Endogenous portfolios and risk premia

Adrien Auclert

NBER heterogeneous-agent workshop, 2025

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  - eg, stocks and bonds, nominal and real bonds, short and long term bonds, home and foreign bonds
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- **Now**: what if we let households trade these different assets directly?
  - MIT shock world: portfolios are **indeterminate**
    - can feed in distribution of portfolios from the data
  - Expected shocks world: portfolios are **determinate**!

What does second order perturbation give us?

Orders in aggregate risk

2nd order

1st order

0th order

Steady state

Exog. portfolios

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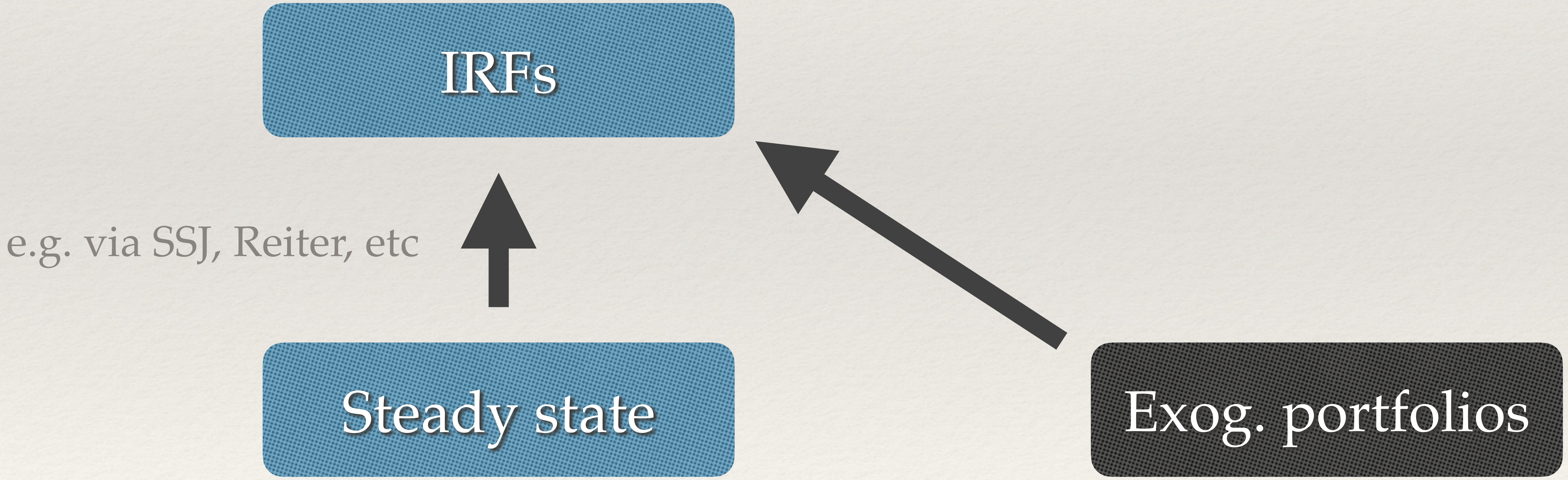

