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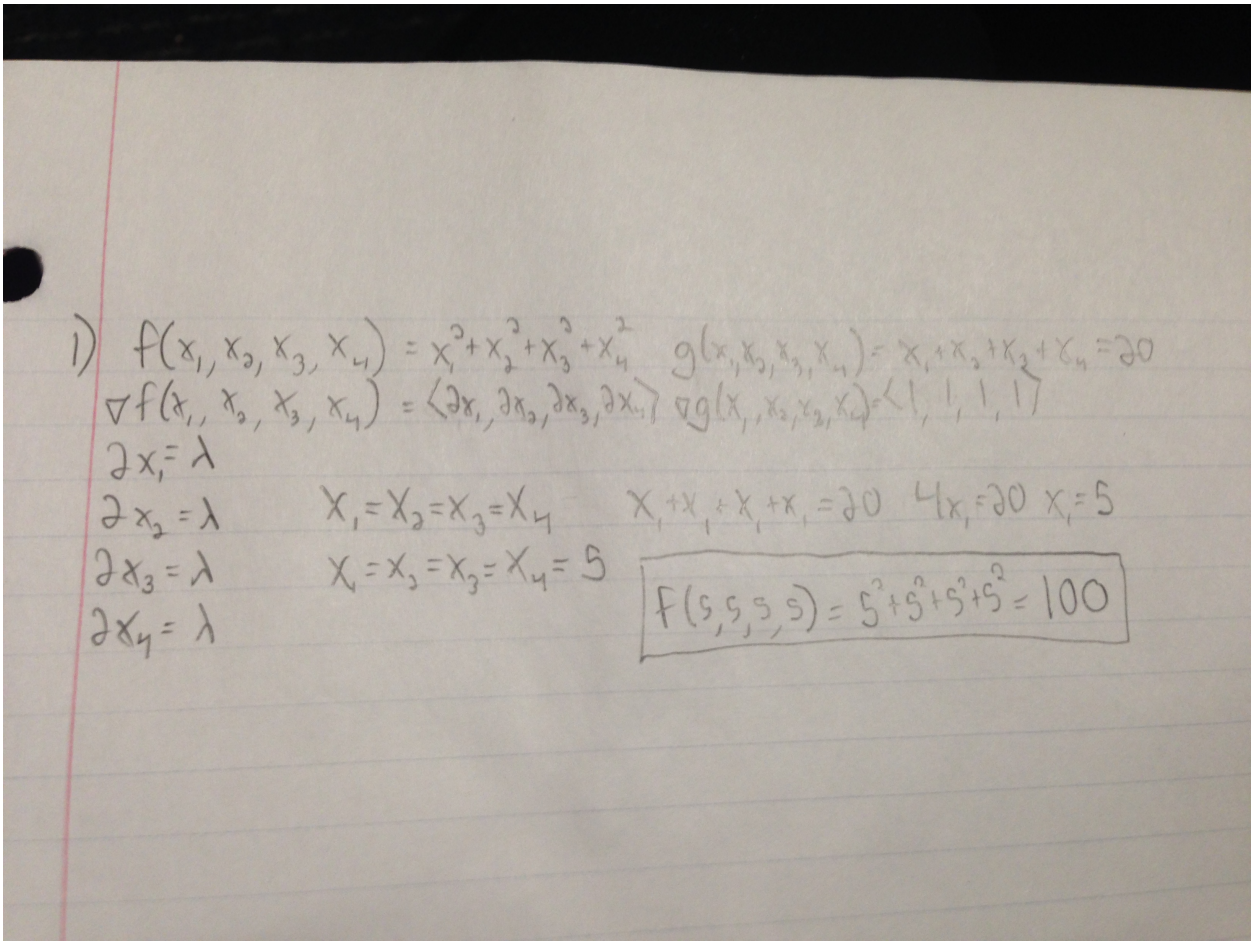
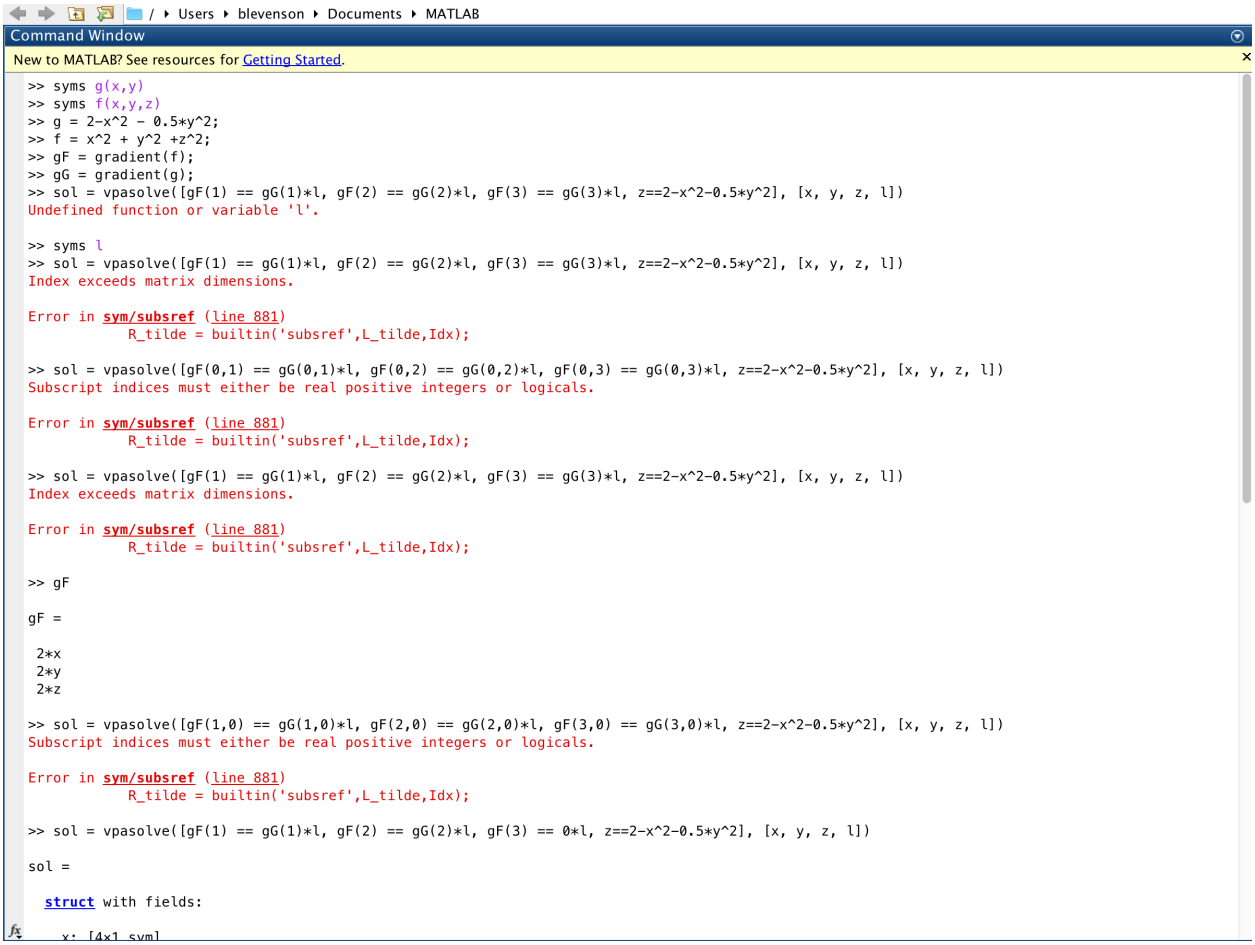
