Your name: ~~Partial~~ Solution Key

EECS 215.
Final Exam
April 25, 2016

This text consists of 8 problems with points as indicated to total 90 points.
Please note that Laplace tables are attached at the end of the exam.

Read through the entire exam before beginning.
Show all work (on the pages provided in this booklet) to earn partial credit.

No credit will be given if no work is shown.

Briefly explain major steps, include units, and write your final answers in the areas provided. Do not unstaple the pages.

Exam policies

- The College of Engineering Honor Code is followed. Please write and sign the honor code pledge ("I have neither given nor received unauthorized aid on this examination, nor have I concealed any violations of the Honor Code.") in the box below.
- Only scientific calculators are allowed – no exceptions.
- No communication of any kind is allowed. No use of cell phones, computers, or any devices besides *scientific* calculators.
- Three sides of 8.5x11 inch notes pages are allowed. No books allowed (closed book exam).

Write and sign the honor pledge:

Signed:

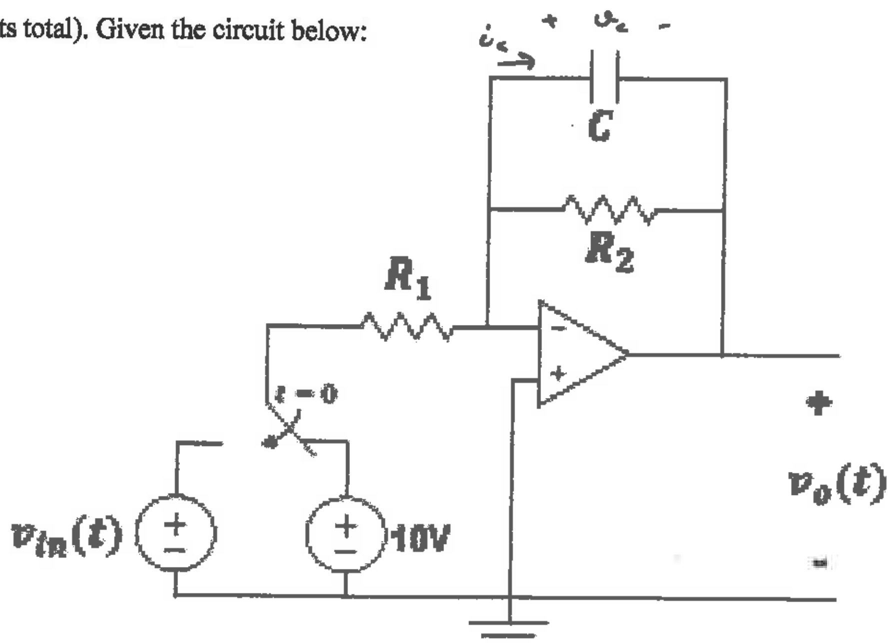
No credit will be given for this exam without a signed honor pledge.

Do not write in this space

Problem 1: [ ]/10	Problem 5: [ ]/5
Problem 2: [ ]/10	Problem 6: [ ]/15
Problem 3: [ ]/15	Problem 7: [ ]/10
Problem 4: [ ]/10	Problem 8: [ ]/15

Total score [ ]/90

1. (10 points total). Given the circuit below:



a. (2 points). Assuming switch has been connected to the 10V source for a long time prior to switching at $t = 0$, determine the initial value for the voltage $v_o(0^+)$.

b. (8 points). Derive (but do not solve) the differential equation describing $v_o(t), t \geq 0$.

Part a)

The capacitor looks like an open-circuit in steady-state.

Therefore, at $t = 0^-$

$$\frac{0-10}{R_1} + \frac{v_c(0^-)}{R_2} = 0$$

$$\Rightarrow v_c(0^-) = \frac{R_2}{R_1} \times 10 = v_c(0^+)$$

Also, $v_c = -v_o \Rightarrow v_c(0^+) = -v_o(0^+)$

$$\Rightarrow v_o(0^+) = -\frac{R_2}{R_1} \times 10 \quad \text{volts}$$

Write your answer here:

a. $v_o(0^+) =$ $-\frac{R_2}{R_1} \times 10$ volts

b. Differential equation for $v_0(t)$: $\frac{dv_o}{dt} + \frac{1}{R_2 C} v_o = -\frac{1}{R_1 C} v_{in}(t)$

Problem 1 score: [ ]/10

Part b)

