Digital Logic

Lecture 3

2nd Stage

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Topics covered

- ✧ Signed Numbers
- ✧ Binary Subtraction

Signed Numbers

$$(-12)_{10} = (?)_2$$

Signed Numbers

Representation of Signed Numbers

1. Sign-Magnitude System
2. First Complement System
3. Second Complement System

1. Sign-Magnitude System

- Use fixed length binary representation
- Represent the decimal number as binary
- Use left-most bit (called *most significant bit* or MSB) for sign:

    0 for positive

    1 for negative

Example: $(+18)_{10} = (00010010)_2$

$(-18)_{10} = (10010010)_2$

1. Sign-Magnitude System

- Ex. 4-bit signed magnitude
  - 1 bit for sign
  - 3 bits for magnitude

	$+N$	$-N$
0	0000	1000
1	0001	1001
2	0010	1010
3	0011	1011
4	0100	1100
5	0101	1101
6	0110	1110
7	0111	1111

1. Sign-Magnitude System

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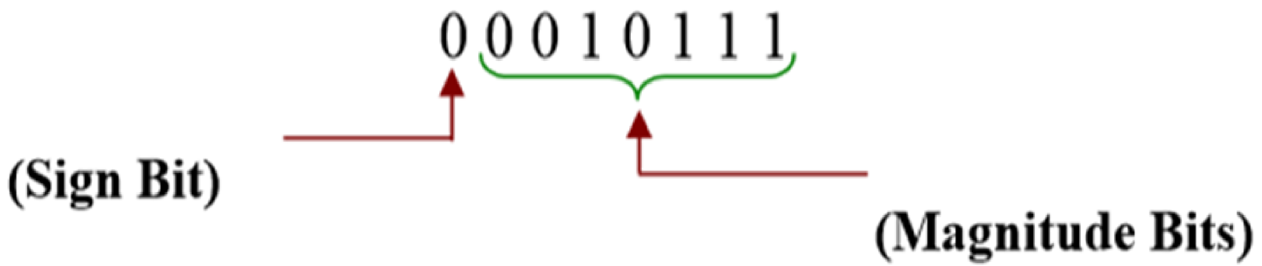
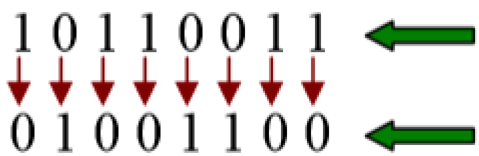
1. Sign-Magnitude System



2. First Complement System

- Method: Invert the ones and zeros



- 11_{10} = 00001011
- -11_{10} = 11110100

- 0 in MSB implies positive
- 1 in MSB implies negative

	$+N$	$-N$
0	0000	1111
1	0001	1110
2	0010	1101
3	0011	1100
4	0100	1011
5	0101	1010
6	0110	1001
7	0111	1000

2. First Complement System

$\underline{0}\,0\,0\,1\,0\,1\,1\,1 \longleftarrow (+23)$

$\underline{1}\,1\,1\,0\,1\,0\,0\,0 \longleftarrow (-23)$

- 26 = ???

+ 26 = 0001 1010

1110 0101

	$+N$	$-N$
0	0000	1111
1	0001	1110
2	0010	1101
3	0011	1100
4	0100	1011
5	0101	1010
6	0110	1001
7	0111	1000

3. Second Complement

- Method: Take the one's complement and add 1

$$
\begin{array}{rl}
11= & 00001011 \\
 & \downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow \\
 & 11110100 \\
 & \underline{\quad\quad +1} \\
-11= & 11110101
\end{array}
$$

one's comp

Add 1

two's comp

	$+N$	$-N$
0	0000	0000
1	0001	1111
2	0010	1110
3	0011	1101
4	0100	1100
5	0101	1011
6	0110	1010
7	0111	1001

3. Second Complement

$$\begin{array}{rl} 5 = & 00000101 \\ & \downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow \\ & 11111010 \\ & \underline{+1} \\ -5 = & 11111011 \end{array}$$

Complement Digits

Add 1

$$\begin{array}{rl} -13 = & 11110011 \\ & \downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow \\ & 00001100 \\ & \underline{+1} \\ 13 = & 00001101 \end{array}$$

Complement Digits

Add 1

Summary of methods for representing signed integers.