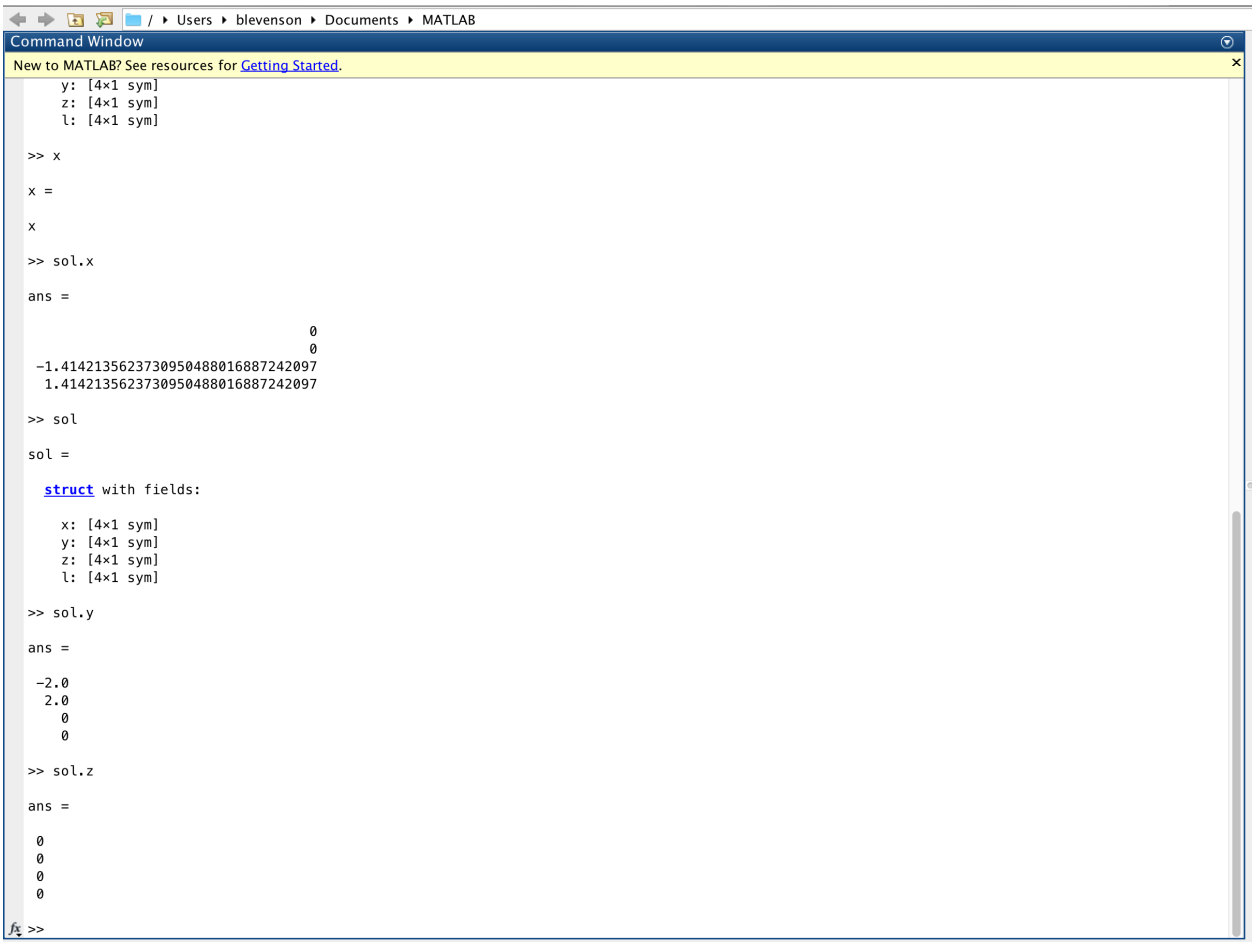
Lab 3