KCL at the op-amp − terminal, for $t \geq 0$

$$\frac{0 - v_{in}(t)}{R_1} + \frac{0 - v_o(t)}{R_2} + i_c = 0$$

Also,

$$i_c = C\frac{dv_c}{dt} = -C\frac{dv_o}{dt}$$

Therefore,

$$\frac{-v_{in}(t)}{R_1} + \frac{-v_o(t)}{R_2} - C\frac{dv_o(t)}{dt} = 0$$

$$\Rightarrow \quad C\frac{dv_o}{dt} + \frac{v_o}{R_2} = -\frac{v_{in}(t)}{R_1}$$

$$\Rightarrow \quad \frac{dv_o}{dt} + \frac{1}{R_2 C} v_o = -\frac{1}{R_1 C} v_{in}(t)$$

2. (10 points total). A circuit voltage is governed by the following differential equation and initial conditions:

$$\frac{d^2v}{dt^2} + 4\frac{dv}{dt} + 3v = -10e^{-5t}, \qquad v(0) = 3, \qquad \left.\frac{dv}{dt}\right|_{t=0} = v'(0) = 5$$

a. (8 points). Use the Laplace transform method to solve for $V(s)$. Express your answer as the ratio of two polynomials.

b. (2 points). Is this circuit critically damped, underdamped, or overdamped?

a) $s^2V(s) - s\,\overset{3}{v(0)} - \overset{5}{v'(0)} + 4\left(sV(s) - \overset{3}{v(0)}\right) + 3V(s) = \frac{-10}{s+5}$

$(s^2+4s+3)V(s) - 3s - 5 - 12 = -10/s+5$

$$V(s) = \frac{\frac{-10}{s+5} + \frac{(3s+17)(s+5)}{s+5}}{s^2+4s+3}$$

$$\boxed{V(s) = \frac{3s^2+32s+75}{(s^2+4s+3)(s+5)}}$$

b) $\alpha = \frac{4}{2} = 2$, $\omega_0 = \sqrt{6} = \sqrt{3}$; $\alpha > \omega_0 \Rightarrow$ $\boxed{\text{overdamped}}$

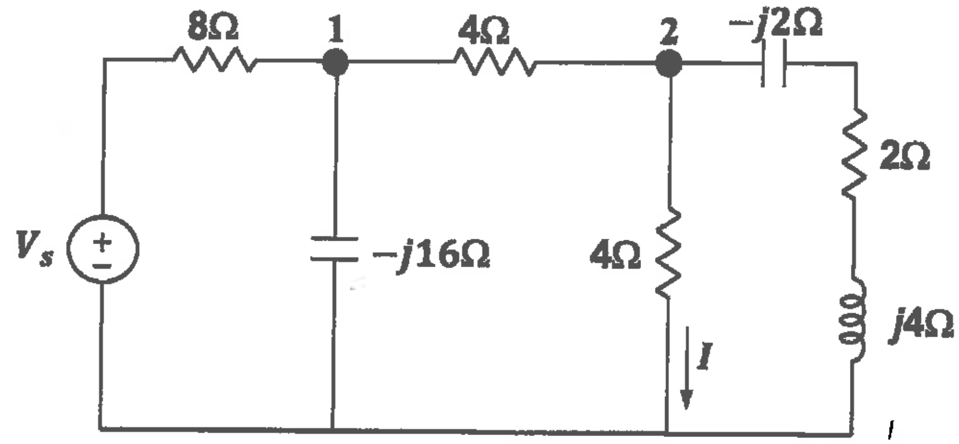
Write your answer here:

a. $V(s) =$ ______________________

b. Which type of damping (circle one): critically damped, underdamped, overdamped

Problem 2 score: [ ]/10

3. (15 points total). Given the circuit below:



a. (9 points). Establish the nodal equations for nodes 1 and 2, and express the equations in matrix form. Simplify the equations so that each element of the matrix is a complex number in rectangular form. (See part b for an example of the required form.)

b. (6 points). The nodal matrix equation for the same circuit but with some different component values is given below. (The 4Ω resistor from node 2 to ground is unchanged from the above figure.) Find the voltage at node 1 (V_1) when the 4Ω resistor has a current $I = 3\angle 45°$ A.

$$\begin{bmatrix} 3+2j & -5 \\ -2-3j & 1+2j \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} (5+2j)V_S \\ 0 \end{bmatrix}$$

