5. To what extent do you agree with the claim “all models are wrong, but some are useful” (attributed to George Box)? Discuss with reference to mathematics**and one other** area of knowledge.

Word count:1600

‘All models are wrong; some are useful.’ Statistician George Box’s thought-provoking statement reveals the positive characteristics-simplicity and practicality of models as knowledge-creation tools. Models are simplified representations of reality that are built to explain, predict or analyse some parts of the existing world. Through repeated observations, researchers identify commonalities, ensure validity via ‘independent repeatability,’ and infer universal patterns to build models. The use of models plays an important role in knowledge exploration, helping scientists discover or refine basic laws, that is how models are useful. However, instead of exactly describing and predicting complex phenomena in the real world, models are wrong as they assume that a model that is valid in a certain time or space is also valid in other situations, which is not the truth. In the process of solving problems, humans inevitably overlook ‘trivial’ details, thus enabling problems to be solved within a certain historical and cultural framework. However, the effectiveness of any model has spatiotemporal boundaries, and once these boundaries are crossed, the previously established laws need to be critically evaluated carefully.

In the AOK of mathematics, while most mathematical models rely on assumptions and fail to account for real conditions exactly, they can still be useful tools for prediction and optimisation. That’s why, despite the inherent limitations of mathematical models, they remain essential tools for understanding and solving complex problems. The Bayes model is a simple probability classifier based on Bayes' theorem and the assumption of feature independence. Although this assumption rarely holds true in reality, the Bayes model performs well in problems such as text classification and spam filtering. For example, to check whether there are spam emails in our mailbox, the simplest method is to make a judgment based on whether certain keywords exist in the email. An assumption is made: the frequency of keywords appearing in spam emails is regular. Under this assumption, by measuring how often certain keywords appear in all emails, how frequently spam emails occur overall, and how often these keywords appear in spam emails, we can calculate and classify the probability that a given email is spam. Bayesian models help to understand the probability of certain intrinsic properties of things through certain easily computable probabilities and allow people to infer the essence of things through phenomena by observing indicators of certain phenomena in reality, which is quite useful. But the reality is that not all emails involving a certain keyword are spam emails, and in practical situations, there are many features interacting with each other, determining whether an email is spam and the probability of all possible keywords appearing should be considered, considering this, the model is oversimplified and wrong.

However, the prediction of the future situation or trend of the real world by models is based on the induction of rules from past facts, and the models infer the unknown parts of the future of things. Then, what is the impact of future variables on the effectiveness of a model? Take the Black-Scholes model, for instance. It was a useful statistical model invented in 1973 to help us understand the basic dynamics of the market and was quickly adopted into widespread usage in quantitative investing - investment through mathematical models (Shanghai Stock Exchange Options House, 2024). However, as more and more companies began adopting the formula and creating their own versions of it, they failed to account for the limitations of the model: it did not account for changes in market conditions such as price volatility. Although this limitation did not cause any large issues when the model was released, it contributed to the global 2008 financial crisis (Tankov, 2023), when there were drastic price changes, the model outputted unreliable data. Being unable to adapt to variables in real-life situations can be a fatal flaw and wrong for models sometimes.

Therefore, mathematical modelling provides us with a tool that enables us to transform complex problems into solvable mathematical problems. Through this approach, we can better understand and predict various phenomena, from simple text classification to extensive financial investments, which is useful. Nevertheless, the failure to consider the complicated and ever-changing reality results in the ineffectiveness of models and sometimes even causes disasters.

Moving on to natural science, models that are later proven to be wrong are commonly used as important frameworks for developing an improved understanding of natural phenomena over time. Thus, one may argue that some historical models are ultimately "wrong" but that they nonetheless have a crucial function for the development of scientific knowledge. The phlogiston theory was a striking example, prevalent in scientific thought in the 17th and 18th centuries. Later debunked, it nonetheless acted as an advance post for the later development of modern chemistry and thermodynamics, showing that even a wrong model can be extraordinarily helpful. According to the phlogiston theory, a substance phlogiston was released during combustion. The model of this type was widely accepted and even guided the early scientific experiments because it outlined the pattern of classification of materials on the basis of their combustibility (Qin, 2007). This theory, however, could not explain why some metals increased in mass when they were burned. However, it was eventually disproven by Antoine Lavoisier, who showed that combustion was about oxygen and thus taught what we know about oxidation and reactions. Although the phlogiston theory did not stand the test of time, it was valuable, for it inspired experimental methods and systematic inquiry into the law of combustion, even if the result was merely a more refined misunderstanding of it (Gu, 2005). It shows how incorrect models can also be propitious for advancement by requiring improvements and alterations.

Similarly, in physics, even though Newtonian mechanism has been overthrown by Einstein’s theory of relativity when applied to objects that are microscopic or travel faster than light speed or in strong gravitational fields (Yin & Hu, 2014), it still applies to macroscopic objects and low-speed moving objects, etc. and can explain many natural phenomena, such as collisions of objects, and the causes of tides. This supports the statement that while some models are imperfect, their imperfections and limitations justify the need for further information. Therefore, microscopic particles such as electrons, protons, etc., were discovered in the early 20th century, whose motion laws cannot be explained by Newtonian mechanics in many cases (Zhang, 1982). Then, in the 1920s, quantum mechanics was established, which could well describe the laws of microscopic particle motion and played an important role in modern science and technology (Zhang, 1982). Natural science models are constantly refined when new cases emerge, indicating they are no longer universally valid. However, Newtonian mechanics will not be negated by new scientific developments but will be included as a special case under certain conditions in new scientific achievements. So, the usefulness of a model does not solely depend on its accuracy, but it depends upon its ability to aid in the discovery of new knowledge, contribute to the development of scientific knowledge and provide practical applications.