Exercise One



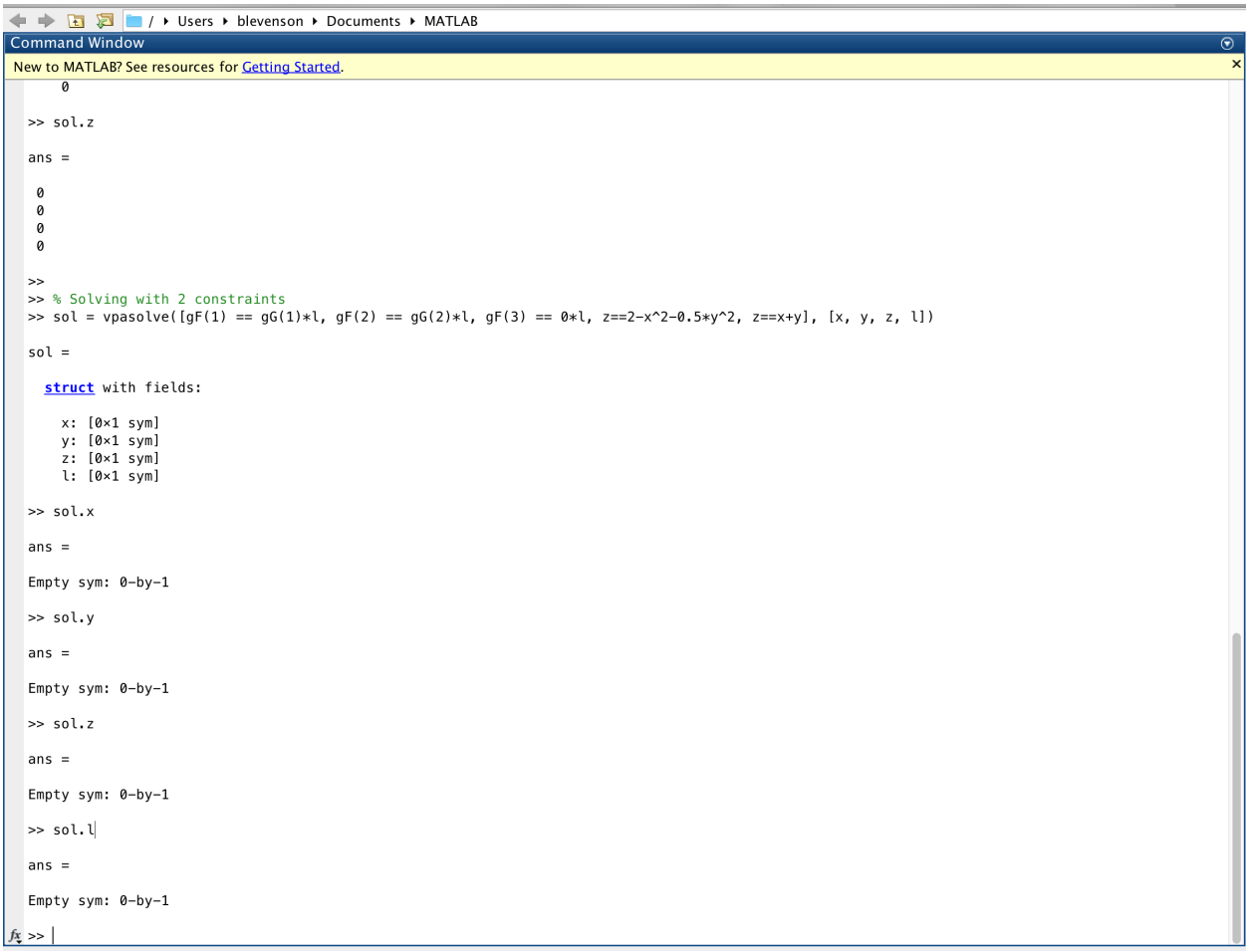
Exercise Two





The solutions that work are: $(0,-2,0),(0,0,0),(-1.4142,-2,0),(-1.4142,0,0),(1.4142,-2,0)$.