Part a,

KCL node 1,

$$\frac{V_1 - V_s}{8} + \frac{V_1}{-j16} + \frac{V_1 - V_2}{4} = 0$$

$$\Rightarrow V_1\left(\frac{1}{8} + j\frac{1}{16} + \frac{1}{4}\right) - \frac{1}{4}V_2 = \frac{1}{8}V_s$$

Multiply throughout by 16,

$$\Rightarrow V_1(2 + j + 4) - 4V_2 = 2V_s$$

$$\Rightarrow V_1(6 + j) - 4V_2 = 2V_s$$

Write your answer here:

a. Please clearly circle your solution, written in matrix form, above or on the next page.

b. $V_1 =$ $7.4421 \angle 52.125°$ Volts

Problem 3 score: [ ]/15

KCL node 2,

$$\frac{V_2 - V_1}{4} + \frac{V_2}{4} + \frac{V_2}{2+j2} = 0$$

$$\Rightarrow \frac{V_2 - V_1}{4} + \frac{V_2}{4} + \frac{V_2(2-j2)}{8} = 0$$

Multiply throughout by 4,

$$\Rightarrow V_2 - V_1 + V_2 + V_2(1-j) = 0$$

$$\Rightarrow -V_1 + V_2(3-j) = 0$$

In matrix form,

$$\begin{bmatrix} 6+j & -4 \\ -1 & 3-j \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2V_s \\ 0 \end{bmatrix}$$

<u>Part b</u>

$$V_2 = 4I = 12 \angle 45^\circ$$

$$(-2-3j)V_1 + (1+2j)V_2 = 0$$

$$\Rightarrow V_1 = \frac{1+2j}{2+3j} \times V_2 = \frac{(1+2j)(2-3j)}{4+9} \times V_2 = \frac{2+6+j(4-3)}{13} \times V_2$$

$$= \frac{8+j}{13} \times 12 \angle 45^\circ$$

$$= \frac{\sqrt{65} \times 12}{13} \angle 45^\circ + \tan^{-1}\left(\frac{1}{8}\right)$$

$$= 7.4421 \angle 52.125^\circ \text{ Volts}$$

4. (10 points total). An inductive load connected to a 220 V power supply draws a current of 7.6 A. (Both voltage and current are RMS values.) The average power delivered to the load is 1317 W.

   a. (4 points). Find the apparent power, the reactive power, and the power factor of the load. (Don't forget to specify whether the power factor is leading or lagging.)

   b. (4 points). The system frequency is $\omega = 377$ rad/sec. Determine the capacitance of a parallel capacitor that will result in a power factor of 1 (unity power factor) for the combined load and capacitor.

   c. (2 points). What is the value of the current drawn from the power supply after the capacitor is installed?

Part a

Apparent power, $|S| = |V| \times |I| = 220 \times 7.6 = 1672$ VA

Power factor, $pf = \frac{P}{|S|} = \frac{1317}{1672} = 0.7877$

- the load is inductive so the power factor is lagging

Reactive power: $|S|^2 = P^2 + Q^2$

$$\Rightarrow Q = \sqrt{|S|^2 - P^2} = \sqrt{1672^2 - 1317^2} = 1030.1 \text{ VAr}$$

Part b

To achieve unity power factor, the capacitor must fully compensate the reactive power drawn by the load. Therefore,

$$Q_{cap} = \frac{|V|^2}{X_c} = 1030.1 \quad \Rightarrow X_c = \frac{220^2}{1030.1} = 47\,\Omega$$

$$X_c = \frac{1}{\omega C} \Rightarrow C = \frac{1}{\omega X_c}$$

$$= \frac{1}{377 \times 47} = 56.45\,\mu F$$

Part c

In this case, $|S| = P = |V| \times |I|$

$$\Rightarrow |I| = \frac{P}{|V|} = \frac{1317}{220} = 6.0 \text{ Amps}$$

Write your answer here:

a. Apparent power: 1672 VA
   Reactive power: 1030.1 VAr
   Power factor: 0.7877 lagging

b. Capacitance: 56.45 μF

c. Current: 6.0 Amps

Problem 4 score: [ ]/10

5. (5 points total). A load impedance with $S = 500 - j200$ VA is supplied from a voltage source $V_s = 10\angle 20^o$ volts (RMS).

   a. (2 points). Compute the current drawn by the load, expressed as an RMS phasor in polar form.
   b. (2 points). Determine the apparent power and the power factor of the load.
   c. (1 points). Sketch the complex power triangle, clearly labelling P, Q and S.

