HW 8

* 8.5
* 8.16
* 8.24
* 8.29 (a)
* 8.33
* 8.49
* 8.55

8.5

(a) $\because$ Voltage across a capacitor and current through an inductor don't change instantaneously

$$v_c(0^-) = 0 = v_c(0^+)$$

$$i_L(0^-) = 0 = i_L(0^+)$$

$\because$ Voltage across 4Ω resistor $(V_R) = 4i = v_c(0^+)$
$= 0\,V$

$\therefore$ $i(0^+) = 0A$

Also, $\because$ current through the inductor
$i_L(0^+) = 0A.$

$\therefore$ $V = 6 i_L(0^+) = 0V.$

(b) $$\frac{di(0^+)}{dt} = \frac{1}{R}\frac{d}{dt}V_R(0^+) = \frac{1}{4}\frac{d}{dt}v_c(0^+)$$

$$= \frac{1}{4}\frac{i_c(0^+)}{C}$$

$$= \frac{1}{4}\frac{4}{(1/4)} = 4\,A/s$$

$$\frac{dv(0^+)}{dt} = 6\frac{d}{dt}i_L(t) = \frac{6\,V_L(0^+)}{L}$$

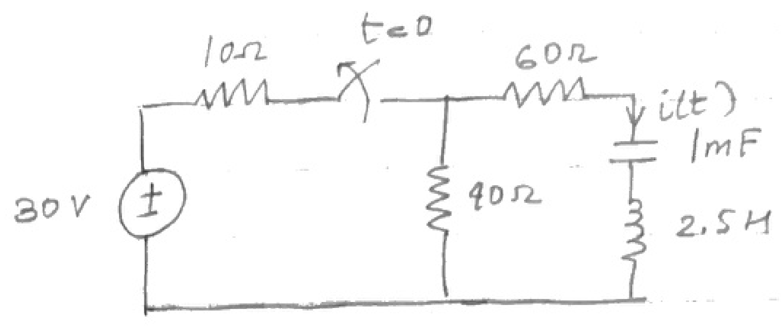
$$= 0.\ V/s.$$

(c) At steady state, the capacitor is an open ckt and inductor is a short ckt.

$$i(\infty) = \frac{6}{10} \times 4 = 2.4\,A$$

$$v(\infty) = \frac{4}{10} \times 4 \times 6 = 9.6\,V$$

8.16



At $t=0$, inductor is a short ckt, while the capacitor is an open ckt.

$\therefore\ i(0^-) = i(0^+) = 0A$ — (1)

and $v_c(0^-) = v_c(0^+) = \frac{40}{50} \times 30 = 24V$ — (2)

Now, $\alpha = \frac{R}{2L} = \frac{100}{2\times(2.5)} = 20$

$\omega_d = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-4}\times 25}} = \frac{1}{0.05} = 20.$

$\therefore\ \alpha = \omega_d \Rightarrow$ critically damped

$\therefore\ i(t) = (A+Bt)e^{-20t}$ — (3) (for a critically damped system)

$i(0) = A$ (from (3))

$= 0$ (from (1))

— (4)

$\therefore$ From (3) & (4)

$i(t) = Bt\,e^{-20t}$ — (5)

$\Rightarrow \frac{d}{dt} i(t) = Be^{-20t} - 20Bt\,e^{-20t}$

$\Rightarrow \frac{d}{dt} i(0) = B.$ — (6)

Also,

$$\frac{di_L(0)}{dt} = \frac{di(0)}{dt} = \frac{-1}{L}\left[100\, i(0) + v_c(0)\right]$$

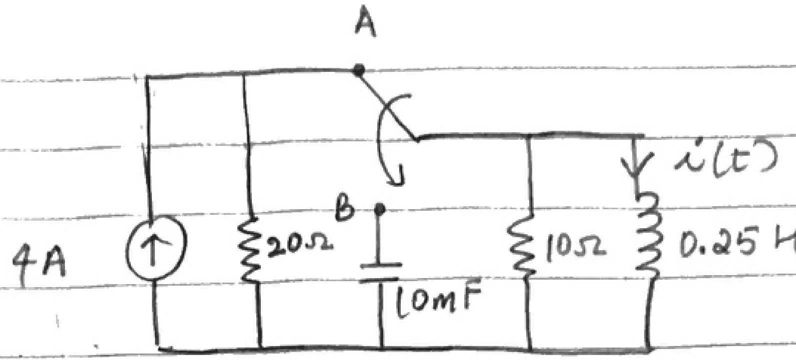
$$= \frac{-1}{2.5}\left[0 + 24\right] \quad [\text{From } 2]$$

$$= -9.6\,V. \quad - 7$$

From 5 & 7 :

$$i(t) = -9.6\, t\, e^{-20t} \ A$$

8.24



When switch is at A, the inductor is shorted,

$\therefore i_L(0^-) = 4A$ and $v_L(0^-) = 0V$ — (1)

