Name: ________ANSWER KEY____________

EXAM 2
EECS 215
Introduction to Electronic Circuits
Wednesday, November 16, 2017, 6:00pm-8:00pm

Lecture Section (circle one):	**001 Finelli**	**002 Lahiji**

This test consists of 6 problems with points as indicated to total 90 points.

Read through the entire exam before beginning.
Show all work (on the pages provided in this booklet) to earn partial credit.
Briefly explain major steps, include units, and write your final answers in the areas provided.
Do not unstaple the pages.

No credit will be given if no work is shown.

Exam Policies

- No food allowed during exam.
- No books allowed (closed book exam).
- One, 8.5 x 11 inch notes page (TWO SIDED) allowed
- Only scientific calculators allowed (**graphing calculators not permitted**).
- No communication of any kind is allowed. No use of cell phones, computers, or any devices besides calculators. Violation of this will be treated as an honor code violation.
- No credit will be given for this exam without a signed honor pledge.

Write out the honor pledge and sign below.

"I have neither given nor received unauthorized aid on this examination, nor have I concealed any violations of the Honor Code"

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Signature: ____

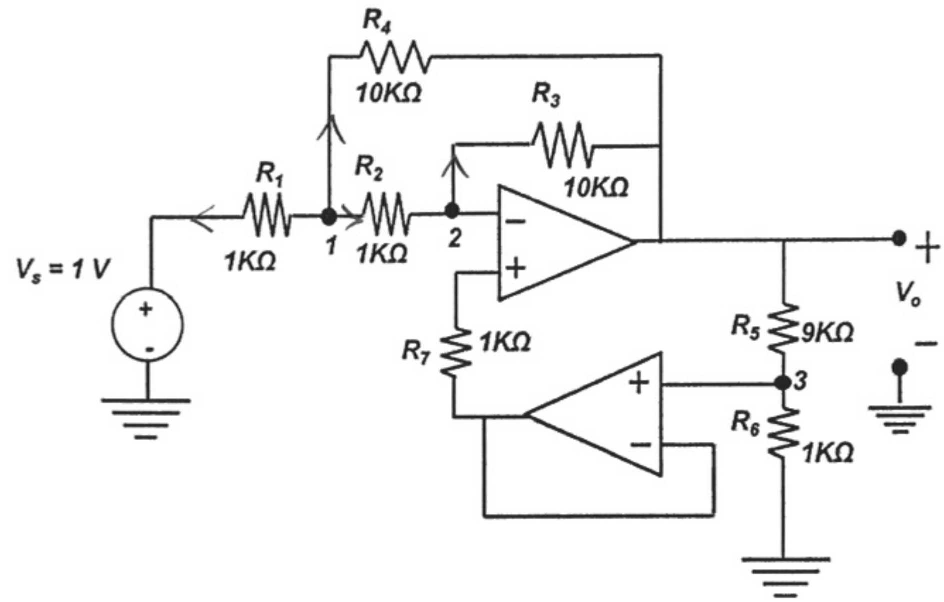
Do not write in this space

Problem 1: [ ]/15	Problem 4: [ ]/10
Problem 2: [ ]/15	Problem 5: [ ]/15
Problem 3: [ ]/15	Problem 6: [ ]/20

Total score [ ]/90

Problem 1. **(15 points total)** Assuming the OpAmp in the circuit is ideal, find the value of the output voltage V_o. We recommend the following series of steps:

- Apply KCL at node 1
- Apply KCL at node 2
- Apply the voltage divider relationship at node 3
- Determine the relationship between the voltage at nodes 2 and 3
- Combine the equations and solve for V_o



$V_o =$ $-5.58\,V$ **(15 points)**

(a) KCL at node 1:

