HW6
Solutions

* 6.8
* 6.13
* 6.22
* 6.42
* 6.47
* 6.76

6.8

(a) $i = C \frac{dv}{dt}$

$i(t) = 0 \qquad t \leq 0$

$i(t) = -100\,ACe^{-100t} - 600\,BCe^{-600t} \qquad t \geq 0$

$i(0) = -100\,AC - 600\,BC = 2A$ (given)

$\because C = 4mF$

$\Rightarrow -A - 6B = 5 \quad — (1)$

Also, $v(0^+) = v(0^-)$

Now, $v(0^-) = 50V$ (given) — (2)

$v(0^+) = Ae^{-100(0)} + Be^{-600(0)}$ (given). — (3)

$= A + B$

$\therefore$ From (2) & (3)

$A + B = 50 \quad — (4)$

Solving (1) and (4)

$A = 61 \quad , \quad B = -11$

(b) Energy $= \frac{1}{2} C \cdot v(0)^2$

$= \frac{1}{2} (4 \times 10^{-3}) (50)^2 = 5J$

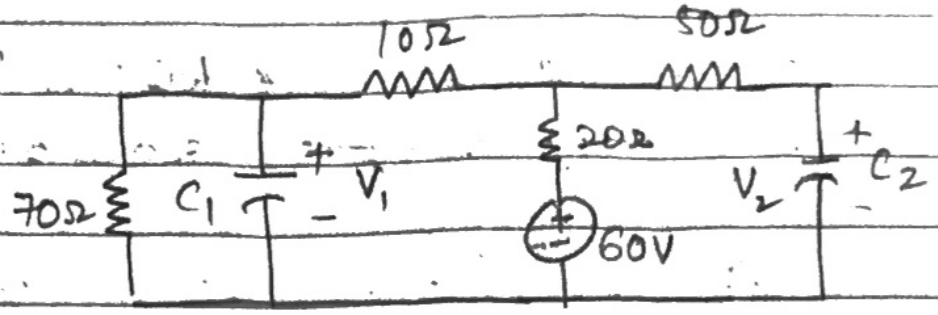
(c)

$$i(t) = -100AC e^{-100t} - 600BC e^{-600t} \quad t \geq 0$$

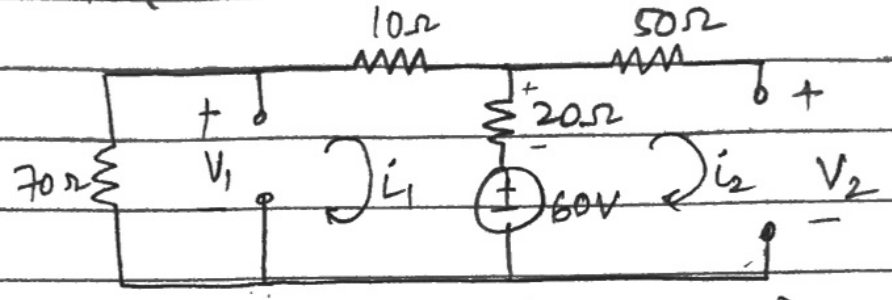
Substituting $C = 4mF$, $A = 61$, $B = -11$

$$i(t) = -24.4 e^{-100t} - 26.4 e^{-600t} \text{ A} \quad t \geq 0.$$

6.B



Under D.C conditions, capacitors are open circuited.



$i_2 = 0$ .... (open ckt) — (1)

Applying KVL in loop 1:

$(70+10+20)\, i_1 + 60 = 0$

$\therefore i_1 = -0.6\,A$ — (2)

V_1: voltage across 70Ω resistor.

$\therefore V_1 = -i_1 \times 70$

$= 42\,V$ (From (2))

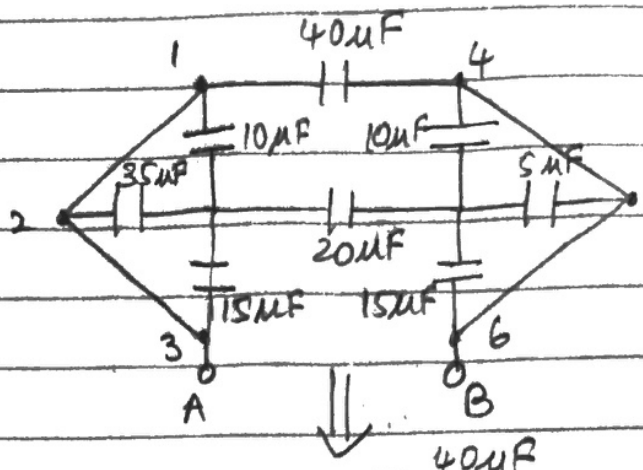
Now, applying KVL in loop 2:

