Digital Logic

Lecture 8

2nd Stage

Computer Science Department

Faculty of Science

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Topics covered

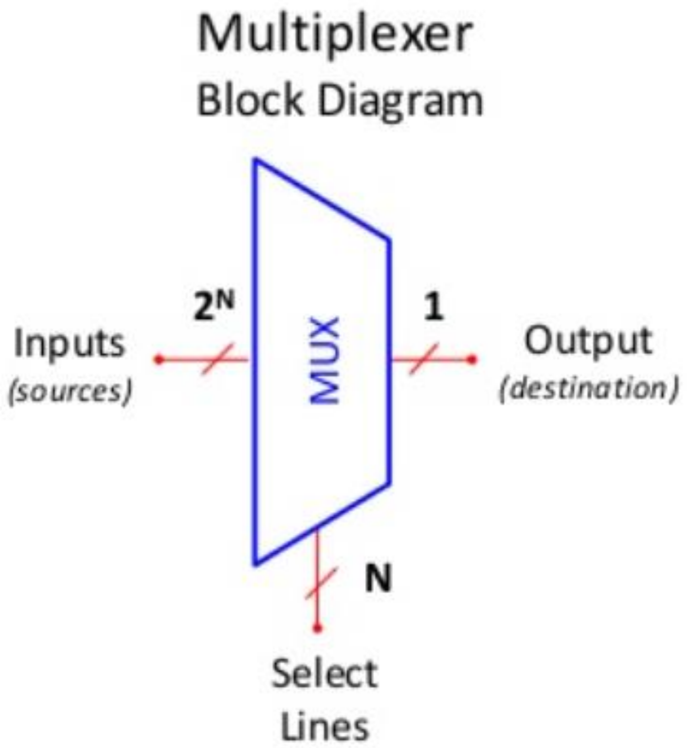
- Multiplexers
- The methods for implementing a circuit using multiplexers.
  - Karnaugh Map Method
  - Algebraic Manipulation
- Logic Expression From Multiplexers

Multiplexers

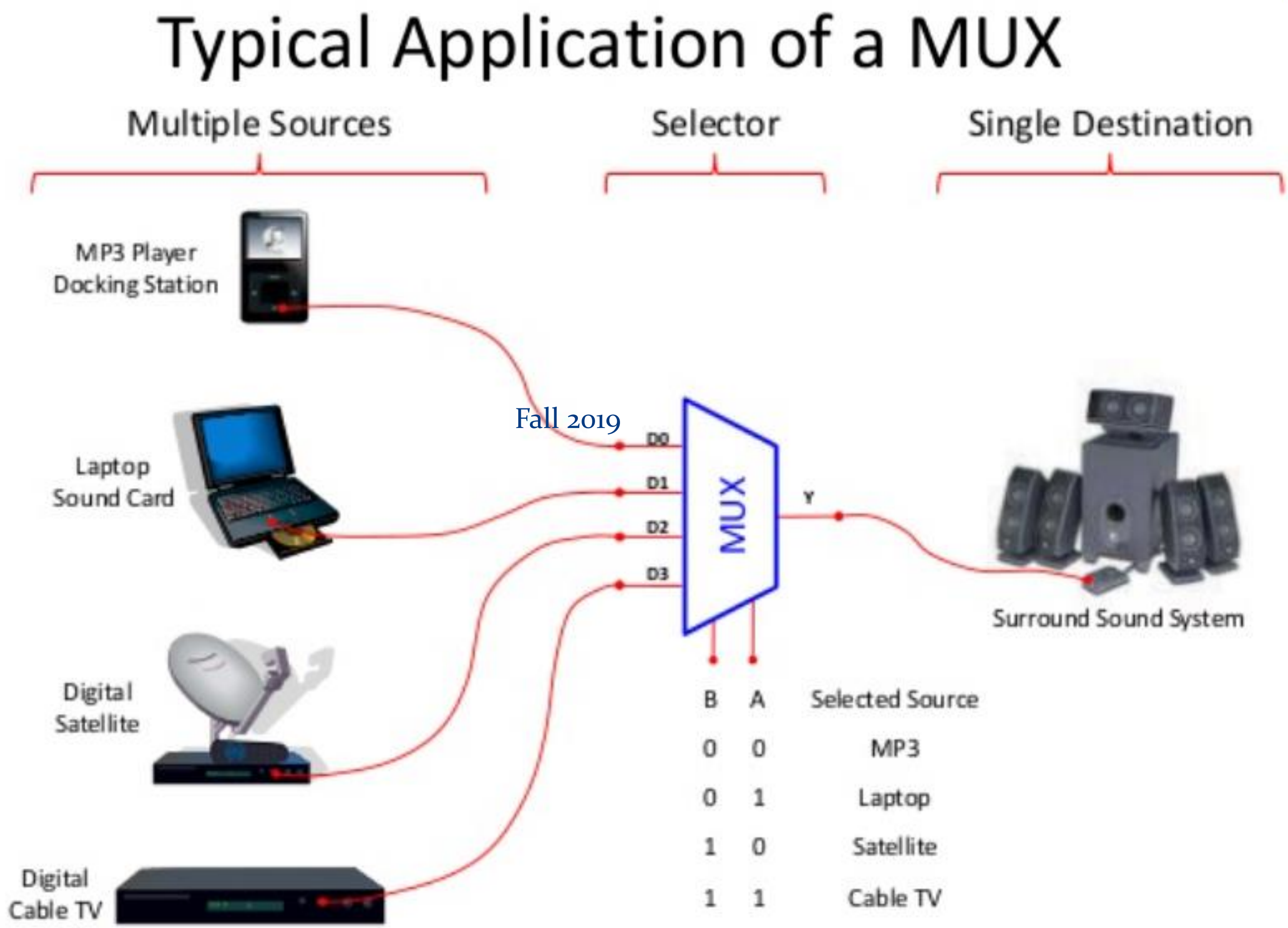
The multiplexer is a combinational logic circuit designed to switch one of several input lines to a single common output line.

- The *multiplexer*, shortened to "**MUX**" or "**MPX**".

- In digital electronics, multiplexers are also known as **data selectors** because they can "**select**" each input line.

