<u>EECS 215 Equation and Procedure Sheet</u>

First/Second Order Circuits

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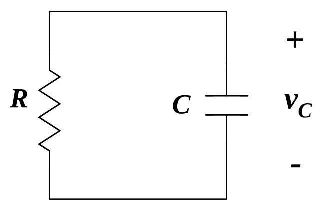
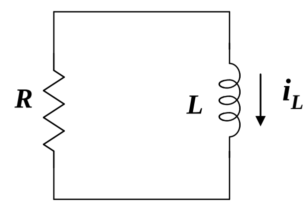
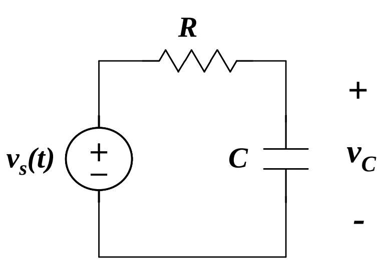
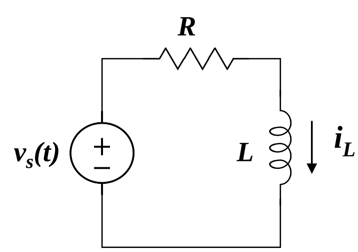
<u>First Order Circuits</u>

Source-free RC (v_s=0)	**Source-free RL (v_s=0)**
	
$v_C(t) = v_C(0)e^{-t/RC} = v_C(0)e^{-t/\tau}$	$i_L(t) = i_L(0)e^{-tR/L} = i_L(0)e^{-t/\tau}$

Step response RC (v_s=V_S *u(t)*)	**Step response RL (v_s=V_S *u(t)*)**
	
$v_C(t) = V_S + \left(v_C(0) - V_S\right)e^{-t/RC}$	$i_L(t) = \frac{V_S}{R} + \left(i_L(0) - \frac{V_S}{R}\right)e^{-tR/L}$

General procedure for first order circuits:

1) Find Thevenin/Norton equivalent

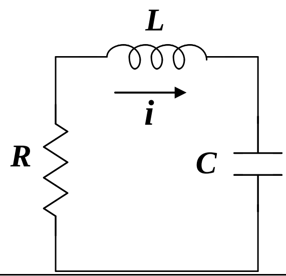
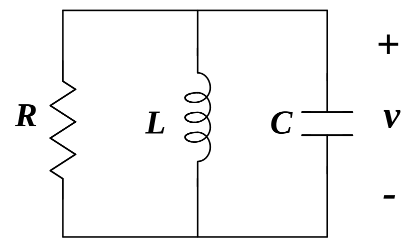
2) Write differential equation for simple circuit – variable on left side of equation and forcing function on right.

3) Determine natural solution (forcing function = 0). Common trial function is Ke^{st}, determine s by inserting into differential equation.

4) Determine forced solution. Use trial function that resembles forcing function. Insert trial function into differential equation to determine unknown constants. If the differential equation does not work, use a different trial function.

5) Write complete solution (natural + forced). Apply initial/boundary condition to determine unknown constant (for example, the value K in the natural solution)

Second Order Circuits

Source-free Series RLC (v_s=0) **(or natural solution for series RLC)**	**Source-free Parallel RLC (v_s=0)** **(or natural solution for parallel RLC)**
	
Differential Equation: $\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$	Differential Equation: $\frac{d^2 v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{v}{LC} = 0$
Characteristic Equation: $s^2 + s\frac{R}{L} + \frac{1}{LC} = 0$	Characteristic Equation: $s^2 + s\frac{1}{RC} + \frac{1}{LC} = 0$
Roots: $s_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$ $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$ $\alpha = \frac{R}{2L} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$	Roots: $s_{1,2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$ $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$ $\alpha = \frac{1}{2RC} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$
Solutions: Overdamped, $\alpha > \omega_0$ $i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ Critically damped, $\alpha = \omega_0$ $i(t) = (A_1 t + A_2) e^{-\alpha t}$ Underdamped, $\alpha < \omega_0$ $i(t) = (B_1 \cos \omega_d t + B_2 \sin \omega_d t) e^{-\alpha t}$ $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$	Solutions: Overdamped, $\alpha > \omega_0$ $v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ Critically damped, $\alpha = \omega_0$ $v(t) = (A_1 t + A_2) e^{-\alpha t}$ Underdamped, $\alpha < \omega_0$ $v(t) = (B_1 \cos \omega_d t + B_2 \sin \omega_d t) e^{-\alpha t}$ $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$

General procedure for second order circuits:

1) Determine differential equation using nodal analysis or other circuit technique. Arrange differential equation with variables on left, forcing function on right. The differential equation will look like the following

$$\frac{d^2v}{dt^2} + 2\alpha\frac{dv}{dt} + \omega_0^2 v = f(t)$$

where $f(t)$ is the forcing function. Note that if you are solving for current, the differential equation will be in terms of i instead of v.

2) Begin determination of the natural solution (forcing function = 0) by determining coefficients in the characteristic equation. In class, we had used the trial function is Ke^{st} and determined s by inserting into differential equation, resulting in the following characteristic equation.

$$s^2 + 2\alpha s + \omega_0^2 = 0$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

Determine the value of α and ω_0 given the differential equation in 1).

3) Determine if circuit is overdamped, critically damped, or underdamped based on values of α and ω_0. Determine the natural solution based on this result.

   Overdamped, $\alpha > \omega_0$

   $$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

   Critically damped, $\alpha = \omega_0$

   $$v(t) = (A_1 t + A_2)e^{-\alpha t}$$

   Underdamped, $\alpha < \omega_0$

   $$v(t) = (B_1 \cos\omega_d t + B_2 \sin\omega_d t)e^{-\alpha t}, \ \omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

   Do not solve for unknown coefficients (A_1, A_2, B_1, B_2, etc) until you have the complete solution!

4) Determine forced solution. Use trial function that resembles forcing function. Insert trial function into differential equation to determine unknown constants. If the differential equation does not work, use a different trial function.

5) Write the complete solution (natural + forced). Apply initial/boundary conditions to determine unknown constants (A_1, A_2, B_1, B_2, etc). Note that you will need two initial/boundary values. These will typically be $v(0^+)$ and $dv(0^+)/dt$, or $v(0^+)$ and $v(\infty)$ for the case of a forcing function applied at t=0.