$-60 + 20(\cancel{i_2}^{0} - i_1) + 50\cancel{i_2}^{0} + V_2 = 0$

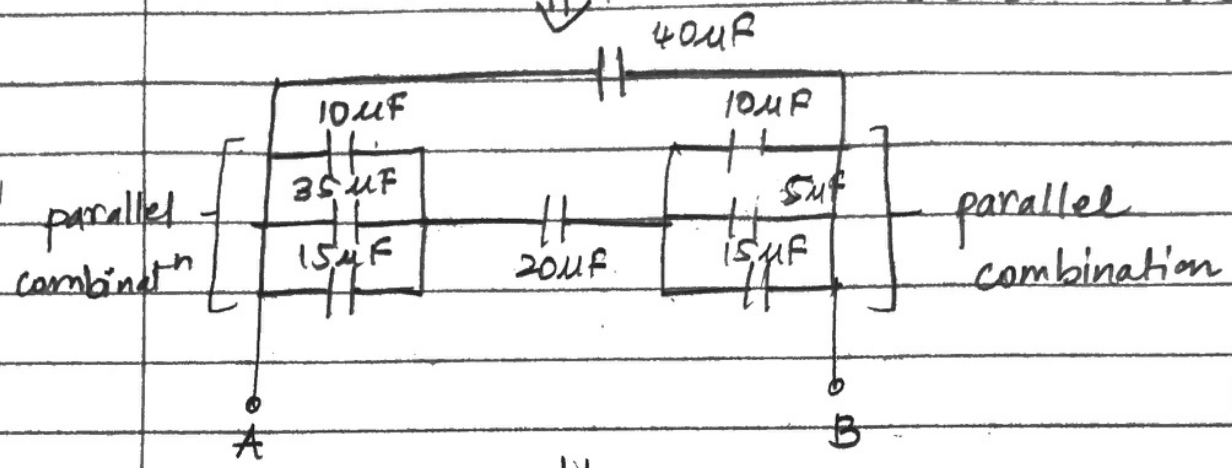
$\Rightarrow V_2 = 60 - 12 = 48V$

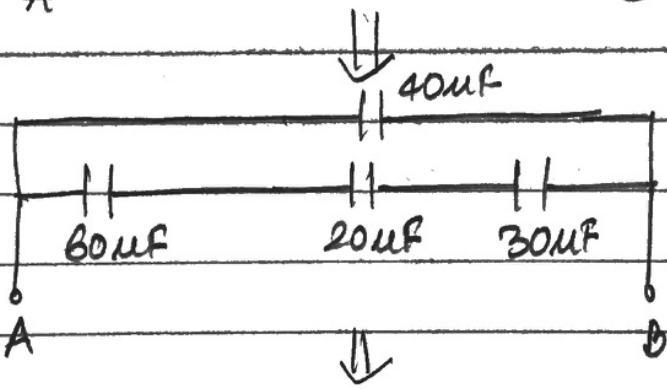
$$\boxed{V_1 = 42V \qquad V_2 = 48V}$$

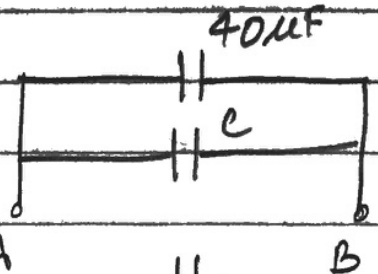
6.22



*Note: 1, 2 and 3 are essentially the same node
ly 4, 5 and 6 are the same node

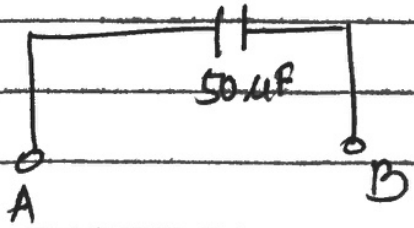






$$C = \left(\frac{1}{60} + \frac{1}{20} + \frac{1}{30}\right)^{-1}$$

$$= 10\,\mu F$$



6.42

Current through the inductor:

$$i_L(t) = \frac{1}{L}\int_{t_0}^{t} v(t)\cdot dt + i_L(0) \quad L = 5H \;\&\; i(0) = -1A$$

