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Lab 4

Exercise One

```
>> answer_squared = integral2(@(r, t) exp(-(r).^2).*r, 0, 100, 0, 2.*pi);
>> sqrt(answer_squared)

ans =

    1.7725

>> sqrt(pi)

ans =

    1.7725
```

Exercise one

Exercise Two

```
>> integral2(@(u,v) (1-(tan(u).^(2).*tan(v).^(2)))./(1-((sin(u)./cos(v)).^(2).*(sin(v)./cos(u)).^(2))), 0, pi./2, 0, @(u)(pi./2)-u)

ans =

    1.2337

>> (pi^2)/8

ans =

    1.2337
```

Exercise two

Exercise Three

```
>> integral3(@(r, p, t) r.^2 .* sin(p), 0, 1, 0, pi,0,2.*pi)

ans =

    4.1888

>> % Finding volume of sphere with radius = 1
>> (4/3)*pi*(1^2)

ans =

    4.1888
```

Exercise three, part one

```
>> syms r p t
>> f = r * sin(p)*cos(t)

f =

r*cos(t)*sin(p)

>> g = r * sin(p)*sin(t);
>> h = r * cos(p);
>> gradient(f, g, h)
Error using sym/gradient
Too many input arguments.

>> gradient(f)

ans =

  r*cos(p)*cos(t)
    cos(t)*sin(p)
 -r*sin(p)*sin(t)

>> gradient(g)

ans =

 r*cos(p)*sin(t)
   sin(p)*sin(t)
 r*cos(t)*sin(p)

>> gradient(h)

ans =

 -r*sin(p)
    cos(p)

>> J = det([gradient(f), gradient(g), gradient(h)])

J =

- r^2*cos(t)^2*sin(p)^3 - r^2*sin(p)^3*sin(t)^2 - r^2*cos(p)^2*cos(t)^2*sin(p) - r^2*cos(p)^2*sin(p)*sin(t)^2
```

Exercise three, part two

```
J =

- r^2*cos(t)^2*sin(p)^3 - r^2*sin(p)^3*sin(t)^2 - r^2*cos(p)^2*cos(t)^2*sin(p) - r^2*cos(p)^2*sin(p)*sin(t)^2

>> M = [[cos(t)*sin(p), r*cos(p)*cos(t), -r*sin(p)*sin(t)], [sin(p)*sin(t), r*cos(p)*sin(t), r*cos(t)*sin(p)], [cos(p), -r*sin(p), 0]]

M =

[ cos(t)*sin(p), r*cos(p)*cos(t), -r*sin(p)*sin(t), sin(p)*sin(t), r*cos(p)*sin(t), r*cos(t)*sin(p), cos(p), -r*sin(p), 0]

>> det(M)
Error using sym/det (line 12)
Matrix must be square.

>> M

M =

[ cos(t)*sin(p), r*cos(p)*cos(t), -r*sin(p)*sin(t), sin(p)*sin(t), r*cos(p)*sin(t), r*cos(t)*sin(p), cos(p), -r*sin(p), 0]

>> M = [cos(t)*sin(p), r*cos(p)*cos(t), -r*sin(p)*sin(t); sin(p)*sin(t), r*cos(p)*sin(t), r*cos(t)*sin(p); cos(p), -r*sin(p), 0]

M =

[ cos(t)*sin(p), r*cos(p)*cos(t), -r*sin(p)*sin(t)]
[ sin(p)*sin(t), r*cos(p)*sin(t),  r*cos(t)*sin(p)]
[        cos(p),       -r*sin(p),                0]

>> det(M)

ans =

r^2*cos(p)^2*cos(t)^2*sin(p) + r^2*cos(p)^2*sin(p)*sin(t)^2 + r^2*cos(t)^2*sin(p)^3 + r^2*sin(p)^3*sin(t)^2

>> abs(ans)

ans =

abs(r^2*cos(p)^2*cos(t)^2*sin(p) + r^2*cos(p)^2*sin(p)*sin(t)^2 + r^2*cos(t)^2*sin(p)^3 + r^2*sin(p)^3*sin(t)^2)

>> % Simplifying this we get:
>> r^2 * sin(p)

ans =

r^2*sin(p)

>>
```