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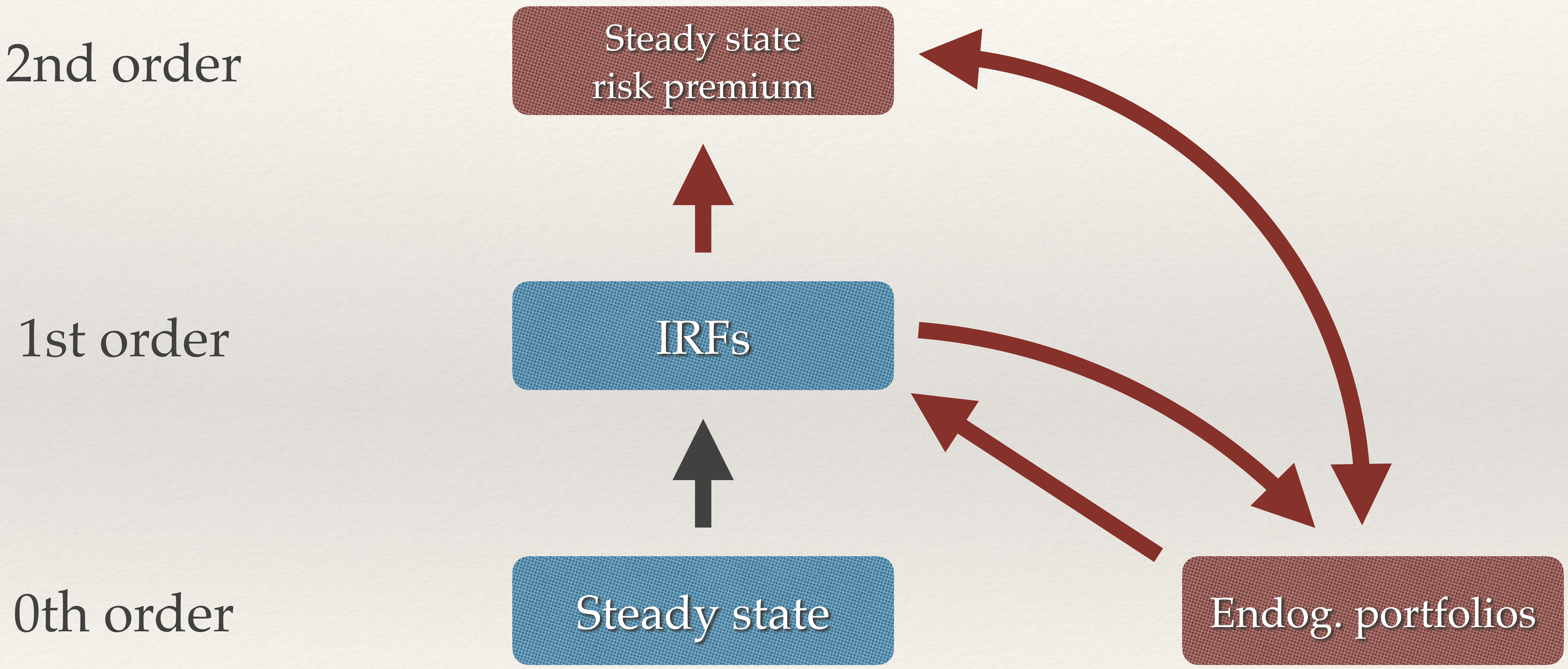

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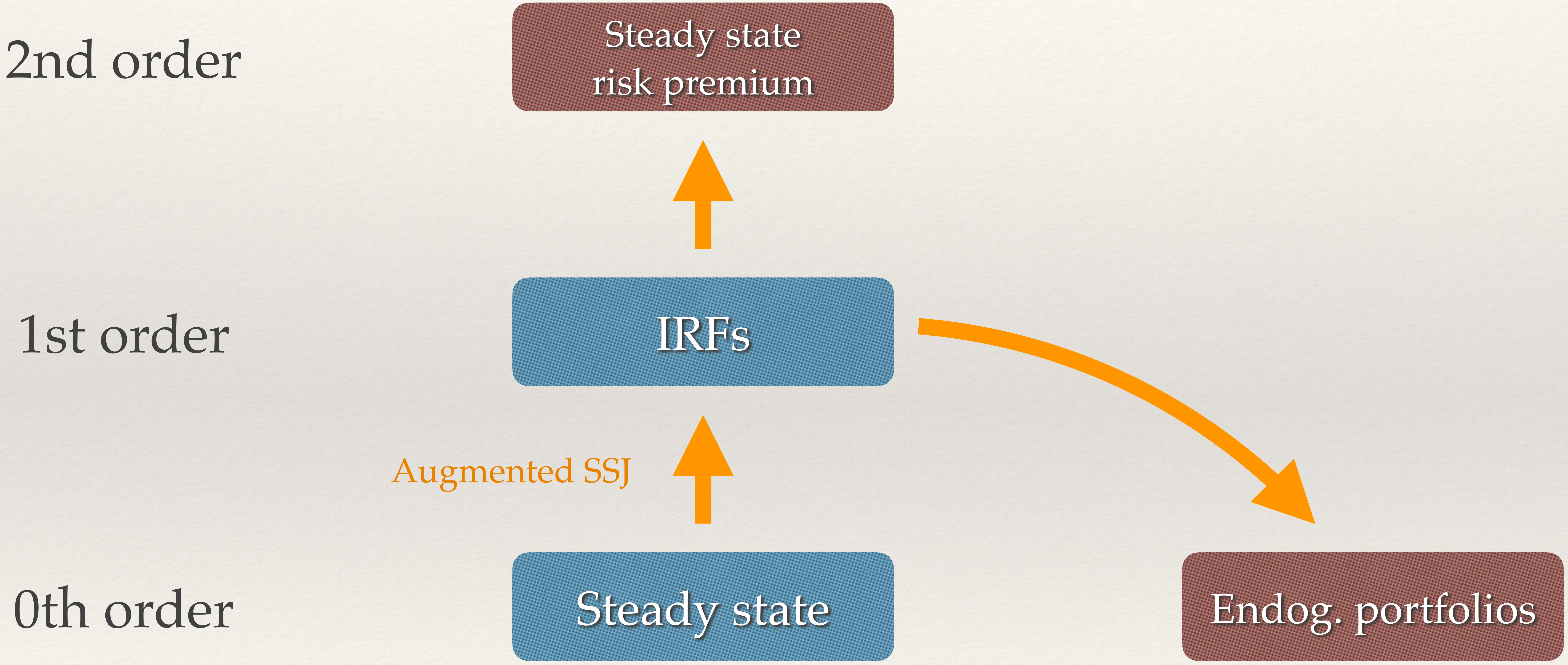

