



# Digital Logic

## Lecture 2

2<sup>nd</sup> Stage

Computer Science Department

Faculty of Science

Soran University

# Topics covered

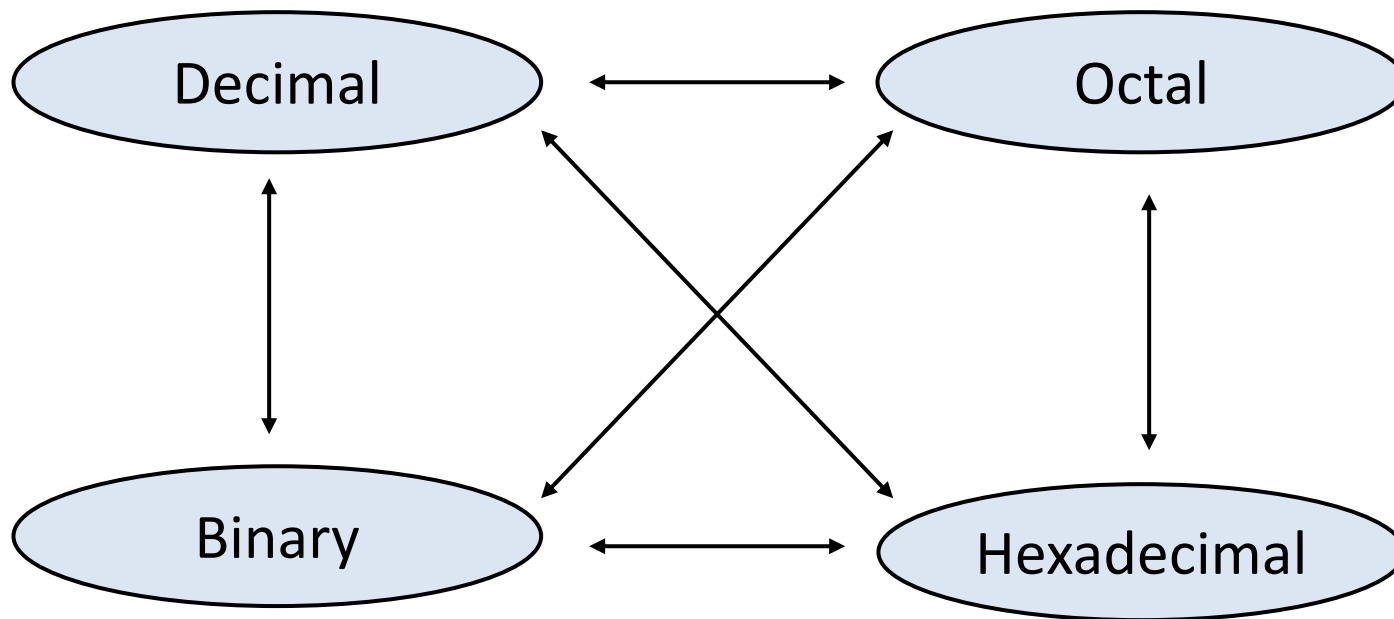
- ✧ Fractional conversion among bases
- ✧ Binary arithmetic operations

# Common Number Systems

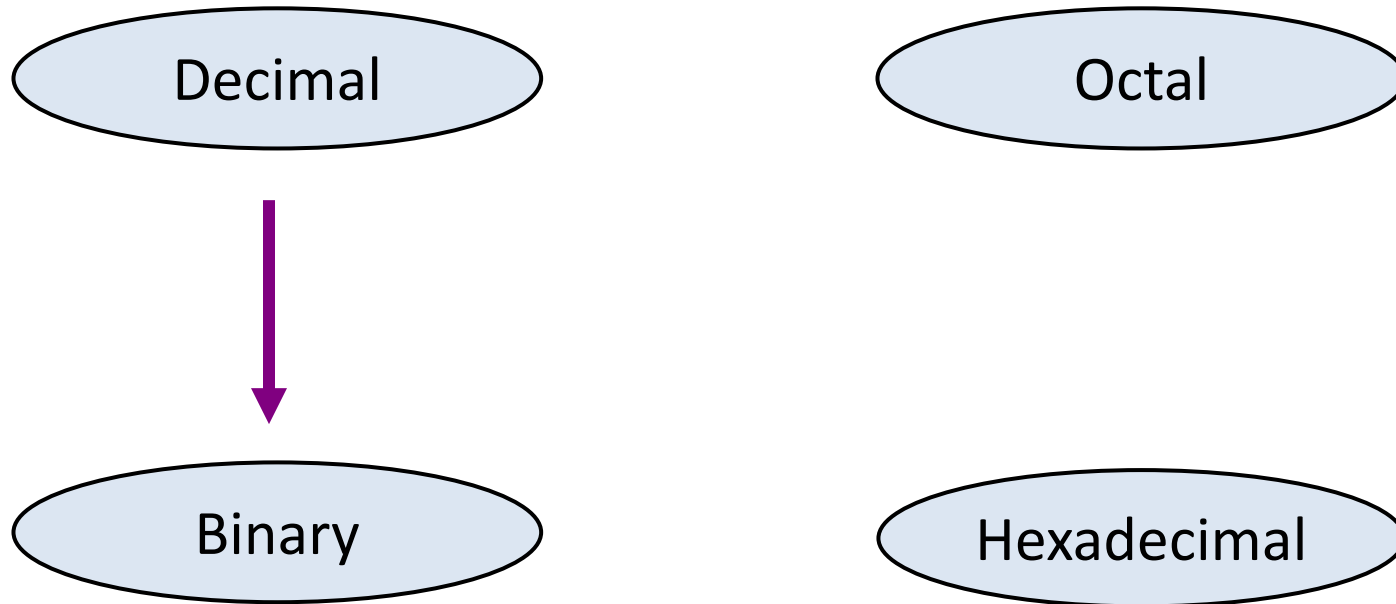


System	Base	Symbols
Decimal	10	0, 1, ... 9
Binary	2	0, 1
Octal	8	0, 1, ... 7
Hexa- decimal	16	0, 1, ... 9, A, B, ... F

