LAB 1: VECTORS AND GEOMETRY, PART B

1. Objectives and Expectations for Lab 1, Part B

- Part B of Lab 1 contains less guidance.
- You are expected to answer problems that are more challenging than the ones in Part A.
- The material covered and the assignments in Part A is instructive for completing Part B.

2. Matlab Commands

2.1. **[a:h:b].** The command `[a:h:b]` generates an ordered list of numbers $(a, a+h, a+2h, a+3h, \dots, a+nh)$ where n is the largest positive number such that $a + nh \leq b$. Examples:

```
>> [1:0.2:2.4]
```

which gives the list `[1.00 1.20 1.40 1.60 1.80 2.00 2.20 2.40]`, and

```
>> [1:0.4:2]
```

which gives `[1.0000 1.4000 1.8000]`. This command is useful for creating equally spaced set of parameters or coordinates.

2.2. **linspace(a,b,h).** `linspace[a,b,n]` creates n equally spaced (linearly spaced) list of numbers between the points a and b, including a and b. Example:

```
>> tset = linspace(0,2,9)
```

which sets `tset` to be the list `[0 0.25 0.5...2]`.

2.3. **plot(x,y).** The command `plot(x,y)` takes as its arguments a vector `x` and another vector `y`, and plots successive pairs of (x, y) coordinates. Thus,

```
>> plot( [0, 1, 2, 3, 4], [0, 1, 4, 9, 16] )
```

will plot five points along the parabola $y = x^2$. Most functions and operations in *MATLAB* are vector-aware, so that they accept a vector as an argument. Thus we can plot five points on the parameteric curve $\mathbf{r}(t) = \langle 1 + 3t, \sin(t) \rangle$ with

```
>> t = [0,1,2,3,4]; plot( 1+3*t, sin(t), 'o' )
```

(the 'o' uses a circle as the plot symbol), and can plot a smooth function (with a blue line) with

```
>> x = [-1:0.01:1]; plot(x, acos(x), '-b')
```

2.4. **plot3(x,y,z).** The command `plot3(x,y,z)` plots in three dimensions the curve whose x, y, and z-coordinates are given by the vectors `x`, `y`, and `z`. Thus

```
>> t = [0:.02:12]; plot3( 2*cos(t), 2*sin(t), t, '--r' )
```

plots (with a dashed red line) a helix extending up along the z-axis.

3. Exercises

3.1. **Dot product and cross product of vectors.** Consider the 1-parameter family of vectors $\mathbf{v}(t)$ given by

$$\mathbf{v}(t) = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} t(1-\sin(t))\cos(t) \\ (1-t\cos(t))\sin(t) \\ \frac{1}{7}(3\sin(t)+2t\cos(t)-5t\sin(t)\cos(t)) \end{bmatrix}, \quad t \in \mathbb{R}.$$

Observe that $\mathbf{v}(0) = (0,0,0)$. We will obtain numerical evidence for the fact that the family of vectors $\{\mathbf{v}(t)\}_{t\in\mathbb{R}}$ lie on a plane.

Exercise 1: Pick a nonzero vector $\mathbf{u}$ from the family by setting $\mathbf{u} = \mathbf{v}(t_1)$ for some $t_1 \in \mathbb{R}$. Compute the **normal vector**s

$$\mathbf{n}(t) = \mathbf{u} \times \mathbf{v}(t)$$

of the planes determined by $\mathbf{u}$ and $\mathbf{v}(t)$ for values of t between 0.1 and 1000 to numerically verify that changing t does not rotate $\mathbf{n}(t)$, but scales $\mathbf{n}(t)$. Use the values of t generated by the *MATLAB* command
`>> tlist=[0.1:0.01:1000];`
Plot the tip of the vector $\mathbf{n}(t)$ as t varies in this range.

3.2. **Projections.** Data that are obtained at successive instants in time are called *time-series data*. Often such data are generated from a sensor or sensors, and in any real-life application the sensor readings will have some error, or news. In this exercise we consider how we might work with such data. For the problem, consider set of points

$$\mathbf{S} = \Big((3,7.4),(4.5,9),(1.32,5.43),(9.18,14.8),$$
$$(1.369,5.05),(7.847,13.922),(6.7765,12.385),(3.1162,7.688)\Big) \tag{1}$$

and suppose that it is data received from a sensor. We can stored these in a sequence of 2-dimensional vectors $\mathbf{x}_1 = \langle 3, 7.4\rangle$, $\mathbf{x}_2 = \langle 4.5, 9\rangle$, $\mathbf{x}_3 = \langle 1.32, 5.43\rangle$, $\mathbf{x}_4 = \langle 9.68, 14.6\rangle$, ..., $\mathbf{x}_8 = \langle 3.4162, 7.988\rangle$.

Exercise 2: Plot these points (vectors). What type of trend do you see?

Exercise 3: Suppose that all of the data points $\mathbf{x} = \langle x_1, x_2\rangle$ should actually lie on the line

$$\mathbf{r}(t) = \langle 1, 5\rangle + t\langle 0.16, 0.2\rangle, \quad t \geq 0. \tag{2}$$

Plot the data points and the line on which we expect the points to lie (to plot both together, you can use the *MATLAB* `plot` command with multiple arguments: `plot( x1, y1, 'ok', x2, y2, '-b' )` will plot the x coordinates in the list `x1` against the y coordinates in `y1` with black circles, and the x coordinates in `x2` against the y coordinates in `y2` with a blue line). Observe that the data points do not lie on the line.

Exercise 4: Remove the noise from this data set. You can to this by mapping each data point $\mathbf{x}_j$ onto a point $\bar{\mathbf{x}}_j$ on the line where the distance of the point $\mathbf{x}_j$ from the line is minimal. (*Hint:* you can use orthogonal projections, first subtracting the vector $\langle 1, 5\rangle$ from the data set, then compute the orthogonal projection of these vectors onto the direction vector of the line, then add $\langle 1, 5\rangle$ back to find the de-noised data. Think about what this will do for one point.)

3.3. **Equations of lines and planes.**

Exercise 5: Given the planes

$$4.2(x - 3.32) - 5.28(y - 1) + (z + 2) = 0$$

and

$$(x + 2.42) + 2(y - 7.34) - 1.1(z - 2) = 0,$$

compute the angle between them.

References

[JS] Stewart, James. Calculus. Eighth edition. Boston, MA:Cengage, 2016.