Problem 1 (20 points)

Assuming the op-amps are ideal, determine v_{out}/v_{in}.

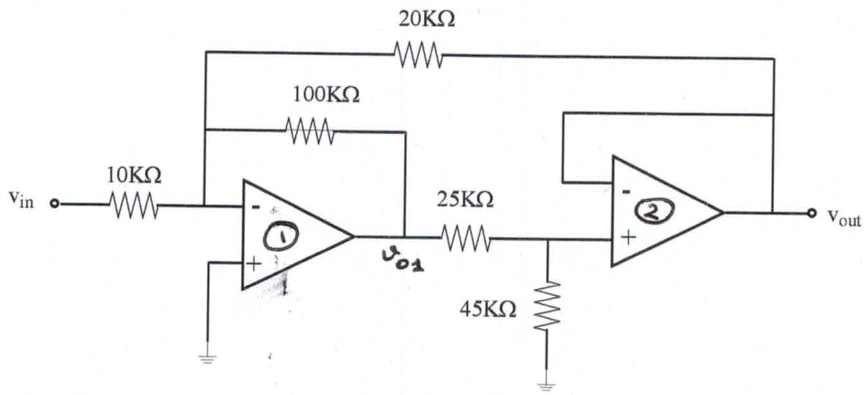


Figure 1: Circuit for Problem 1.

Write your answer here:

$v_{out}/v_{in} =$ $-\frac{90}{59} = -1.5254$

$$v_{out} = v_{n2} = v_{p2} = \frac{45k}{45k+25k} \times v_{o1}$$

$$\Rightarrow v_{o1} = \frac{14}{9} v_{out}$$

$$v_{p1} = v_{n1} = 0$$

KCL at 1 negative terminal :

$$\frac{v_{in}}{10k} + \frac{v_{o1}}{100k} + \frac{v_{out}}{20k} = 0$$

$$\Rightarrow \frac{v_{in}}{10k} + \frac{14}{9}\frac{v_{out}}{100k} + \frac{v_{out}}{20k} = 0$$

$$\Rightarrow v_{out} = -\frac{1}{\frac{14}{9} \times \frac{1}{100k} + \frac{1}{20k}} \times \frac{1}{10k} \times v_{in}$$

$$= -\frac{1}{\frac{14}{90} + \frac{1}{2}} \times v_{in}$$

$$= -\frac{90}{59} v_{in} = -1.5254\, v_{in}$$

Problem 2 (10 points)

1. Determine C_{eq} in terms of C_1, C_2 and C_3. Express your answer as the ratio of standard polynomials of C_1, C_2 and C_3, i.e. $C_{eq} = \frac{num(C_1,C_2,C_3)}{den(C_1,C_2,C_3)}$.
2. Determine L_{eq} in terms of L_1, L_2 and L_3. Express your answer as the ratio of standard polynomials of L_1, L_2 and L_3, i.e. $L_{eq} = \frac{num(L_1,L_2,L_3)}{den(L_1,L_2,L_3)}$.

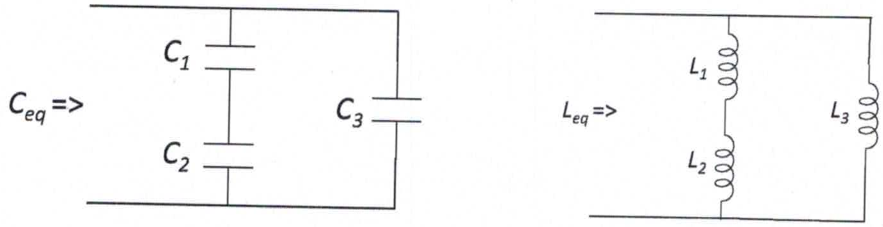


Figure 2: Circuits for Problem 2.