Canonical model with locally complete markets

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$$\mathbb{E}_\epsilon \left[R^k(\epsilon) \frac{\partial V_i}{\partial a}(\epsilon) \right] = \gamma_i \quad \forall i, k$$

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  - 2nd-order perturbation to ϵ shows we need $K \geq \dim\epsilon$ + spanning condition
  - "0th order portfolios" a_i^k; well-defined in the limit as $\epsilon \to \epsilon_{ss}$

    [eg Tille-van Wincoop 2010, Devereux-Sutherland 2011, Coeurdacier-Rey 2013, etc.]

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- Consider now standard one-account model with idiosyncratic risk e + aggregate risk

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$$c + \sum_{k=0}^{K} a^{k\prime} = \sum_{k=0}^{K} R_t^k a^k + W_t e \, ; \quad \sum_{k=0}^{K} a^{k\prime} \geq \underline{a}$$

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- Say all agg. risk realized at date 0. For $t \geq 1$: same return R_t, only $a \equiv \sum a_k$ matters

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- Can use this to **test** for portfolio optimality and **solve** for optimal portfolios
- Equivalently: solve for "transfers" $t_0(a_{ss}, e_-, \epsilon)$ relative to baseline portfolios
  - Later, figure out what portfolios decentralize these transfers

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➡ Complete markets ~**undo the heterogeneous consumption effects of shocks**

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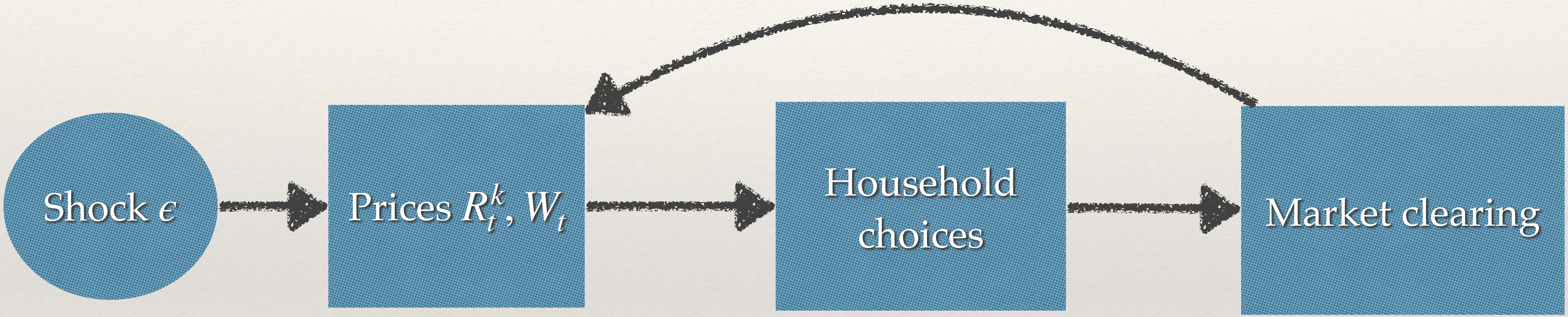