# Fractional Conversion Among Bases



# Decimal to Binary



# Decimal to Binary

$$(25.625)_{10} = (?)_2$$

$$(25)_{10} = ( )_2$$

2	25	1
2	12	0
2	6	0
2	3	1
2	1	1
	0	



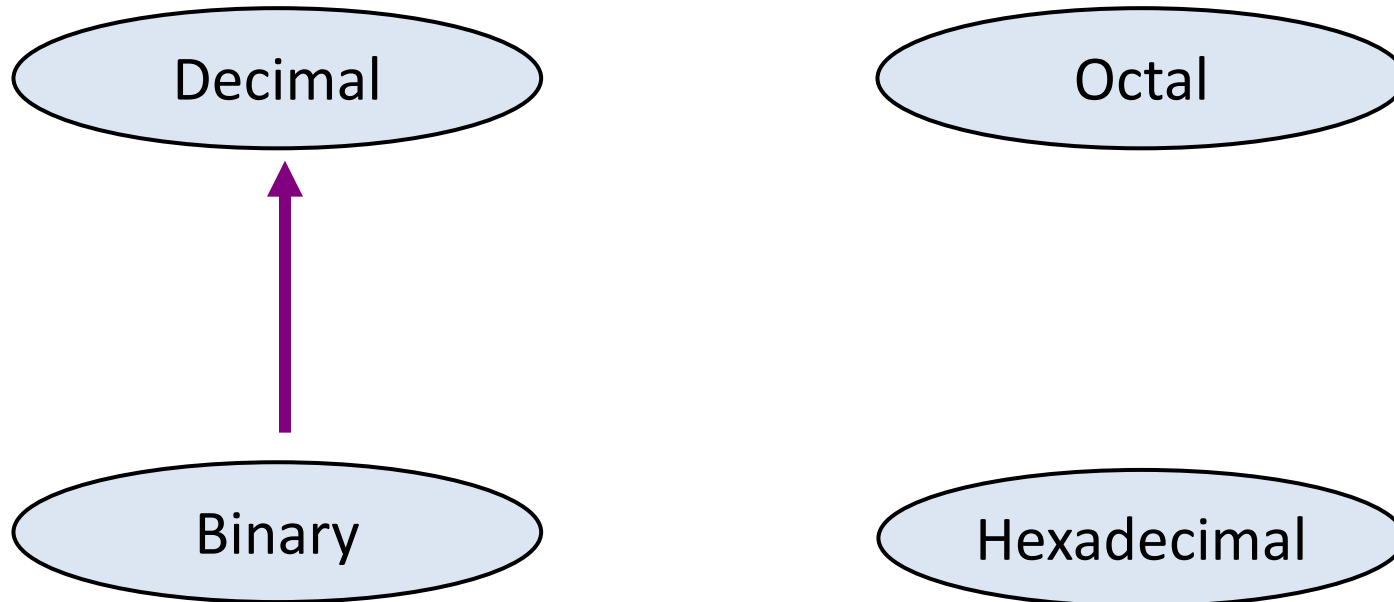
$$(0.625)_{10} = ( )_2$$

0.625 * 2	1.25
0.25 * 2	0.50
0.50 * 2	1.00
0.00 * 2	0.00



$$(25.625)_{10} = (11001.101)_2$$

# Binary to Decimal



# Binary to Decimal

$$(10101.11)_2 = ( )_{10}$$

$$(10101)_2 \Rightarrow 1 \times 2^0 = 1$$

$$0 \times 2^1 = 0$$

$$1 \times 2^2 = 4$$

$$0 \times 2^3 = 0$$

$$1 \times 2^4 = 16$$

$$(21)_{10}$$

$$(0.11)_2 \Rightarrow 1 \times 2^{-1} = 0.5$$