$$i_L(t) = \frac{1}{5}\int_0^t v(t)\cdot dt - 1 \qquad \underline{0<t<1}$$

$$= 2\int_0^t dt - 1$$

$$= 2\,[t-0] - 1$$

$$= 2t - 1 \qquad -(1)$$

$$i_L(t) = \frac{1}{5}\int_1^t 0\cdot dt + i_L(1) \qquad \underline{1<t<2}$$

$$= 1A \quad \{\text{From } (1)\ i_L(1) = 1A\} \quad -(2)$$

$$i_L(t) = \frac{1}{5}\int_2^t 10\,dt + i_L(2) \qquad \underline{2<t<3}$$

$$= 2\,[t-2] + 1 \quad \{\text{From } (2)\ i_L(2) = 1A\}$$

$$= 2t - 3 \ A \qquad -(3)$$

$$i_L(t) = \frac{1}{5}\int_3^t 0\cdot dt + i_L(3) \qquad \underline{3<t<4}$$

$$= 3A \quad \{\text{From } (3)\ i_L(3) = 3A\} \quad -(4)$$

$$i_L(t) = \frac{1}{5}\int_4^t 10\,dt + i_L(4) \qquad \underline{4<t<5}$$

$= 2(t-4) + i_L(4)$

$= 2t - 8 + 3$ {From 4, $i_L(4) = 3A$}

$= 2t - 5$ A — (5)

$$i_L(t) = \frac{1}{5}\int_5^t v(t)\,dt + i_L(5) \quad t > 5$$

$$= \frac{1}{5}\int_5^t 0 \cdot dt + 5$$ {From 5, $i_L(5) = 5A$}

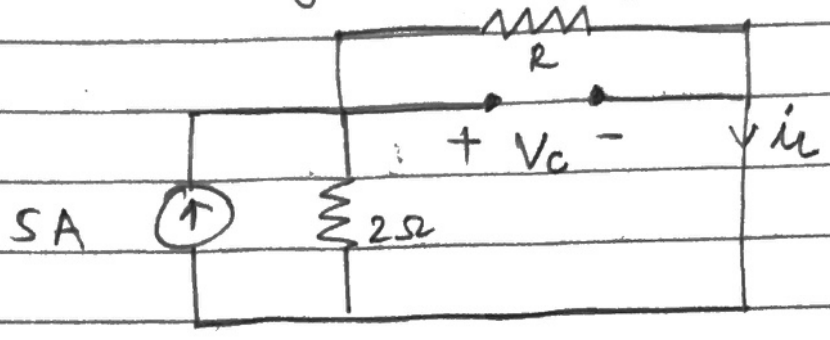
$= 5A \qquad t > 5$

Thus,

$$i_L(t) = \begin{cases} 2t-1 \text{ A} & 0 < t < 1 \\ 1A & 1 < t < 2 \\ 2t-3 \text{ A} & 2 < t < 3 \\ 3A & 3 < t < 4 \\ 2t-5 \text{ A} & 4 < t < 5 \\ 5 \text{ A} & 5 < t \end{cases}$$

6.47

Under D.C. conditions, the capacitor is an open ckt, while the inductor is shorted.
∴ Resulting ckt diagram:



Energy stored in capacitor $(W_C) = \frac{1}{2} C V_C^2 = 80 V_C^2$ μJ

Energy stored in inductor $(W_L) = \frac{1}{2} L i_L^2$

$= 2 i_L^2$ mW.

Now if

$W_C = W_L$

$\Rightarrow \frac{40 V_C^2}{1000} = i_L^2$

$\Rightarrow \frac{V_C}{5} = i_L$

$\Rightarrow V_C = 5 i_L$ — (1)

Now by inspection $V_C = R i_L$ — (2)

From (1) & (2)

$R = 5\Omega$.

6.76

For a differentiator:

$$v_o = -RC \frac{dv_i}{dt}$$

Here $RC = 50 \times 10^3 \times 10 \times 10^{-6}$

$= 0.5s$

$\therefore v_o = -0.5 \frac{dv_i}{dt}$.

From given data,

Slope $= \frac{dv_i}{dt} = 20$ V/s $\quad 0 < t < 5$ ms

$= -10$ V/s $\quad 5$ ms $< t < 15$ ms.

$\therefore v_o = -10$ V $\quad 0 < t < 5$ ms

$= 5$ V $\quad 5$ ms $< t < 15$ ms.