Quantum Phase estimation

Friday, 21 May 2021 16:31

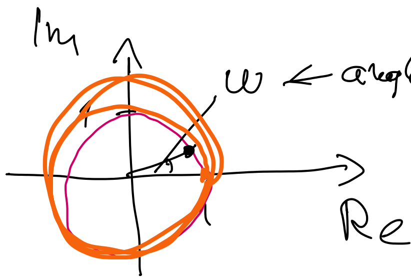
U unitary matrix:

eigenvalue equation $\qquad U|u\rangle = \lambda|u\rangle = e^{i\omega} = e^{2\pi i\varphi}$

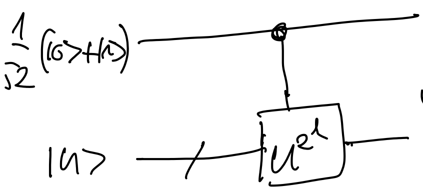
u qubits

eigenvalues $\quad |\lambda| = 1$

2^u distinct eigenvalues

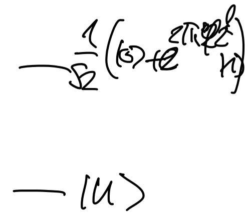


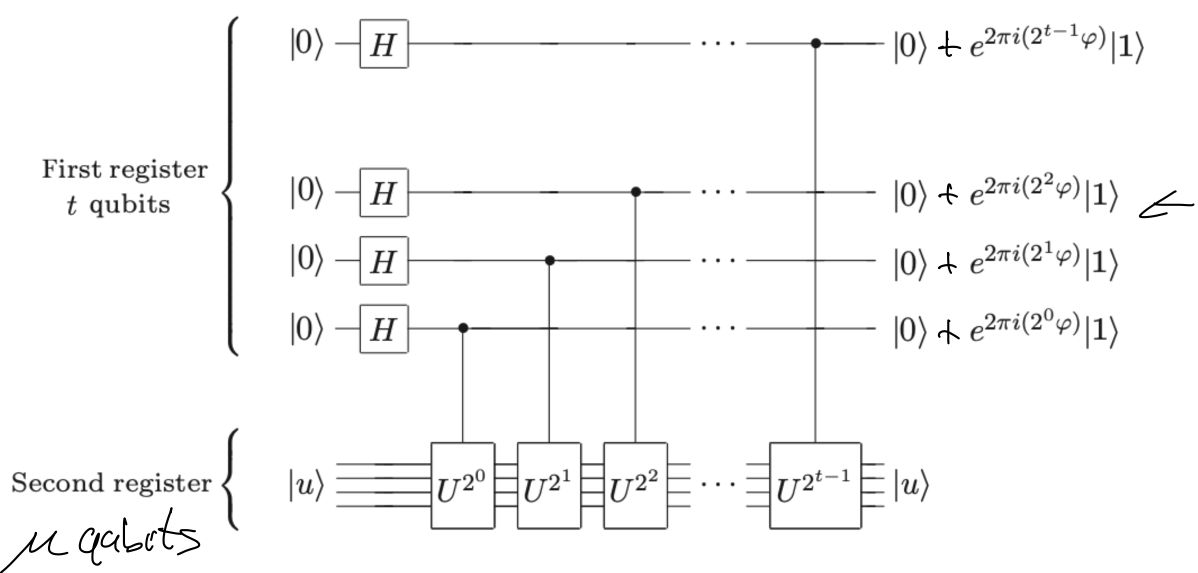
controlled U^{2^l}



$$\frac{1}{\sqrt{2}}\left(|0\rangle|u\rangle + |1\rangle e^{2\pi i\varphi 2^l}|u\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(|0\rangle + e^{2\pi i\varphi 2^l}|1\rangle\right)|u\rangle$$