Conversely, improper models can serve as useful stepping stones in natural science but can preclude scientific progress if they become institutionally rooted or are ignored for their limitations. One such example was the caloric theory of heat that persisted in ongoing for over a century before modern thermodynamic principles replaced it (Liu & Li, 1992). It also shows how flawed theories can sometimes slow scientific progress rather than promote it. In the 18th century the caloric theory was developed that heat was a weightless, invisible fluid called caloric that flowed from hot to cold objects. While it explained some heat transfer phenomena, it did not satisfy experiments on mechanical work generating heat by Benjamin Thompson (Solar Element, 2023). While his observation that heat was generated from friction posed a threat to the caloric model, scientific institutions refused to give it up. As the Caloric Theory of Heat not only qualitatively explains many thermal phenomena, but also quantitatively describes heat conduction using mathematics. The experiment only exposed the difficulties of the Caloric Theory of Heat, which is almost negligible compared to the precise laws established based on it. The reluctance to accept emerging evidence delayed the development of thermodynamics and the acceptance of heat as a form of energy, which did not arrive until the work of James Joule and Rudolf Clausius in the 19th century (Sina Blog, n.d.). The caloric theory is an example of how institutional biases and the inertia of science can keep flawed models around past their usefulness (Kuhn, 2003). While the phlogiston theory had produced productive experimentation, the caloric model held back progress by leaving little room for alternative ideas. This reveals that if wrong models are already ingrained in scientific thought, then the shift to better theories can be slow. Therefore, it follows that wrong models are nonetheless useful Only if they are subjected to critical analysis. A model can become a hindrance to further knowledge if its limitations are ignored or if it is kept in spite of contradictory evidence. This drives this point home: Models must be constantly questioned to ensure they serve as tools for progress, not barriers to truth.

Finally, the examples suggest that some models are much more ´perfect´ and subject to less revision than others. No model can perfectly replicate reality, and they may miss certain key factors or overly simplify certain details in reality. Although models may not be perfect, they can still provide us with enormous value as they capture the main features of phenomena and provide us with meaningful insights by predicting or describing a general understanding of the future, as exemplified in mathematics and by facilitating refinement or discovery of new knowledge and helping the development of scientific knowledge, as exemplified in natural sciences.

# Bibliography

1. lack, Fischer, and Myron Scholes. "The Pricing of Options and Corporate Liabilities." Journal of Political Economy, vol. 81, no. 3, 1973, pp. 637–654. https://people.dm.unipi.it/pratelli/Finanza/Black-Scholes.pdf. Accessed 1 Feb. 2025.
2. artington, J. R. A History of Chemistry: Volume 3. Macmillan, 1962. https://archive.org/details/historyofchemist0003part. Accessed 1 Feb. 2025.
3. ardwell, D. S. L. From Watt to Clausius: The Rise of Thermodynamics in the Early Industrial Age. Cornell University Press, 1971. https://archive.org/details/fromwatttoclausi0000card. Accessed 1 Feb. 2025.
4. ienberg, Stephen E. Bayesian Methods in the Social Sciences. CRC Press, 2006. https://students.aiu.edu/submissions/profiles/resources/onlineBook/n5y9y2\_Bayesian\_Methods\_Social\_and\_Behavioral.pdf. Accessed 1 Feb. 2025.
5. How was the B-S model born. Shanghai Stock Exchange Options House. Shang Hai. 2024, 12, 27. https://mp.weixin.qq.com/s/aq5aSLXpVnltZNGFaVE6Lw
6. Peter Tankov. Black Scholes Formula: The Founder of Wall Street Formulas. Aerial View of Innovation. Bei Jing. 2023, 9, 20. https://mp.weixin.qq.com/s/xsPgJ0x2xeDUS1VHa\_xrDg
7. Qin Fangming. "The establishment and collapse of the 'phlogiston theory'" [J]. Invention and Innovation (Student Edition), 2007, (07):15.
8. Gu Xin. "From phlogiston to oxygen" [J]. Middle School Student Encyclopedia, 2005, (13):34-36.
9. Yin Ye, Hu Suhui. "Analysis and Comparison of the Theoretical Foundations of Newtonian Mechanics and Relativity" [J]. Frontier Science, 2014, 8(02): 33-41.
10. Zhang Jingxun. "A Brief History of the Development of Quantum Mechanics" [J]. Journal of Northwest University (Natural Science Edition), 1982, (04):71-84.
11. Liu Fangxin, Li Zongmin. "Thermodynamics and Early Thermodynamics" [J]. Physics, 1992, (03):186-191.
12. A Brief Review of Thermodynamics. Solar element. Guang Dong, 2023, 4, 19. https://mp.weixin.qq.com/s/22XHle0wBiPxuOHV7xY72A
13. A Brief Review of Thermodynamics. http://blog.sina.com.cn/u/1113509
14. The Structure of Scientific Revolution [M]. (American) by Thomas S. Kuhn; Translated by Jin Wulun and Hu Xinhe. Peking University Press, 2003.