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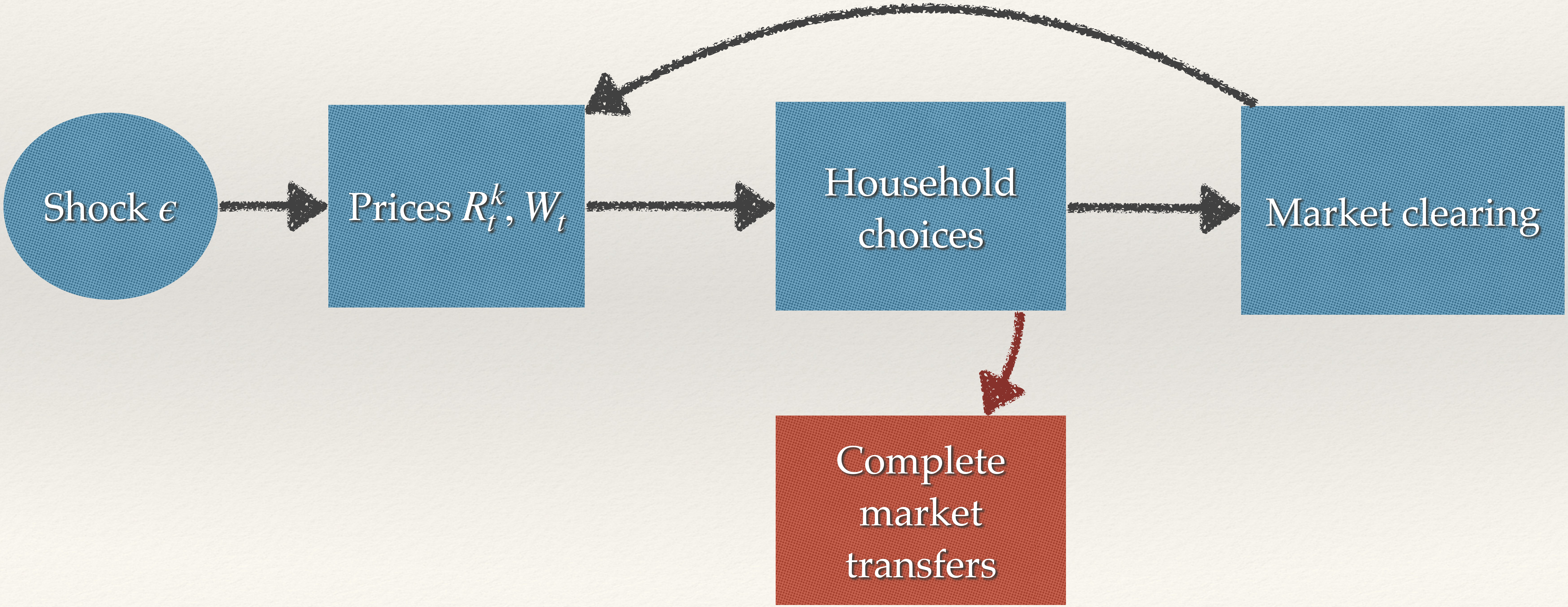

