Digital Logic

Lecture 6

2nd Stage

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Topics covered

- Karnaugh Maps (K-Maps)
  - 2, 3, and 4 variable maps
  - Simplification using K-Maps

Karnaugh Maps (K-map)

- **A K-map is a collection of squares**
  - Each square represents a minterm
  - The collection of squares is a graphical representation of a Boolean function
  - Adjacent squares differ in the value of one variable
  - Alternative algebraic expressions for the same function are derived by recognizing patterns of squares

- **The K-map can be viewed as**
  - A reorganized version of the truth table

Some uses of K-maps

- **Provide a means for:**
  - Finding optimum or near optimum
    - SOP and POS canonical forms, and
    - two-level AND/OR and OR/AND circuit implementations for functions with small numbers of variables.
  - Visualizing concepts related to manipulating Boolean expressions.
  - Demonstrating concepts used by computer-aided design programs to simplify large circuits

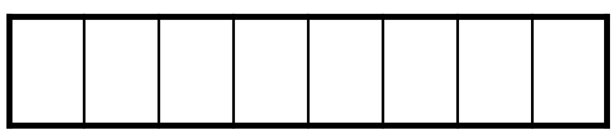
Two Variable Maps

- **A 2-variable Karnaugh Map:**
  - Note that minterm **m0** and minterm **m1** are "adjacent" and differ in the value of the variable **y**
  - Similarly, minterm **m0** and minterm **m2** differ in the **x** variable.
  - Also, **m1** and **m3** differ in the **x** variable as well.
  - Finally, **m2** and **m3** differ in the value of the variable **y**

y LSB

x MSB

x \ y	y = 0	y = 1
x = 0	$m_0 = \bar{x}\,\bar{y}$	$m_1 = \bar{x}\,y$
x = 1	$m_2 = x\,\bar{y}$	$m_3 = x\,y$



MSB
(Most significant bit)

LSB
(Least significant bit)

K-Map and Truth Tables

- The K-Map is just a different form of the truth table.
- Example – Two variable function:
  - We choose a,b,c and d from the set {0,1} to implement a particular function, F(x,y).

Function Table

Input Values x	Input Values y	Function Value F(x,y)
0	0	a
0	1	b
1	0	c
1	1	d

K-Map

x \ y	y = 0	y = 1
x = 0	a	b
x = 1	c	d

K-Map Function Representation

- Example: F(x,y) = x

LSB

MSB

x \ y	y = 0	y = 1
x = 0	0	0
x = 1	1	1

- For function F(x,y), the two adjacent cells containing 1's can be combined using the Minimization Theorem:

$$F(x, y) = x\,\overline{y} + x\,y = x$$

K-Map Function Representation

- Example: G(x,y) = x + y

LSB

MSB

x \ y	y = 0	y = 1
x = 0	0	1
x = 1	1	1

Three Variable Maps

- A three-variable K-map:

YZ / X	yz=00	yz=01	yz=11	yz=10
x=0	m_0	m_1	m_3	m_2
x=1	m_4	m_5	m_7	m_6

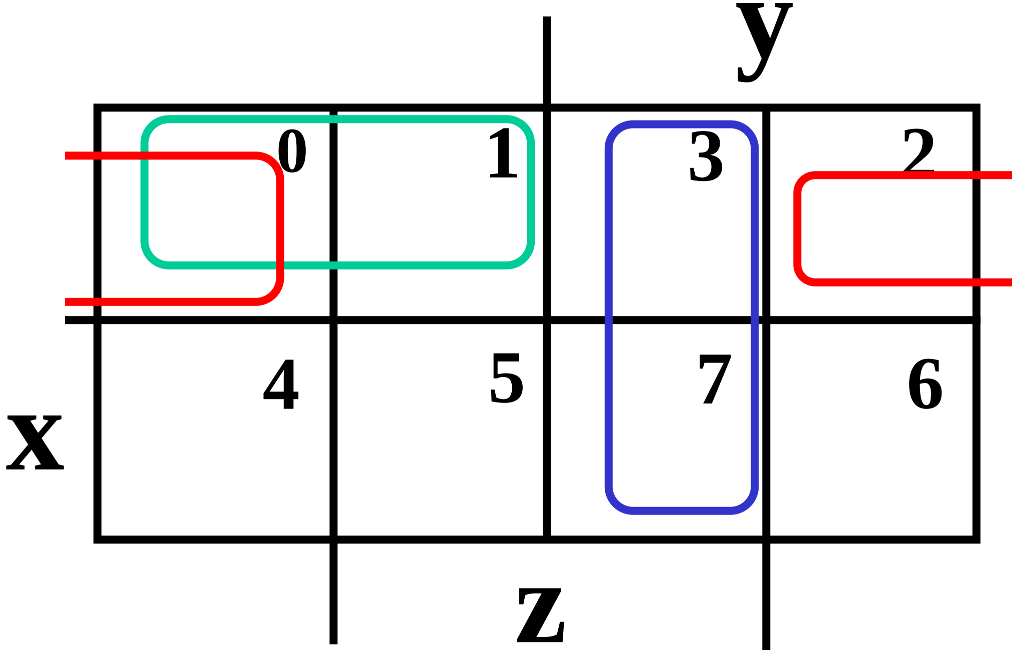
- Where each minterm corresponds to the product terms:

YZ / X	yz=00	yz=01	yz=11	yz=10
x=0	$\bar{x}\,\bar{y}\,\bar{z}$	$\bar{x}\,\bar{y}\,z$	$\bar{x}\,y\,z$	$\bar{x}\,y\,\bar{z}$
x=1	$x\,\bar{y}\,\bar{z}$	$x\,\bar{y}\,z$	$x\,y\,z$	$x\,y\,\bar{z}$

- Note that if the binary value for an index differs in one bit position, the minterms are adjacent on the K-Map

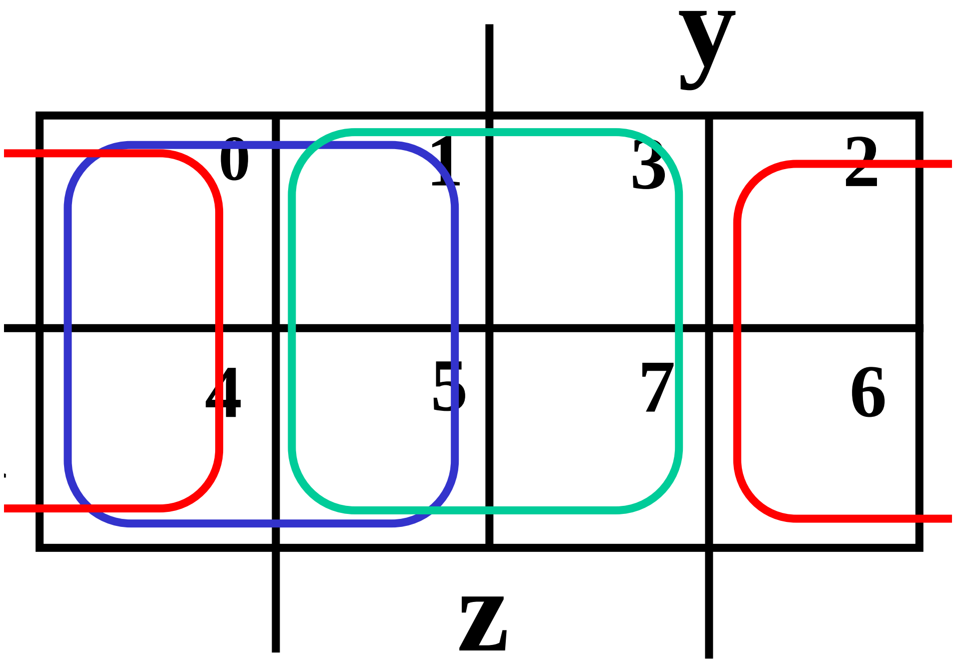
Three-Variable Maps

- Example Shapes of 2-cell Rectangles:



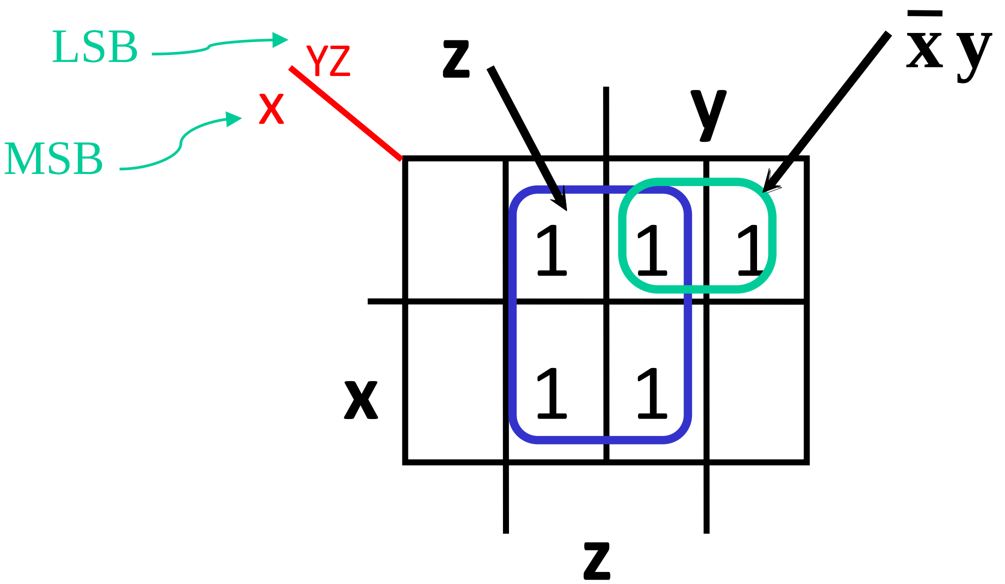
Three-Variable Maps

- Example Shapes of 4-cell Rectangles:



Three Variable Maps

- K-Maps can be used to simplify Boolean functions by systematic methods. Terms are selected to cover the "1s"in the map.
- Example: Simplify $\mathbf{F(x, y, z) = \Sigma_m(1,2,3,5,7)}$



$$F(x, y, z) = z + \bar{x}\, y$$

Example Functions

- **By convention, we represent the minterms of F by a "1" in the map**

- **Example:**

$$F(x, y, z) = \Sigma_m(2,3,4,5)$$

LSB

MSB

x \ yz

0	1	3 **1**	2 **1**
4 **1**	5 **1**	7	6

- **Example:**

$$G(x, y, z) = \Sigma_m(3,4,6,7)$$

0	1	3 **1**	2
4 **1**	5	7 **1**	6 **1**

Combining Squares

- By combining squares, we reduce number of literals in a product term, reducing the literal cost, thereby reducing the other two cost criteria
- On a 3-variable K-Map:
  - One square represents a minterm with three variables
  - Two adjacent squares represent a product term with two variables
  - Four “adjacent” terms represent a product term with one variable
  - Eight “adjacent” terms is the function of all ones (no variables) = 1.

Example: Combining Squares

- Example: Let

  $F = \Sigma m(2,3,6,7)$

			y	y
	0	1	3 **1**	2 **1**
x	4	5	7 **1**	6 **1**
		z	z	

- Applying the Minimization Theorem three times:

$$
\begin{aligned}
F(x, y, z) &= \bar{x}\,y\,z + x\,y\,z + \bar{x}\,y\,\bar{z} + x\,y\,\bar{z} \\
&= yz + y\bar{z} \\
&= y
\end{aligned}
$$

- Thus the four terms that form a 2 × 2 square correspond to the term "y".