N	$+n$	signedMag $-n$	1sComp $-n$	2sComp $-n$
0	0000	1000	1111	0000
1	0001	1001	1110	1111
2	0010	1010	1101	1110
3	0011	1011	1100	1101
4	0100	1100	1011	1100
5	0101	1101	1010	1011
6	0110	1110	1001	1010
7	0111	1111	1000	1001

Binary Subtraction

$$
\begin{array}{rr}
 & 9 \\
+ & 5 \\
\hline
 & 14
\end{array}
\quad
\begin{array}{c}
\longrightarrow \\
\longrightarrow \\
\\
\longleftarrow
\end{array}
\quad
\begin{array}{rr}
 & 1001 \\
+ & 0101 \\
\hline
 & 1110
\end{array}
$$

$$
\begin{array}{rr}
 & 9 \\
+ & -5 \\
\hline
 & 4
\end{array}
\qquad\qquad
\begin{array}{rr}
 & 9 \\
- & 5 \\
\hline
 & 4
\end{array}
$$

Binary Subtraction

8-bit subtraction using 2's complement

$$\begin{array}{r} 9 \\ +\ (-5) \\ \hline 4 \end{array} \qquad \begin{array}{r} 00001001 \\ +\ 11111011 \\ \hline \cancel{1}]00000100 \end{array}$$

8^{th} Bit = 0: Answer is Positive
Disregard 9^{th} Bit

$$\left.\begin{array}{r} 00001101 \\ \downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow \\ 11111010 \\ +1 \\ \hline 11111011 \end{array}\right\}$$

2's Complement Process

Binary Subtraction

8-bit subtraction using 2's complements

```
  (-9)        11110111
+   5       + 00000101
------      ----------
   -4         11111100
```

8^{th} Bit = 1: Answer is Negative

2's Complement Process

```
00001001
↓↓↓↓↓↓↓↓
11110110
      +1
--------
11110111
```

To Check: Perform 2's Complement On Answer

```
11111100
↓↓↓↓↓↓↓↓
00000011
      +1
--------
00000100
```

Binary Subtraction

8-bit subtraction using 2's complements

$$
\begin{array}{rcr}
(-9) & \longrightarrow & 11110111 \\
+\ (-5) & \longrightarrow & +\ 11111011 \\
\hline
-\boxed{14} & \longleftarrow & \cancel{1}]\ 11110010
\end{array}
$$

2’s Complement Numbers, See Conversion Process In Previous Slides

8^{th} Bit = 1: Answer is Negative
Disregard 9^{th} Bit

To Check: Perform 2’s Complement On Answer

$$
\begin{array}{r}
11110010 \\
\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow \\
00001101 \\
+1 \\
\hline
00001110
\end{array}
$$

Classwork - 2

Perform the following operations in binary, using the 2’s complement method, and show your work:

A) $(-9)_{10} + (-7)_{10}$

B) $(-12)_{10} - (-6)_{10}$

Exercises - 2

Perform the following operations in binary, using the first and second complement methods, and show your work:

A) $(-18)_{10} + (71)_{10}$

B) $(-21)_{10} + (-1)_{10}$

C) $(2)_{10} + (-8)_{10}$

Deadline: October 15, 2022 @ 11:59 PM

Homework 3

Perform the following operations in binary, using the first and second complement methods, and show your work:

A) $(-9)_{10} + (6)_{16}$

B) $(-5)_{10} - (16)_{8}$

C) $(-14)_{10} + (7)_{8}$

Deadline: October 21, 2022 @ 11:59 PM