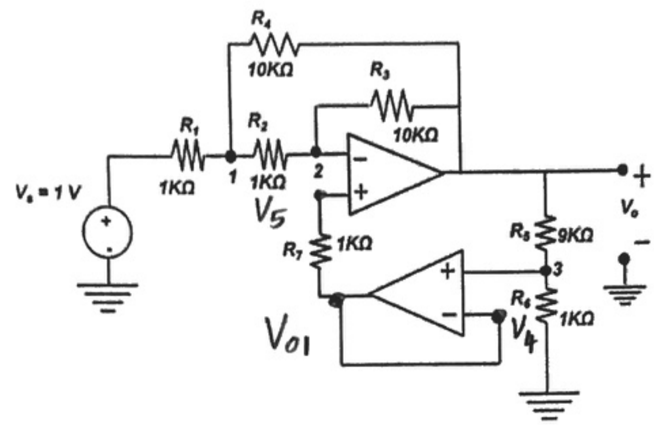
$$\frac{V_1 - 1}{1k} + \frac{V_1 - V_o}{10k} + \frac{V_1 - V_2}{1k} = 0$$

or $\quad 21V_1 - 10V_2 - V_o = 10 \quad - (1)$

(b) KCL at node 2:

$$-\left(\frac{V_1 - V_2}{1k}\right) + \frac{V_2 - V_o}{10k} = 0 \quad \text{OR} \quad 10V_1 - 11V_2 + V_o = 0$$

$- (2)$



(Circuit redrawn for your convenience)

(c) Voltage Divider at 3 :

$$V_3 = \frac{V_o}{10}$$

$$\therefore \quad V_{01} = V_4 = V_3 = \frac{V_o}{10} \quad - \text{(3)}$$

(d) $\because$ Current through 1k resistor between V_5 & $V_{01} = 0A$,

$$\therefore V_5 = V_{01} = \frac{V_o}{10} \qquad (\text{From (3)})$$

$$\therefore V_2 = V_5 = \frac{V_o}{10} = V_3 \quad - \text{(4)}$$

$$\therefore V_5 = V_3$$

(e) Combining (1), (2) + (4)

Linear eq^n. :

$$\left.\begin{array}{llr} (1) & 21V_1 - 2V_o = & 10 \\ (2) & 100V_1 \quad - V_o = & 0 \end{array}\right\} \qquad \boxed{V_o = -5.58V}$$

Problem 2. **(15 points total)** A 5 mF capacitor is initially uncharged, and the current flowing through the capacitor is defined as:

$$i_C(t) = \begin{cases} 0, & t \le 0\ sec \\ 20t\ A, & 0 \le t \le 0.025\ sec \\ 0.5\ A, & t \ge 0.025\ sec \end{cases}$$

Determine the voltage $v_C(t)$ for $t \ge 0$ and write your answer here:

$$v_C(t) = \begin{cases} \underline{2\times10^3 t^2\ V} \ \textbf{(7 points)}, & 0 \le t \le 0.025\ sec \\ \underline{100t - 1.25\ V} \ \textbf{(8 points)}, & t \ge 0.025\ sec \end{cases}$$

$$V_c(t) = \frac{1}{C}\int_{t_0}^{t} i_c(t) + V_c(t_0)$$

For : $0 \le t \le 0.025\,s$

$$V_c(t) = \frac{1}{5\times10^{-3}}\int_0^t 20t\cdot dt + \cancel{V_c(0)}^{\,0\,V}$$