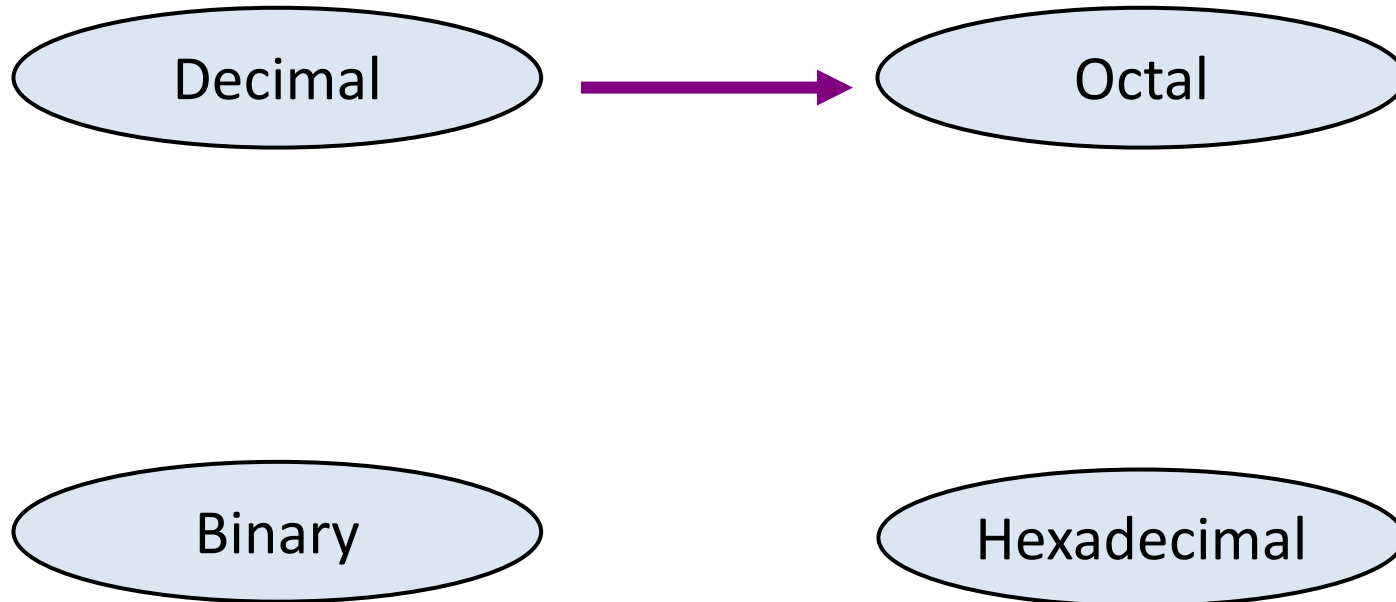
$$1 \times 2^{-2} = 0.25$$

$$(0.75)_{10}$$

$$(10101.11)_2 = (21.75)_{10}$$



# Decimal to Octal



# Decimal to Octal

$$(25.625)_{10} = (?)_8$$

$$(25)_{10} = ( )_8$$

8	25	1
8	3	3
	0	



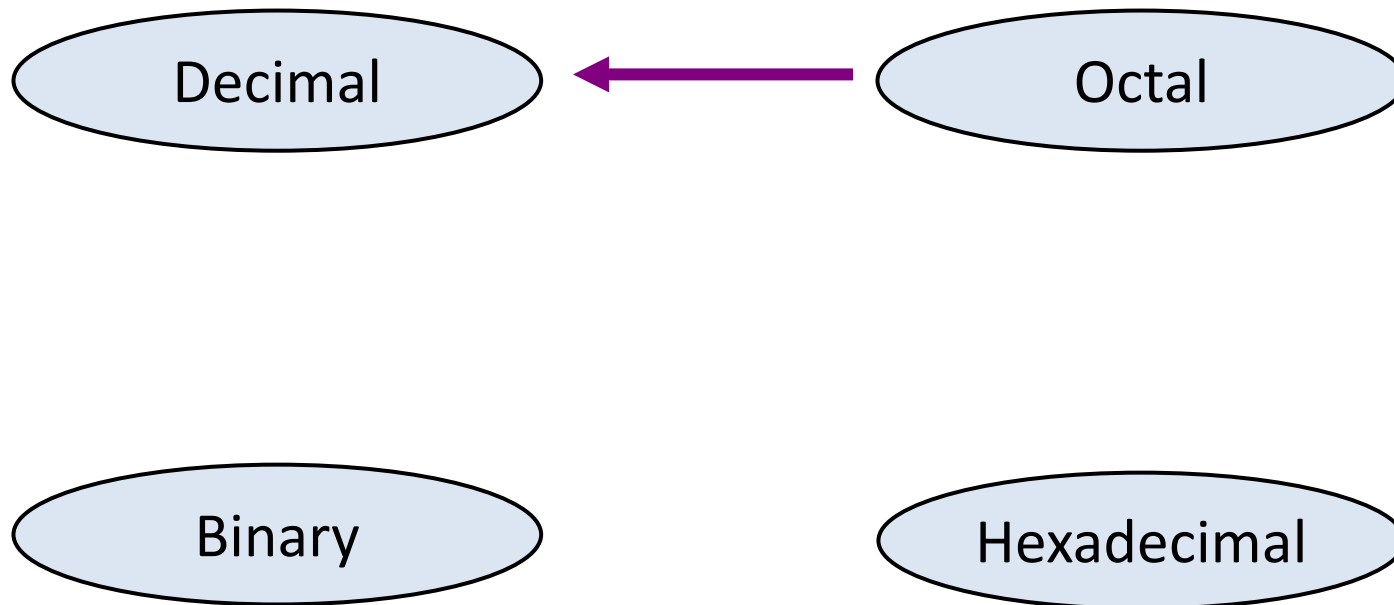
$$(0.625)_{10} = ( )_8$$

0.625 * 8	5.000
0.000 * 8	0.000



$$(25.625)_{10} = (31.50)_8$$

# Octal to Decimal



# Octal to Decimal

$$(4507.44)_8 = ( )_{10}$$

$$(4507)_8 \Rightarrow$$

$$7 \times 8^0 = 7$$

$$0 \times 8^1 = 0$$

$$5 \times 8^2 = 320$$

$$4 \times 8^3 = 2048$$

$$(2375)_{10}$$

$$(0.44)_8 \Rightarrow$$