Exercise Three



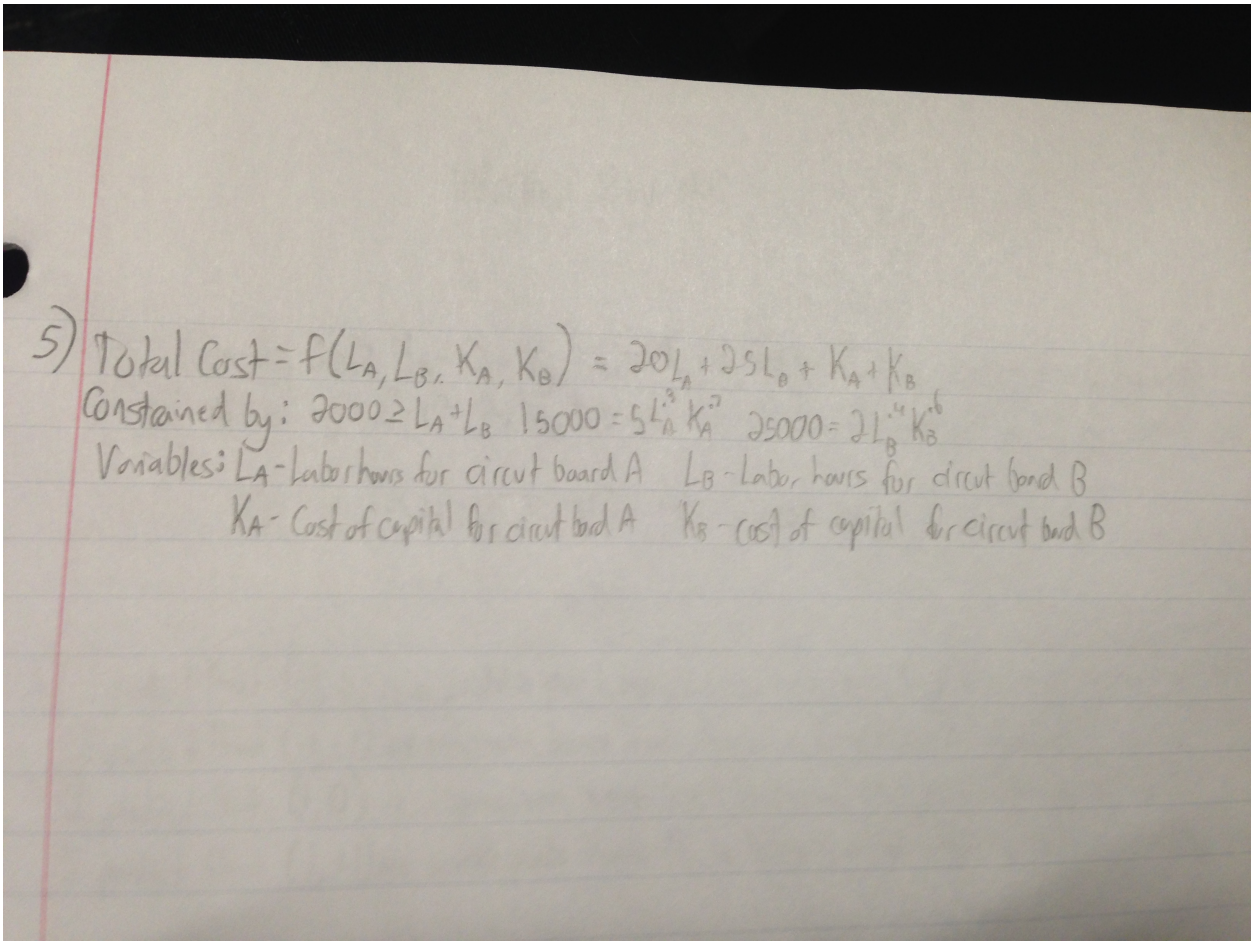
When solving the system with two constraints, there are no critical points of $f(x, y, z)$ constrained by the boundary. This is because the space has been overrefined, preventing any points from fulfilling both the constraints and the LaGrange multiplier equations. Because there are no points in the boundary, we are unable to determine if the gradients are linearly independent at every point on the boundary.

