Name: SOLUTIONS.

# EXAM 1
# EECS 215
## Introduction to Electronic Circuits
**Wednesday, October 11, 2017, 6:00pm-8:00pm**

| **Lecture Section (circle one):** | **001 Finelli** | **002 Lahiji** |
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**This test consists of 6 problems with points as indicated to total 100 points.**

Read through the entire exam before beginning.
**Show all work** (on the pages provided in this booklet) to earn partial credit.
Briefly explain major steps, include units, and write your final answers in the areas provided.
Do not unstaple the pages.

**No credit will be given if no work is shown.**

**Exam Policies**

- No food allowed during exam.
- No books allowed (closed book exam).
- One, 8.5 x 11 inch notes page (ONE SIDED) allowed
- Only scientific calculators allowed (**graphing calculators not permitted**).
- No communication of any kind is allowed. No use of cell phones, computers, or any devices besides calculators. Violation of this will be treated as an honor code violation.
- No credit will be given for this exam without a signed honor pledge.

**Write out the honor pledge and sign below.**

"I have neither given nor received unauthorized aid on this examination, nor have I concealed any violations of the Honor Code"

I have neither given nor received any unauthorized aid on this examination, nor have I concealed any violations of the Honor Code.

Signature: XYZ

Do not write in this space

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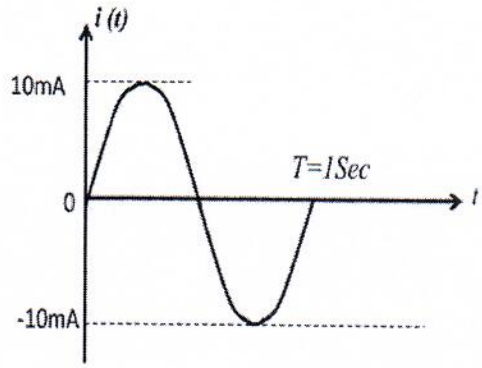
|---|---|
| Problem 1: [ ]/15 | Problem 4: [ ]/15 |
| Problem 2: [ ]/20 | Problem 5: [ ]/15 |
| Problem 3: [ ]/15 | Problem 6: [ ]/20 |

**Total score [ ]/100**

Problem 1. (**15 points total**) Electrical current flowing through an electrical element is given by a sinusoidal signal having a period of $T = 1$ second:

$$i(t) = 10\sin(2\,\pi t)\text{ mA}$$




**a.** Assuming there is no initial charge in the element, find the total amount of charge that has entered the element at $t = T/2 = 0.5$ sec.

$q(0) = 0C \qquad T = 1sec$

$q(t) = \int_0^t i(t).dt$

$q = \int_0^{T/2} 10\ sen(2\pi t).dt$

$q = -\frac{10}{2\pi}\left[\cos(2\pi t)\right]_0^{T/2}$

$= \frac{-10}{2\pi}[-1 - ] = 3.18\ mC$

Total charge = 3.18 mC **(5 points)**

$\frac{5di}{dt}$

**b.** If voltage across the element is $v(t) = 5i(t)/dt$ V, find the power delivered to the device at $t = T/8 = 0.125$ sec.

$$V(t) = 5\frac{di(t)}{dt} = 5 \cdot 20\pi \cos 2\pi t = 100\pi \cos 2\pi t \text{ mV}$$

$$P = V \cdot i = 100\pi \times 10^{-3} \cos 2\pi\left(\frac{\pi}{8}\right) \cdot 10 \times 10^{-3} \sin\left[2\pi\left(\frac{\pi}{8}\right)\right]$$

$$= 1000\pi \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} \times 10^{-6} = 500\pi \ \mu W$$

