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October 31, 2017

Lab 3

Exercise One

1) $f(x_1, x_2, x_3, x_4) = x_1^2 + x_2^2 + x_3^2 + x_4^2$ $\quad g(x_1, x_2, x_3, x_4) = x_1 + x_2 + x_3 + x_4 = 20$

$\nabla f(x_1, x_2, x_3, x_4) = \langle 2x_1, 2x_2, 2x_3, 2x_4 \rangle$ $\quad \nabla g(x_1, x_2, x_3, x_4) = \langle 1, 1, 1, 1 \rangle$

$2x_1 = \lambda$

$2x_2 = \lambda$ $\quad x_1 = x_2 = x_3 = x_4$ $\quad x_1 + x_1 + x_1 + x_1 = 20$ $\quad 4x_1 = 20$ $\quad x_1 = 5$

$2x_3 = \lambda$ $\quad x_1 = x_2 = x_3 = x_4 = 5$ $\quad \boxed{f(5,5,5,5) = 5^2 + 5^2 + 5^2 + 5^2 = 100}$

$2x_4 = \lambda$

Exercise Two

```
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Command Window
New to MATLAB? See resources for Getting Started.

>> syms g(x,y)
>> syms f(x,y,z)
>> g = 2-x^2 - 0.5*y^2;
>> f = x^2 + y^2 +z^2;
>> gF = gradient(f);
>> gG = gradient(g);
>> sol = vpasolve([gF(1) == gG(1)*l, gF(2) == gG(2)*l, gF(3) == gG(3)*l, z==2-x^2-0.5*y^2], [x, y, z, l])
Undefined function or variable 'l'.

>> syms l
>> sol = vpasolve([gF(1) == gG(1)*l, gF(2) == gG(2)*l, gF(3) == gG(3)*l, z==2-x^2-0.5*y^2], [x, y, z, l])
Index exceeds matrix dimensions.

Error in sym/subsref (line 881)
            R_tilde = builtin('subsref',L_tilde,Idx);

>> sol = vpasolve([gF(0,1) == gG(0,1)*l, gF(0,2) == gG(0,2)*l, gF(0,3) == gG(0,3)*l, z==2-x^2-0.5*y^2], [x, y, z, l])
Subscript indices must either be real positive integers or logicals.

Error in sym/subsref (line 881)
            R_tilde = builtin('subsref',L_tilde,Idx);

>> sol = vpasolve([gF(1) == gG(1)*l, gF(2) == gG(2)*l, gF(3) == gG(3)*l, z==2-x^2-0.5*y^2], [x, y, z, l])
Index exceeds matrix dimensions.

Error in sym/subsref (line 881)
            R_tilde = builtin('subsref',L_tilde,Idx);

>> gF

gF =

 2*x
 2*y
 2*z

>> sol = vpasolve([gF(1,0) == gG(1,0)*l, gF(2,0) == gG(2,0)*l, gF(3,0) == gG(3,0)*l, z==2-x^2-0.5*y^2], [x, y, z, l])
Subscript indices must either be real positive integers or logicals.

Error in sym/subsref (line 881)
            R_tilde = builtin('subsref',L_tilde,Idx);

>> sol = vpasolve([gF(1) == gG(1)*l, gF(2) == gG(2)*l, gF(3) == 0*l, z==2-x^2-0.5*y^2], [x, y, z, l])

sol =

  struct with fields:

    x: [4x1 sym]
```