Write your answers here:

$C_{eq} = \dfrac{C_1C_2 + C_1C_3 + C_2C_3}{C_1 + C_2}$

$L_{eq} = \dfrac{L_1L_3 + L_2L_3}{L_1 + L_2 + L_3}$

$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} + C_3$$

$$= \frac{C_1C_2}{C_1 + C_2} + C_3$$

$$= \frac{C_1C_2 + C_1C_3 + C_2C_3}{C_1 + C_2}$$

$$L_{eq} = (L_1 + L_2) \| L_3$$

$$= \frac{(L_1 + L_2)L_3}{L_1 + L_2 + L_3}$$

$$= \frac{L_1L_3 + L_2L_3}{L_1 + L_2 + L_3}$$

Problem 3 (20 points)

For the given circuit, determine $v_c(t)$ for $t \geq 0$. (Note that the left-hand $3A$ source has been on from $t = -\infty$ whereas the right-hand source turns on at $t = 0$.)

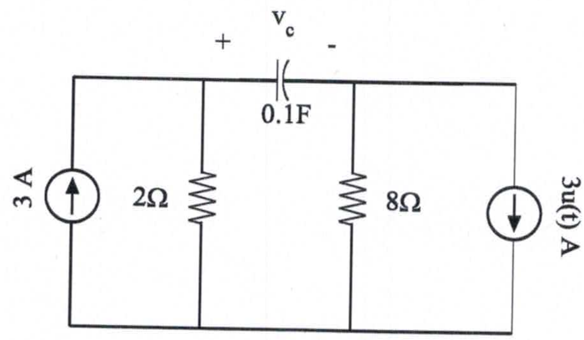
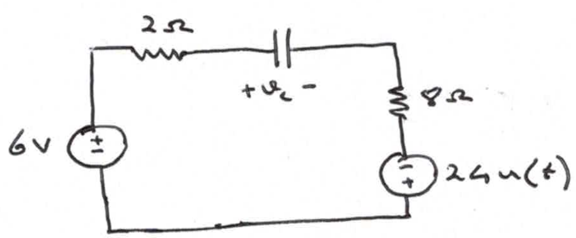


Figure 3: Circuit for Problem 3.

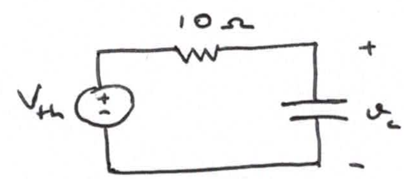
Write your answer here:

$v_c(t) =$ $30 - 24e^{-t}$ volts (for $t \geq 0$)

Source transformation gives:



Thevenin equivalent ⇒



$V_{th} = 6 + 24u(t)$ volts

$v_c(0^-) = 6$ volts

$v_{cf} = 30$ volts , $v_{cn} = Ae^{-t/\tau}$

$\tau = RC = 10 \times 0.1 = 1$ sec

$\Rightarrow v_c(t) = 30 + Ae^{-t}$

$v_c(0) = 30 + A = 6 \Rightarrow A = -24$

$\Rightarrow v_c(t) = 30 - 24e^{-t}$ volts $t \geq 0$

Problem 4 (25 points)

Assume the switch has been open from $t = -\infty$ and closes at $t = 0$. Determine $v_c(t)$ for $t \geq 0$.

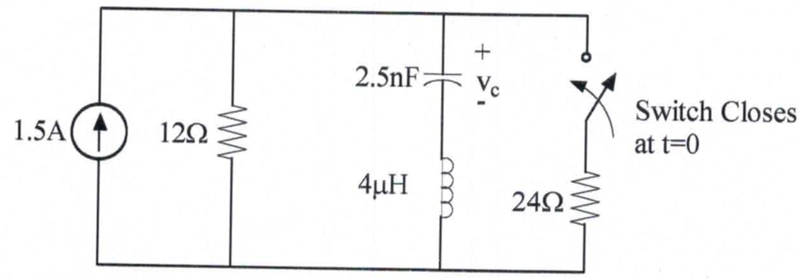


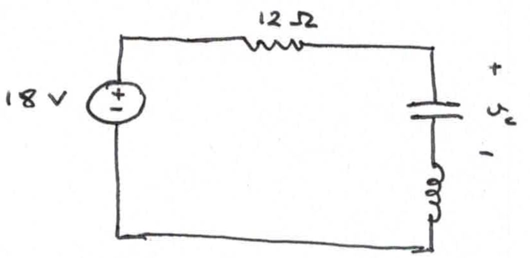
Figure 4: Circuit for Problem 4.