$$= 1.57 \text{ mW}$$

Power delivered = 1.57 mW **(5 points)**

**c.** Find total energy delivered to the element after one period of the input signal.

$$E = \int_0^t p(t) \cdot dt$$

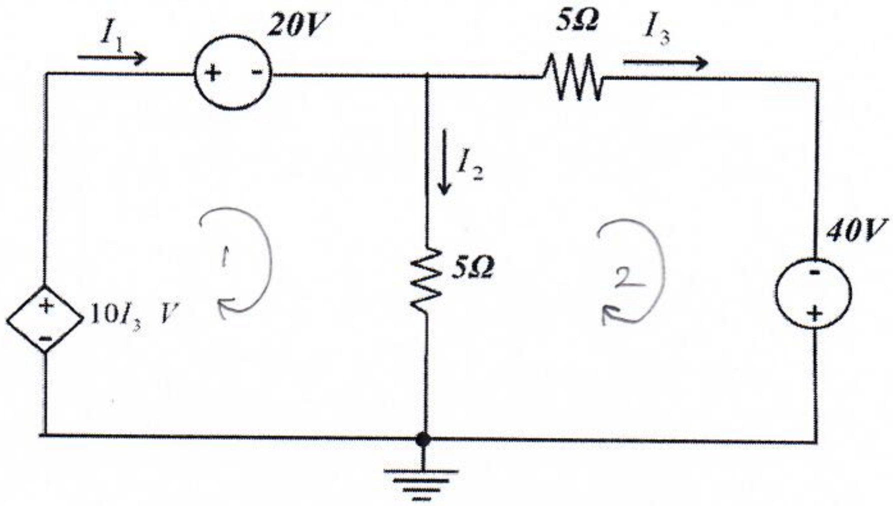
$$= \int_0^T 1000\pi \times 10^{-6} \cos 2\pi t \cdot \sin 2\pi t \cdot dt$$

$$= \int_0^{1\text{sec}} \frac{\pi \times 10^{-3}}{2} \sin 4\pi t \ dt$$

$$= -\frac{\pi \times 10^{-3}}{2 \times 4\pi} \left[\cos 4\pi t\right]_0^{1\text{sec}} = 0 \text{J}$$

Total energy = 0 J **(5 points)**

Problem 2. (**20 points total**) For the following circuit:




**a.** Use KVL/KCL to find the currents $I_1$, $I_2$, and $I_3$.

By writing KVL in loop 1:

$-10I_3 + 20 + 5I_2 = 0 \quad - (1)$

By writing KVL in loop 2:

$-40 - 5I_2 + 5I_3 = 0 \quad - (2)$

Adding eq^n (1) + (2):

$-5I_3 = 20$

$\Rightarrow I_3 = -4A \quad - (3)$

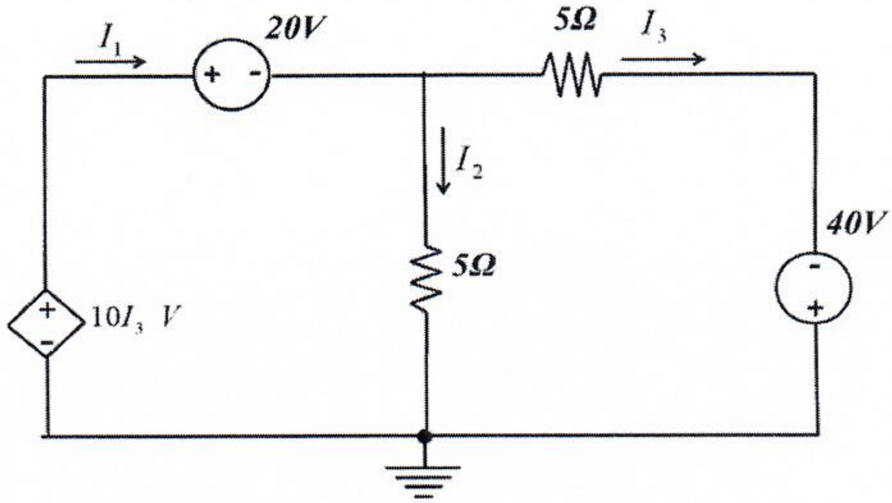
Subsituting (3) in eq^n (1),

$I_2 = -12A$

$I_1 = I_2 + I_3 = -16A$

$I_1 =$ <u>$-4A$</u>, $I_2 =$ <u>$-12A$</u>, $I_3 =$ <u>$-16A$</u> **(10 points total)**

**b.** For the same circuit (redrawn here for convenience), determine the power absorbed by each element and show that power is conserved in the circuit.