$$= 2\times10^3 t^2 \quad V$$

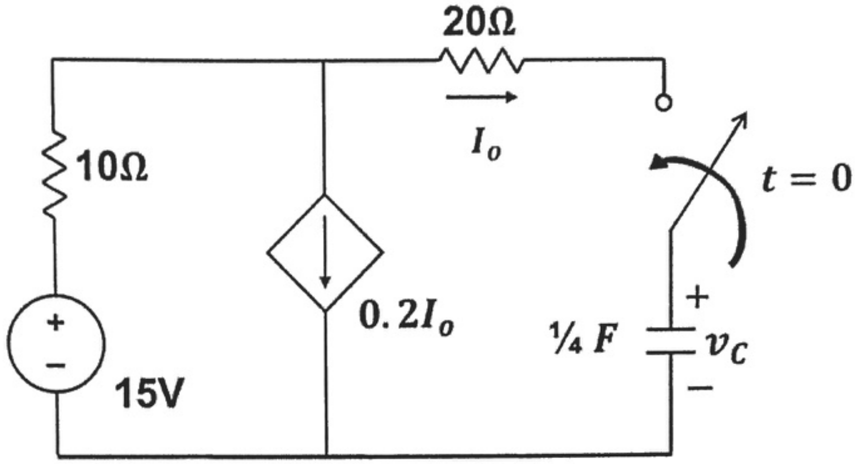
For $t \ge 0.025\,s$

$$v_c(t) = \frac{1}{5\times10^{-3}}\int_{0.025}^{t} 0.5\,dt + v_c(0.025)$$

$$= 100\quad(t - 0.025) + (2000)(0.025)^2$$

$$= 100t - 1.25\ V$$

Problem 3. **(15 points)** The switch in the circuit below has been **open** for a very long time, and it **closes** at $t = 0$. Find an expression for $v_C(t), t \geq 0$

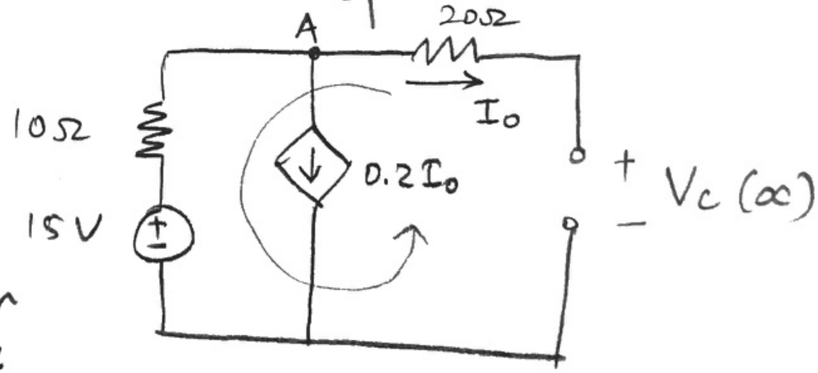


$v_C(t) =$ $15 - 15e^{-t/8}$ V **(15 points)**

(*) At $t = 0$

$v_c(0^-) = 0V = v_c(0^+)$

(*) $t \to \infty$, capacitor acts as an open ckt:

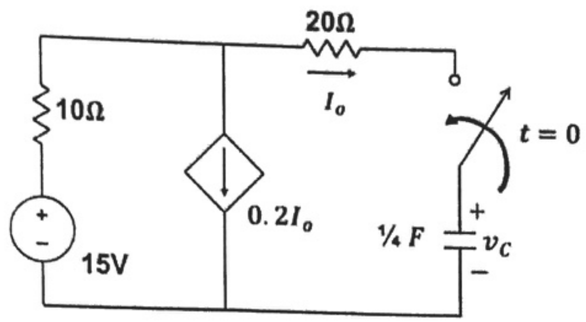


$I_o = 0A$

∴ Currents in all branches = 0A

Applying KVL in outermost loop:

$15V = V_c(\infty)$



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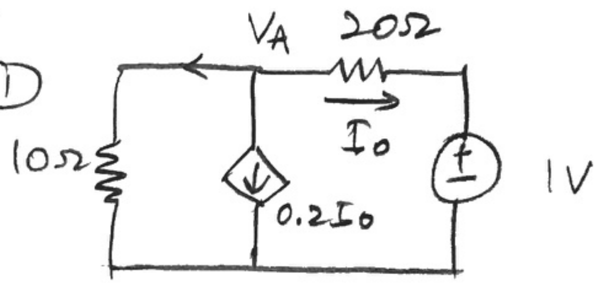
Finding $\tau = R_{Th} C$



$$I_o = \frac{V_A - 1V}{20} \quad - (1)$$

KCL at node A :

$$\frac{V_A}{10} + \frac{0.2(V_A - 1V)}{20} + \frac{V_A - 1V}{20} = 0$$

(Using (1))