Three-Variable K-Maps

$f = \sum(0,4) = \overline{B}\,\overline{C}$

A \ BC	00	01	11	10
0	1	0	0	0
1	1	0	0	0

$f = \sum(4,5) = A\,\overline{B}$

A \ BC	00	01	11	10
0	0	0	0	0
1	1	1	0	0

$f = \sum(0,1,4,5) = \overline{B}$

A \ BC	00	01	11	10
0	1	1	0	0
1	1	1	0	0

$f = \sum(0,1,2,3) = \overline{A}$

A \ BC	00	01	11	10
0	1	1	1	1
1	0	0	0	0

$f = \sum(0,4) = \overline{A}\,C$

A \ BC	00	01	11	10
0	0	1	1	0
1	0	0	0	0

$f = \sum(4,6) = A\,\overline{C}$

A \ BC	00	01	11	10
0	0	0	0	0
1	1	0	0	1

$f = \sum(0,2) = \overline{A}\,\overline{C}$

A \ BC	00	01	11	10
0	1	0	0	1
1	0	0	0	0

$f = \sum(0,2,4,6) = \overline{C}$

A \ BC	00	01	11	10
0	1	0	0	1
1	1	0	0	1

Three-Variable K-Maps

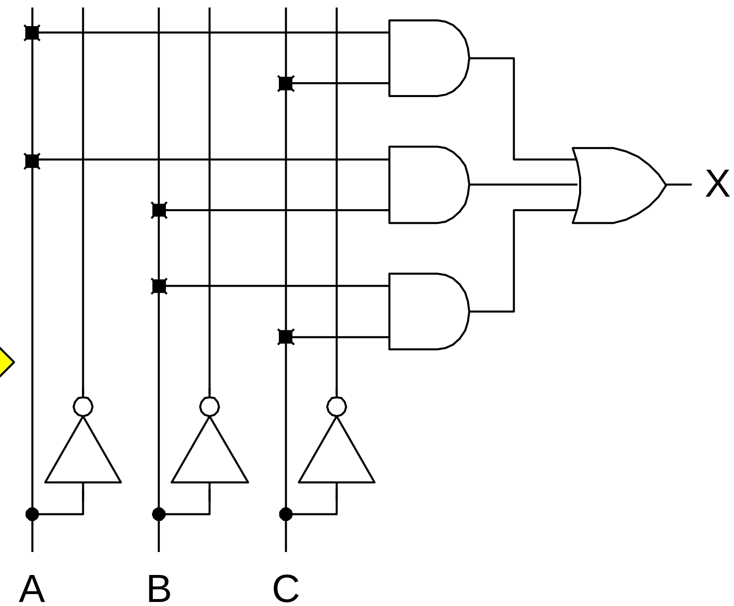
- Example: Design a 3-input (A,B,C) digital circuit that will give at its output (X) a logic 1 only if the binary number formed at the input has more ones than zeros.

	Inputs A	B	C	Output X
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	1
6	1	1	0	1
7	1	1	1	1