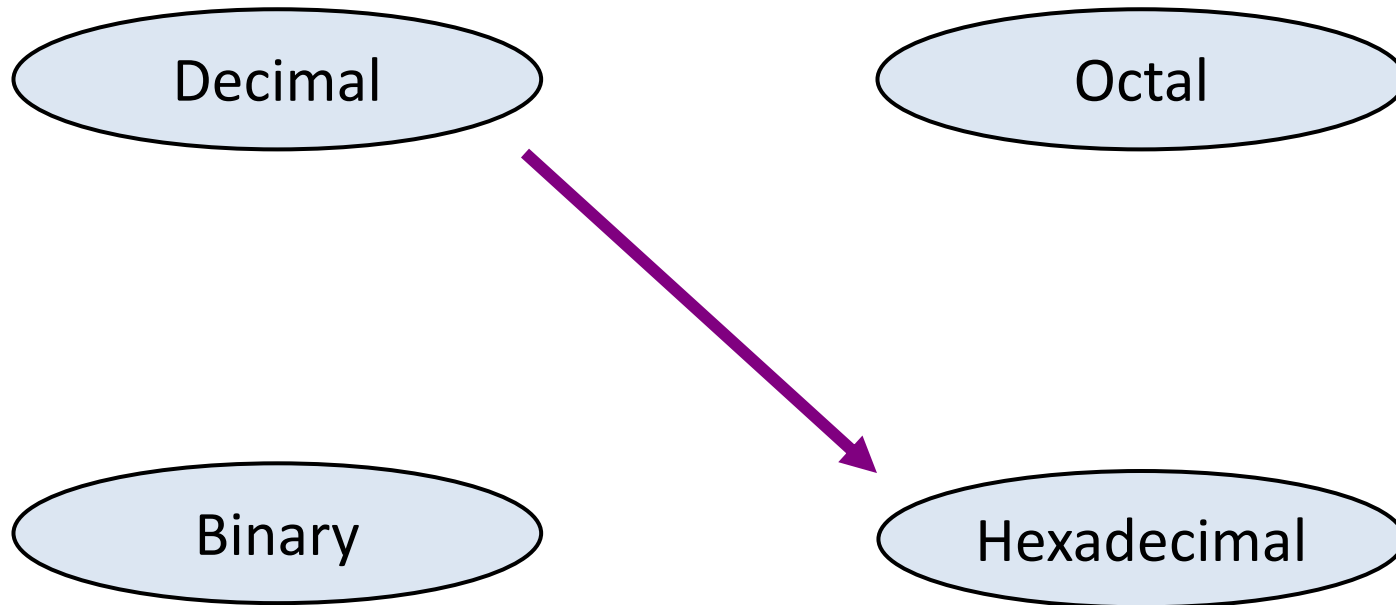
$$4 \times 8^{-1} = 0.5$$

$$4 \times 8^{-2} = 0.0625$$

$$(0.5625)_{10}$$

$$(4507.44)_8 = (2375.5625)_{10}$$

# Decimal to Hexadecimal



# Decimal to Hexadecimal

$$(25.625)_{10} = (?)_{16}$$

$$(25)_{10} = ( )_{16}$$

16	25	9
16	1	1
	0	



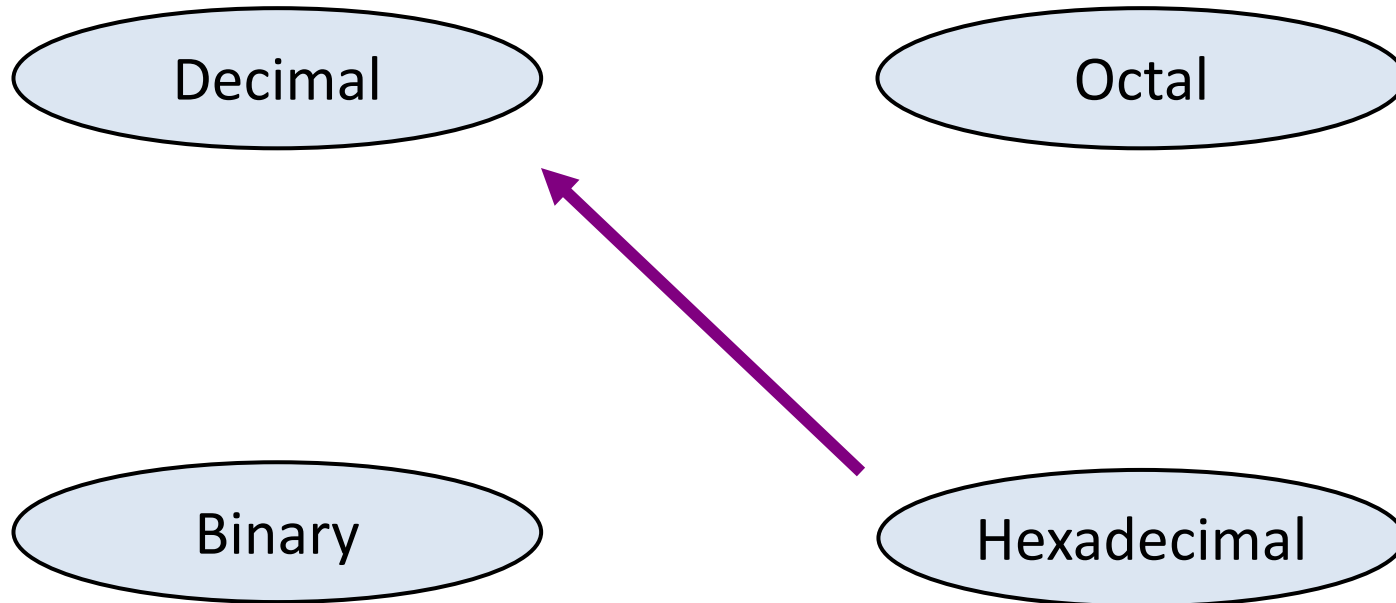
$$(0.625)_{10} = ( )_{16}$$

0.625 * 16	10.000 = A
0.000 * 16	0.000



$$(25.625)_{10} = (19.A)_{16}$$

# Hexadecimal to Decimal



# Hexadecimal to Decimal

$$(57.4)_{16} = ( )_{10}$$

$$(57)_{16} \Rightarrow$$

$$7 \times 16^0 = 7$$

$$5 \times 16^1 = 80$$