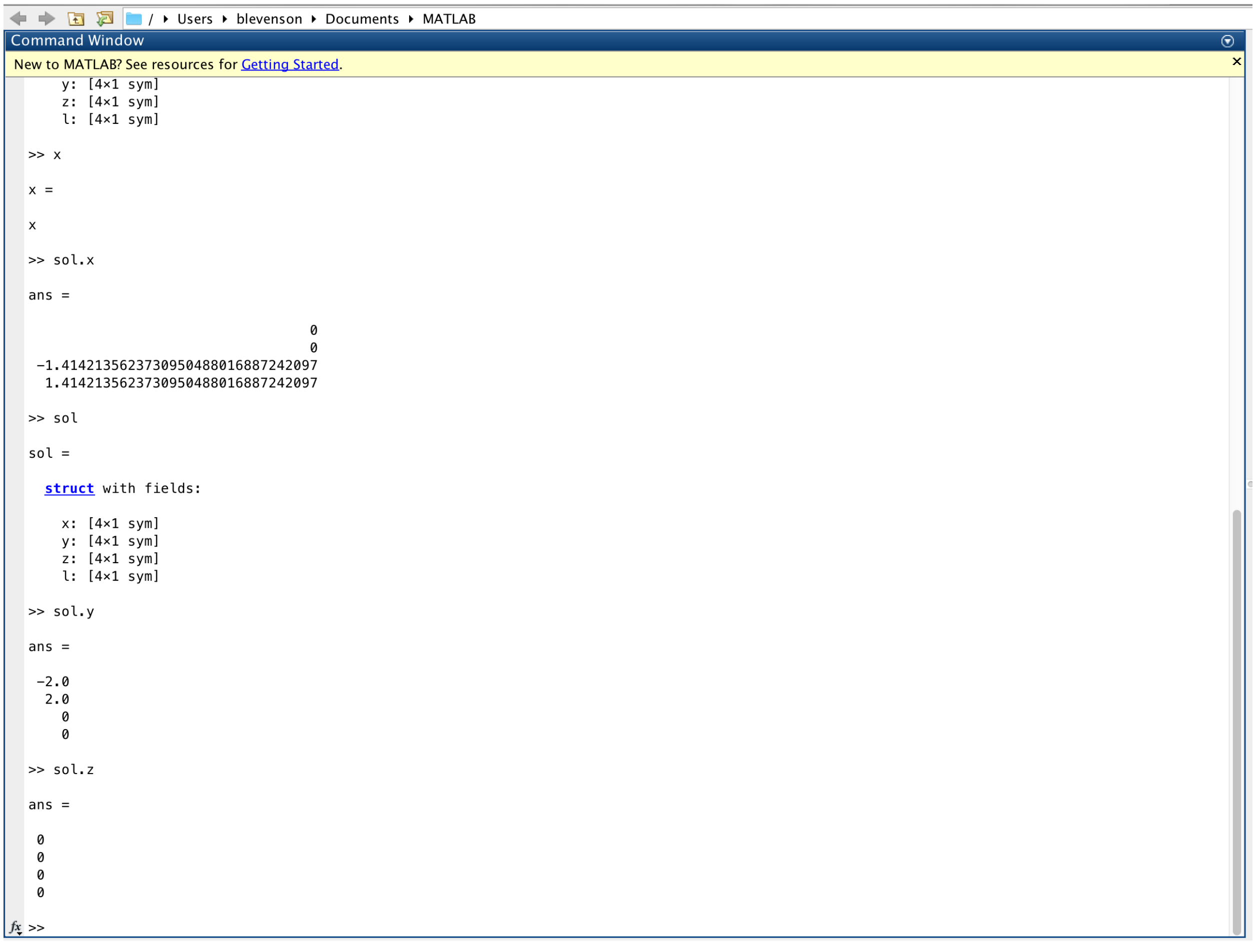

```
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New to MATLAB? See resources for Getting Started.
    y: [4×1 sym]
    z: [4×1 sym]
    l: [4×1 sym]

>> x

x =

x

>> sol.x

ans =

                                  0
                                  0
 -1.4142135623730950488016887242097
  1.4142135623730950488016887242097

>> sol

sol =

  struct with fields:

    x: [4×1 sym]
    y: [4×1 sym]
    z: [4×1 sym]
    l: [4×1 sym]

>> sol.y

ans =

 -2.0
  2.0
    0
    0

>> sol.z

ans =

 0
 0
 0
 0

>>
```

The solutions that work are: $(0,-2,0), (0,0,0), (-1.4142,-2,0), (-1.4142,0,0), (1.4142,-2,0)$.

