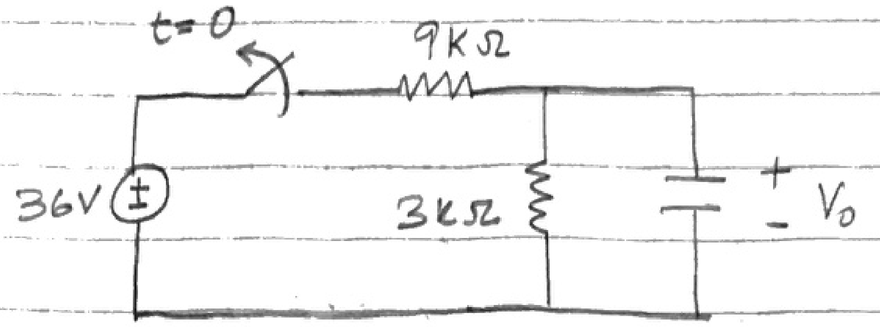
HW 7

* 7.10
* 7.19
* 7.24 (b) - write $i(t)$ in terms of unit step functions
* 7.24 (c) - write $x(t)$ in terms of ramp function
* 7.26 (c) - write $v_3(t)$ in terms of step function
* 7.26 (d) - write $v_4(t)$ in terms of ramp function and step functions
* 7.45
* 7.60
* 7.70.

7.10



For $t<0$ (Using voltage divider)

$$V_o(0^-) = \frac{3}{12} \times 36V$$

$$= 9V$$

For $t>0$ (for a source-free RC circuit)

$$\tau = RC = 3\times10^3 \times 20\times10^{-6}$$

$$= 0.06 \text{ s.}$$

$$V_o(t) = 9e^{-16.67t}$$

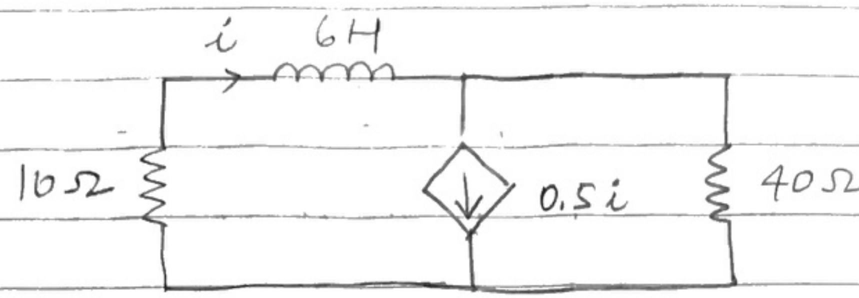
When V_o reduces to $\frac{1}{3}V_o(0^-) = 3V$,
let the time be t_o sec.

$$\therefore V_o(t_o) = 9e^{-16.67t_o} = 3V.$$

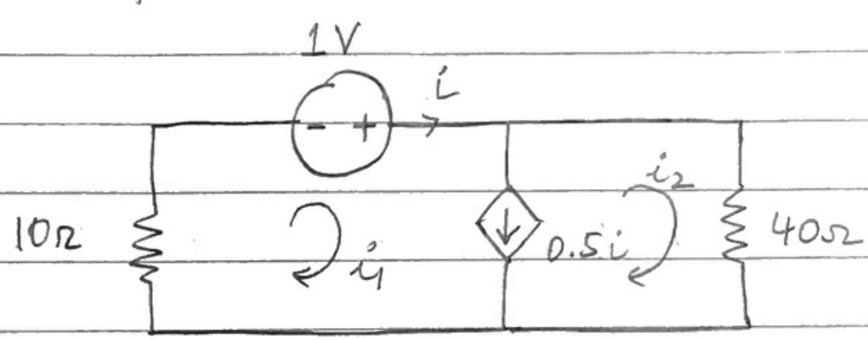
$$\therefore t_o = \ln(3)/16.67$$

$$= 0.0659 \text{ sec.}$$

7.19



To find the time constant $\tau = \frac{L}{R_{Th}}$



$i_1 = i$ — (1)

loops 1 and 2 form a supermesh :

$10 i_1 - 1V + 40 i_2 = 0$ — (2)

Constraint eqn. :

$i_1 - i_2 = 0.5 i = 0.5 i_1$ (From (1))

$\therefore 0.5 i_1 = i_2$ — (3)

From (2) & (3) :