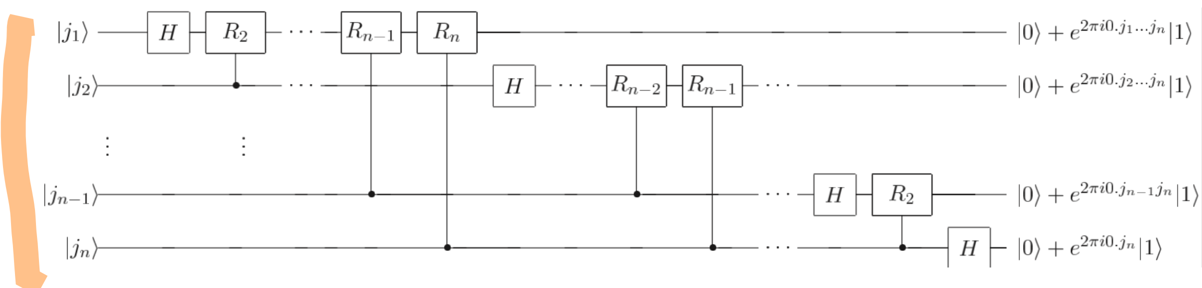
Phase estimation:

→ task: estimate the eigenvalue $e^{2\pi i\varphi}$ of U

to use QPE: we need to know the corresponding eigenstate

$|u\rangle$

Quantum Fourier transform : QFT



$|j\rangle = |j_n j_{n-1} \dots j_2 j_1\rangle$

$$\frac{1}{2^{n/2}} \sum_{k=1}^{N-1} e^{2\pi i j k 2^{-n}} |k\rangle$$

output from first half of QPE circuit

k^{th} qubit: $\frac{1}{\sqrt{2}}\left(|0\rangle + e^{2\pi i \varphi 2^k}\right)$

$0 \leq \varphi < 1$

binary fraction :

$\varphi \approx 0.\varphi_1 \varphi_2 \varphi_3 \varphi_4 \dots$

(decimal fractions: $\frac{1}{3} = 0.33\dots$)

$\varphi 2^k = \underbrace{\varphi_1 \varphi_2 \dots \varphi_{k-1}}_{\text{integer part}} . \underbrace{\varphi_k \varphi_{k+1} \dots}_{\text{fraction}}$

it was in the exponent:

$$e^{2\pi i \varphi 2^k} = e^{2\pi i 0.\varphi_k \varphi_{k+1} \dots}$$

$$e^{2\pi i l} = 1$$

integer

Qiskit $\longleftrightarrow$ Cirq

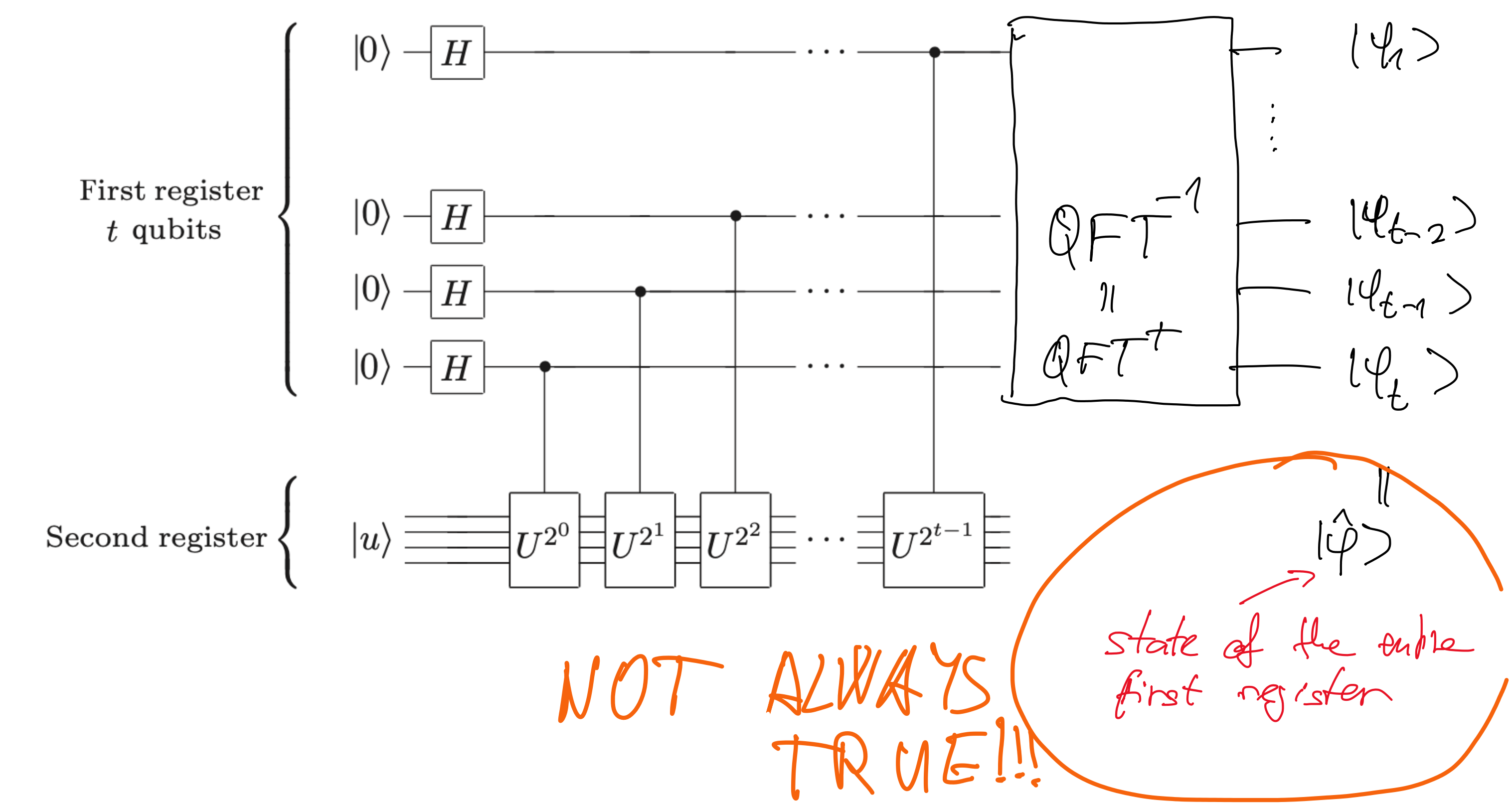
machine representation of integers

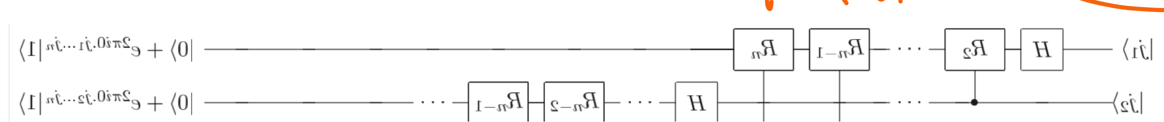
little endian representation: $4_{10} = 100_2$ ← least significant digit is at the end

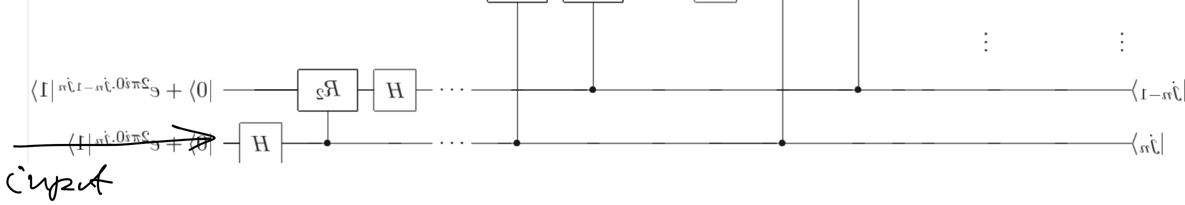
mainframes (Alpha DEC): big endian representation

$4_{10} = 001$ ↖ most significant digit that's at the end

Quantum Phase Estimation Circuit







$$H^{+} = H \qquad R_k^{+} = R_{-k} = R^{-1}$$

what are these "phis"?