$P_{10I_3} = -I_1 \cdot V_{10I_3} = -(-16)(10\cdot(-4))$
$= -640\,W.$

$P_{20V} = I_1 \cdot 20\,V = -16\cdot 20 = -320\,W$

$P_{5\Omega} = 5 \cdot I_2^2 = 5\cdot(-12)^2 = 5\times 144 = 720\,W.$

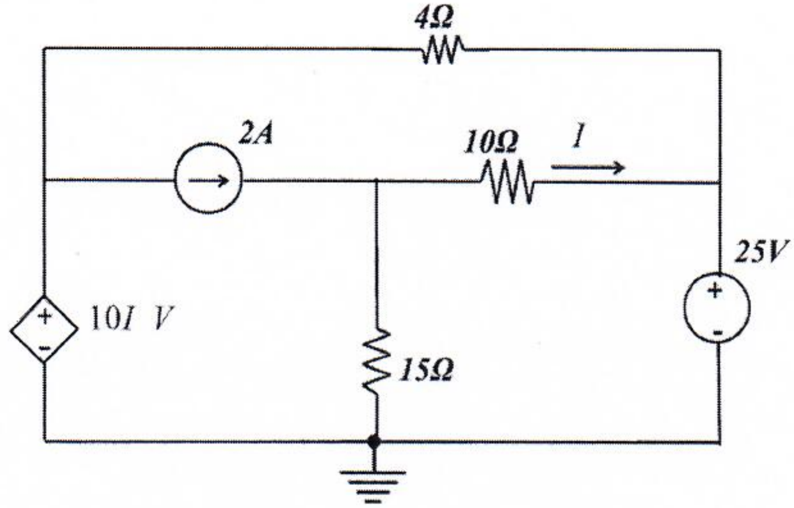
$P_{5\Omega} = 5 \cdot I_3^2 = 5\cdot(-4)^2 = 80\,W$

$P_{40V} = -(I_3 \cdot 40\,V) = 160\,W.$

$\Sigma$ Power $= P_{10I_3} + P_{20V} + P_{5\Omega} + P_{5\Omega} + P_{40V}$
$= -640W - 320W + 720W + 80W + 160W$
$= 0\,W$

Power absorbed by: dependent source= -640W, 20 V source= - 320W,
middle resistor = 720 W, top resistor = 80 W, 40 V source = 160W,
Is power conserved? Yes **(10 points total)**

Problem 3. (**15 points**) In this circuit, use superposition to determine the value of current $I$ in 10Ω resistor

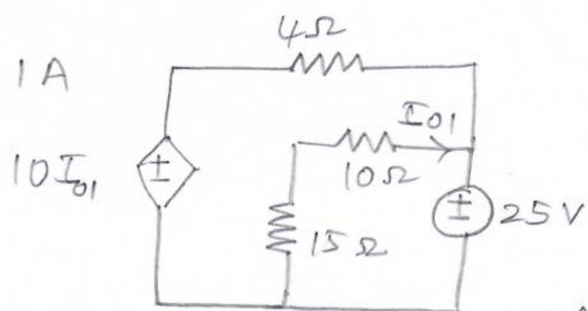



$I = I_{01} + I_{02}$ → due to 2A current source.