$30 i_1 = 1V$

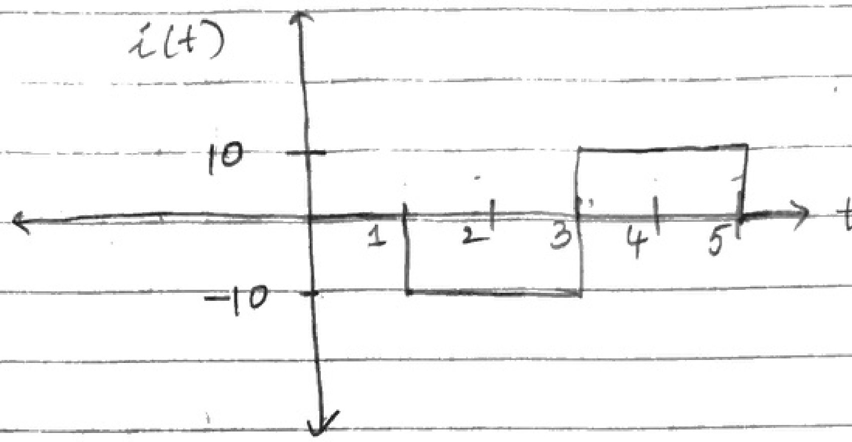
$\Rightarrow i_1 = 1/30$

$\therefore R_{Th} = \frac{1V}{i_1} = 30\Omega$

$\therefore \tau = \frac{L}{R_{Th}} = \frac{6}{30} = 0.2 \text{ sec}$

$i(t) = i(0) e^{-t/\tau} = 5 e^{-5t}$ A. $t > 0$.

7.24 (b)



$$i(t) = -10\left(u(t-1) - u(t-3)\right) + 10\left(u(t-3) - u(t-5)\right)$$

$$= -10\,u(t-1) + 20\,u(t-3) + 10\,u(t-5)$$

(c)

$$i(t) = (t-1)\left[u(t-1) - u(t-2)\right] + \left[u(t-2) - u(t-3)\right] + (4-t)\left[u(t-3) - u(t-4)\right]$$

$$= (t-1)\,u(t-1) - (t-2)\,u(t-2) - (t-3)\,u(t-3) + (t-4)\,u(t-4)$$

$$= r(t-1) - r(t-2) - r(t-3) + r(t-4)$$

7.26

(c) $V_3(t) = 2[u(t-2) - u(t-4)] + 4[u(t-4) - u(t-6)]$

$= 2u(t-2) + 2u(t-4) - 4u(t-6)$

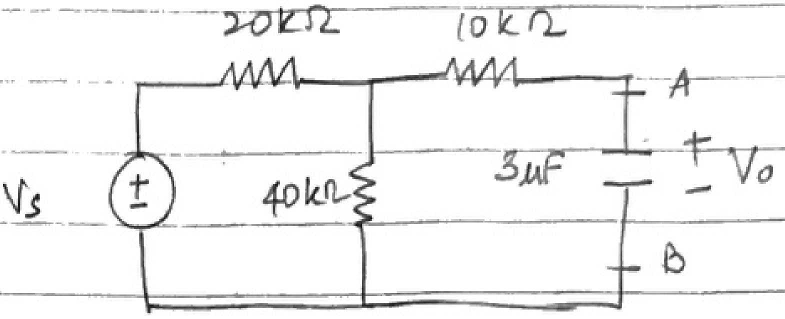
(d) $V_4(t) = -t[u(t-1) - u(t-2)]$

$= -(t-1+1)u(t-1) + (t+2-2)u(t-2)$

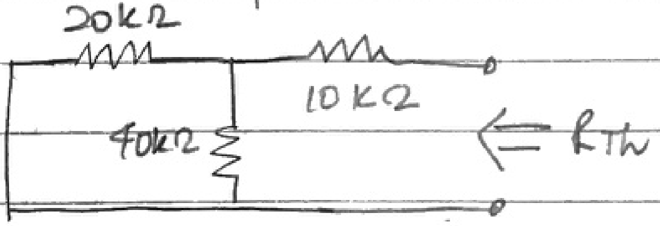
$= -(t-1)u(t-1) + (t-2)u(t-2) - u(t-1) + 2u(t-2)$

$= -r(t-1) + r(t-2) - u(t-1) + 2u(t-2)$

7.45



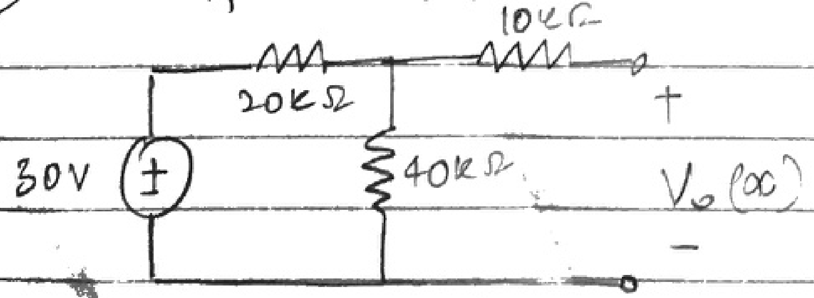
① To first find the time constant : $\tau = R_{Th}C$, we have to find R_{Th} across A and B



$$R_{Th} = (20k \parallel 40k) + 10k$$
$$= \frac{70}{3} k\Omega.$$

$$\therefore \tau = \frac{70}{\cancel{3}} \times \cancel{3} \times 10^{-6} \times 10^{3} = 70 ms$$

