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Lab 4

Exercise One

```
>> answer_squared = integral2(@(r, t) exp(-(r).^2).*r, 0, 100, 0, 2.*pi);
>> sqrt(answer_squared)

ans =

    1.7725

>> sqrt(pi)

ans =

    1.7725
```

Exercise one

Exercise Two

```
>> integral2(@(u,v) (1-(tan(u).^2).*tan(v).^2))./(1-((sin(u)./cos(v)).^2).*(sin(v)./cos(u)).^2)), 0, pi./2, 0, @(u) (pi./2)-u)

ans =

    1.2337

>> (pi^2)/8

ans =

    1.2337
```

Exercise two

Exercise Three

Exercise Four

4) Volume of Mantle = Volume using radius of mantle - Volume using radius of core

Core + Mantle: $0 \leq r \leq 6375$ $0 \leq \theta \leq 2\pi$ $0 \leq \phi \leq \pi$

Volume = $\int_0^{2\pi} \int_0^\pi \int_0^{6375} r^2 \sin \phi \, dr \, d\phi \, d\theta$

$= \int_0^{2\pi} d\theta \int_0^\pi \sin \phi \, d\phi \int_0^{6375} r^2 \, dr = 2\pi [-\cos \phi]_0^\pi \left[\frac{1}{3} r^3 \right]_0^{6375} = 2\pi (2) (8.636 \times 10^{10}) = 1.085 \times 10^{12} \text{ km}^3$

Core: $0 \leq r \leq 3475$ $0 \leq \theta \leq 2\pi$ $0 \leq \phi \leq \pi$

Volume = $\int_0^{2\pi} \int_0^\pi \int_0^{3475} r^2 \sin \phi \, dr \, d\phi \, d\theta = \int_0^{2\pi} d\theta \int_0^\pi \sin \phi \, d\phi \int_0^{3475} r^2 \, dr = 2\pi [-\cos \phi]_0^\pi \left[\frac{1}{3} r^3 \right]_0^{3475} = 2\pi (2) (1.399 \times 10^{11})$

$= 1.758 \times 10^{11} \text{ km}^3$

Core + Mantle - Core = Mantle = $1.085 \times 10^{12} - 1.758 \times 10^{11} = 9.092 \times 10^{11} \text{ km}^3$

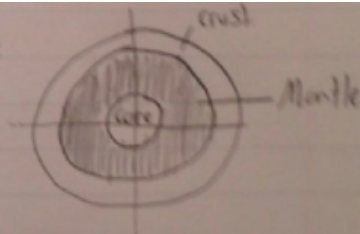


Figure 1: Hand calculation for exercise four

```
>> integral3(@(u,v,w) u.^2.*(sin(v)), 3475, 6375, 0, pi, 0, 2.*pi)

ans =

    9.0948e+11
```

Figure 2: Matlab calculation for exercise four

Exercise Five

5) Volume of Sphere = $\frac{4}{3}\pi(13)^3 = 9202.77 \text{ mm}^3$
 Volume of cylinder inside sphere = the integral of the hemisphere over region of intersection

Hemisphere = $z = \sqrt{13^2 - x^2 - y^2} = \sqrt{169 - r^2}$
 Region D = $x^2 + y^2 \leq 25$ $r = 5$ $0 \leq r \leq 5$ $0 \leq \theta \leq 2\pi$

Volume of cylinder in head = $2 \iint_D \sqrt{13^2 - x^2 - y^2} dx dy = 2 \int_0^{2\pi} \int_0^5 r \sqrt{169 - r^2} dr d\theta$
 $= 2 \int_0^{2\pi} \left[-\frac{1}{3} (169 - r^2)^{3/2} \right]_0^5 d\theta = 2 \int_0^{2\pi} \left(-\frac{1}{3} (1738) + \frac{1}{3} (2197) \right) d\theta = 2 (2\pi) \left(\frac{459}{3} \right) = \frac{1876}{3} \pi \approx 1964.54 \text{ mm}^3$

Volume of head = $9202.77 - 1964.54 = \boxed{7238.23 \text{ mm}^3}$

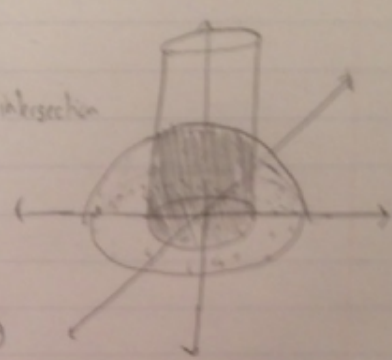


Figure 3: Hand calculation for exercise five

```
>> integral2(@(r,t) sqrt(13^(2)-r.(2))-(-sqrt(13^(2)-r.(2))), 5, 13, 0, 2*pi)

ans =

    871.7598
```

Figure 4: Matlab calculation for exercise five