$I_{01}$ → due to 25V source

$I_{01}$: Open ckt 2A current source

$$I_{01} = \frac{-25V}{10+15} = -1A$$

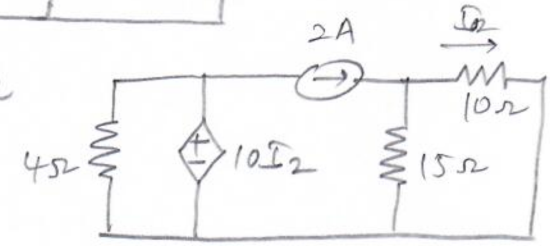



$I_{02}$: short circuit 25V source

$$I_{02} = 2A \cdot \left(\frac{15 \| 10}{10}\right)$$

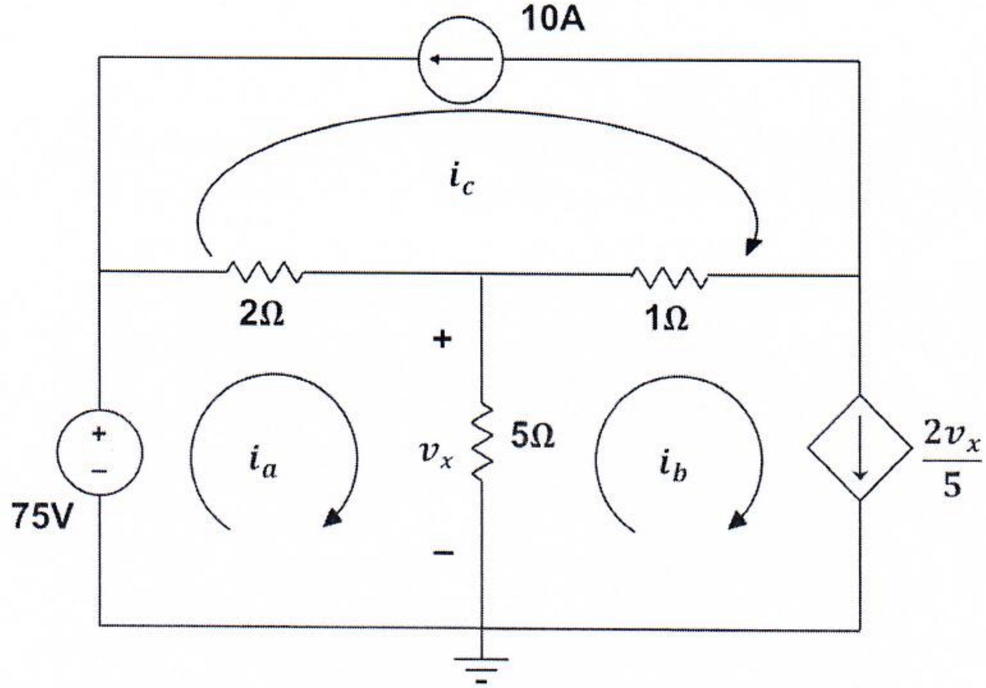
$$= 2A \cdot \left(\frac{15}{25}\right)$$

$$= 1.2A$$




$\therefore \; I = I_{01} + I_{02} = 0.2A$ $\qquad I =$ ____0.2A____ (**15 points**)

Problem 4. (**15 points**) For this circuit, apply mesh analysis to find the currents $i_a$, $i_b$, and $i_c$.




KVL around mesh a:

$-75 + 2(i_a - i_c) + 5(i_a - i_b) = 0$ $\boxed{7i_a - 5i_b - 2i_c = 75}$ (1)

KVL around mesh b not needed since $i_b = 2^{v_x}/_5$ constraint eqn $v_x = 5(i_a - i_b)$ $\Big\}$ $i_b = 2(i_a - i_b)$ $\boxed{i_a = 3/2\, i_b}$ (2)

KVL around mesh c not needed sinc $\boxed{i_c = -10A}$ (3)

combine equations (1) + (3)

$7i_a - 5i_b + 20 = 75 \Rightarrow$ $\boxed{7i_a - 5i_b = 55}$ (4)

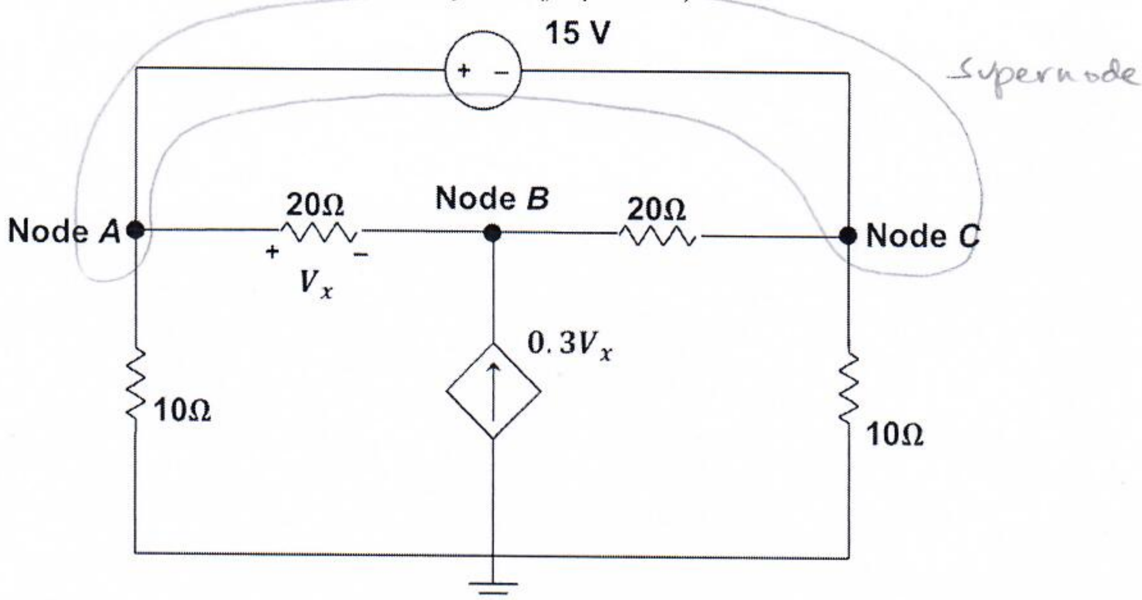
combine equations (4) and (2)