$\therefore$ $V_A = 0.375$

$I_o = -0.0312A$

$\therefore R_{Th} = \frac{1V}{-I_o}$

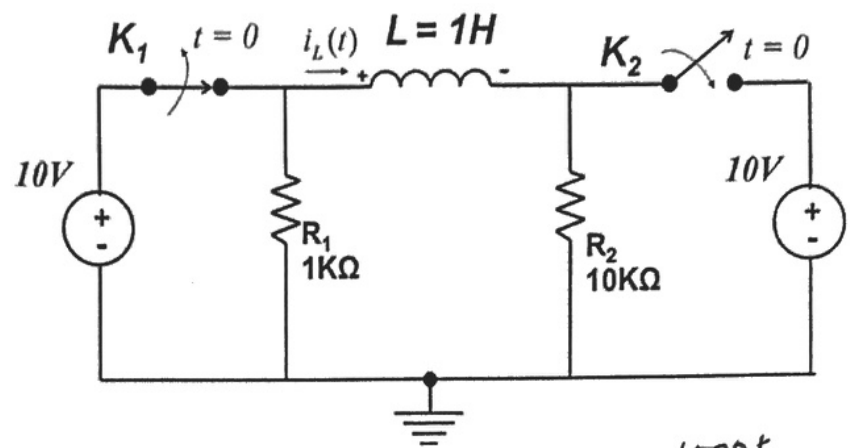
$= 32\Omega$

$\therefore \tau = 8s$

$$v_c(t) = V_c(\infty) + [V_c(0) - V_c(\infty)] e^{-t/\tau}$$

$$= 15 - 15 e^{-t/8} \text{ V}$$

Problem 4. **(10 points).** Switches K_1 and K_2 after being in position shown in the circuit for a long time, are moved to the new position at $t = 0$. With the given parameters in this circuit, find $i_L(t), t > 0$



$i_L(t) =$ $-10 + 11e^{-1000t}$ **(10 points)**

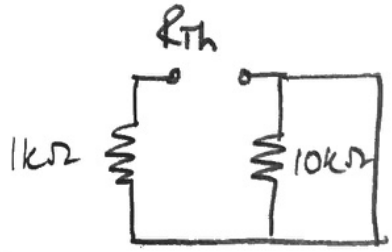
$t = 0^-$

$i_L(0^-) = \dfrac{10V}{10k} = 1mA$

$\therefore i_L(0^+) = 1mA$

$t \to \infty,\ i_L(\infty) = \dfrac{-10V}{1k} = -10mA$

$\tau = \dfrac{L}{R_{Th}}$



$\therefore R_{Th} = 1k\Omega$

($\because$ 10k resistor is shorted)

$\therefore \tau = 1/1k = 1ms$

$i_L(t) = i_L(\infty) + [\,i_L(0) - i_L(\infty)\,]\,e^{-t/\tau}$

$= -10 + [11]e^{-1000t}$ mA

Problem 5. **(15 points total)** For the circuit below, the switch has been in position A for a long time, then it moves to position B at $t = 0$. Determine the initial and final values as listed below (don't forget to include units):