When switch is at B, there is a parallel RLC ckt:

$$\alpha = \frac{1}{2RC} = \frac{1}{10 \times 10^{-3} \times 10 \times 2} = 5$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-2} \times 0.25}} = 20$$

$\because \alpha < \omega_0$, for an underdamped system:

$S_{1,2} = -5 \pm j\,19.37$

$\therefore i(t) = e^{-5t}(A_1 \cos 19.37t + A_2 \sin 19.37t)$ — (2)

Using initial values,

$i(0) = A_1$ (From (2))

$= 4$ (From (1))

$\Rightarrow A_1 = 4.$ — (3)

Also,

$$V = L \frac{d\, i_L(0)}{dt}$$

$$\Rightarrow \frac{di_L(0)}{dt} = 0 \qquad (\text{From } 1) \quad - 4$$

Now, $$\frac{di}{dt} = -5e^{-5t}(A_1 \cos 19.37t + A_2 \sin 19.37t) + e^{-5t}(-A_1(19.37)\sin 19.37t + A_2(19.37)\cos 19.37t)$$

$$\therefore \frac{di(0)}{dt} = -5(A_1) + (19.37 A_2)$$

$$= 0 \qquad (\text{From } 4)$$

$$\therefore A_2 = \frac{20}{19.37} = 1.03 \quad - 5$$

$\therefore$ From 2, 3 & 5 :

$$i(t) = e^{-5t}[4\cos(19.37t) + 1.03\sin(19.37t)]$$

8.29(a) $\frac{d^2v}{dt^2} + 4v = 12$

Characteristic eq^n : $s^2 + 4s = 0$

$\therefore \quad \alpha = 0$

$\omega_0 = 2.$

$\therefore S_{1,2} = \pm 2j$

$\Rightarrow v_{nt}(t) = A\cos 2t + B\sin 2t$

Under steady stati cond^n :

$4 V_{ss}(t) = 12$

$\therefore v_{ss}(t) = 3 + A\cos 2t + B\sin 2t$

$\therefore \quad v(t) = 3 + A \qquad + Be^{-j2t}$

Now using intial cond^n :

(1) $v(0) = 3 + A = 0$

$\Rightarrow A = -3$

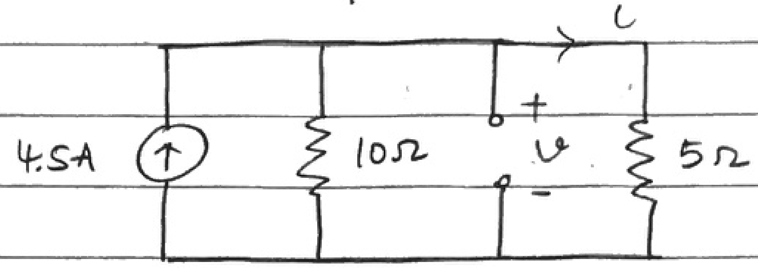
(2) $\frac{d\,v(0)}{dt} = -2A\sin(2\times 0) + 2B\cos(2\times 0)$
$= 2$

$\Rightarrow B = 1$

$\therefore$ Complete eq^n : $3 - 3\cos 2t + \sin 2t$

8.32

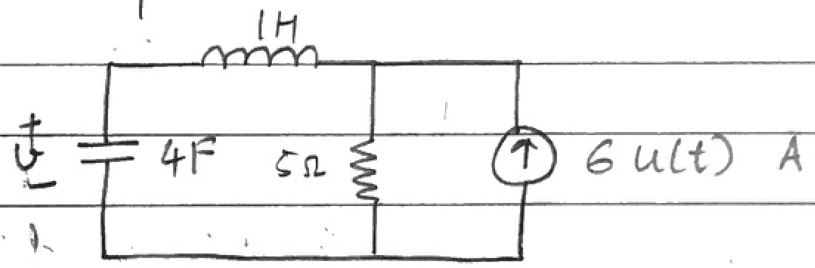
For $t<0$: equivalent ckt :