$$(87)_{10}$$

$$(0.4)_{16} \Rightarrow$$

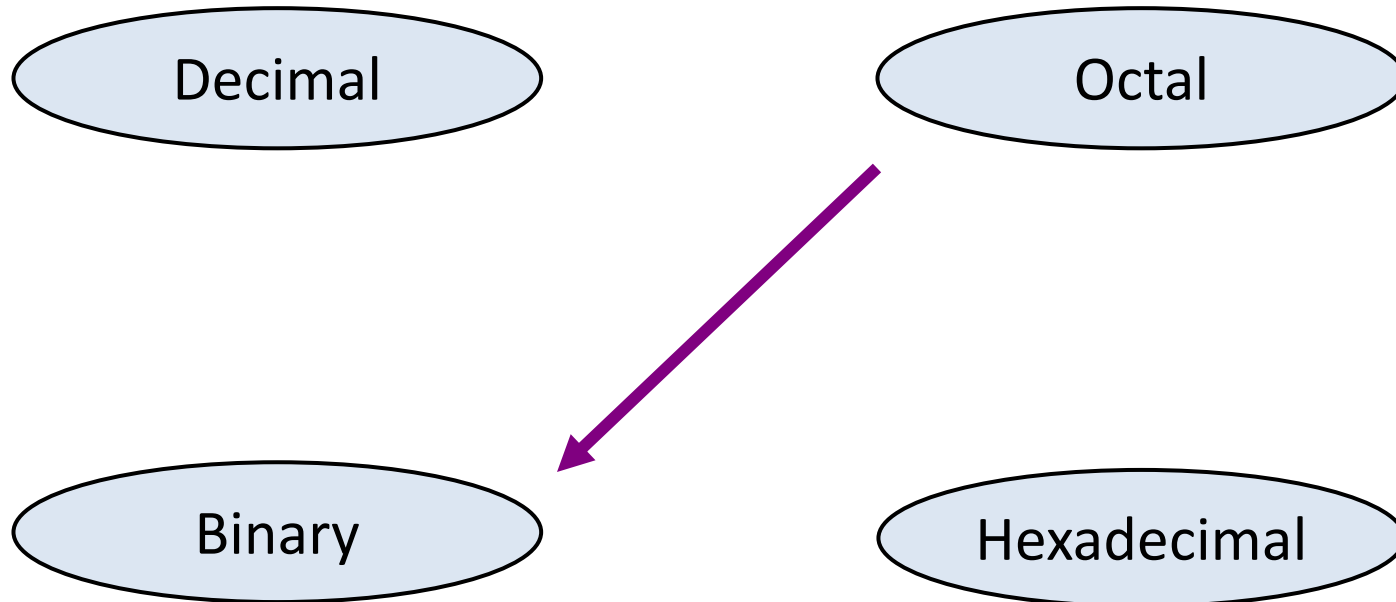
$$4 \times 16^{-1} = 0.25$$

$$(0.25)_{10}$$

$$(57.4)_{16} = (87.25)_{10}$$



# Octal to Binary



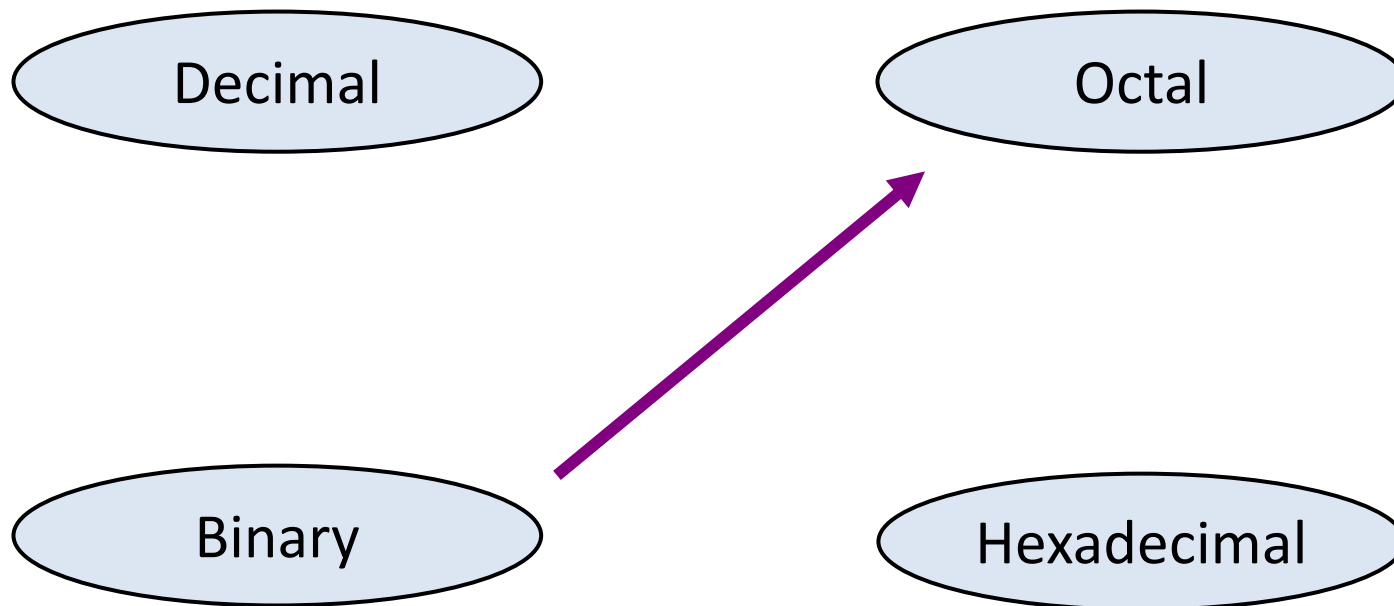
# Octal to Binary

$$(37.45)_8 = (?)_2$$

3	7	.	4	5
↓	↓		↓	↓
011	111	.	100	101

$$(37.45)_8 = (011111.100101)_2$$

# Binary to Octal



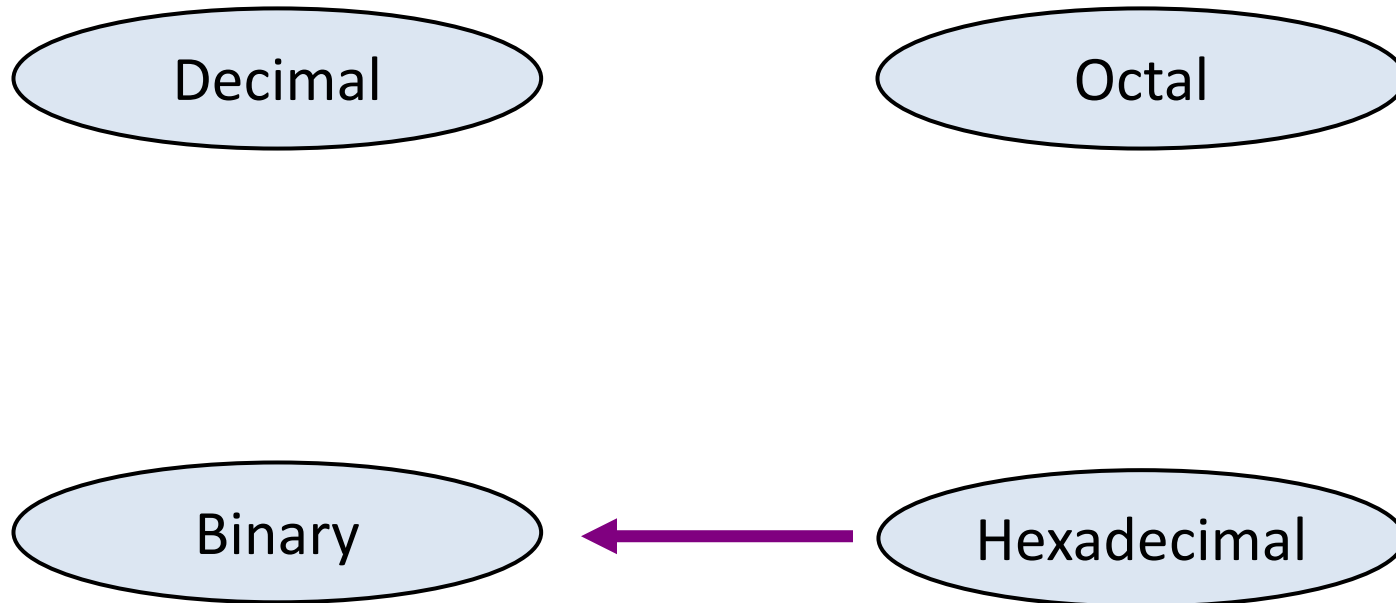
# Binary to Octal

$$(10110.11)_2 = (?)_8$$

$$\begin{array}{ccc} \underline{0}10 & 110 & . \quad 11\underline{0} \\ \downarrow & \downarrow & \downarrow \\ 2 & 6 & . \quad 6 \end{array}$$