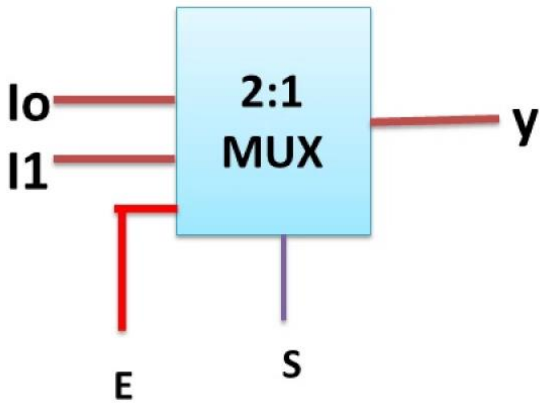


Multiplexers



Multiplexers

2 : 1 Multiplexer



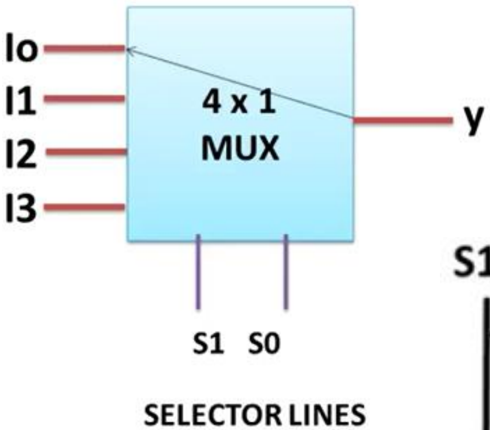
E	S	Y
0	X	0
1	0	I0
1	1	I1

$$Y = E.S'.I_0 + E.S.I_1$$

$$Y = E\,(S'.I_0 + S.I_1)$$

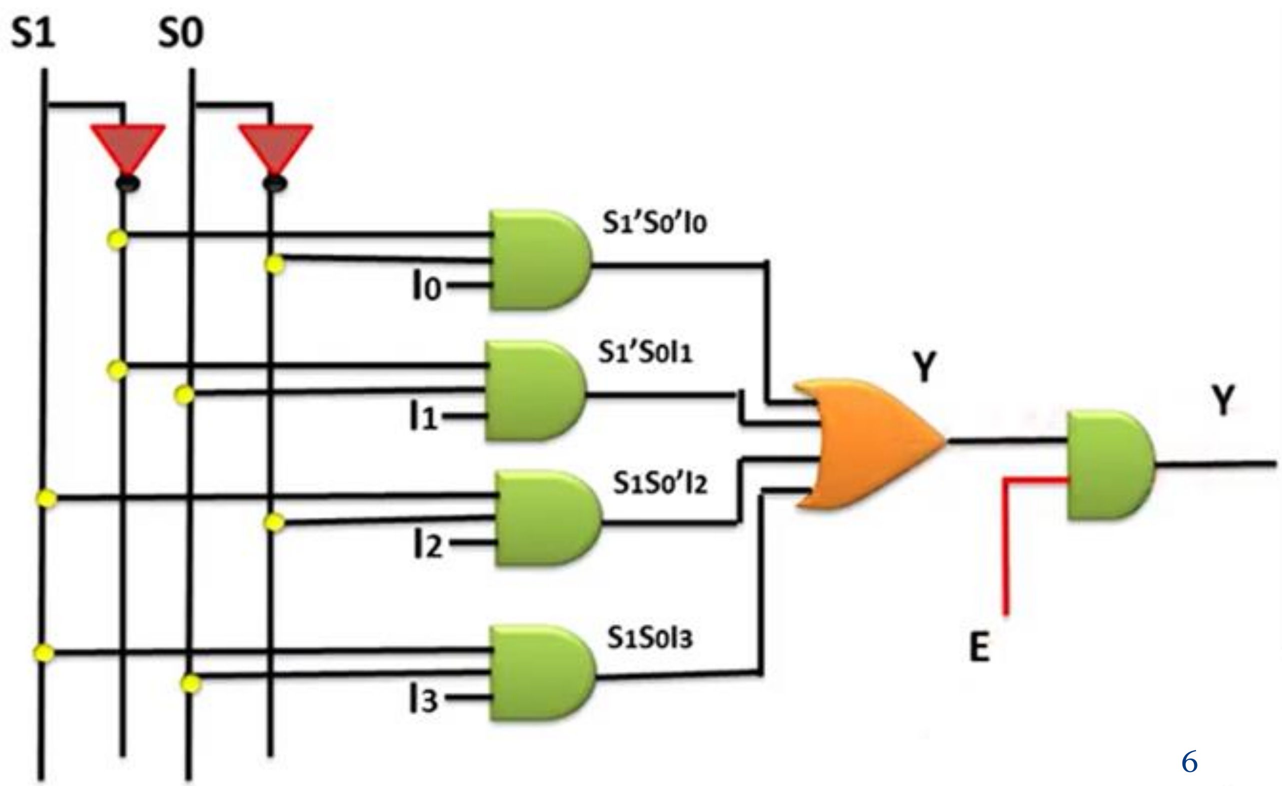
Multiplexers

4 : 1 Multiplexer



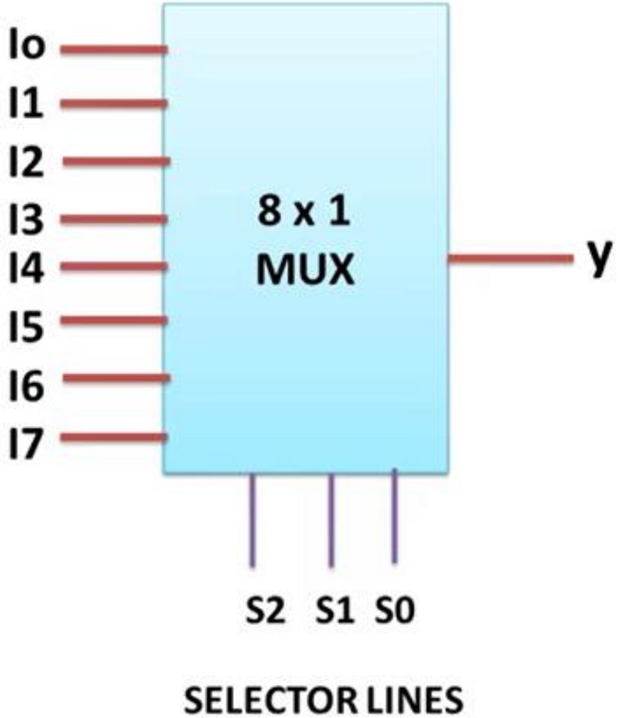
$$Y = S_1'S_0'I_0 + S_1'S_0I_1 + S_1S_0'I_2 + S_1S_0I_3$$