Exercise three, part three

Exercise Four

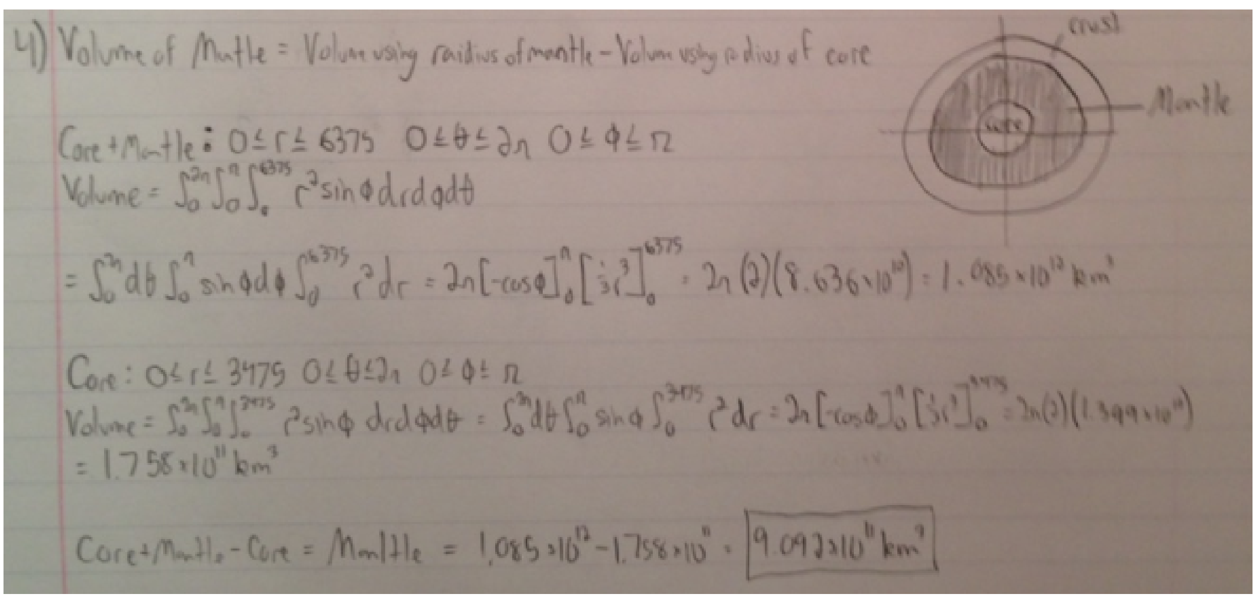


Figure 1: Hand calculation for exercise four

```
>> integral3(@(u,v,w) u.^(2).*(sin(v)), 3475, 6375, 0, pi, 0, 2.*pi)

ans =

   9.0948e+11
```

Matlab calculation for exercise four

Exercise Five

5) Volume of Shpere $= \frac{4}{3}\pi(13)^3 = 9202.77\ mm^3$

Volume of cylinder inside sphere = the integel of the hemisphere over region of intersection

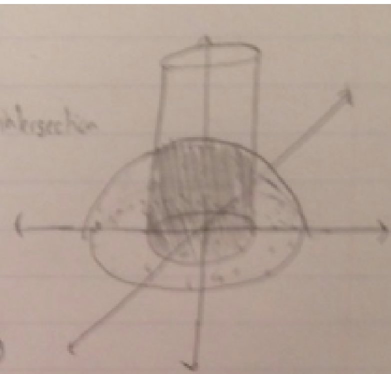
Hemisphere $= z = \sqrt{25 - x^2 - y^2} = \sqrt{169 - r^2}$

Region $D = x^2 + y^2 = 25 \quad r = 5 \quad 0 \le r \le 5 \quad 0 \le \theta \le 2\pi$

Volume of cylinder in bead $= 2\iint_D \sqrt{25 - x^2 - y^2}\, dx\, dy = 2\int_0^{2\pi}\int_0^5 r\sqrt{169 - r^2}\, dr\, d\theta$

$= 2\int_0^{2\pi}\left[-\frac{1}{3}(169 - r^2)^{3/2}\right]_0^5 d\theta = 2\int_0^{2\pi}\left(-\frac{1}{3}(1728) + \frac{1}{3}(2197)\right) d\theta = 2(2\pi)\left(\frac{469}{3}\right) = \frac{1876}{3}\pi \approx 1964.54\ mm^3$

Volume of bead $= 9202.77 - 1964.54 = \boxed{7238.23\ mm^3}$



Hand calculation for exercise five

```
% Exercise 5
syms a b c;
fxn5 = @(a,b,c) a;
cmin = @(a,b) -sqrt(13.^2-a.^2);
cmax = @(a,b) sqrt(13.^2-a.^2);
bead = integral3(fxn5,5,13,0,2.*pi,cmin,cmax);

mantle =

   9.0948e+11

>> bead

bead =

   7.2382e+03
```

Matlab calculation for exercise five