$$(10110.11)_2 = (26.6)_8$$

# Hexadecimal to Binary



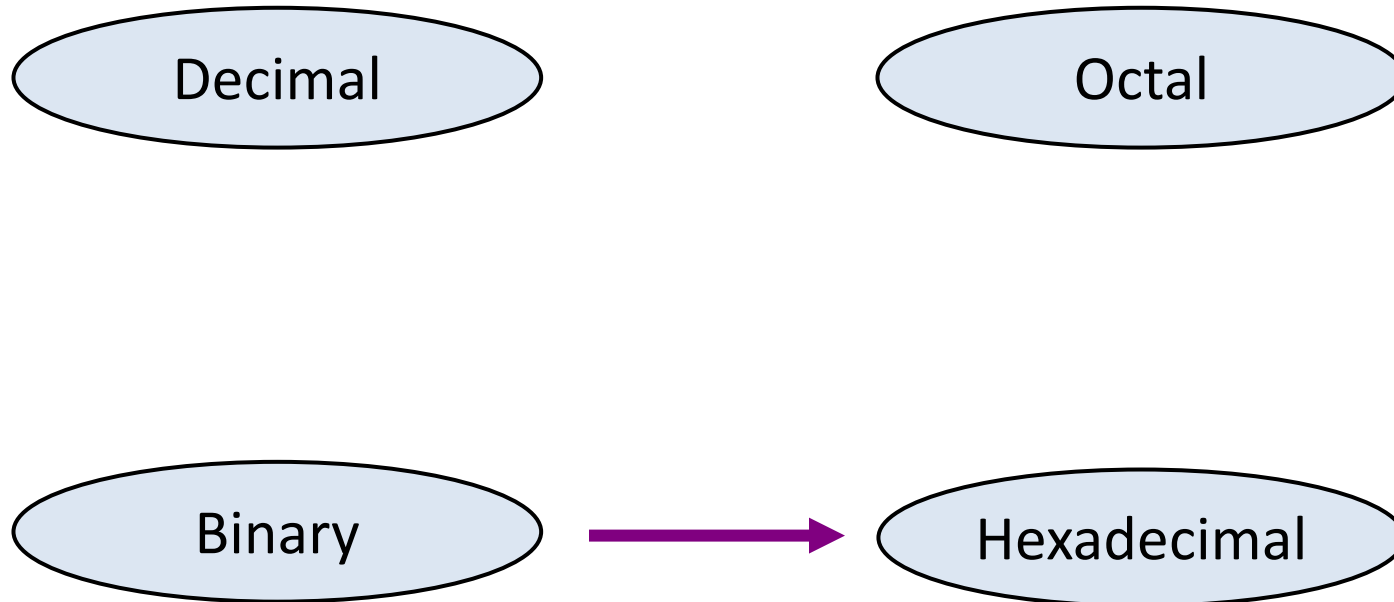
# Hexadecimal to Binary

$$(CAFE.3D)_{16} = (?)_2$$

C	A	F	E	.	3	D
↓	↓	↓	↓		↓	↓
1100	1010	1111	1110	.	0011	1101

$$(CAFE.3D)_{16} = (110010101111110.00111101)_2$$

# Binary to Hexadecimal



# Binary to Hexadecimal

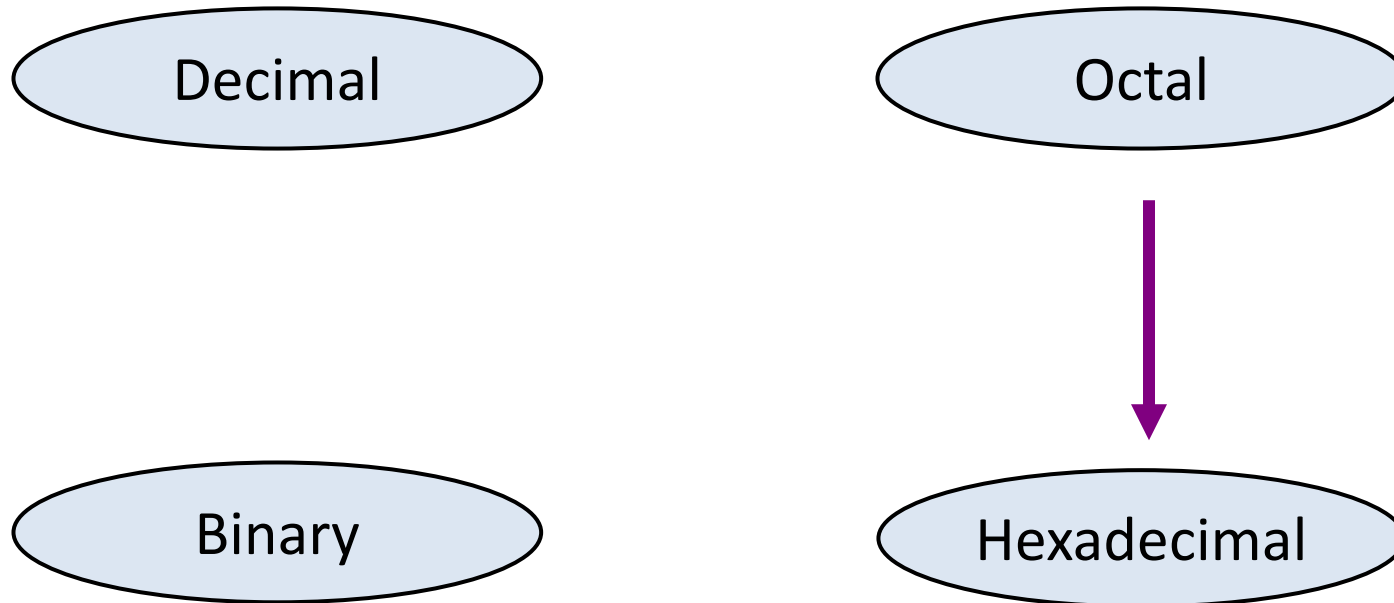
$$(10001001.11)_2 = ( \quad )_{16}$$

1000	1001	.	11	<u>00</u>
↓	↓		↓	
8	9		C	

$$(10001001.11)_2 = (89.C)_{16}$$

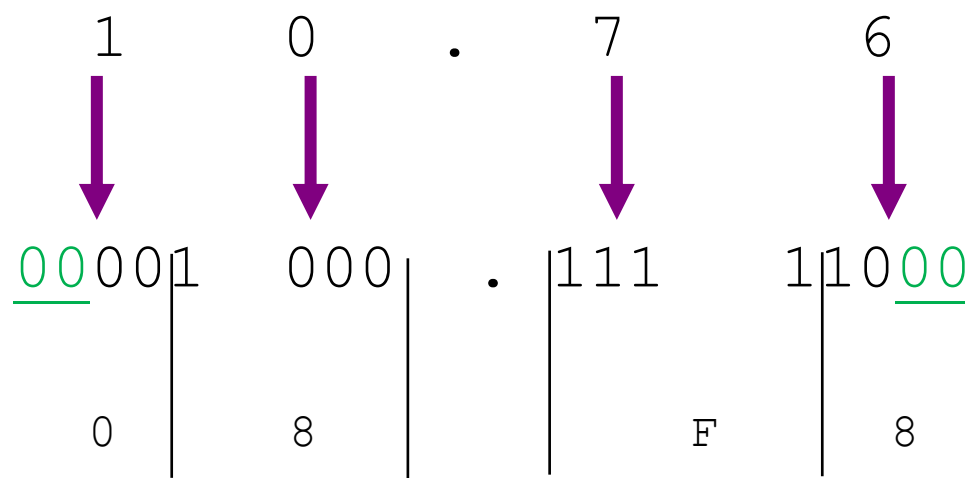


# Octal to Hexadecimal



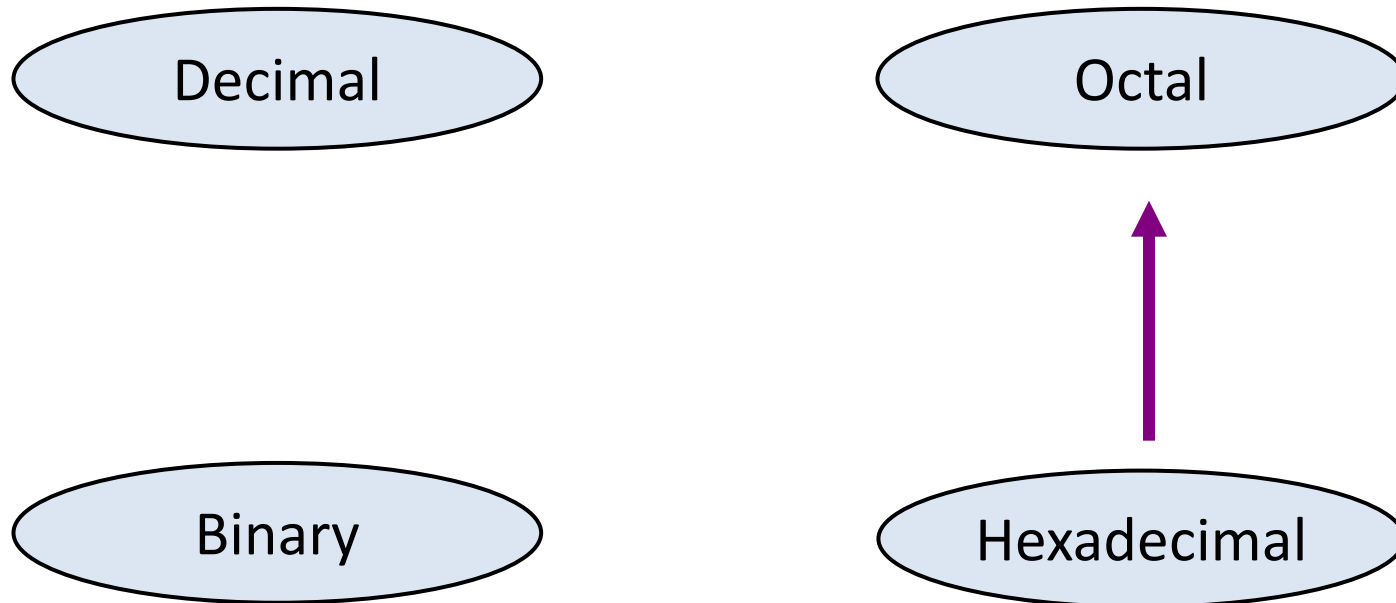
# Octal to Hexadecimal

$$(10.76)_8 = (?)_{16}$$



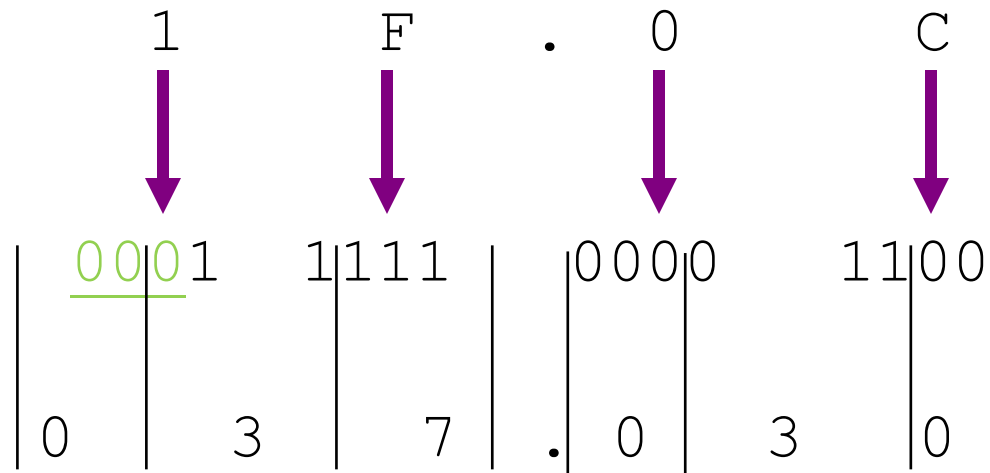