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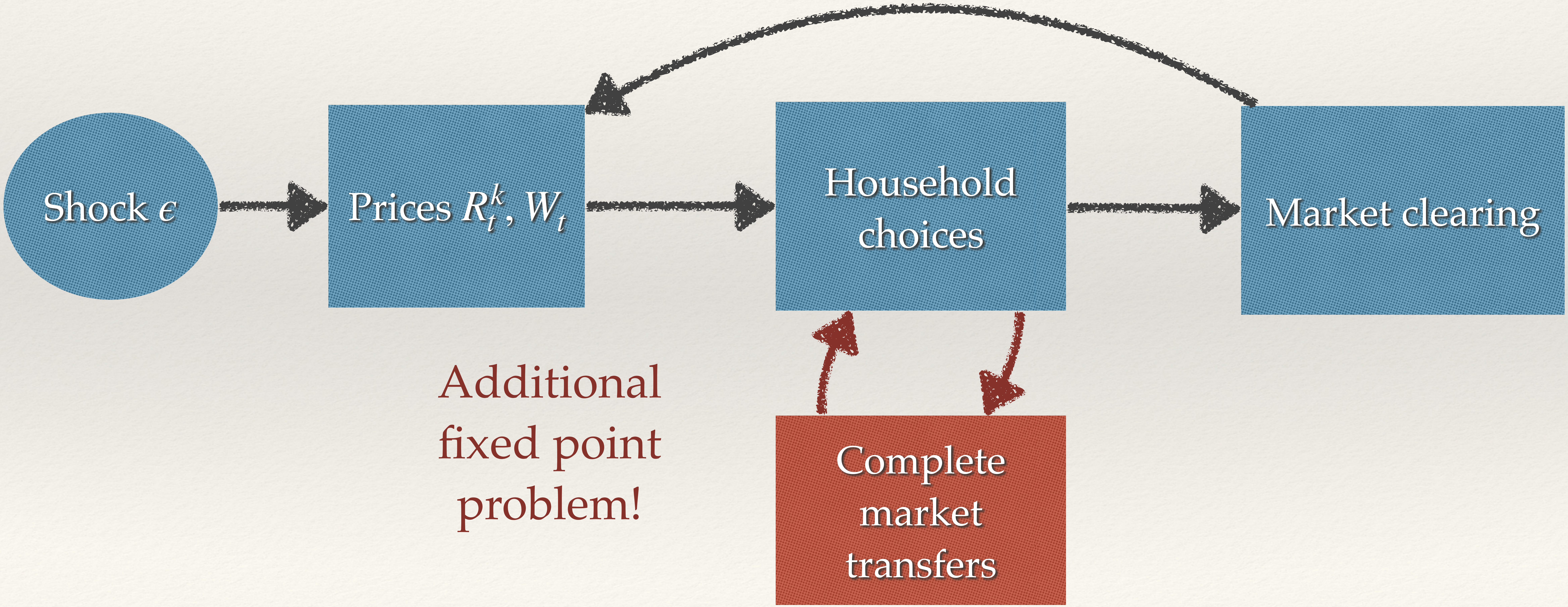

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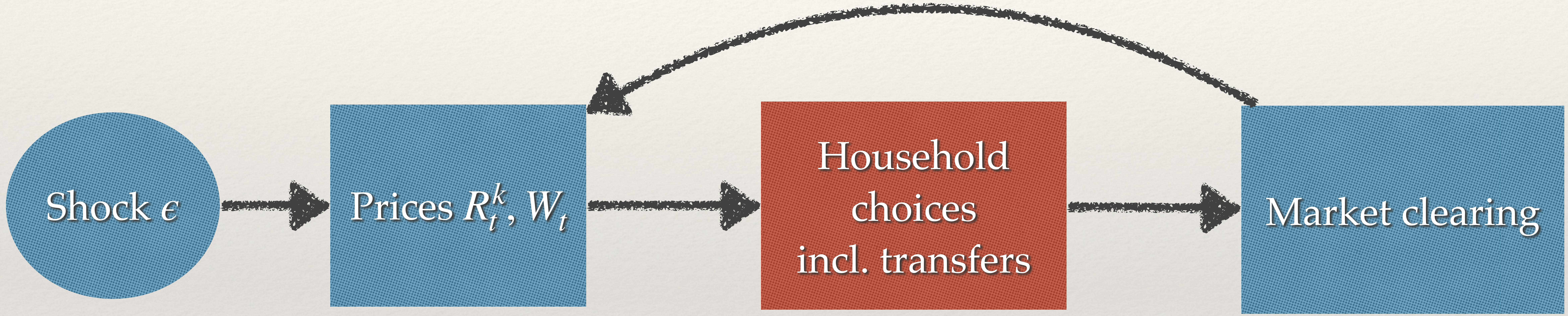