Write your answer here:

$v_c(t) = 12 + \left[6 \cos \omega_d t + \frac{2}{\sqrt{11}} \sin \omega_d t \right] e^{-\alpha t}$ (for $t \geq 0$)

where $\omega_d = 3 \times \sqrt{11} \times 10^6$ rad/sec $\approx 9.95 \times 10^6$, $\alpha = 10^6 \ s^{-1}$, $\frac{2}{\sqrt{11}} \approx 0.603$

For $t < 0$, source transformation gives:

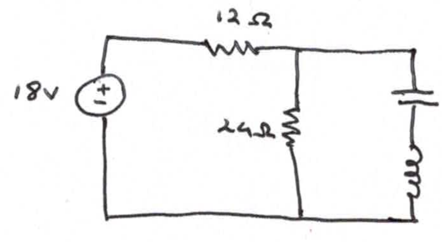


Initial conditions:

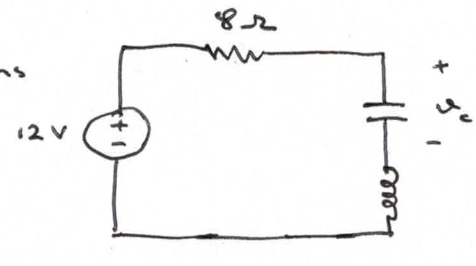
$v_c(0) = 18$ V

$i_L(0) = i_c(0) = 0$A

For $t \geq 0$



Source transformations $\Rightarrow$



Series RLC circuit

Problem 4 (continued)

$$\alpha = \frac{R}{2L} = \frac{8}{2\times 4\mu} = 1\times 10^{6}$$

$$\omega_o = \left(\frac{1}{LC}\right)^{\frac{1}{2}} = \left(\frac{1}{4\times10^{-6}\times 2.5\times10^{-9}}\right)^{\frac{1}{2}} = 1\times10^{7}$$

$\alpha < \omega_o \Rightarrow$ underdamped

$$\omega_d = (\omega_o^2 - \alpha^2)^{\frac{1}{2}} = (99\times10^{12})^{\frac{1}{2}} \approx 9.95\times10^{6} \quad (= 3\times\sqrt{11}\times10^{6})$$

From the series RLC circuit,

$$v_{cf} = 12 \text{ V}$$

$$v_{cn}(t) = [B_1 \cos\omega_d t + B_2 \sin\omega_d t]\, e^{-\alpha t}$$

$$v_c(t) = v_{cf} + v_{cn}(t)$$

$$v_c(0) = 18 = 12 + B_1 \Rightarrow B_1 = 6$$

$$i_c(0) = C\left.\frac{dv_c}{dt}\right|_{t=0} = C[-\alpha B_1 + \omega_d B_2] = 0$$

$$\Rightarrow B_2 = \frac{\alpha}{\omega_d} B_1 = \frac{10^6}{3\times\sqrt{11}\times10^6}\times 6 = \frac{2}{\sqrt{11}} = 0.603$$

$$\Rightarrow v_c(t) = 12 + \left[6\cos\omega_d t + \frac{2}{\sqrt{11}}\sin\omega_d t\right] e^{-\alpha t}$$

Problem 5 (25 points)

Assume the switch has been closed from $t = -\infty$ and opens at $t = 0$.