Part a

$$S = VI^* \Rightarrow I = \left(\frac{S}{V}\right)^* = \frac{500 + j200}{10\angle -20} = \frac{538.5165\angle 21.80^\circ}{10\angle -20^\circ}$$

$$\Rightarrow I = 53.8516\angle 41.80^\circ \text{ Amps}$$

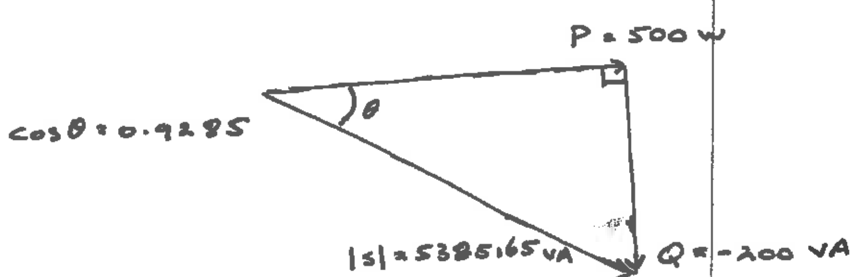
Part b

Apparent power $= |S| = 538.5165$ VA (from above).

Power factor $= \frac{P}{|S|} = \frac{500}{538.5165} = 0.9285$ leading

(Power factor is leading because the current phasor leads the voltage phasor.)

Part c



Write your answer here:
a. Current: $53.8516\angle 41.80^\circ$ Amps
b. Apparent power: 538.5165 VA
Power factor: 0.9285 leading
c. Please circle your clearly labeled power triangle above or on the next page.
Problem 5 score: [ ]/5

6. (15 points total). A circuit is characterized by the following transfer function:

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{5s + 2}{(s^2 + 2s + 1)} = \frac{5s+2}{(s+1)^2}$$

Apply partial fraction expansion to find the output $v_o(t)$ if $v_i(t) = 10\,u(t)$ Volts. $\Rightarrow \quad V_i(s) = \frac{10}{s}$

$$V_o(s) = \frac{10(5s+2)}{s(s+1)^2} = \frac{A}{s} + \frac{B}{(s+1)^2} + \frac{C}{s+1}$$

$$A = 10\left(\frac{5s+2}{(s+1)^2}\right)\Bigg|_{s=0} = 20$$

$$B = 10\left(\frac{5s+2}{s}\right)\Bigg|_{s=-1} = \frac{-3}{-1} = 30$$

$$C = 10\left(\frac{d}{ds}\left(\frac{5s+2}{s}\right)\Bigg|_{s=-1}\right) = 10\left(\frac{s5-(5s+2)}{s^2}\right)\Bigg|_{s=-1} = 10\left(\frac{-5-(-5+2)}{1}\right)$$

$$= -20$$

$$v_o(t) = \mathcal{L}^{-1}\left\{\frac{20}{s} + \frac{30}{(s+1)^2} + \frac{-20}{s+1}\right\}$$

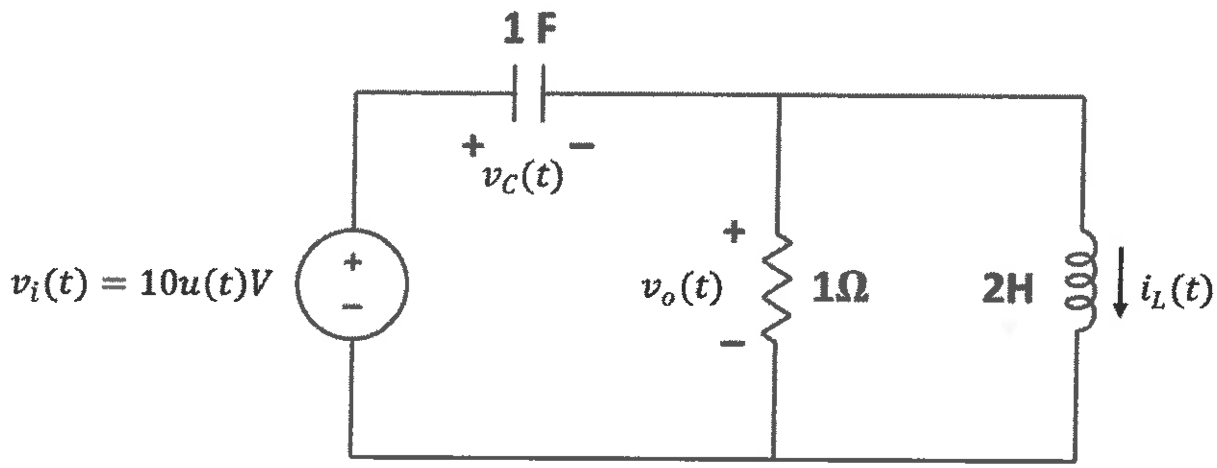
$$\boxed{v_o(t) = \left(20 + 30t\,e^{-t} - 20\,e^{-t}\right)u(t)}$$