② To find $V_o(\infty)$:



Using voltage divider rule :

$$\therefore V_o(\infty) = \frac{40k}{\cancel{60k}} \times \cancel{30} = 20V$$

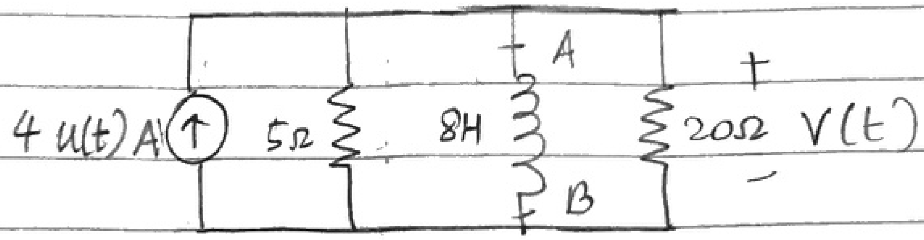
(3) Complete equat^n for $V_o(t)$:

$$V_o(t) = V_o(\infty) + [V_o(0) - V_o(\infty)]e^{-t/RC}$$

$$= 20 + [5 - 20]\, e^{-t/0.07}$$

$$= 20 - 15\, e^{-14.28t}\ V$$

7.60



(1) v(t) across the 20Ω resistor is same as that across the inductor.

$$V_L(t) = L\frac{di_L(t)}{dt}.$$

$\therefore$ we need to first find $i_L(t)$.

(2) $t<0$, $u(t)=0 \Rightarrow i_L(t)=0$

(3) $t>0$, we need to first find the $\tau = \frac{L}{R_{Th}}$, where R_{Th} is the equivalent resistance across A & B.

$\therefore R_{Th} = (5 \| 20)\,\Omega = 4\,\Omega$

$\therefore \tau = \frac{8}{4}$ sec. $= 2s$

(4) To find $i_L(\infty)$:

At $t\to\infty$, the inductor is a short circuit :

$i_L(\infty) = 4A$

(5) Complete eq^n for $i_L(t)$:

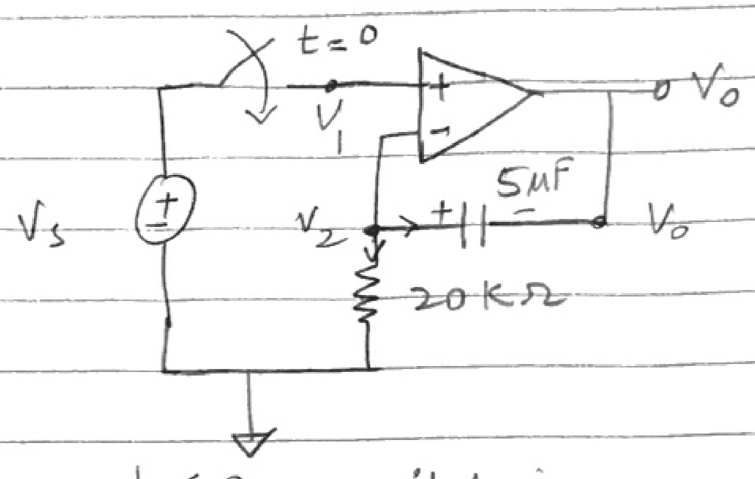
$$i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)]e^{-t/2}$$
$$= 4 + [0 - 4]e^{-t/2}$$
$$= 4(1 - e^{-0.5t})$$

(6) Find $V_L(t)$:

$$V_L(t) = L\frac{di_L(t)}{dt}$$
$$= 8\frac{d}{dt}(4(1-e^{-0.5t}))$$
$$= 8[(4 \times 0.5)e^{-0.5t}]$$
$$= 16e^{-0.5t} \text{ V}$$

$\therefore$ $V(t) = V_L(t) = 16e^{-0.5t}$ V

7.70



$t<0$, switch is open and $V(0)=0V$

$t>0$, switch is closed

→ Voltage across capacitor : $V_c = V_2 - V_o$ — (1)

→ $V_2 = V_1 = V_s = 20mV$ — (2)

→ Applying KCL at node 2 :

$$\frac{V_2}{20K} + C\frac{dV_c(t)}{dt} = 0$$

$$\frac{-V_s}{20k} = C\frac{dV_c(t)}{dt} = 0 \text{ (From (2))}$$

$$\therefore V_c(t) = \frac{-1}{20\times5\times10^{-3}}\int_0^t V_s\,dt$$

$$= \frac{-tV_s}{10^{-1}}$$

$$= -10tV_s = -0.2t\ V. \quad \text{— (3)}$$

→ From (1), (2) and (3) :

$$V_o = V_2 - V_c(t) = V_s - V_c(t)$$

$$= 0.02(1+10t)\ V$$