1. Determine the differential equation describing $v_1(t)$ for $t \geq 0$. Do not solve. Express the differential equation in standard form (with the coefficient of the highest derivative equal to 1.)
2. Determine the initial values $v_1(0^+)$ and $\frac{dv_1}{dt}\big|_{t=0^+}$.
3. Determine the steady-state value $v_1(t)$ as $t \to \infty$.

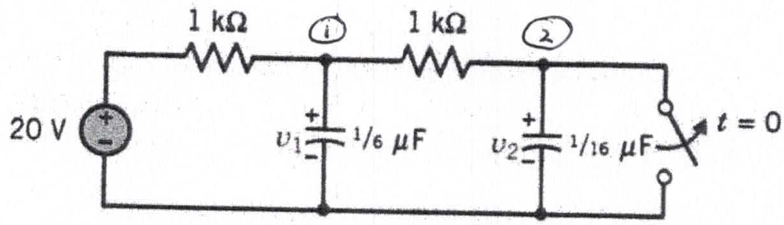


Figure 5: Circuit for Problem 5.

Write your answers here:

D.e. for $v_1(t)$: $\frac{d^2 v_1}{dt^2} + 2.8\times10^4 \frac{dv_1}{dt} + 9.6\times10^7 v_1 = 1.92\times10^7$, $t \geq 0$

$v_1(0^+) =$ 10 volts

$\frac{dv_1}{dt}\big|_{t=0^+} =$ 0 volts/sec

$v_1(t)|_{t\to\infty} =$ 20 volts

For $t > 0$,

KCL at node 1 gives: $\frac{v_1 - 20}{1000} + \frac{1}{6}\times10^{-6}\frac{dv_1}{dt} + \frac{v_1 - v_2}{1000} = 0$

$$\Rightarrow 2v_1 + \frac{1}{6}\times10^{-3}\frac{dv_1}{dt} - 20 = v_2 \qquad (1)$$

At node 2: $\frac{v_1 - v_2}{1000} = \frac{1}{16}\times10^{-6}\frac{dv_2}{dt}$

$$\Rightarrow v_1 - v_2 - \frac{1}{16}\times10^{-3}\frac{dv_2}{dt} = 0 \qquad (2)$$

Problem 5 (continued)

Substitute (1) into (2):

$$v_1 - \left(2v_1 + \frac{1}{6}\times 10^{-3}\frac{dv_1}{dt} - 20\right) - \frac{1}{16}\times 10^{-3}\left(2\frac{dv_1}{dt} + \frac{1}{6}\times 10^{-3}\frac{d^2v_1}{dt^2}\right) = 0$$

$$\Rightarrow -\frac{1}{16}\times 10^{-3}\times\frac{1}{6}\times 10^{-3}\frac{d^2v_1}{dt^2} + \left(-\frac{1}{6}\times 10^{-3} - \frac{1}{8}\times 10^{-3}\right)\frac{dv_1}{dt} - v_1 + 20 = 0$$

$$\Rightarrow 1.0417\times 10^{-8}\frac{d^2v_1}{dt^2} + 0.2917\times 10^{-3}\frac{dv_1}{dt} + v_1 = 20$$

$$\Rightarrow \frac{d^2v_1}{dt^2} + 2.8\times 10^{4}\frac{dv_1}{dt} + 9.6\times 10^{7}v_1 = 1.92\times 10^{9}$$

Directly from the circuit, as $t\to\infty$, $v_f = 20$ Volts

For $t<0$, when the switch is closed, $v_2(0^-) = 0$

$$v_1(0^-) = \frac{1k}{1k+1k}\times 20 = 10 \text{ Volts} = v_1(0^+)$$

At time $t=0^+$, immediately after the switch opens, (1) is valid, so

$$2v_1(0^+) + \frac{1}{6}\times 10^{-3}\left.\frac{dv_1}{dt}\right|_{t=0^+} - 20 = v_2(0^+) = v_2(0^-) = 0$$

$$\Rightarrow \left.\frac{dv_1}{dt}\right|_{t=0^+} = 20 - 2\times 10 = 0$$