$i_L(0^+) =$ -0.5 A **(2 points)**

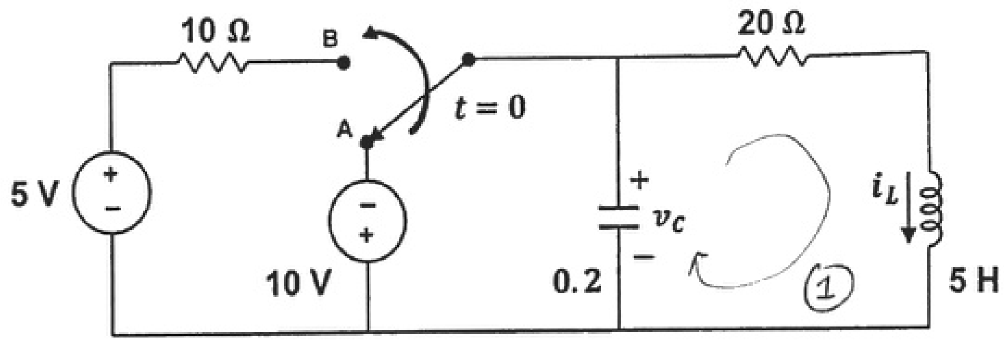
$v_C(0^+) =$ -10 V **(2 points)**

$i_L'(0^+) =$ 0 **(3.5 points)**

$v_C'(0^+) =$ 10 V/s **(3.5 points)**

$i_L(\infty) =$ 0.1667 A **(2 points)**

$v_C(\infty) =$ 3.33 V **(2 points)**



(a) $i_L(0^-) = \frac{-10}{20} = -0.5\text{ A} = i_L(0^+)$

(b) $v_C(0^-) = -10\text{V} = v_C(0^+)$

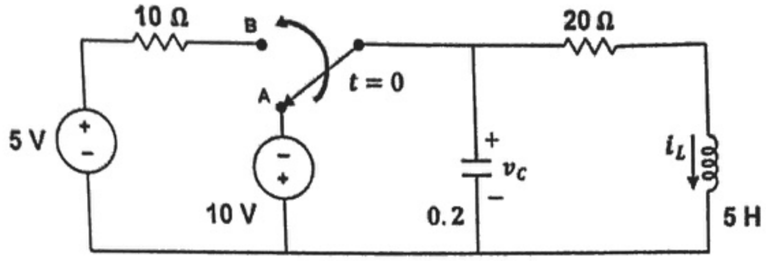
(c) $i_L'(0^+) = \frac{v_L(0^+)}{L}$

Applying KVL around 1 :

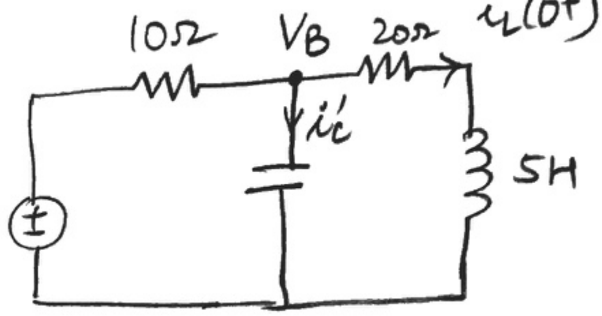
$-V_C(0^+) + 20\, i_L(0^+) + v_L(0^+) = 0$

$\therefore V_L(0^+) = 0\text{ V}$

$\therefore i_L'(0^+) = 0\text{ A/s}$



(Circuit redrawn here)



(d) $V_B = V_c(0^+)$

KCL at B:

$$\frac{V_c(0^+) - 5}{10} + i_c(0^+) + i_L(0^+) = 0$$

5V

$$\therefore \frac{15}{10} + 0.5 = i_c(0^+)$$

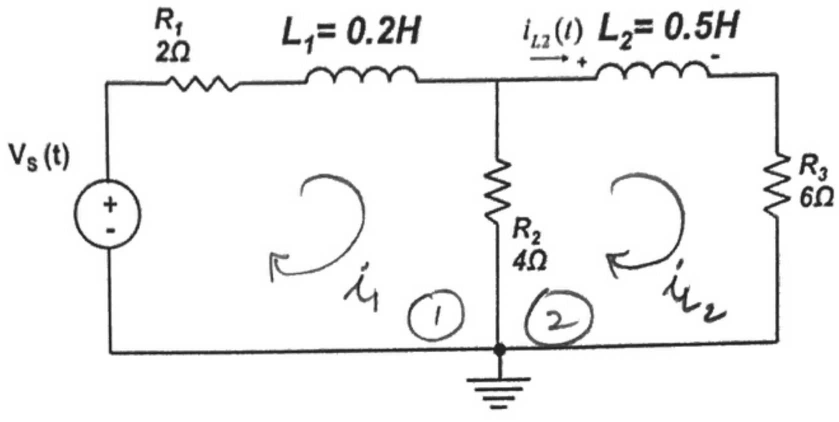
$\therefore$ $i_c(0^+) = 2A$

$\therefore$ $V_c'(0^+) = 10\ V/s$

(e) $i_L(\infty) = \dfrac{5}{10+20} = 0.1667A$

(f) $V_c(\infty) = \dfrac{20}{30} \times 5V = 3.33V.$

Problem 6. (**20 points total**). In the circuit below, $V_s(t) = 10\,u(t)$ V.