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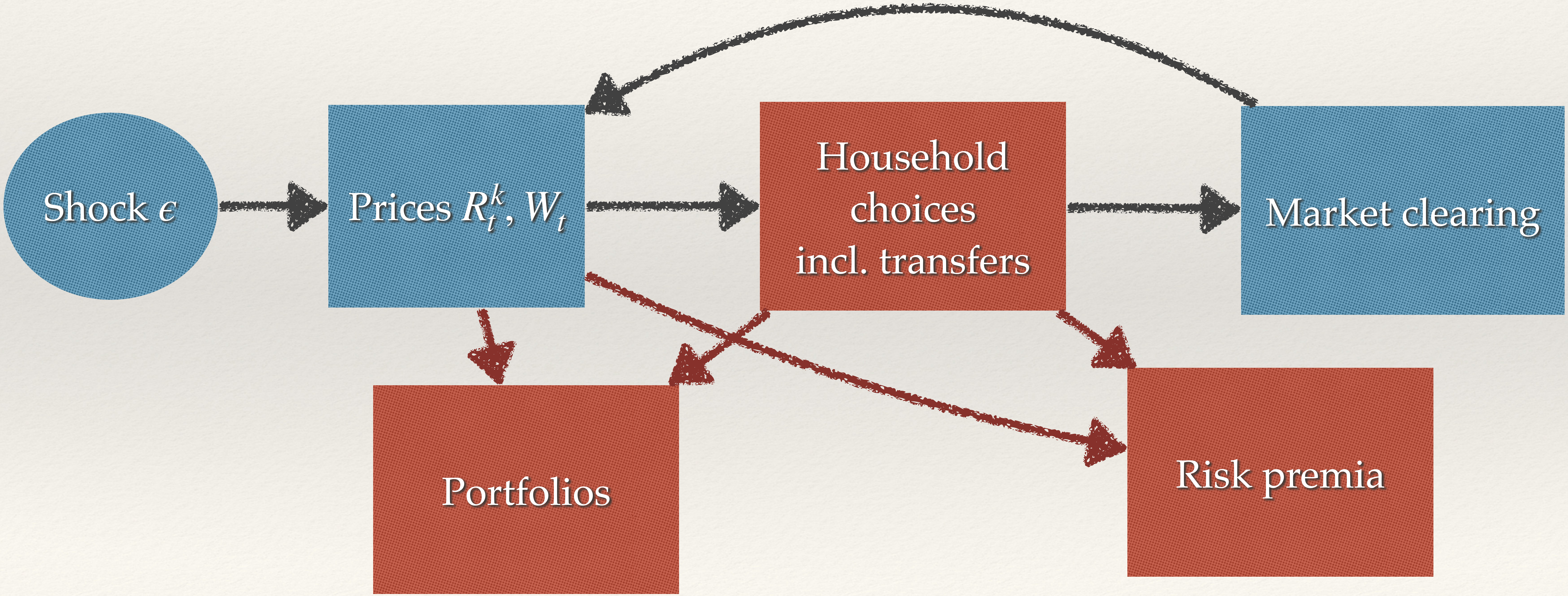

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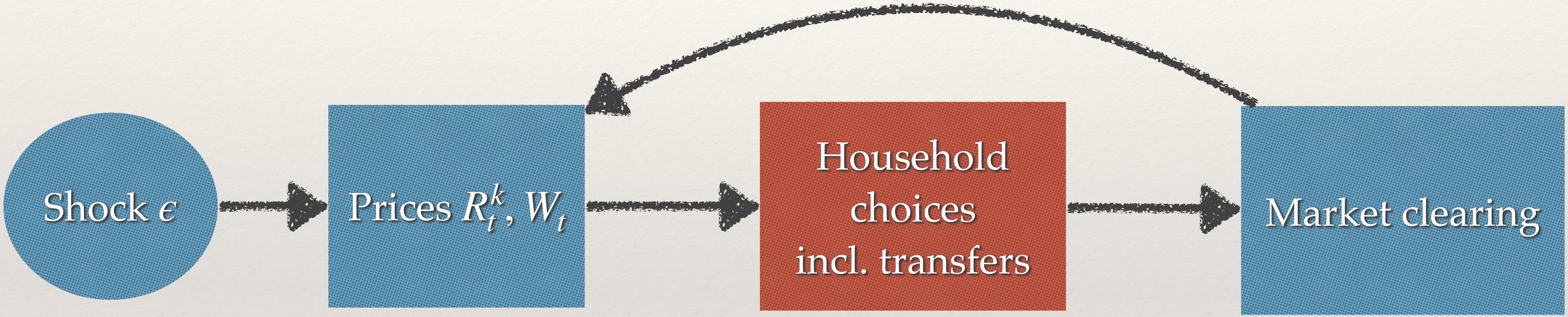

- Just "correct" the household Jacobian for effect of complete markets
- With this correction applied, solve equilibrium as usual in SSJ!
- This correction has simple form!