$\therefore \; v(0) = (10 \parallel 5) \times 4.5 = 15\,V$ — (1)

$i(0) = \frac{2}{3} \times 4.5 = 3\,A$

For $t>0$: equivalent ckt :



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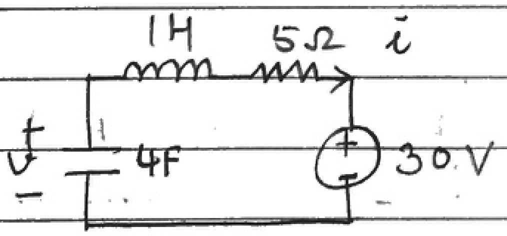


(Series RLC)

$$\alpha = \frac{R}{2L} = \frac{5}{2} = 2.5$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 0.5$$

$\therefore \; \alpha > \omega_0 \Rightarrow$ overdamped response.

$$S_{1,2} = -2.5 \pm \sqrt{2.5^2 - 0.5^2}$$
$$= -2.5 \pm 2.44$$
$$= -4.95, -0.05$$

$$v(t) = V_s + \left[A_1 e^{-4.95t} + A_2 e^{-0.05t}\right]$$

At steady state :

$$v(\infty) = V_s = 30V$$

$$\therefore v(t) = 30 + \left[A_1 e^{-4.95t} + A_2 e^{-0.05t}\right] \quad -(2)$$

Using Initial condⁿ :

* $v(0) = 30 + A_1 + A_2 = 15$ (from (1))

$$\therefore A_1 + A_2 = -15 \quad -(3)$$

* $i(t) = -C\dfrac{dv(t)}{dt} \Rightarrow \dfrac{dv(0)}{dt} = \dfrac{3}{4} = 0.75$

(From (1))

Also $\dfrac{dv(0)}{dt} = -4.95A_1 - 0.05A_2$ (From (2))

$$\therefore 4.95A_1 + 0.05A_2 = 0.75 \quad -(4)$$

Solving (3) & (4) :

$A_1 = 0.306$ and $A_2 = -15.306$ $\quad -(5)$

$\therefore$ From (2) & (5) :

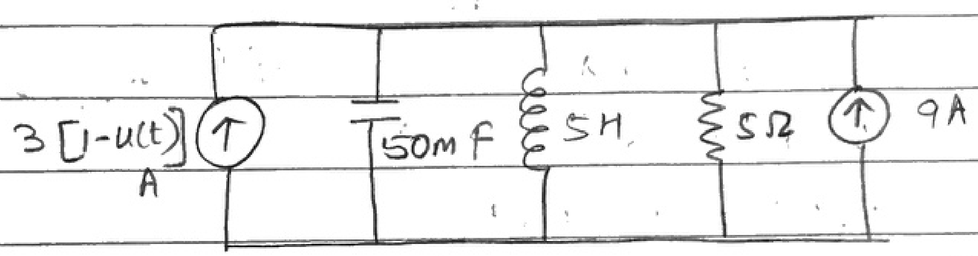
$$v(t) = 30 + \left[0.306e^{-4.95t} - 15.306e^{-0.05t}\right] \text{ V}$$

8.49

For $t<0$, capacitor and inductor are open and short circuits respectively.

$$\therefore \left.\begin{aligned} i(0^-) &= 3A + (45/5)A = 12A \\ v(0^-) &= 0V \end{aligned}\right] - (1)$$

For $t>0$ equivalent ckt :



This is a parallel RLC circuit :

$$\alpha = \frac{1}{2RC} = \frac{1}{(2\times5\times50\times10^{-3})} = 2$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5\times50\times10^{-3}}} = 2.$$

$\Rightarrow \alpha = \omega_0 \Rightarrow$ critically damped.