S1	S0	Y
0	0	I_0
0	1	I_1
1	0	I_2
1	1	I_3



Multiplexers

8 : 1 Multiplexer

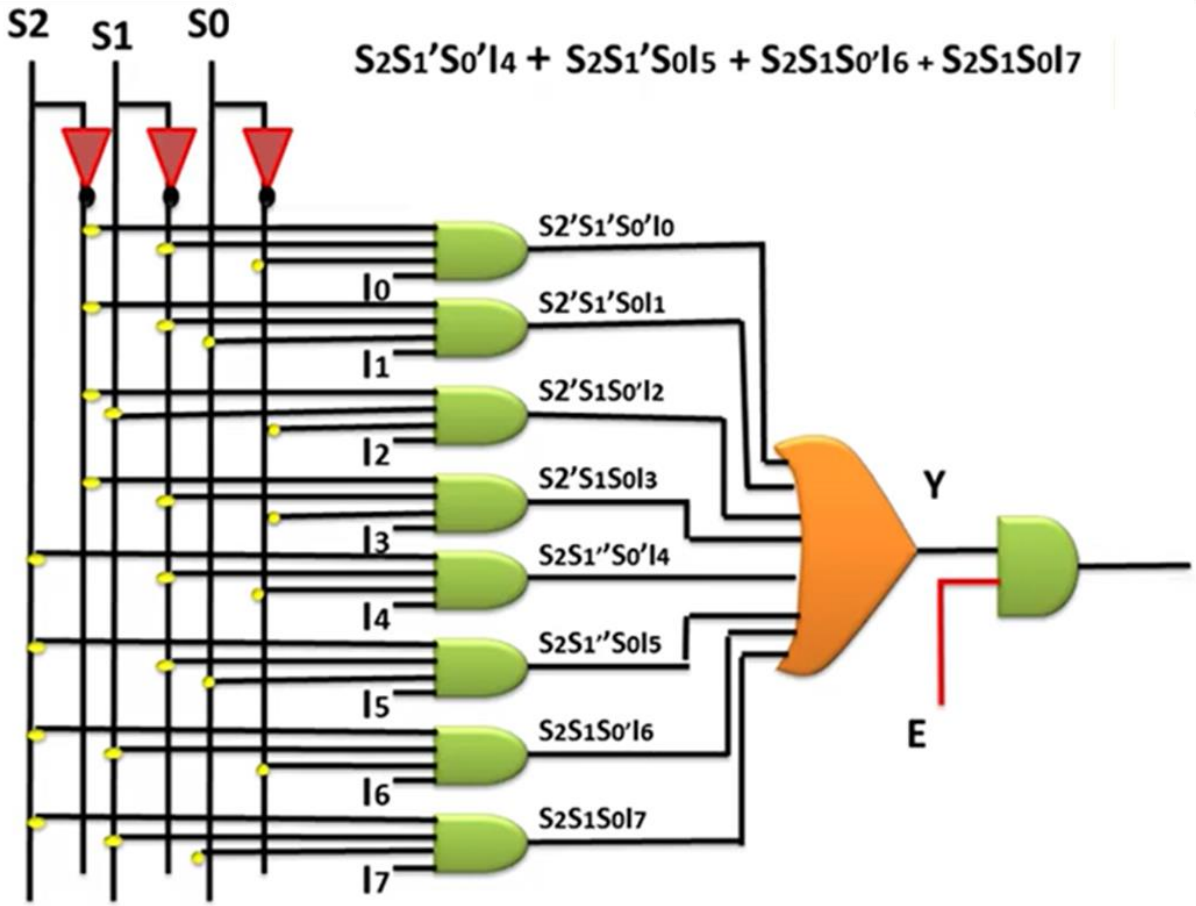


S2	S1	S0	Y
0	0	0	Io
0	0	1	I1
0	1	0	I2
0	1	1	I3
1	0	0	I4
1	0	1	I5
1	1	0	I6
1	1	1	I7

$$Y = S_2'S_1'S_0'I_0 + S_2'S_1'S_0I_1 + S_2'S_1S_0'I_2 + S_2'S_1S_0I_3 + S_2S_1'S_0'I_4 + S_2S_1'S_0I_5 + S_2S_1S_0'I_6 + S_2S_1S_0I_7$$

Multiplexers

$Y = S_2'S_1'S_0'I_0 + S_2'S_1'S_0I_1 + S_2'S_1S_0'I_2 + S_2'S_1S_0I_3 + S_2S_1'S_0'I_4 + S_2S_1'S_0I_5$
$+ S_2S_1S_0'I_6 + S_2S_1S_0I_7$



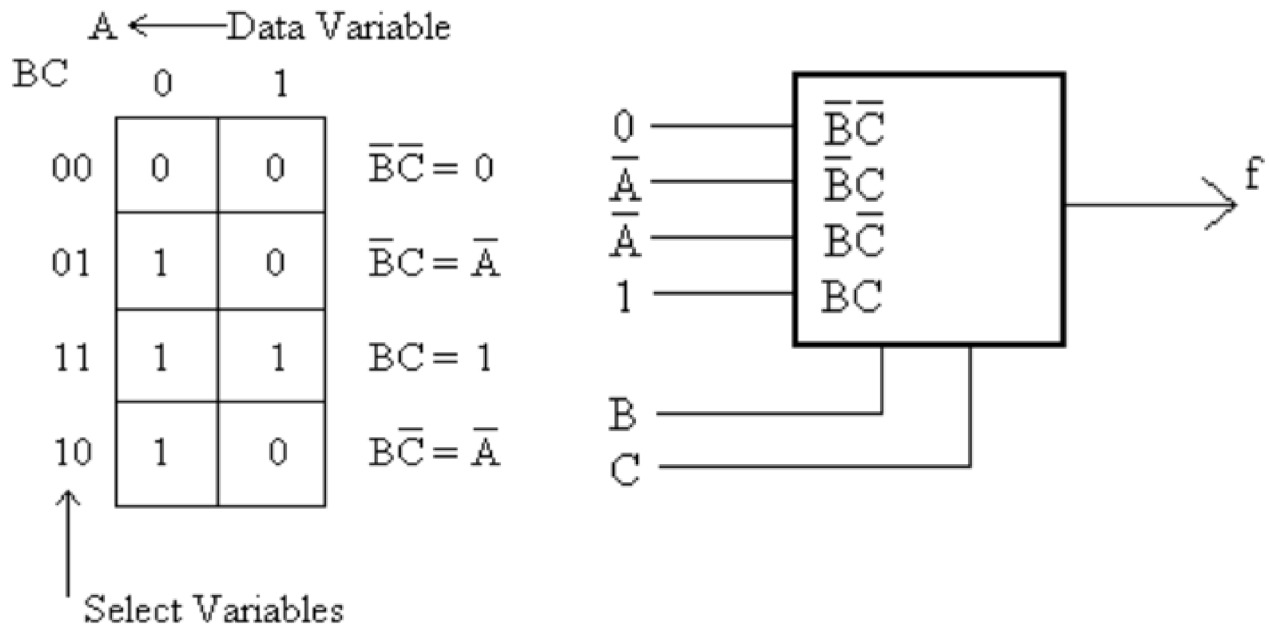
Karnaugh Map Method for Implementation of Multiplexers

Multiplexers using Karnaugh Map

Example 1: Consider the function: $f(A,B,C) = \bar{A}B + BC + \bar{A}C$
the data variable has been suggested although any variable can be chosen as the data variable and the other two as the select variables.