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- See notebook for additional details!

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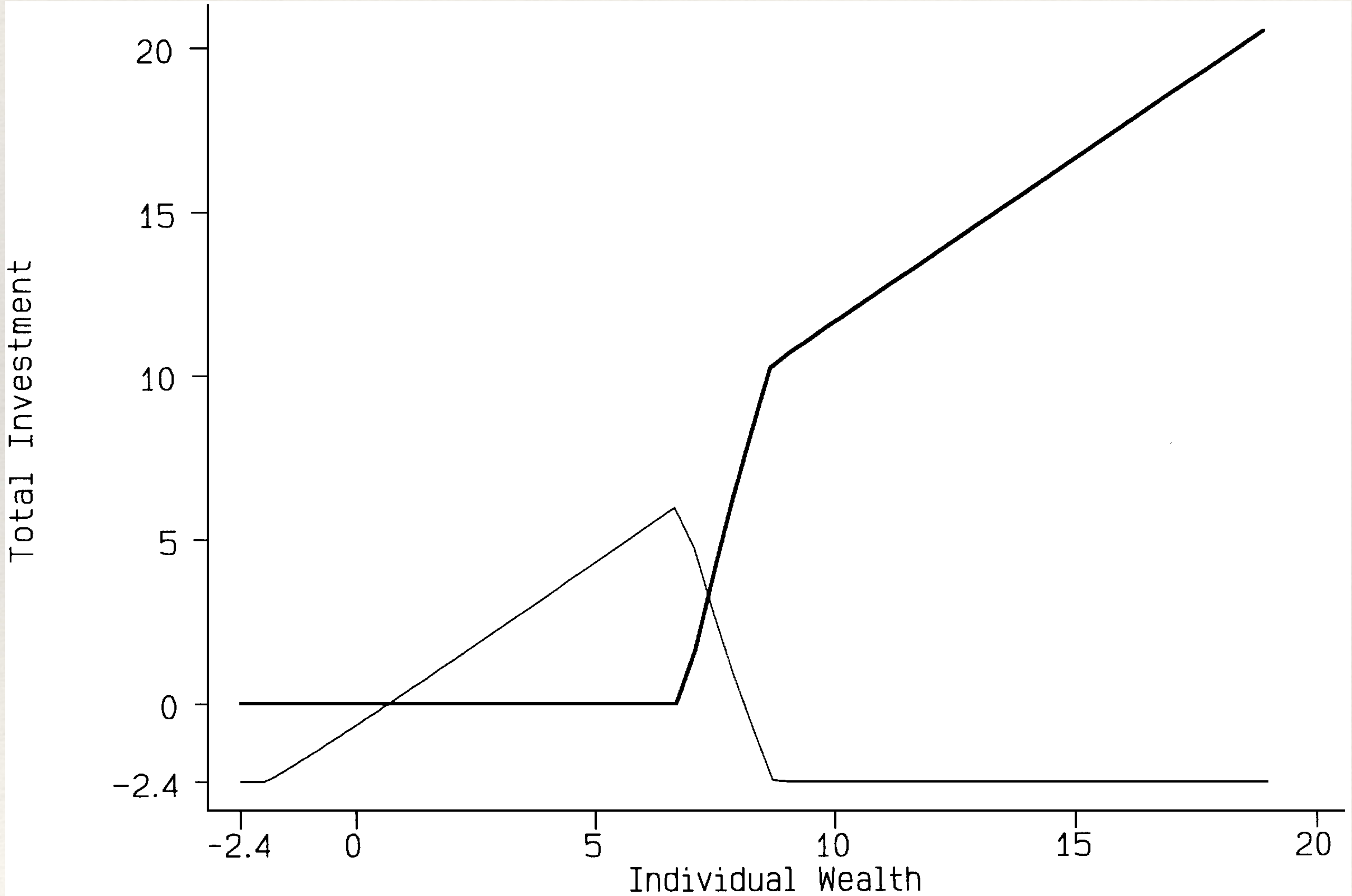

