6/11/2023

Probability Assignment - 2.

1) Flight Delay times:

Delay times : 5.5, 10.5, 13, 22.5, 45, 55.

$\mu = 25.25$

$\sigma = 20.2$ min

Degree of Freedom$(n-1)$ = 5.

Sample size $\boxed{n = 6}$

Setting up hypothesis

Null Hypothesis (H_0) = $\mu \leq 20$.

Alternative Hypothesis (H_a) = $\mu > 20$.

$$t = \frac{\bar{x} - \mu}{\left(\frac{S}{\sqrt{n}}\right)} = \left[\frac{25.25 - 20}{\frac{20.2}{\sqrt{6}}}\right] = \frac{+5.25}{\left(\frac{20.2}{2.44}\right)} = 3.67$$

$\boxed{\cancel{t = +0.573}}$

$\boxed{t = 3.67}$

Chosen Significance level:

$\boxed{\alpha = 0.01}$

$\boxed{\text{p-value} = 0.005}$

p-value is less than $\alpha = 0.01$.

$\therefore$ We reject null hypothesis :

Mean delay is greater than 20 mins.

2) a) Orders delivered after 90 mins ?

$X = 90$ mins

$\mu = 68$ mins

$\sigma = 14$ mins

Z- Score :

$$Z = \frac{(x - \mu)}{\sigma}$$

$$= \frac{90 - 68}{14}$$

$$\boxed{Z = 1.57}$$

$P(Z > 1.57) = \sim 0.0594$.

$$\boxed{P(Z > 1.57) = 5.9\%}$$

b) Promised delivery time, at least 99% orders before time :

Z- score for 99%

~ 2.33.

$$Z = \frac{(x - \mu)}{\sigma}$$

$$\boxed{X = Z * \sigma + \mu}$$

$$X = 2.33 \times 14 + 68$$

$$\boxed{X = 100.62 \text{ mins}}$$

3) Gauges: $\mu = 1.50$, $\sigma = 0.2$.

$P(1.50 - d \leq X \leq 1.50 + d) = 0.95$ (95% of time)

Now, $Z = \frac{(X - \mu)}{\sigma}$

Z - Upper $= (1.50 - d - 1.50)/0.2 = -d/0.2 = -5d$.

Z - Lower $= (1.50 + d - 1.50)/0.2 = d/0.2 = 5d$.

Z-score for 95% ~ 1.96.

To find d:

$-5d \leq 1.96$.

$d \geq -1.96/-5$

$\boxed{d \geq 0.392}$.

4) Car's transmission failure:

Here mean = 100,000.

$$\lambda = \frac{1}{\mu} = \frac{1}{100,000}.$$

$$\boxed{\lambda = 0.00001}$$

$$P(X \leq 50,000) = 1 - e^{-\lambda x}$$

$$= 1 - e^{-0.00001 \times 50,000}$$

$$= 1 - e^{-1}$$

$$= 1 - 0.3678$$

$$= 0.6321$$

Probability that car's transmission will fail during its first 50,000 miles is 63.2%

5. a) Probability that bearing lasts < 6000 hrs.

$$F(x) = 1 - e^{-(x/\alpha)^{\gamma}}$$

Given :

$\alpha = 5000$

$\gamma = 0.5$

$x = 6000$.

$$F(6000) = 1 - e^{-(6000/5000)^{0.5}}$$

$$= 1 - e^{-(1.2)^{0.5}}$$

$$= 1 - e^{-1.0954}$$

$$\boxed{F(6000) = 0.3156.}$$

b) Mean time to failure :

$$\boxed{\mu = \alpha . \Gamma\left(1 + \frac{1}{\gamma}\right)}$$

$$\mu = 5000 . \Gamma\left(1 + \frac{1}{0.5}\right)$$

$$= 5000 . \Gamma(3)$$

$$= 5000 \times 2$$

$$\boxed{\mu = 10{,}000 \text{ hrs.}}$$