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Lab 3

Exercise One

1) $f(x_1, x_2, x_3, x_4) = x_1^2 + x_2^2 + x_3^2 + x_4^2$ $g(x_1, x_2, x_3, x_4) = x_1 + x_2 + x_3 + x_4 = 20$
 $\nabla f(x_1, x_2, x_3, x_4) = \langle 2x_1, 2x_2, 2x_3, 2x_4 \rangle$ $\nabla g(x_1, x_2, x_3, x_4) = \langle 1, 1, 1, 1 \rangle$
 $2x_1 = \lambda$
 $2x_2 = \lambda$ $x_1 = x_2 = x_3 = x_4$ $x_1 + x_1 + x_1 + x_1 = 20$ $4x_1 = 20$ $x_1 = 5$
 $2x_3 = \lambda$ $x_1 = x_2 = x_3 = x_4 = 5$
 $2x_4 = \lambda$
 $f(5, 5, 5, 5) = 5^2 + 5^2 + 5^2 + 5^2 = 100$

Exercise Two

```
Command Window
New to MATLAB? See resources for Getting Started.

>> syms g(x,y)
>> syms f(x,y,z)
>> g = 2-x^2 - 0.5*y^2;
>> f = x^2 + y^2 + z^2;
>> gF = gradient(f);
>> gG = gradient(g);
>> sol = vpasolve([gF(1) == gG(1)*l, gF(2) == gG(2)*l, gF(3) == gG(3)*l, z==2-x^2-0.5*y^2], [x, y, z, l])
Undefined function or variable 'l'.

>> syms l
>> sol = vpasolve([gF(1) == gG(1)*l, gF(2) == gG(2)*l, gF(3) == gG(3)*l, z==2-x^2-0.5*y^2], [x, y, z, l])
Index exceeds matrix dimensions.

Error in sym/subsref (line 881)
    R_tilde = builtin('subsref',L_tilde,Idx);

>> sol = vpasolve([gF(0,1) == gG(0,1)*l, gF(0,2) == gG(0,2)*l, gF(0,3) == gG(0,3)*l, z==2-x^2-0.5*y^2], [x, y, z, l])
Subscript indices must either be real positive integers or logicals.

Error in sym/subsref (line 881)
    R_tilde = builtin('subsref',L_tilde,Idx);

>> sol = vpasolve([gF(1) == gG(1)*l, gF(2) == gG(2)*l, gF(3) == gG(3)*l, z==2-x^2-0.5*y^2], [x, y, z, l])
Index exceeds matrix dimensions.

Error in sym/subsref (line 881)
    R_tilde = builtin('subsref',L_tilde,Idx);

>> gF

gF =

    2*x
    2*y
    2*z

>> sol = vpasolve([gF(1,0) == gG(1,0)*l, gF(2,0) == gG(2,0)*l, gF(3,0) == gG(3,0)*l, z==2-x^2-0.5*y^2], [x, y, z, l])
Subscript indices must either be real positive integers or logicals.

Error in sym/subsref (line 881)
    R_tilde = builtin('subsref',L_tilde,Idx);

>> sol = vpasolve([gF(1) == gG(1)*l, gF(2) == gG(2)*l, gF(3) == 0*l, z==2-x^2-0.5*y^2], [x, y, z, l])

sol =

    struct with fields:
        x: [4x1 sym]
```

```
Command Window
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y: [4x1 sym]
z: [4x1 sym]
l: [4x1 sym]

>> x
x =
x

>> sol.x
ans =
0
0
-1.4142135623730950488016887242097
1.4142135623730950488016887242097

>> sol
sol =
struct with fields:
    x: [4x1 sym]
    y: [4x1 sym]
    z: [4x1 sym]
    l: [4x1 sym]

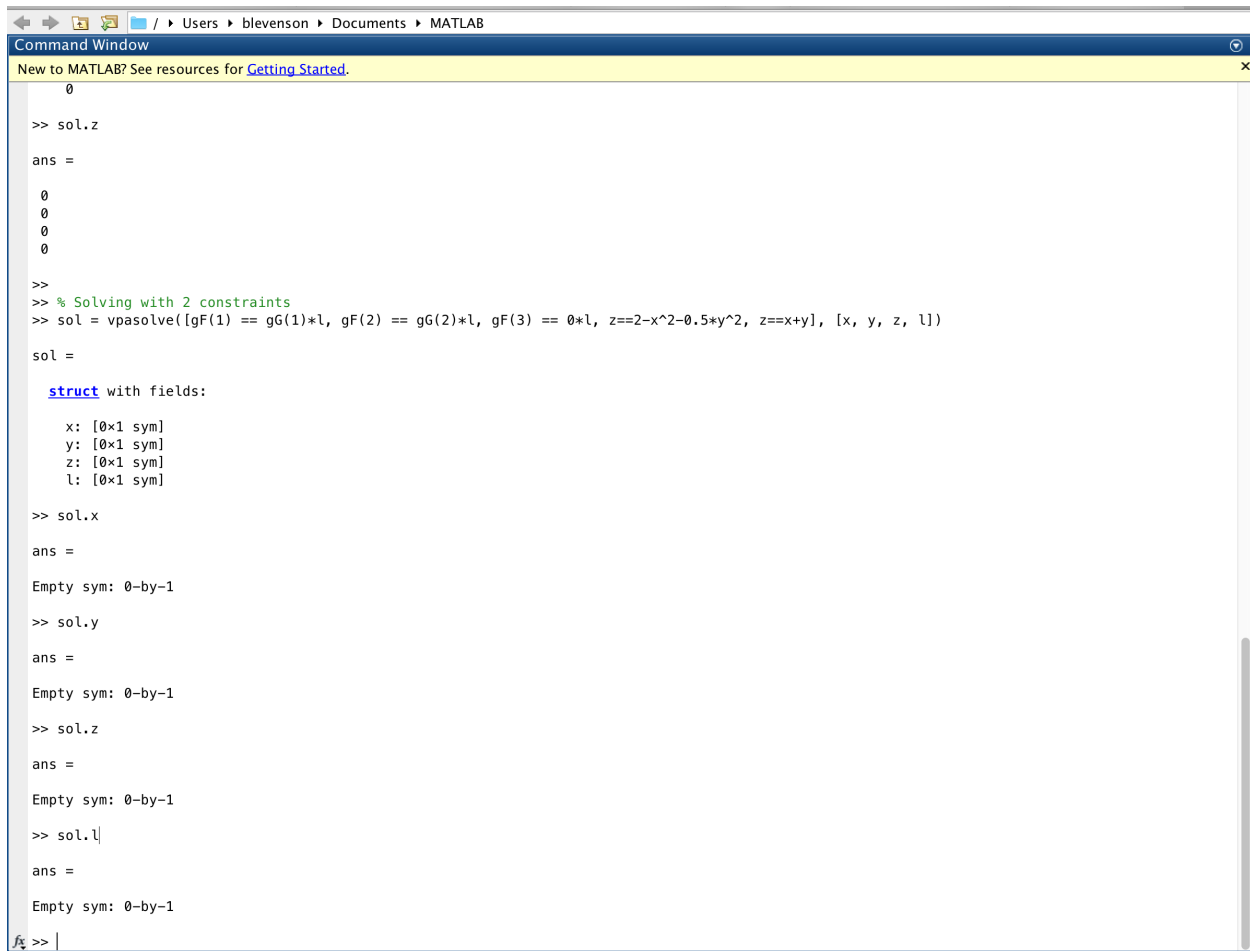
>> sol.y
ans =
-2.0
2.0
0
0

>> sol.z
ans =
0
0
0
0

fx >>
```