φ: angle ($0 \leq \varphi < 1$)

binary fraction representation $\varphi \simeq 0.\varphi_1 \varphi_2 \cdots \varphi_{t-1} \varphi_t$

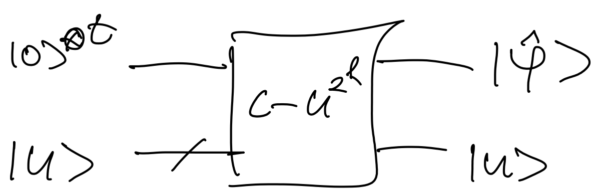
binary digits

another symbol: $\hat{\varphi}$ ← integer

$$\hat{\varphi} = \varphi_1 \varphi_2 \cdots \varphi_{t-1} \varphi_t$$

if: $\varphi = 0.\varphi_1 \varphi_2 \ldots \varphi_{t-1} \varphi_t$ EXACTLY

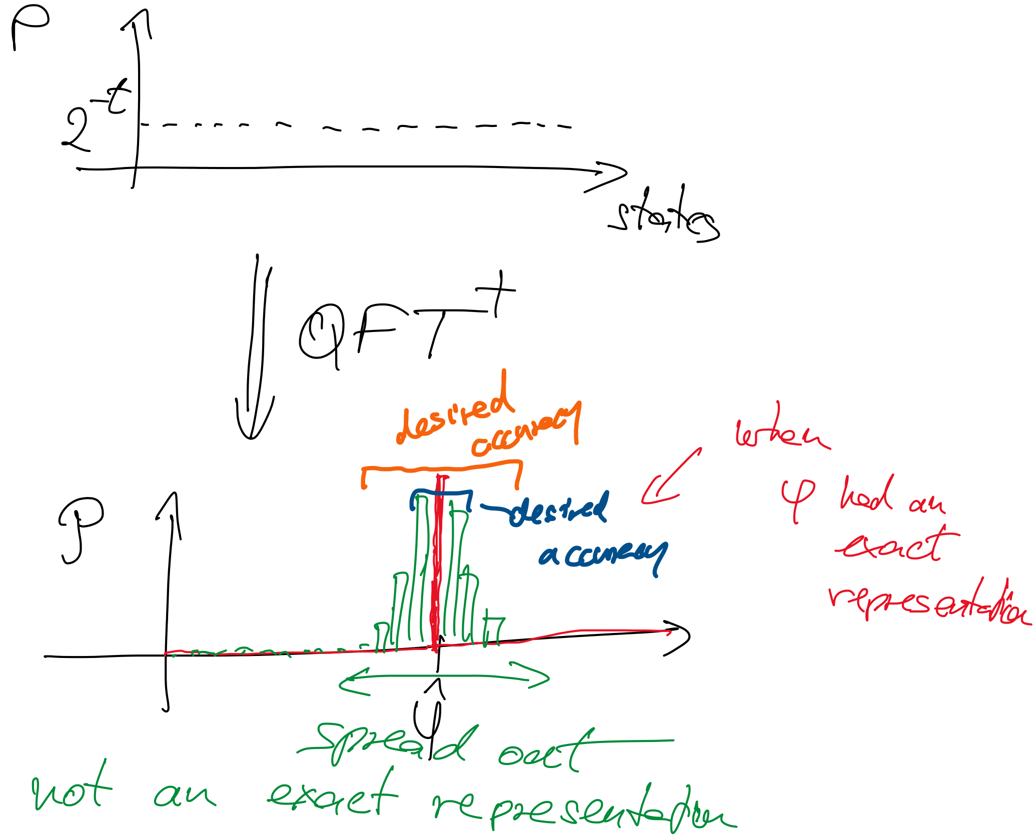
QPE circuit



QFT does to the amplitudes what DFT would do to the vector holding the amplitudes

power spectrum: squared modulus ⟺ probability distribution

probability distribution after 1st half of QPE



width: depend on size of first register t

We measure after QPE circuit

we obtain: $|\tilde{\varphi}\rangle$ (not "hat", but "tilde")

To obtain $\tilde{\varphi}$ accurate to n binary digits
with probability of success $p \geq 1-\varepsilon$

$\Rightarrow$ we need to take t registers

$$t = n + \left\lceil \log_2\left(2 + \frac{1}{2\varepsilon}\right) \right\rceil$$