Write your answer here:

$v_o(t) =$ ____

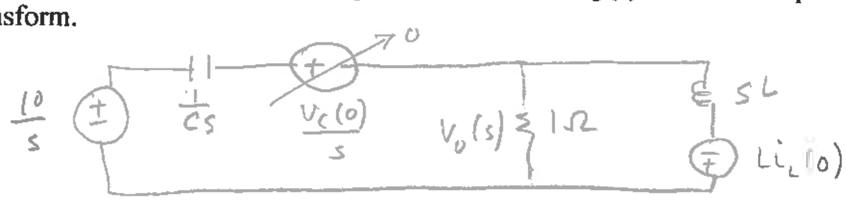
Problem 6 score: [ ]/15

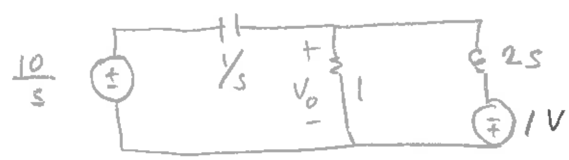
7. (10 points). Given the circuit below with initial conditions $v_C(0) = 0$ and $i_L(0) = \frac{1}{2}A$.



a. (3 points). Draw the s-domain circuit.

b. (7 points). Solve for $V_o(s)$, the Laplace transform of $v_o(t)$. Do not compute the inverse Laplace transform.





KCL

$$\frac{V_o - 10/s}{1/s} + \frac{V_o}{1} + \frac{V_o + (1)}{2s} = 0$$

$$2s\left(sV_o - 10 + V_o + \frac{1}{2s}V_o + \frac{1}{2s} = 0\right)$$

$$(2s^2 + 2s + 1)V_o = 20s - 1$$

$$\boxed{V_o(s) = \frac{20s - 1}{2s^2 + 2s + 1}}$$

Write your answer here:

a. Please circle the s-domain circuit above or on the next page

b. $V_o(s)$ = ______________________________

Problem 7 score: [ ]/10

8. (15 points total). A circuit has a voltage transfer function as follows:

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{500s}{s^2 + 1000s + 250{,}000}$$

$H(jw) = \frac{500jw}{1000jw + 250{,}000 - w^2}$

a. (2 points). What is the resonant frequency ω_o for the filter?

b. (2 points). What is M_o, the magnitude of the transfer function at the resonant frequency?

c. (3 points). Write $H(s)$ in standard form for Bode plots.

d. (5 points). Sketch the straight-line approximation to the magnitude response using the semilog paper on the next page.

e. (2 points). Indicate the location of ω_o on the Bode plot.

f. (1 point). What type of filter does this circuit represent?

a) w_o when $|H(jw)|$ real ⇒ when denominator purely imag.

$w_o^2 = 250{,}000 \Rightarrow \boxed{w_o = 500 \frac{rad}{sec}}$

b) $M_o = \left| \frac{500j(500)}{1000j(500)} \right| = \boxed{\frac{1}{2}}$

c) $H(s) = \frac{500s}{250{,}000\left(1 + 2j\underbrace{\zeta}_{\zeta=1}\frac{s}{500} + \left(\frac{s}{500}\right)^2\right)} = \frac{s/500}{(1 + s/500)^2}$