Solution:

Suppose one were to take A as the data variable. and B, C to be the select variables. The corresponding Karnaugh map is then:



Multiplexers using Karnaugh Map

Example 2: Design multiplexer implementations for the following functions using the Karnaugh map method.

- For the first problem, use A as the data variable and B,C as the select variables.
- For the second problem, use C as the data variable and A,B as the select variables.

$$Z = f(A,B,C) = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}B + AB\overline{C} + AC$$

$$Z = f(A,B,C) = \overline{A}B + \overline{B}C + BC + A\overline{B}\,\overline{C}$$

Multiplexers using Karnaugh Map

Solution 1:

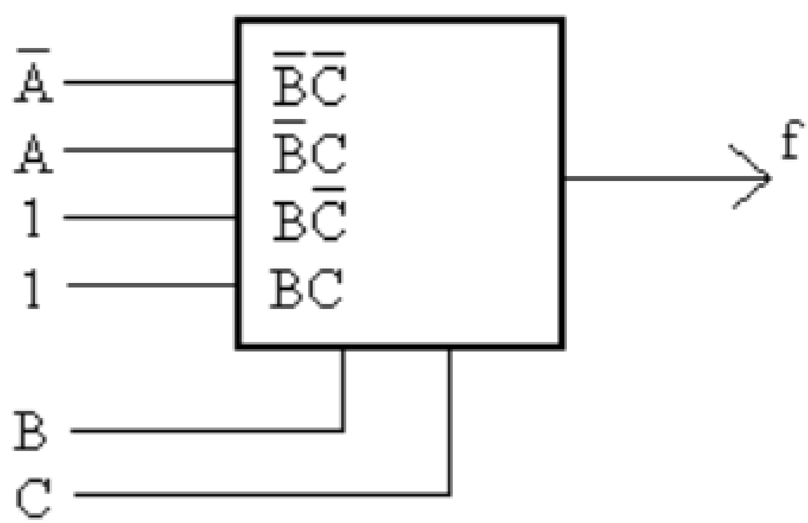
Using A as the data variable and B,C as the select variables:

A ⟵ Data Variable

$$Z = f(A,B,C) = \bar{A}\bar{B}\bar{C} + \bar{A}B + AB\bar{C} + AC$$

BC \ A	0	1	
00	1	0	$\bar{B}\bar{C} = \bar{A}$
01	0	1	$\bar{B}C = A$
11	1	1	$BC = 1$
10	1	1	$B\bar{C} = 1$

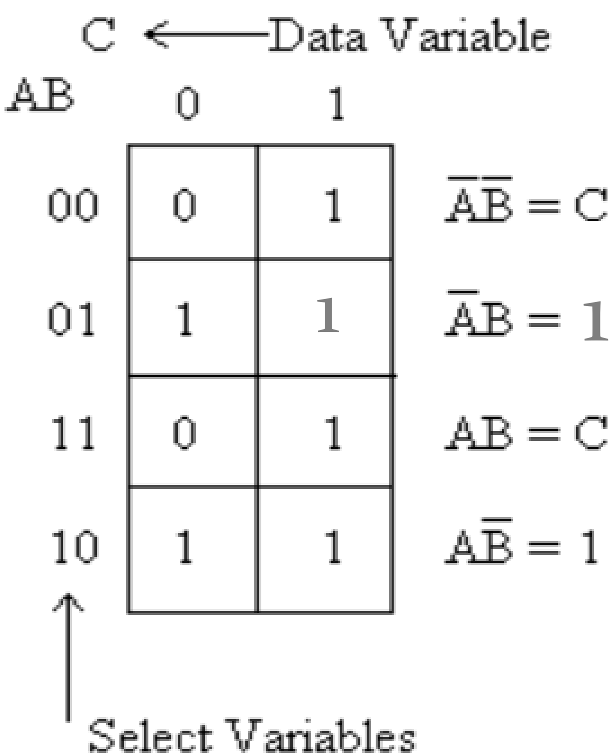
↑ Select Variables



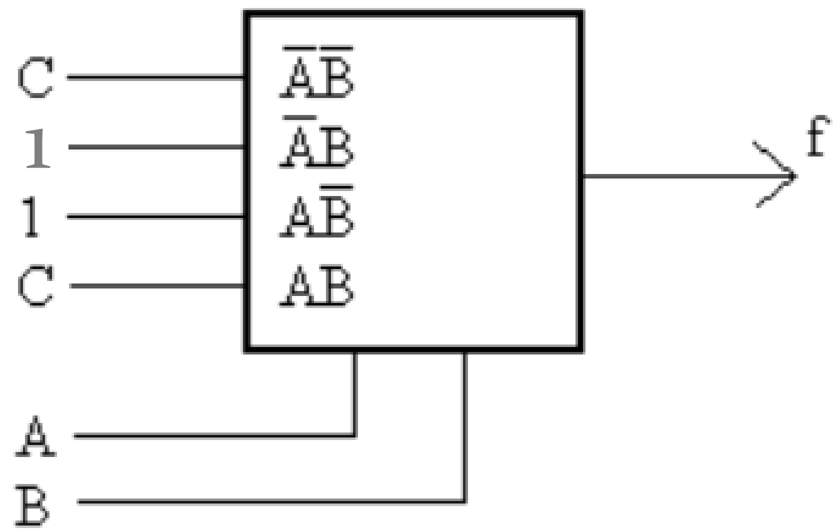
Multiplexers using Karnaugh Map

Solution 2:

Using C as the data variable and A,B as the select variables:



$$Z = f(A,B,C) = \bar{A}B + \bar{B}C + BC + A\bar{B}\bar{C}$$



Algebraic Method for Implementation of Multiplexers

Multiplexers using Algebraic Methods

Example 1: Consider the function: $f(A,B,C) = \bar{A}B + BC + \bar{A}C$

Expanding to canonical sum of products form:

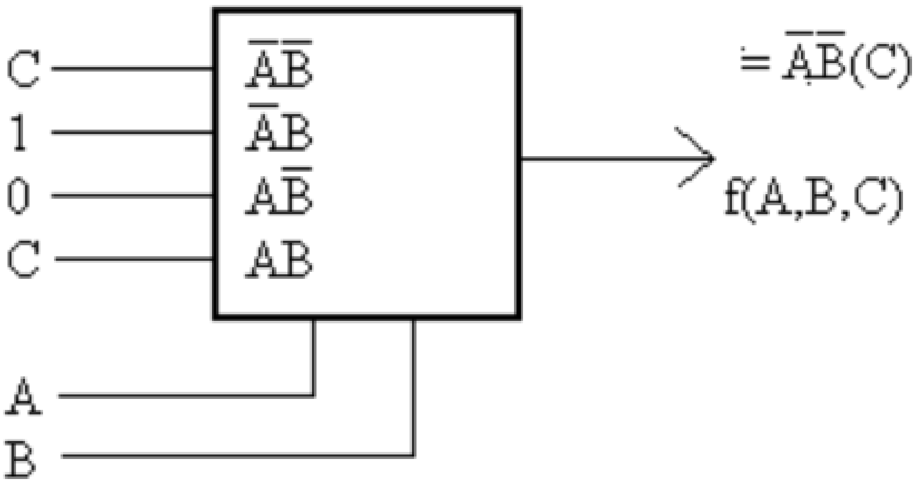
$$f(A,B,C) = \bar{A}B(C+\bar{C}) + BC(A+\bar{A}) + \bar{A}C(B+\bar{B})$$

$$= \bar{A}BC + \bar{A}B\bar{C} + ABC + \cancel{\bar{A}BC} + \cancel{\bar{A}BC} + \bar{A}\bar{B}C$$

$$= \bar{A}BC + \bar{A}B\bar{C} + \bar{A}\bar{B}C + ABC$$

$$= \bar{A}\bar{B}C + \bar{A}B(C+\bar{C}) + ABC$$

$$= \bar{A}\bar{B}(C) + \bar{A}B(1) + A\bar{B}(0) + AB(C)$$

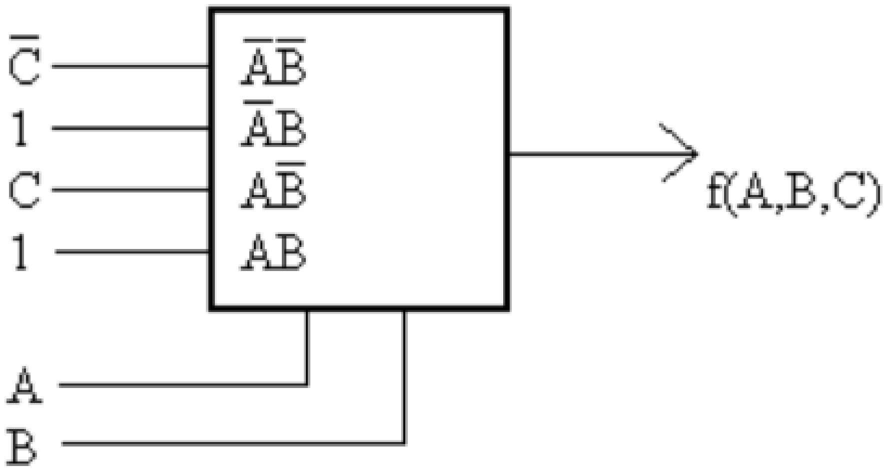


Multiplexers using Algebraic Methods

Example 2: Consider the function below:

Expanding to sum of products form

$$
\begin{aligned}
F(A,B,C) &= \bar{A}\bar{B}\bar{C} + \bar{A}B(C+\bar{C}) + AB\bar{C} + AC(B+\bar{B}) \\
&= \bar{A}\bar{B}\bar{C} + \bar{A}BC + \bar{A}B\bar{C} + AB\bar{C} + ABC + A\bar{B}C \\
&= \bar{A}\bar{B}(\bar{C}) + \bar{A}B(1) + AB(1) + A\bar{B}(C)
\end{aligned}
$$



Multiplexers using Algebraic Methods

Example 3:

Expanding to sum of products form:

$$F(A,B,C) = \bar{A}B(C+\bar{C}) + \bar{B}C(A+\bar{A}) + BC(A+\bar{A}) + A\bar{B}\bar{C}$$

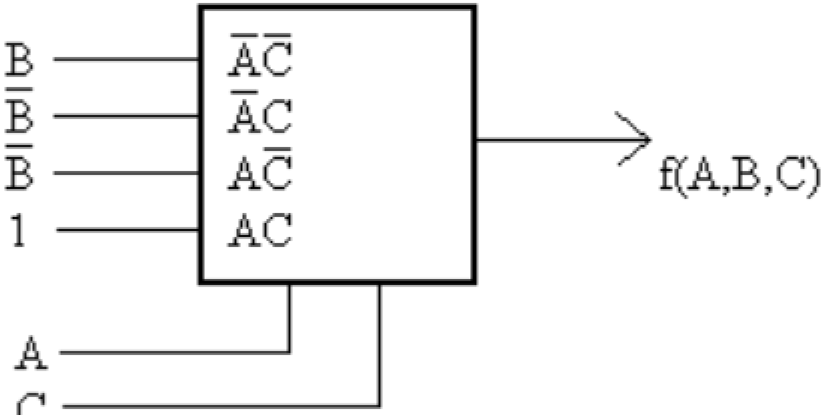
$$= \cancel{\bar{A}BC} + \bar{A}B\bar{C} + A\bar{B}C + \bar{A}\bar{B}C + ABC + \cancel{\bar{A}BC} + A\bar{B}\bar{C}$$

$$= \bar{A}B\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C + ABC$$

$$= \bar{A}B\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + AC(B+\bar{B})$$

$$= \bar{A}\bar{C}(B) + \bar{A}C(\bar{B}) + A\bar{C}(\bar{B}) + AC(1)$$

What is wrong?