$$(10.76)_8 = (08.F8)_{16}$$

# Hexadecimal to Octal



# Hexadecimal to Octal

$$1F.0C_{16} = (?)_8$$



$$(1F.0C)_{16} = (37.03)_8$$

# Classwork-1

Convert the following numbers

A)  $(D.1E)_{16} = ( ? )_2$  (show your work)

B)  $(10.06)_8 = ( ? )_{16}$  (show your work)

C)  $(1101.0101)_2 = ( ? )_{10}$  (show your work)

# Binary Addition

- Two 1-bit values

A	B	A + B
0	0	0
0	1	1
1	0	1
1	1	10

Carry

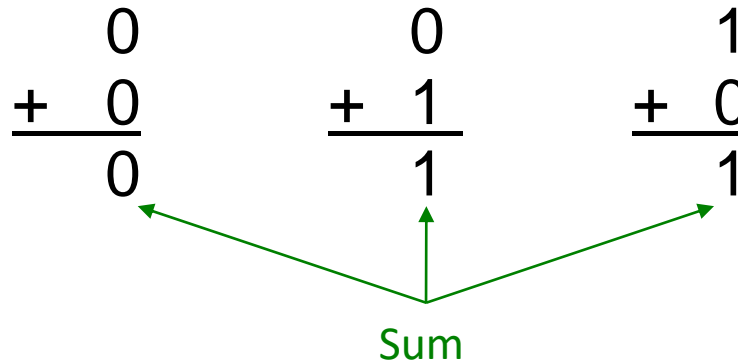


# Binary Addition

- Two 1-bit values

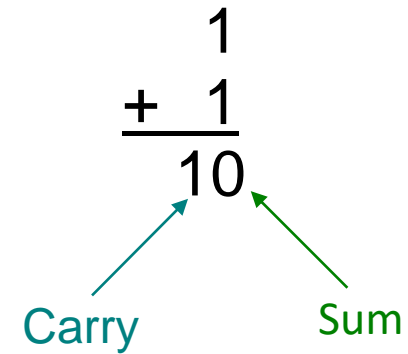
$$\begin{array}{r} 0 \\ + 0 \\ \hline 0 \end{array} \quad \begin{array}{r} 0 \\ + 1 \\ \hline 1 \end{array} \quad \begin{array}{r} 1 \\ + 0 \\ \hline 1 \end{array}$$