• zero at origin, $k = 500 \Rightarrow$

mag = straight line, slope = 20dB/dec crossing origin at $\omega = 500$

20dB/dec

w=500

• double pole at $w_c = 500 \Rightarrow$

mag = flat until $w = 500$, then straight line slope = −40dB/dec

500

−40dB/dec

d,e) see attached

f) bandpass

Write your answer here:

a. $\omega_o =$ ______

b. $M_0 =$ ______

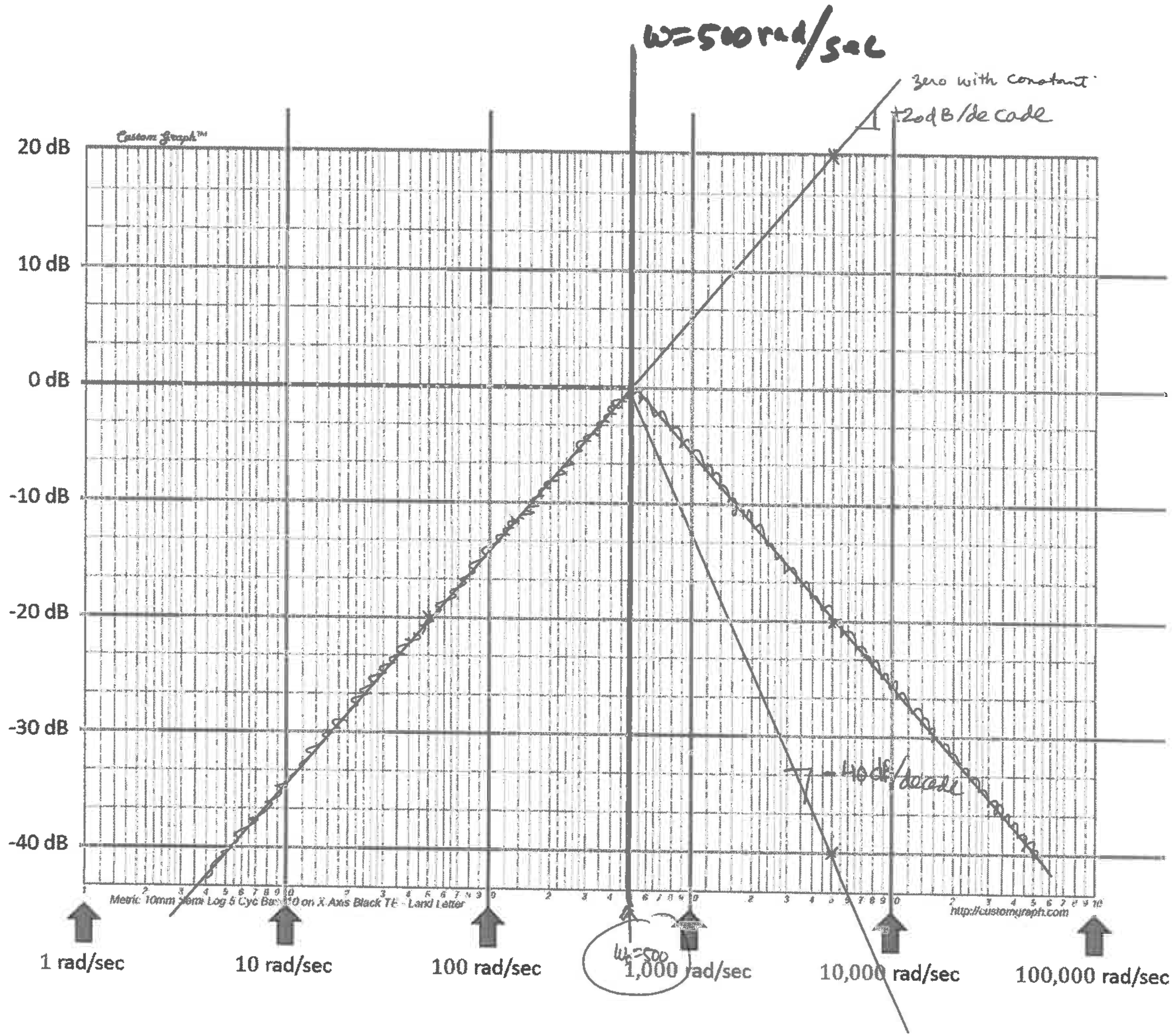
c. $H(s) =$ ______

d. Sketch the magnitude plot on the next page

e. Label ω_o on the sketch.

f. Filter type: ______

Problem 8 score: [ ]/15


w=500 rad/sec
zero with constant
+20dB/decade
-40dB/decade
w=500
Custom Graph™
20 dB
10 dB
0 dB
-10 dB
-20 dB
-30 dB
-40 dB
1 rad/sec
10 rad/sec
100 rad/sec
1,000 rad/sec
10,000 rad/sec
100,000 rad/sec
http://customgraph.com