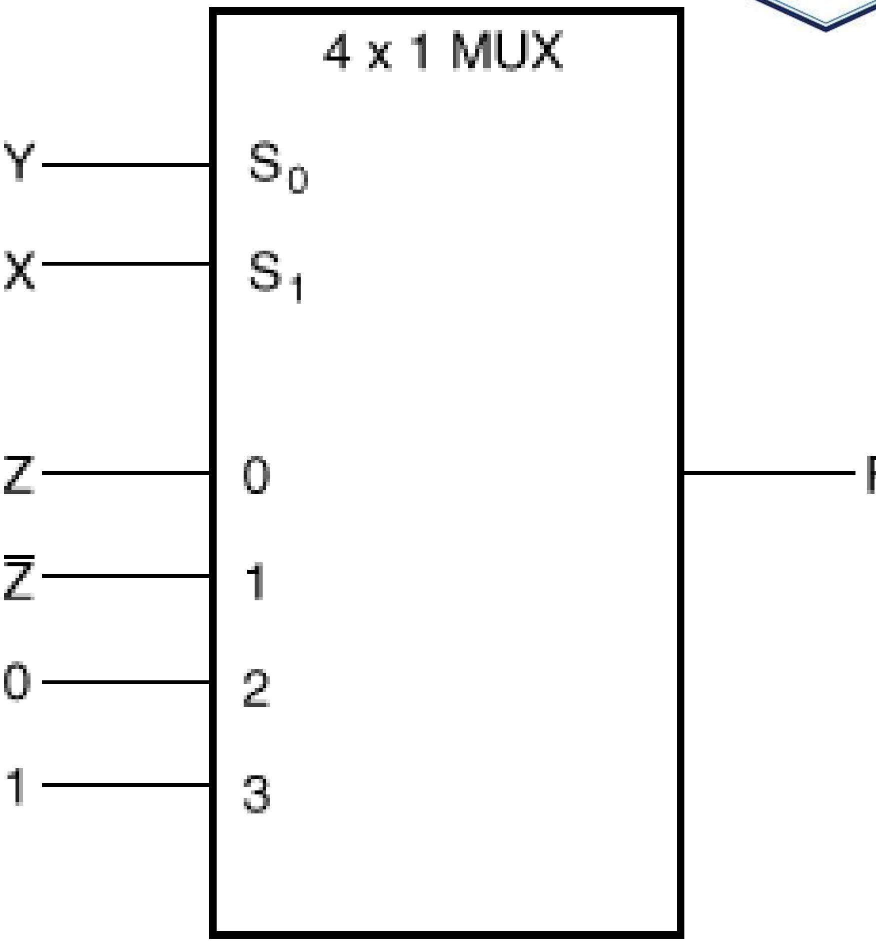


Implementing Boolean Functions with a Multiplexer

$F(x,y,z) = \Sigma m(1, 2, 6, 7)$

X	Y	Z	F	
0	0	0	0	F = Z
0	0	1	1	
0	1	0	1	$F = \overline{Z}$
0	1	1	0	
1	0	0	0	F = 0
1	0	1	0	
1	1	0	1	F = 1
1	1	1	1	

(a) Truth table

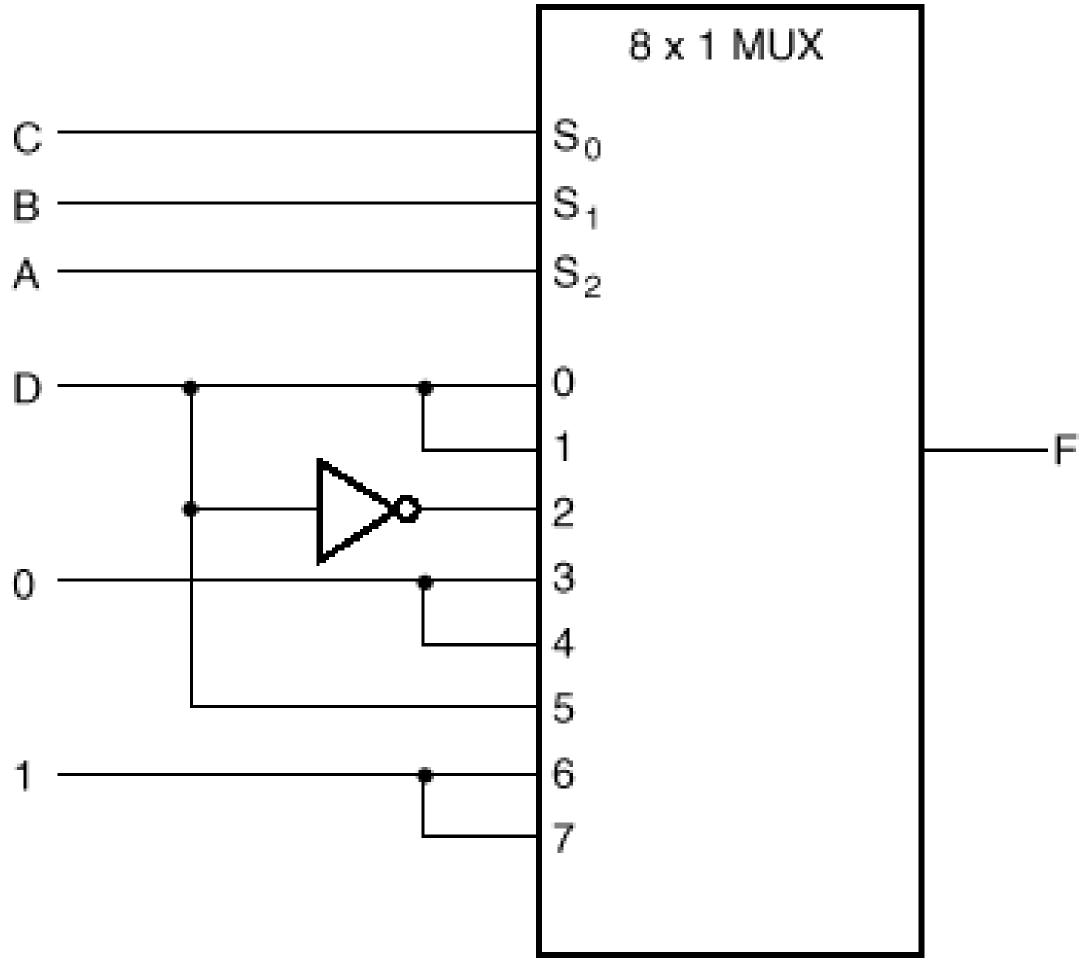


(b) Multiplexer implementation

Implementing Boolean Functions with a Multiplexer

$F(A,B,C,D) = \Sigma m(1,3,4,11,12,13,14,15)$

A	B	C	D	F	
0	0	0	0	0	$F = D$
0	0	0	1	1	
0	0	1	0	0	$F = D$
0	0	1	1	1	
0	1	0	0	1	$F = \overline{D}$
0	1	0	1	0	
0	1	1	0	0	$F = 0$
0	1	1	1	0	
1	0	0	0	0	$F = 0$
1	0	0	1	0	
1	0	1	0	0	$F = D$
1	0	1	1	1	
1	1	0	0	1	$F = 1$
1	1	0	1	1	
1	1	1	0	1	$F = 1$
1	1	1	1	1	