$$X = \sum(3,5,6,7)$$

A \ BC	00	01	11	10
0	0	0	1	0
1	0	1	1	1

$$X = AC + AB + BC$$

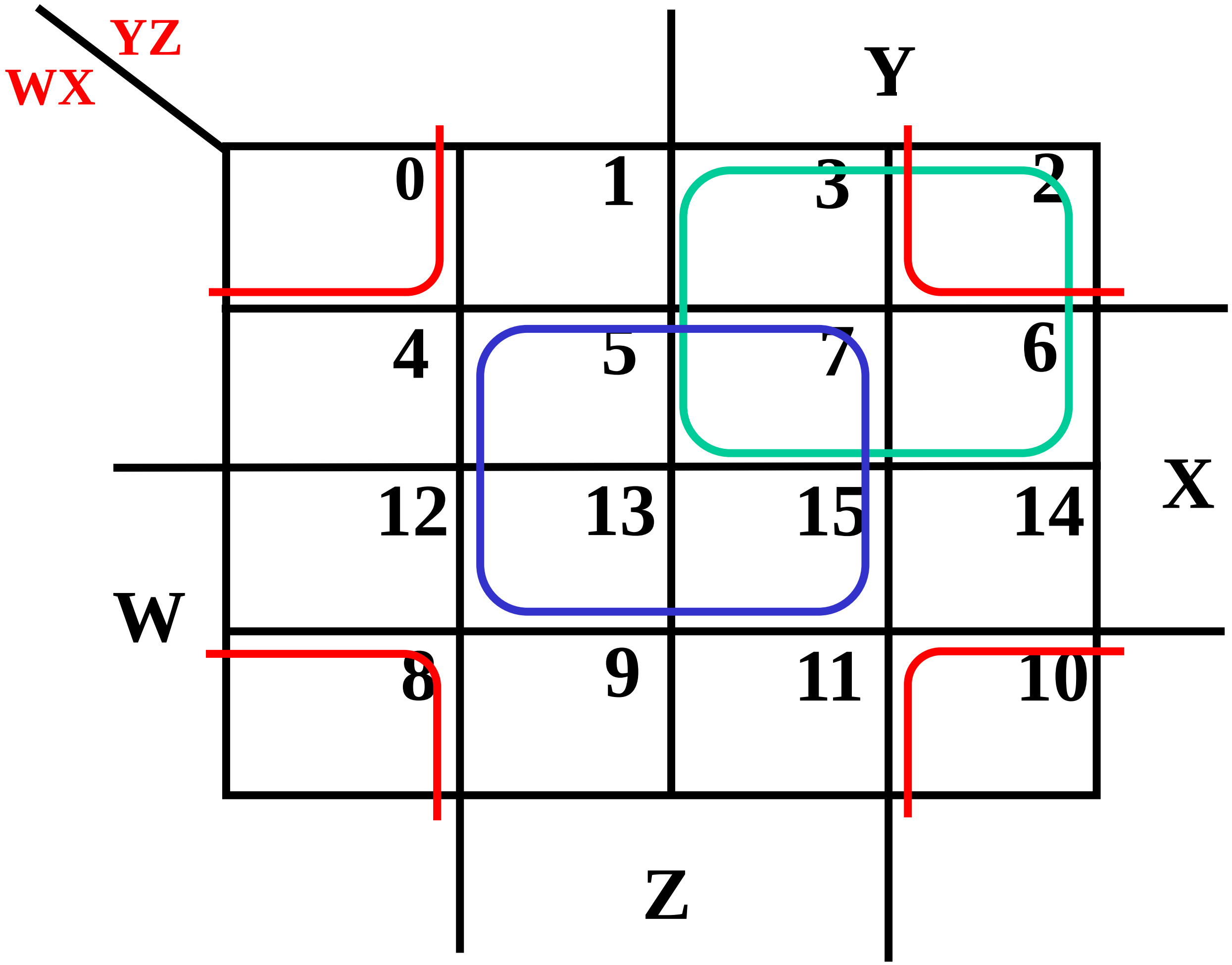


Four Variable Terms

- Four variable maps can have rectangles corresponding to:
  - A single 1 = 4 variables, (i.e. Minterm)
  - Two 1s = 3 variables,
  - Four 1s = 2 variables
  - Eight 1s = 1 variable,
  - Sixteen 1s = zero variables (i.e. Constant "1")