Exercise Three

```
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     0

>> sol.z

ans =

 0
 0
 0
 0

>>
>> % Solving with 2 constraints
>> sol = vpasolve([gF(1) == gG(1)*l, gF(2) == gG(2)*l, gF(3) == 0*l, z==2-x^2-0.5*y^2, z==x+y], [x, y, z, l])

sol =

  struct with fields:

    x: [0×1 sym]
    y: [0×1 sym]
    z: [0×1 sym]
    l: [0×1 sym]

>> sol.x

ans =

Empty sym: 0-by-1

>> sol.y

ans =

Empty sym: 0-by-1

>> sol.z

ans =

Empty sym: 0-by-1

>> sol.l

ans =

Empty sym: 0-by-1

fx >>
```
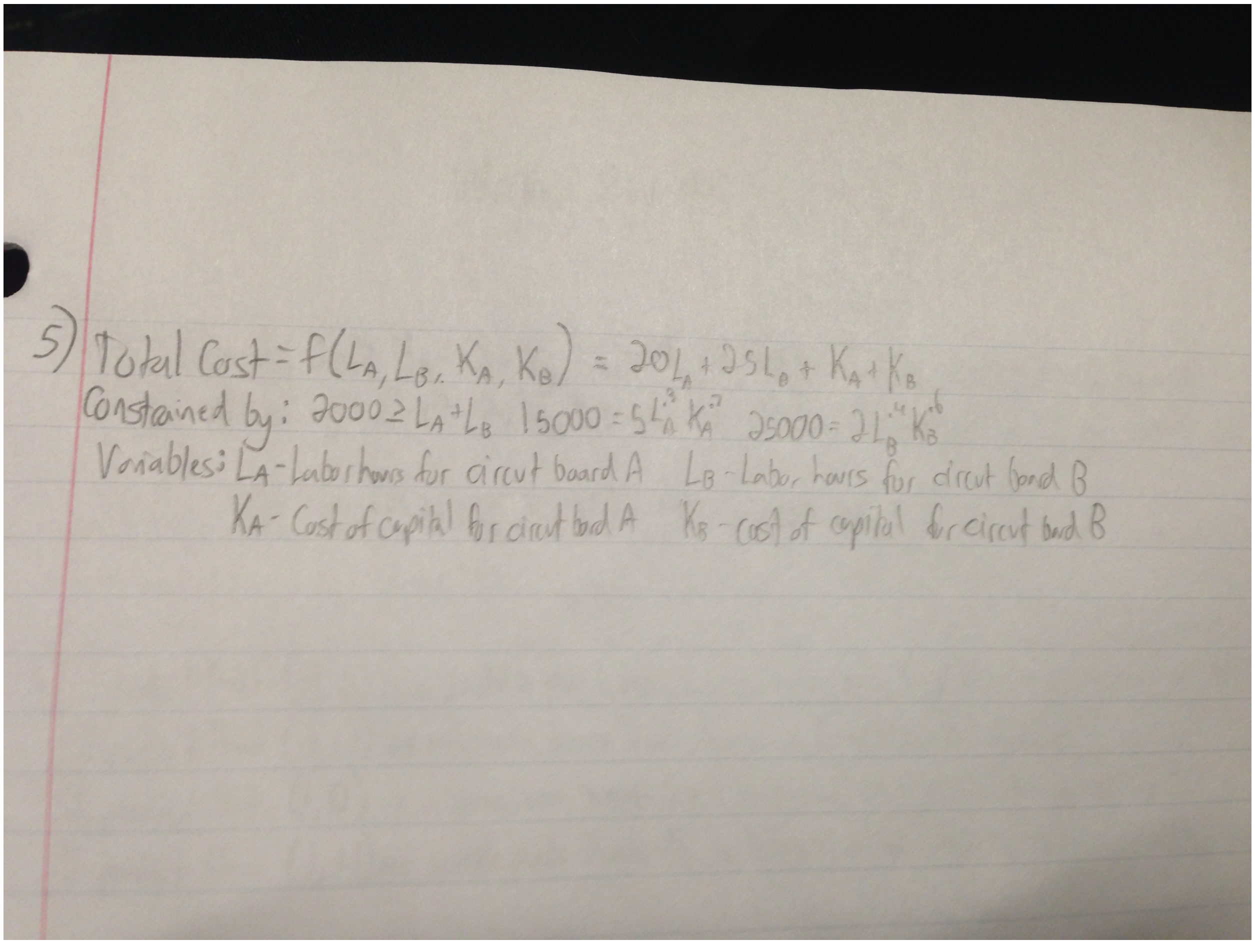

When solving the system with two constraints, there are no critical points of $f(x, y, z)$ constrained by the boundary. This is because the space has been overrefined, preventing any points from fulfilling both the constraints and the LaGrange multiplier equations. Because there are no points in the boundary, we are unable to determine if the gradients are linearly independent at every point on the boundary.