Implementing Boolean Functions with a Multiplexer

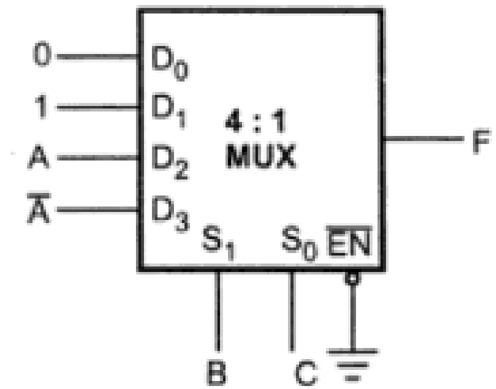
Implement the following Boolean function using 4:1 Multiplexer $F(A,B,C) = \sum m(1,3,5,6)$

Minterm	A	B	C	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

Truth Table

	D_0	D_1	D_2	D_3
$\bar{A}$	0	1	2	3
A	4	5	6	7
	0	1	A	$\bar{A}$

Implementation Table



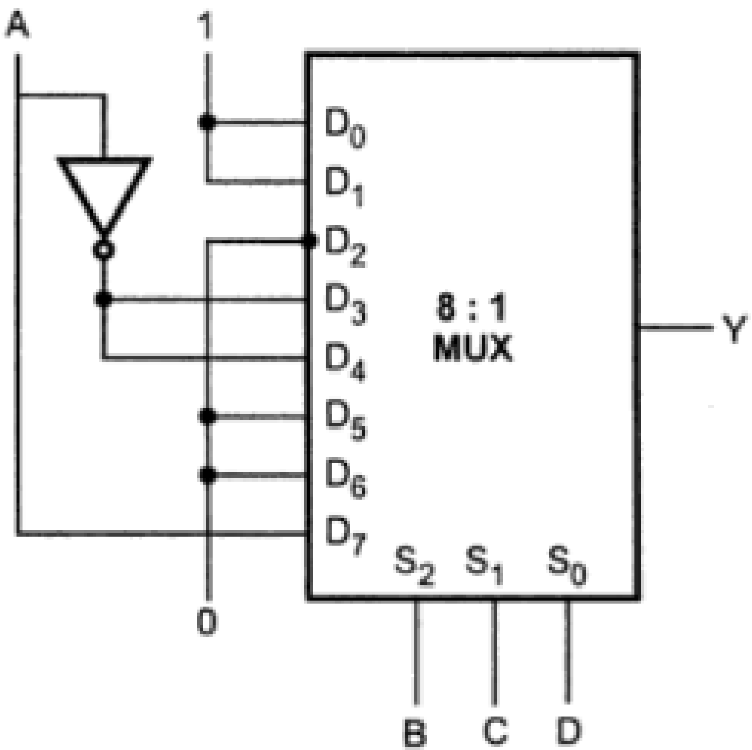
Multiplexer Implementation

Implementing Boolean Functions with a Multiplexer

Implement the following Boolean function using 8:1 Multiplexer
$F(A,B,C,D) = \sum m(0,1,3,4,8,9,15)$

	D_0	D_1	D_2	D_3	D_4	D_5	D_6	D_7
$\bar{A}$	0	1	2	3	4	5	6	7
A	8	9	10	11	12	13	14	15
	1	1	0	$\bar{A}$	$\bar{A}$	0	0	A

Implementation Table

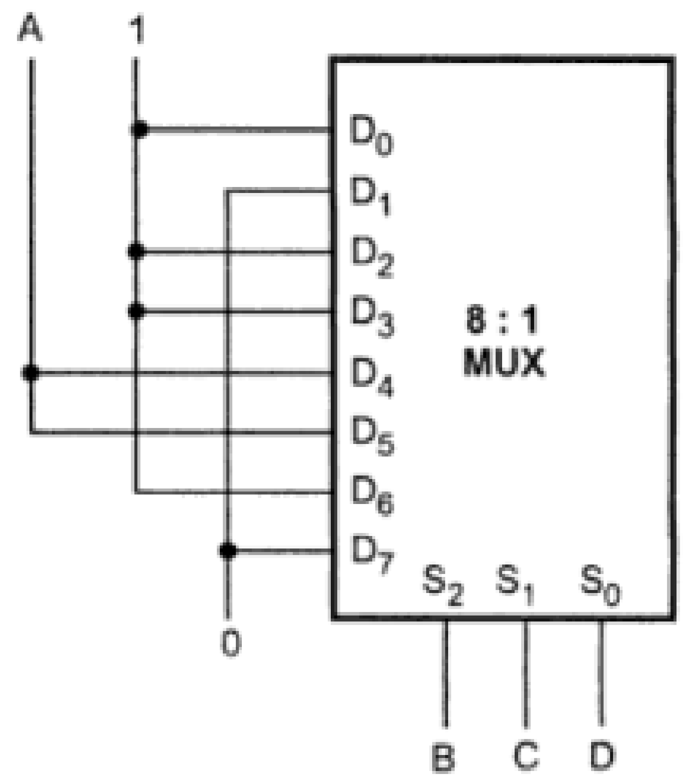


Multiplexer Implementation

Implementing Boolean Functions with a Multiplexer

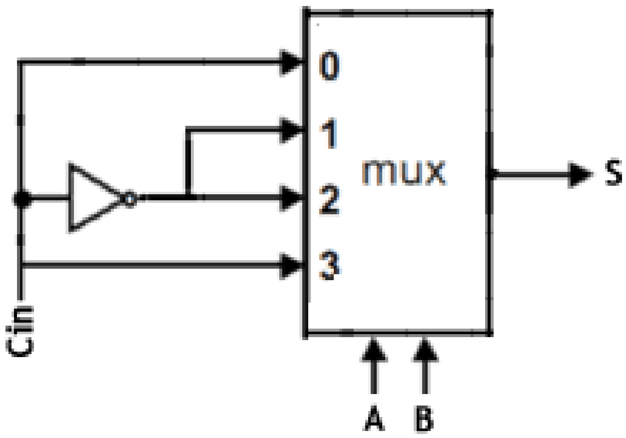
Implement the following Boolean function using 8:1 Multiplexer
$F(A,B,C,D) = \sum m(0,2,3,6,8,10,11,12,13,14)$

