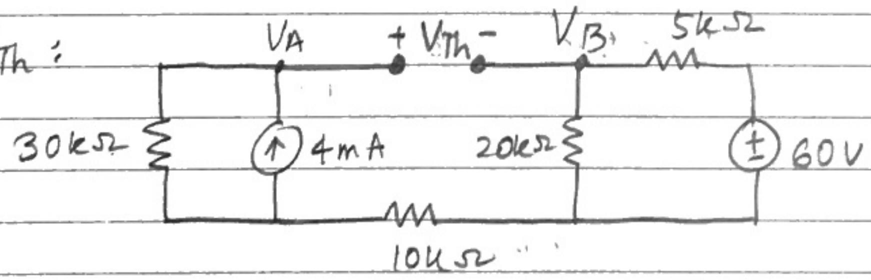


1/2

$$R_{Th} = [(20 \| 5) + 40]$$

$$= 44 \text{ k}\Omega$$

V_{Th} :



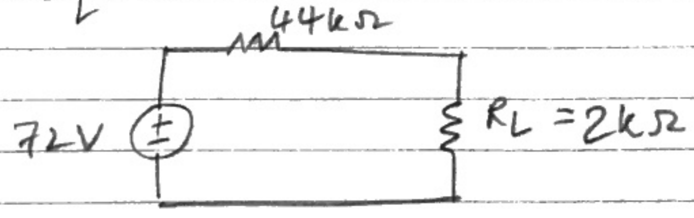
** Since there is an open ckt, no current to/from the left circuit flows from/to the right ckt.

$\therefore$ $V_A = 4 \times 30 = 120 \text{ V}$

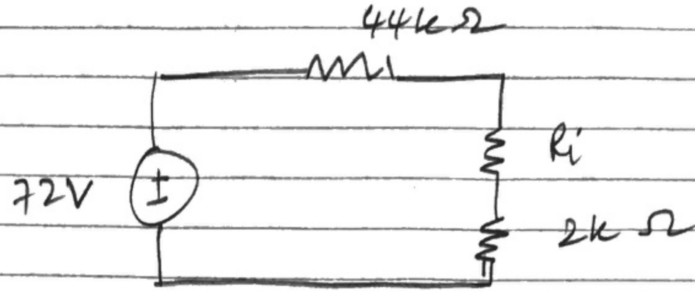
$V_B = \frac{20}{25} \times 60 = 48 \text{ V}$

1 $\therefore$ $V_{Th} = 72 \text{ V}$

$\therefore$ Eq ckt :



Now if we insert ammeter in series with R_L, we get :



1/2

Here we subsitute ammeter by it's internal resistance R_i.

$$\therefore I = \frac{72}{46 + R_i}$$

(a) When $R_i = 500\Omega = 0.5\ k\Omega$

1/2

$$I = \frac{72}{46.5} = 1.548\ mA$$

(b) When $R_i = 0\ k\Omega$

1/2

$$I = \frac{72}{46} = 1.565\ mA$$