Figure 1: The solutions that work are: $(0, -2, 0)$, $(0, 0, 0)$, $(-1.4142, -2, 0)$, $(-1.4142, 0, 0)$, $(1.4142, -2, 0)$.

Exercise Three



```
Command Window
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0
>> sol.z
ans =
0
0
0
0

>>
>> % Solving with 2 constraints
>> sol = vpasolve([gF(1) == gG(1)*l, gF(2) == gG(2)*l, gF(3) == 0*l, z==2-x^2-0.5*y^2, z==x+y], [x, y, z, l])

sol =
struct with fields:
    x: [0x1 sym]
    y: [0x1 sym]
    z: [0x1 sym]
    l: [0x1 sym]

>> sol.x
ans =
Empty sym: 0-by-1

>> sol.y
ans =
Empty sym: 0-by-1

>> sol.z
ans =
Empty sym: 0-by-1

>> sol.l
ans =
Empty sym: 0-by-1

fx >> |
```

Figure 2: When solving the system with two constraints, there are no critical points of $f(x, y, z)$ constrained by the boundary. This is because the space has been overrefined, preventing any points from fulfilling both the constraints and the LaGrange multiplier equations. Because there are no points in the boundary, we are unable to determine if the gradients are linearly independent at every point on the boundary.

Exercise Four

Exercise Five

5) Total Cost = $f(L_A, L_B, K_A, K_B) = 20L_A + 25L_B + K_A + K_B$
Constrained by: $2000 \geq L_A + L_B$ $15000 = 5L_A^3 K_A^2$ $25000 = 2L_B^4 K_B^6$
Variables: L_A - Labor hours for circuit board A L_B - Labor hours for circuit board B
 K_A - Cost of capital for circuit board A K_B - cost of capital for circuit board B

Exercise Six

Exercise Seven