Sum



$$\begin{array}{r} 1 \\ + 1 \\ \hline 10 \end{array}$$

Carry Sum



# Binary Addition



- Two  $n$ -bit values

$$\begin{array}{r} 10101 \\ + 11001 \\ \hline 101110 \end{array}$$

$$\begin{array}{r} 21 \\ + 25 \\ \hline 46 \end{array}$$

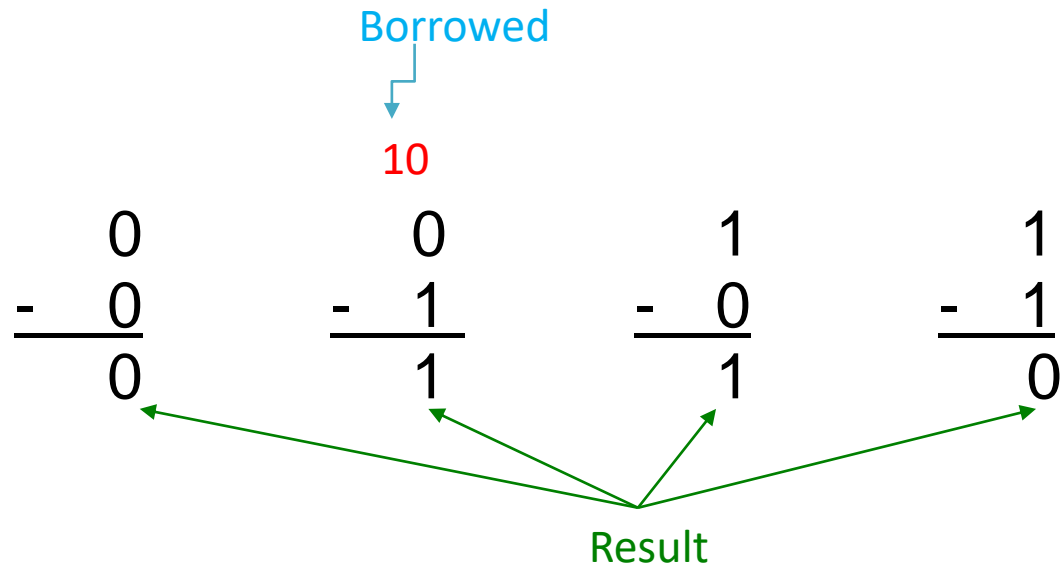
$$\begin{array}{r} 01011011 \\ + 01110010 \\ \hline 11001101 \end{array}$$

$$\begin{array}{r} 10110101 \\ + 01101100 \\ \hline ? \end{array}$$



# Binary Subtraction

- Two 1-bit values



# Binary Subtraction

- Two n-bit values

$$\begin{array}{r} 1110 \\ - 1001 \\ \hline 0101 \end{array}$$

$$\begin{array}{r} 01110101 \\ - 00110010 \\ \hline 01000011 \end{array}$$

$$\begin{array}{r} 10110001 \\ - 01101100 \\ \hline ? \end{array}$$

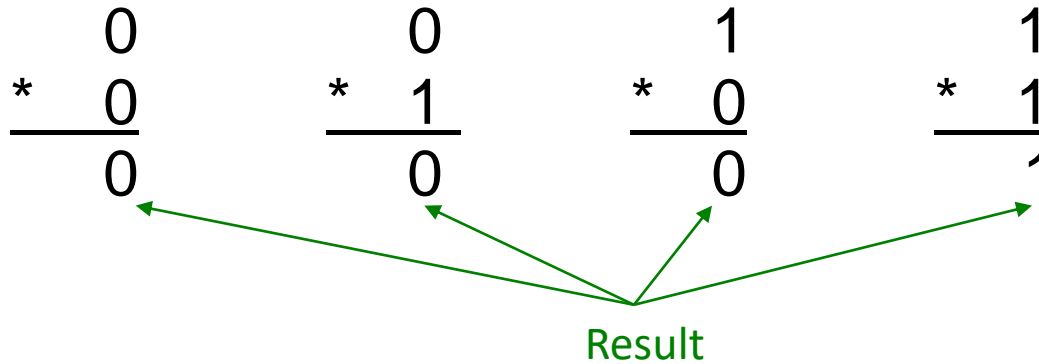
# Binary Multiplication

- Binary, two 1-bit values

A	B	$A \times B$
0	0	0
0	1	0
1	0	0
1	1	1

# Binary Multiplication

- Binary, two 1-bit values



# Binary Multiplication

- Binary, two  $n$ -bit values

$$\begin{array}{r} 1110 \\ \times 1011 \\ \hline 1110 \\ 1110- \\ 0000-- \\ 1110--- \\ \hline 10011010 \end{array}$$

$$\begin{array}{r} 1000 \\ \times 0110 \\ \hline 0000 \\ 1000- \\ 1000-- \\ 0000--- \\ \hline 0110000 \end{array}$$

# Binary Multiplication

- Binary, two  $n$ -bit values

$$\begin{array}{r} 10110001 \\ \times 01101101 \\ \hline 10110001 \\ 00000000- \\ 10110001-- \\ 10110001--- \\ 00000000---- \\ 10110001----- \\ 10110001----- \\ 00000000----- \\ \hline 10010110101101 \end{array}$$

# Exercises - 1

Perform the following operations in binary:

A)  $(178)_{10} + (82)_{10} = ?$  (show your work)

B)  $(138)_{10} - (59)_{10} = ?$  (show your work)

C)  $(48)_{10} * (35)_{10} = ?$  (show your work)

Deadline: October 1, 2022 @ 11:59 PM

# Homework 2



- 1) What are octal and hexadecimal number systems are used for?
- 2) Convert the following numbers, and show your work:
  - A)  $(D7.2E)_{16} = ( ? )_8$
  - B)  $(F16.13)_{16} = ( ? )_2$
  - C)  $(101000.001)_2 = ( ? )_{10}$
  - D)  $(38.04)_{10} = ( ? )_2$
- 3) Perform the following operations in binary, and show your work:
  - A)  $(61)_8 + (61)_{10}$
  - B)  $(14)_{10} - (7)_{10}$
  - C)  $(73)_8 * (AD)_{16}$

Deadline: October 7, 2022 @ 11:59 PM