Exercise Four

```
>> syms g(x,y)
>> syms f(x,y,z)
>> g = 2-x^2 - 0.5*y^2;
>> f = x^2 + y^2 + z^2;
>> gF = gradient(f);
>> gG = gradient(g);
>> syms l
>> sol = vpasolve([gF(1) == gG(1)*l, gF(2) ==gG(2)*l, gF(3) == 0*l, z==2-x^2-0.5*y^2], [x,y,z,l])

sol =

  struct with fields:

    x: [4×1 sym]
    y: [4×1 sym]
    z: [4×1 sym]
    l: [4×1 sym]

>> f = @(x,y,z) (x^2) + (y^2) + (z^2);
>> f(sol.x(1), sol.y(1), sol.z(1))

ans =

4.0

>> f(sol.x(2), sol.y(2), sol.z(2))

ans =

4.0

>>  f = @(x,y,z) (x^2) + (y^2) + (z^2)

f =

  function_handle with value:

    @(x,y,z)(x^2)+(y^2)+(z^2)

>> |
```

The maximum value is 4.0 from the point $(0, 0, 2)$ and the minimum is 1.75 from the point $(-1.2247, 0, 0.5)$.

Exercise Five



5) Total Cost $= f(L_A, L_B, K_A, K_B) = 20L_A + 25L_B + K_A + K_B$

Constrained by: $2000 \geq L_A + L_B$ $\quad 15000 = 5L_A^{.3}K_A^{.7}$ $\quad 25000 = 2L_B^{.4}K_B^{.6}$

Variables: L_A - Labor hours for circut board A $\quad L_B$ - Labor hours for circut board B

K_A - Cost of capital for circut bord A $\quad K_B$ - cost of capital for circut bord B

Exercise Six

```
>> syms La Ka Lb Kb lam mu
>> sol=vpasolve([15000==5*La^0.3*Ka^0.7, 25000==2*Lb^0.4*Kb^0.6, 20==lam*1.5*La^(-0.7)*Ka^0.7, 25==mu*0.8*Lb^(-0.6)*Kb^0.6, 1==lam*3.5*La^0.3*Ka^(-0.3),1==mu*1.2*Lb^0.4*Kb^(-0.4)],[Ka, Kb, La, lam, Lb, mu]);
>> sol.La;
>> sol.Lb;
>> sol.Ka;
>> sol.Kb;
>> Cost=@(La, Lb, Ka, Kb) 20*La+25*Lb+Ka+Kb;
>> Cost(sol.La, sol.Lb, sol.Ka, sol.Kb)

ans =

102366.0188159983158290692432657 9
```

Running the function gave the critical point $(203.62, 1420.7, 9502.2, 53275)$, returning a total cost of \$102,366.02.

Exercise Seven

```
>> syms La Lb Ka Kb lam mu chi
>> sol=vpasolve([15000==5*La^0.3*Ka^0.7, 25000==2*Lb^0.4*Kb^0.6, 2000==La+Lb, 20==1.5*lam*La^(-0.7)*Ka^0.7+chi, 25==0.8*mu*Lb^(-0.6)*Kb^0.6+chi, 1==3.5*lam*La^0.3*Ka^(-0.3), 1==1.2*mu*Lb^0.4*Kb^(-0.4)],[chi, Ka, Kb, La, lam, Lb, mu]);
>> sol.La;
>> sol.Lb;
>> sol.Ka;
>> sol.Kb;
>> Cost=@(La, Lb, Ka, Kb) 20*La+25*Lb+Ka+Kb;
>> Cost(sol.La, sol.Lb, sol.Ka, sol.Kb);
>> Cost(sol.La, sol.Lb, sol.Ka, sol.Kb)

ans =

87630.783104032564276701023377223 - 3592.8057077197124087188518225968i

>> |
```

Running the function returns a critical point consisting of imaginary numbers, meaning there are no critical points on the boundary constraint $La + Lb = 2000$. Therefore the minimum cost is the cost found in Exercise 6, \$102,366.02.