$\frac{21}{2} i_b - 5i_b = 55 \Rightarrow \boxed{i_b = 10A}$

$i_a = 3/2\, i_b \Rightarrow \boxed{i_a = 15A}$

$i_a =$ 15A, $i_b =$ 10A, $i_c =$ −10A (**15 points total**)

Problem 5. (**15 points total**) Derive the three independent equations that would be required to solve for the node voltages by completing the steps outlined below.
(*Note, you are not asked to solve the system of equations*).




**a.** Noting that a voltage source is connected between Nodes A and C, write KCL at the supernode between those two nodes and simplify the equation.

$$\left( \frac{V_A}{10} + \frac{(V_A - V_B)}{20} + \frac{(V_C - V_B)}{20} + \frac{V_C}{10} = 0 \right) 20$$

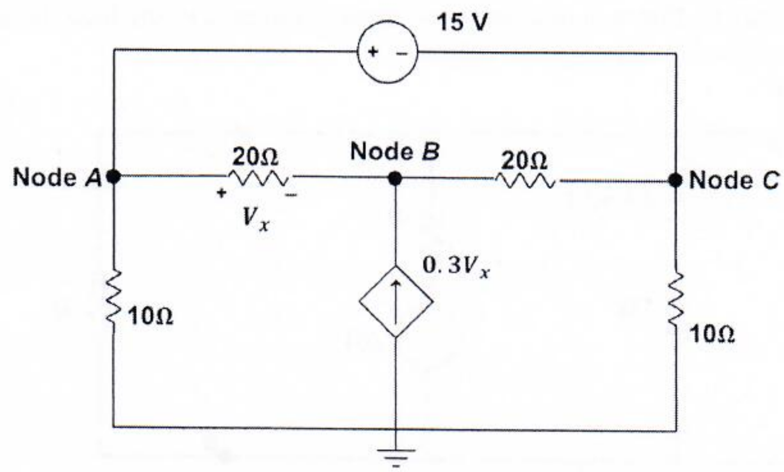
$$2V_A + (V_A - V_B) + (V_C - V_B) + 2V_C = 0$$

$$3V_A - 2V_B + 3V_C = 0$$

$\underline{\ 2\ }\ V_A + \underline{\ -2\ }\ V_B + \underline{\ 3\ }\ V_C = \underline{\ 0\ }$ **(5 points)**