Exercise Four

```
>> syms g(x,y)
>> syms f(x,y,z)
>> g = 2-x^2 - 0.5*y^2;
>> f = x^2 + y^2 + z^2;
>> gF = gradient(f);
>> gG = gradient(g);
>> syms l
>> sol = vpasolve([gF(1) == gG(1)*l, gF(2) ==gG(2)*l, gF(3) == 0*l, z==2-x^2-0.5*y^2], [x,y,z,l])

sol =

  struct with fields:

    x: [4×1 sym]
    y: [4×1 sym]
    z: [4×1 sym]
    l: [4×1 sym]

>> f = @(x,y,z) (x^2) + (y^2) + (z^2);
>> f(sol.x(1), sol.y(1), sol.z(1))

ans =

4.0

>> f(sol.x(2), sol.y(2), sol.z(2))

ans =

4.0

>>  f = @(x,y,z) (x^2) + (y^2) + (z^2)

f =

  function_handle with value:

    @(x,y,z)(x^2)+(y^2)+(z^2)

>>
```

The maximum value is 4.0 from the point $(0, 0, 2)$ and the minimum is 1.75 from the point $(-1.2247, 0, 0.5)$.

Exercise Five



Exercise Six

```
>> syms La Ka Lb Kb lam mu
>> sol=vpasolve([15000==5*La^0.3*Ka^0.7, 25000==2*Lb^0.4*Kb^0.6, 20==lam*1.5*La^(-0.7)*Ka^0.7, 25==mu*0.8*Lb^(-0.6)*Kb^0.6, 1==lam*3.5*La^0.3*Ka^(-0.3),1==mu*1.2*Lb^0.4*Kb^(-0.4)],[Ka, Kb, La, lam, Lb, mu]);
>> sol.La;
>> sol.Lb;
>> sol.Ka;
>> sol.Kb;
>> Cost=@(La, Lb, Ka, Kb) 20*La+25*Lb+Ka+Kb;
>> Cost(sol.La, sol.Lb, sol.Ka, sol.Kb)

ans =

102366.01881599831582906924326579
```

Running the function gave the critical point $(203.62, 1420.7, 9502.2, 53275)$, returning a total cost of \$102,366.02.

Exercise Seven

```
>> syms La Lb Ka Kb lam mu chi
>> sol=vpasolve([15000==5*La^0.3*Ka^0.7, 25000==2*Lb^0.4*Kb^0.6, 2000==La+Lb, 20==1.5*lam*La^(-0.7)*Ka^0.7+chi, 25==0.8*mu*Lb^(-0.6)*Kb^0.6+chi, 1==3.5*lam*La^0.3*Ka^(-0.3), 1==1.2*mu*Lb^0.4*Kb^(-0.4)],[chi, Ka, Kb, La, lam, Lb, mu]);
>> sol.La;
>> sol.Lb;
>> sol.Ka;
>> sol.Kb;
>> Cost=@(La, Lb, Ka, Kb) 20*La+25*Lb+Ka+Kb;
>> Cost(sol.La, sol.Lb, sol.Ka, sol.Kb);
>> Cost(sol.La, sol.Lb, sol.Ka, sol.Kb)

ans =

87630.783104032564276701023377223 - 3592.8057077197124087188518225968i

>>
```

Running the function returns a critical point consisting of imaginary numbers, meaning there are no critical points on the boundary constraint $La + Lb = 2000$. Therefore the minimum cost is the cost found in Exercise 6, \$102,366.02.