$\therefore s_{1,2} = -2$

For a critically damped system :

$$i(t) = i_{ss}(t) + (A_1 + A_2 t)e^{-\alpha t} \quad - (2)$$

For steady state response :

the inductor is a short, all the 9A flows through the inductor.

$$i_{ss} = 9A \quad - (3)$$

$\therefore$ From (2) & (3) :

$$i(t) = 9 + (A_1 + A_2 t) e^{-2t} \quad -(4)$$

Using Initial Conditions :

$t = 0$

* $i(0) = 9 + A_1$ (from (4))

$= 12A$ (From (1))

$\therefore A_1 = 3$ — (5)

* $V(0) = L \left.\frac{di(t)}{dt}\right|_{t=0}$

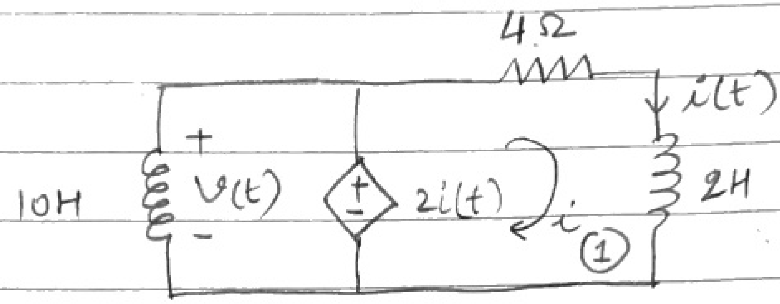
$\therefore \frac{V(0)}{L} = -2(A_1) + A_2$ (From (4))

$= 0$ (From (1))

$\therefore A_2 = 6$ (From (5))

$$\therefore \boxed{i(t) = 9 + (3 + 6t) e^{-2t}}$$

8.55



Applying KVL around loop 1:

$$-2i(t) + 4i(t) + 2\frac{di(t)}{dt} = 0$$

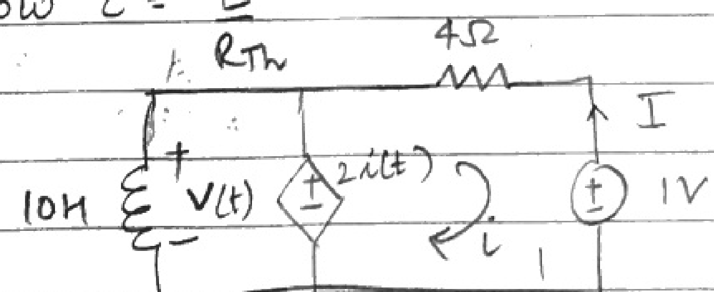
$$\Rightarrow \frac{di(t)}{dt} = -i(t)$$

∴ This is a first order differential eqⁿ:

General solⁿ: $i(t) = \underline{A} + Be^{-t/\tau}$ A — (1)

Now $\tau = \frac{L}{R_{Th}}$

R_{Th}:



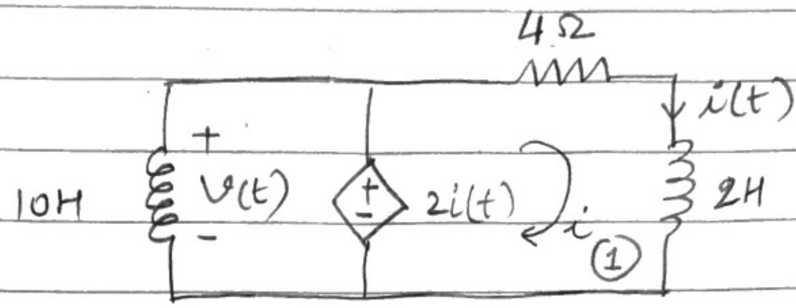
Applying KVL in loop 1:

$$-2i(t) + 4i(t) + 1 = 0$$

$$\Rightarrow i(t) = -\frac{1}{2} = -I$$

$$\therefore R_{Th} = \frac{1V}{I} = 2\Omega$$

8.55



Applying KVL around loop 1:

$$-2i(t) + 4i(t) + 2\frac{di(t)}{dt} = 0$$

$$\Rightarrow \frac{di(t)}{dt} = -i(t).$$

$\therefore$ This is a first order differential eq^n:

General sol^n: $i(t) = i(0)\,e^{-t}$ A — (1)

$\because$ $i(0) = 2A$

$i(t) = 2e^{-t}$

$v(t) = 2i(t)$

$= 4e^{-t}$