*The blanks above should be integers*

Circuit redrawn here for convenience




**b.** Write the constraint equation for the 15V source

$V_A - V_C = 15$

__1__ $V_A$+ __0__ $V_B$+ __-1__ $V_C$ = __15__ **(5 points)**

*The blanks above should be integers*

**c.** Write KCL at node B.

$$\frac{V_B - V_A}{20} - .3V_x + \frac{V_B - V_C}{20} = 0$$

and $V_x = V_{A-B}$

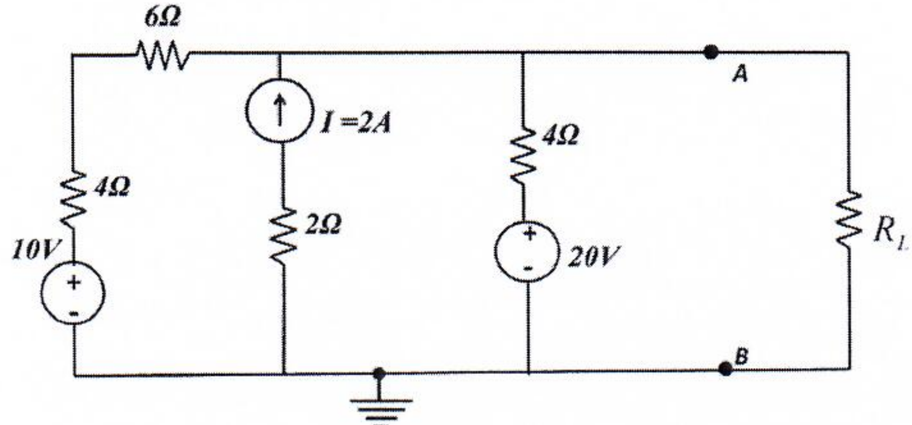
$$\left(\frac{V_B - V_A}{20} - \frac{3}{10}(V_A - V_B) + \frac{V_B - V_C}{20} = 0\right) 20$$

$$V_B - V_A - 6(V_A - V_B) + (V_B - V_C) = 0$$

__-7__ $V_A$+ __8__ $V_B$+ __-1__ $V_C$ = __0__ **(5 points)**

*The blanks above should be integers*

Problem 6. (**20 points total**) For the circuit below, we desire to calculate the value of the load resistor $R_L$ that will maximize the power delivered to the load. In doing this, complete the steps outline below.




**a.** Find the Thevinin equivalent of the circuit at terminals A-B.

Using Nodal Analysis in the ckt with $R_L$ removed or open ckt:

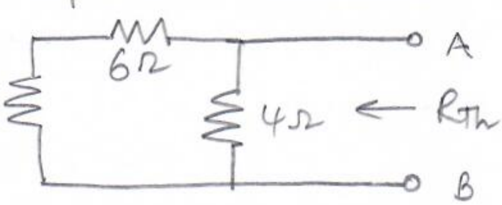
$$-2 + \frac{V_{ab}-10}{10\Omega} + \frac{V_{ab}-20}{4\Omega} = 0$$

$$-2 + \frac{2V_{ab}-20+5V_{ab}-100}{20\Omega} = 0$$

$$7V_{ab} = 160, \quad V_{ab} = \frac{160}{7}\ \text{V}$$

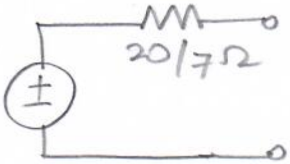
To find $R_{Th}$, we turn off all independent sources:

$$R_{Th} = \frac{10\times 4}{10+4} = \frac{40}{14} = \frac{20}{7}\,\Omega$$




Draw the Thevenin's equivalent circuit here:

$V_{Th} = \frac{160}{7}$V

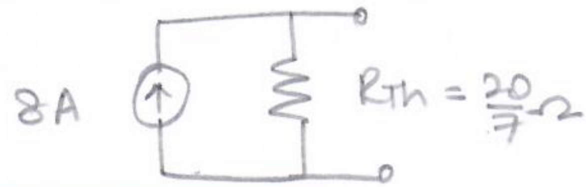



**(10 points)**

**b.** Find the Norton's current and resistance.

$$I_N = \frac{V_{Th}}{R_{Th}} = \frac{160 \times 7}{20 \times 7} = 8\,A$$

Draw the Norton's equivalent circuit here:




**(5 points)**

**c.** Find $R_L$ to extract the maximum power, and also calculate the amount of this maximum power.

For maximum power transfer to load,

$$R_L = R_{Th}$$

$$\Rightarrow R_L = \frac{20}{7}\,\Omega$$

$$P_{max} = \left(\frac{V_{Th}}{(R_{Th}+R_L)}\right)^2 R_L = \frac{V_{Th}^2}{4R_{Th}}$$

$$= \frac{1}{4} \times \frac{160^2}{7^2 \times \frac{20}{7}}$$

$$= 45.715\,W.$$

$R_L =$ <u>20/7 Ω</u>, power= <u>45.715W</u> **(5 points total)**