(a) (**10 points**) Using **mesh analysis**, derive a second order differential equation that describes the current $i_{L2}(t)$ for $t > 0$. Write your answer by filling in the blanks below:

$$\frac{d^2 i_{L2}(t)}{dt^2} + \underline{50}\ \frac{di_{L2}(t)}{dt} + \underline{440}\ i_{L2}(t) = \underline{400} \qquad \textbf{(10 points)}$$

(b) (**10 points**) Suppose the answer for part (a) is as follows (*note, this is NOT the correct answer*):

$$\frac{d^2 i_{L2}(t)}{dt^2} + 10\frac{di_{L2}(t)}{dt} + 20 i_{L2}(t) = 15\ A$$

Solve the equation to find $i_{L2}(t), t \geq 0$

$i_{L2}(t) =$ ________________ (**10 points**)

(a) For $t>0$ KVL in loop 1:

$$-10 + 2i_1 + 0.2\frac{di_1}{dt} + 4(i_1 - i_2) = 0$$

$$6i_1 + 0.2\frac{di_1}{dt} - 4i_{L2} - 10 = 0 \quad -①$$

KVL in loop 2:

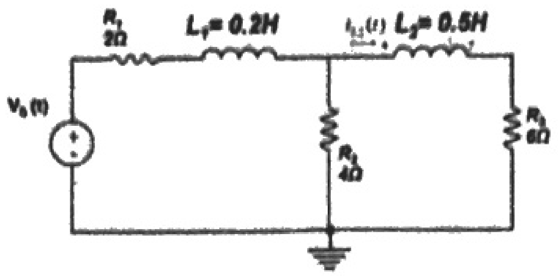
$$-4(i_1 - i_{L2}) + 0.5\frac{di_{L2}(t)}{dt} + 6i_{L2}(t) = 0$$

$$\therefore i_1 = \frac{1}{4}\left[10\, i_{L_2} + 0.5 \frac{d}{dt} i_{L_2}(t)\right] \quad - ②$$

Subsitute ② in ① :

$$0.025 \frac{d^2 i_{L_2}(t)}{dt^2} + 1.25 \frac{d}{dt} i_{L_2}(t) + 11\, i_{L_2}(t) - 10 = 0$$

$$\therefore \frac{d^2 i_{L_2}(t)}{dt^2} + 50 \frac{d}{dt} i_{L_2}(t) + 440\, i_{L_2}(t) = 400$$

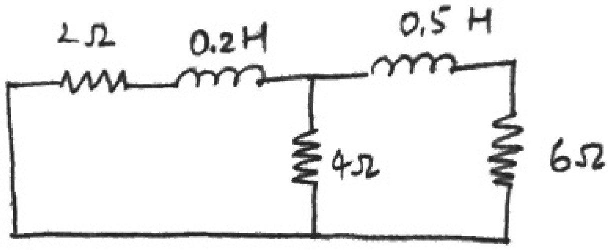


(Circuit redrawn for your convenience)

Q6 (b). Need $i_{L_2}(0^+)$ and $\frac{d}{dt} i_{L_2}(t) \Big|_{t=0} = \frac{v_{L_2}(t)}{L}$

$t = 0^-$

$\therefore i_{L_2}(0^+) = i_{L_2}(0^-) = 0A$



$\because$ No current flows through the ckt at $t = 0^+$,

$v_{L_2}(0^+) = 0V \Rightarrow \frac{d}{dt} i_{L_2}(t) = 0$ A/s

eq^n : $\frac{d^2}{dt^2} i_{L_2}(t) + 10 \frac{d}{dt} i_{L_2}(t) + 20 i_{L_2}(t) = 15$

roots : $-2.76, -7.76$

natural response : $A_1 e^{-2.76t} + A_2 e^{-7.76t} = i_{L_2}(t)$

steady state response $i_{L_2}(t) = \frac{15}{20} = 0.75$

Complete Sol^n :

$i_{L_2}(t) = A_1 e^{-2.76t} + A_2 e^{-7.76t} + 0.75$.

Using Initial Condⁿ :

$i_{L_2}(0) = A_1 + A_2 + 0.75 = 0$

$i'_{L_2}(0) = -2.76 A_1 - 7.76 A_2 = 0$

$\therefore \quad A_1 = -1.164 \qquad A_2 = 0.414$

$\therefore \; i_{L_2}(t) = -1.164 e^{-2.76t} + 0.414 e^{-7.76t} + 0.75 \text{ A}$