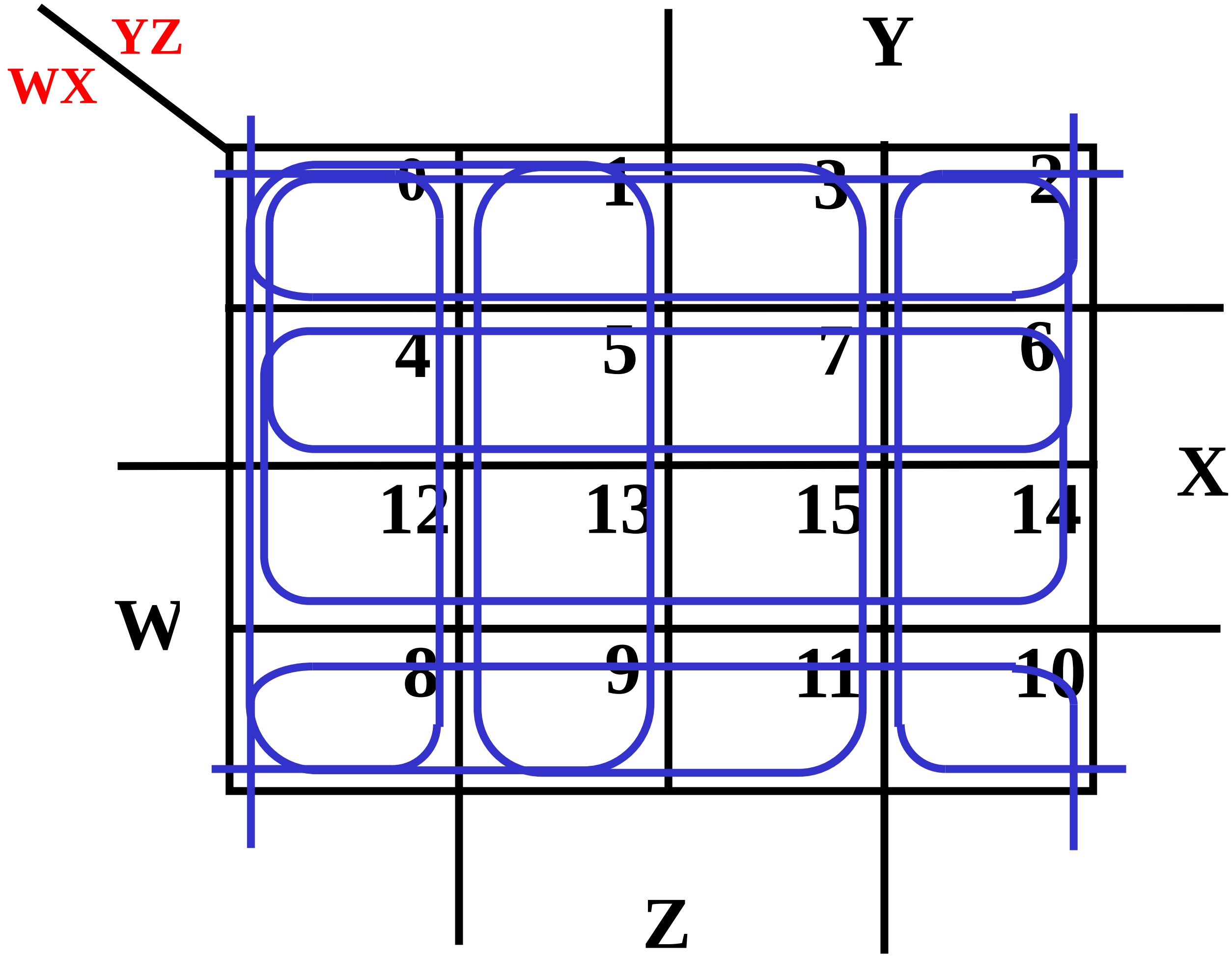
Four-Variable Maps

- Example Shapes of Rectangles:



Four-Variable Maps

- Example Shapes of Rectangles:



Four-Variable Maps

AB \ CD	00	01	11	10
00	1	0	0	0
01	0	0	0	0
11	0	0	0	0
10	1	0	0	0

$$f = \sum(0,8) = \overline{B} \bullet \overline{C} \bullet \overline{D}$$

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	1	0	0
11	0	1	0	0
10	0	0	0	0

$$f = \sum(5,13) = B \bullet \overline{C} \bullet D$$

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	1	1	0
10	0	0	0	0

$$f = \sum(13,15) = A \bullet B \bullet D$$

AB \ CD	00	01	11	10
00	0	0	0	0
01	1	0	0	1
11	0	0	0	0
10	0	0	0	0

$$f = \sum(4,6) = \overline{A} \bullet B \bullet \overline{D}$$

AB \ CD	00	01	11	10
00	0	0	1	1
01	0	0	1	1
11	0	0	0	0
10	0	0	0	0

$$f = \sum(2,3,6,7) = \overline{A} \bullet C$$

AB \ CD	00	01	11	10
00	0	0	0	0
01	1	0	0	1
11	1	0	0	1
10	0	0	0	0

$$f = \sum(4,6,12,14) = B \bullet \overline{D}$$

AB \ CD	00	01	11	10
00	0	0	1	1
01	0	0	0	0
11	0	0	0	0
10	0	0	1	1

$$f = \sum(2,3,10,11) = \overline{B} \bullet C$$

AB \ CD	00	01	11	10
00	1	0	0	1
01	0	0	0	0
11	0	0	0	0
10	1	0	0	1

$$f = \sum(0,2,8,10) = \overline{B} \bullet \overline{D}$$

More Examples

Try:

Use Karnaugh Maps to simplify the following Boolean functions to a minimum number of literals.

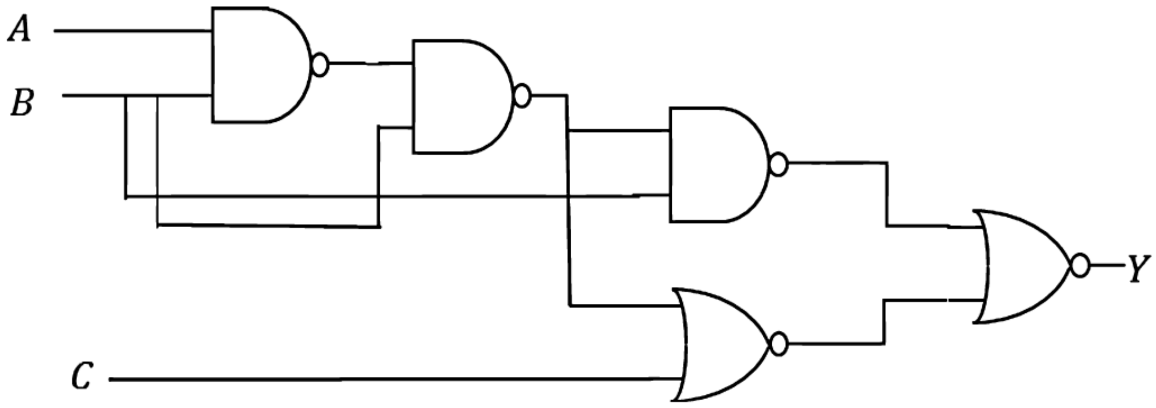
1) $F(x, y, z) = xyz + \bar{x}\, y + xy\bar{z}$

2) $Y = \overline{(A \oplus B)}.\,(\bar{A} \otimes \bar{B}\,)$

3) $Y = A + (\bar{B} + C) + \overline{(B.\bar{C}) + \bar{A}}$

Exercise -5

Use Karnaugh Map method to simplify the following logic diagram.



Deadline: November 8, 2022 @ 11:59 PM

Homework 6

Use Karnaugh Map method to simplify each of the following to a minimum number of literals.

1) $F = \overline{(C.(B\bar{A}))\ \overline{(B \oplus \bar{C})}}$

2) $F(A,B,C,D) = \Sigma m(0,2,6,8,10,14)$

3)

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

Deadline: November 11, 2022 @ 11:59 PM