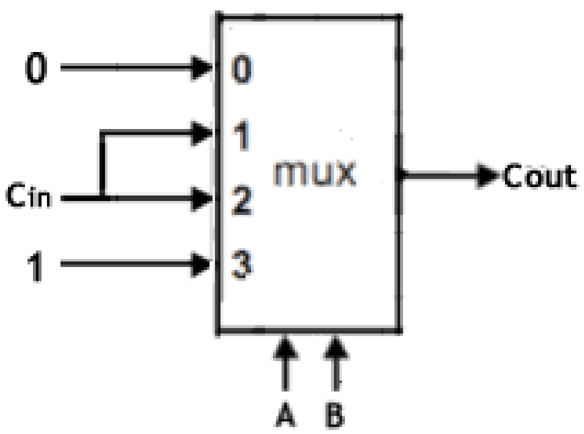
	D_0	D_1	D_2	D_3	D_4	D_5	D_6	D_7
$\bar{A}$	0	1	2	3	4	5	6	7
A	8	9	10	11	12	13	14	15
	1	0	1	1	A	A	1	0



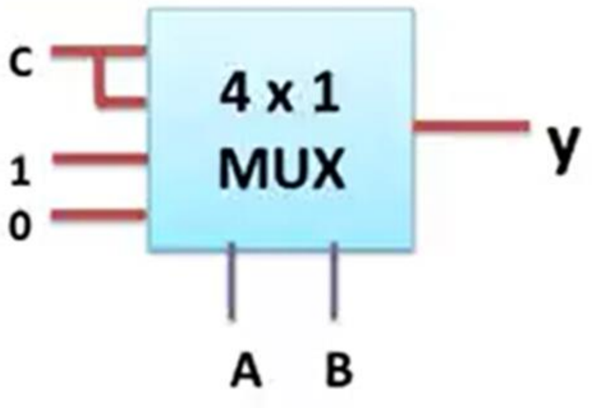
Multiplexer as a Full-Adder

INPUTS			OUTPUTS	
A	B	Cin	Sum	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1





Logic Expression From Multiplexers



A	B	Y
0	0	I_0
0	1	I_1
1	0	I_2
1	1	I_3

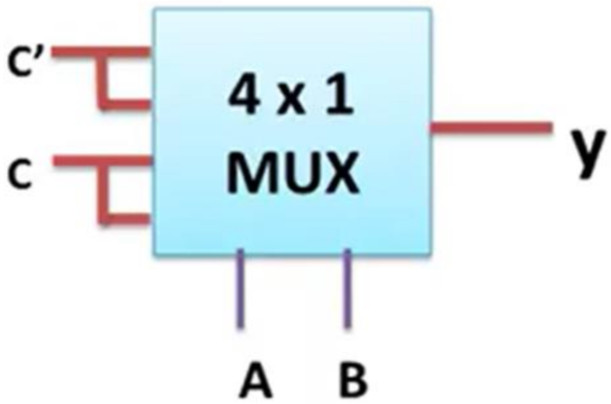
A	B	Y
0	0	C
0	1	C
1	0	1
1	1	0

$$Y = A'B'C + A'BC + AB'1 + AB0$$

$$Y = A'C\,(B + B') + AB'$$

$$Y = A'C + AB'$$

Logic Expression From Multiplexers



A	B	Y
0	0	I_0
0	1	I_1
1	0	I_2
1	1	I_3

A	B	Y
0	0	C’
0	1	C’
1	0	C
1	1	C

$$Y = A'C' + AC$$

Try

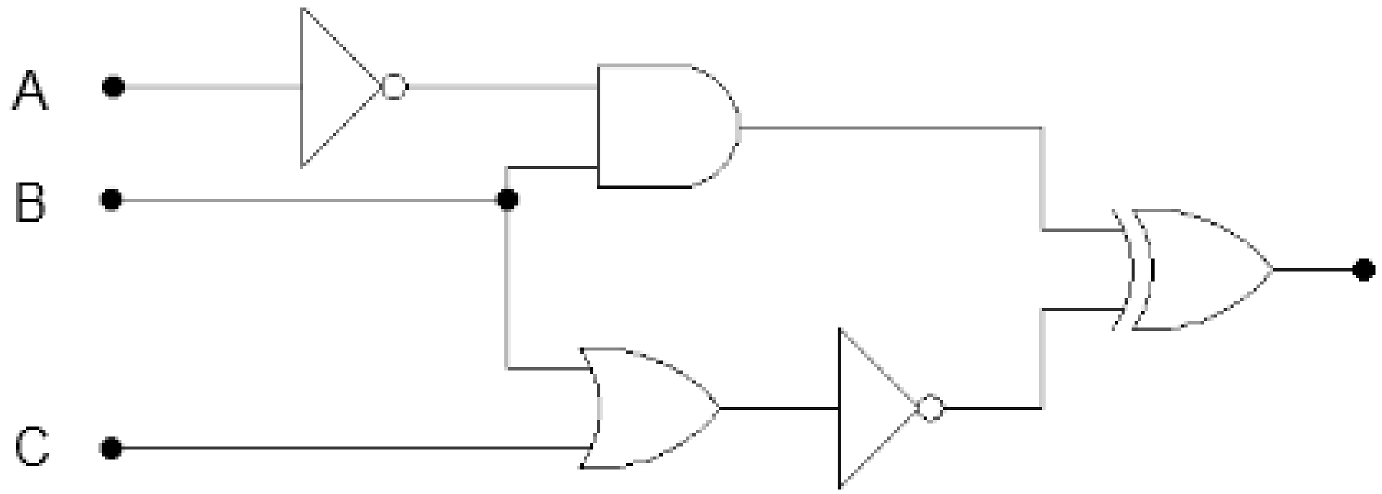
Implement the following Boolean function using a Multiplexers.

$$F(A,B,C,D) = \sum m(0, 4, 5, 6, 7, 9, 13, 15)$$

i. Draw the Multiplexer Block diagram
ii. Draw the Multiplexer using logic gates.
iii. Fully label all inputs and outputs

Try

Construct the equivalent multiplexer for the following logic diagram, use B as the data variable and A,C as the select variables.



Homework 9

Q1) Implement a full subtractor with two 4:1 multiplexers.

Q2) Implement the following Boolean function with two 4:1 and one 2:1 multiplexers.

$$F(w, x, y, z) = \Pi M(3, 10, 11)$$

Deadline: December 16, 2022 @ 11:59 PM