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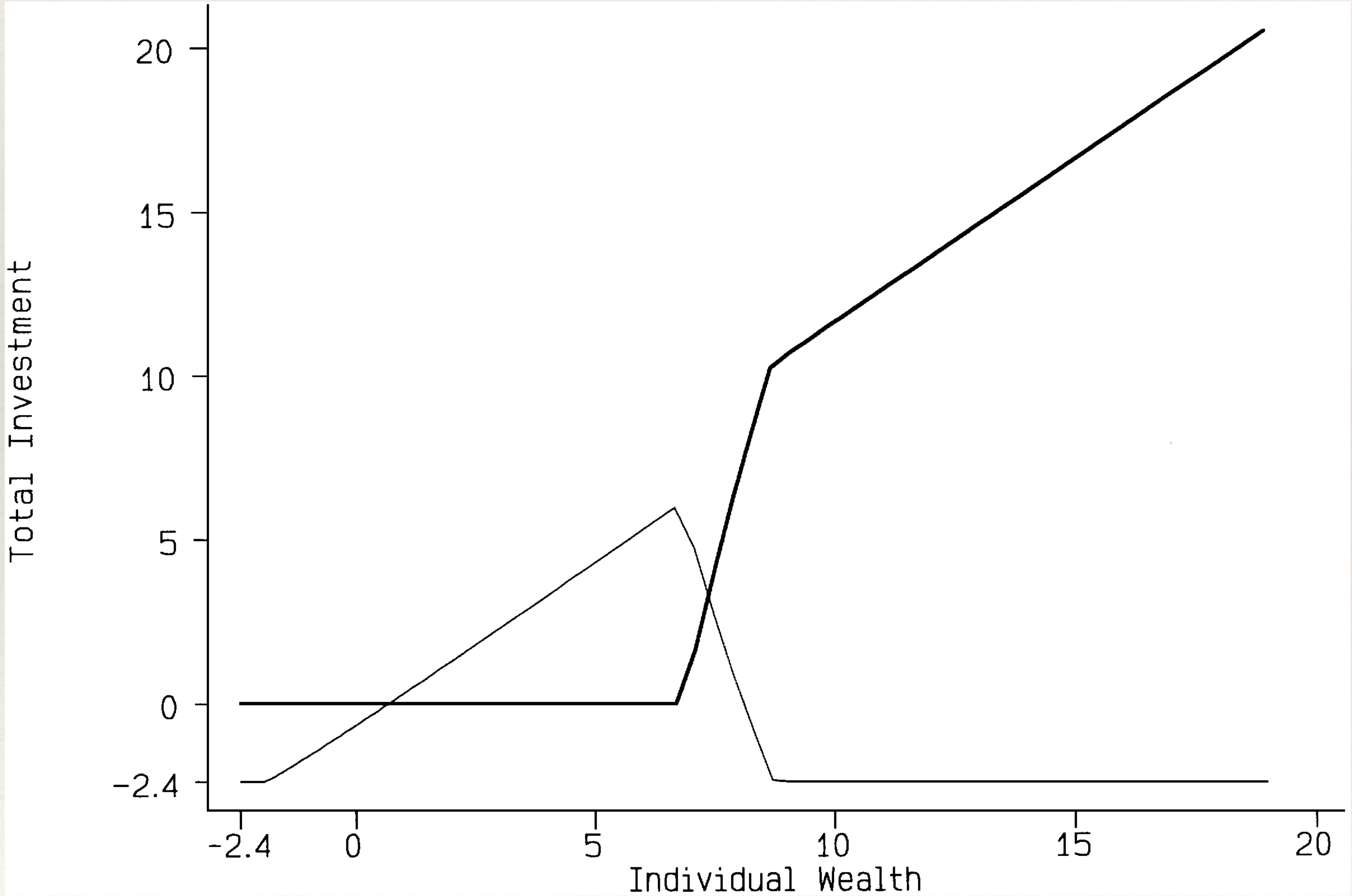

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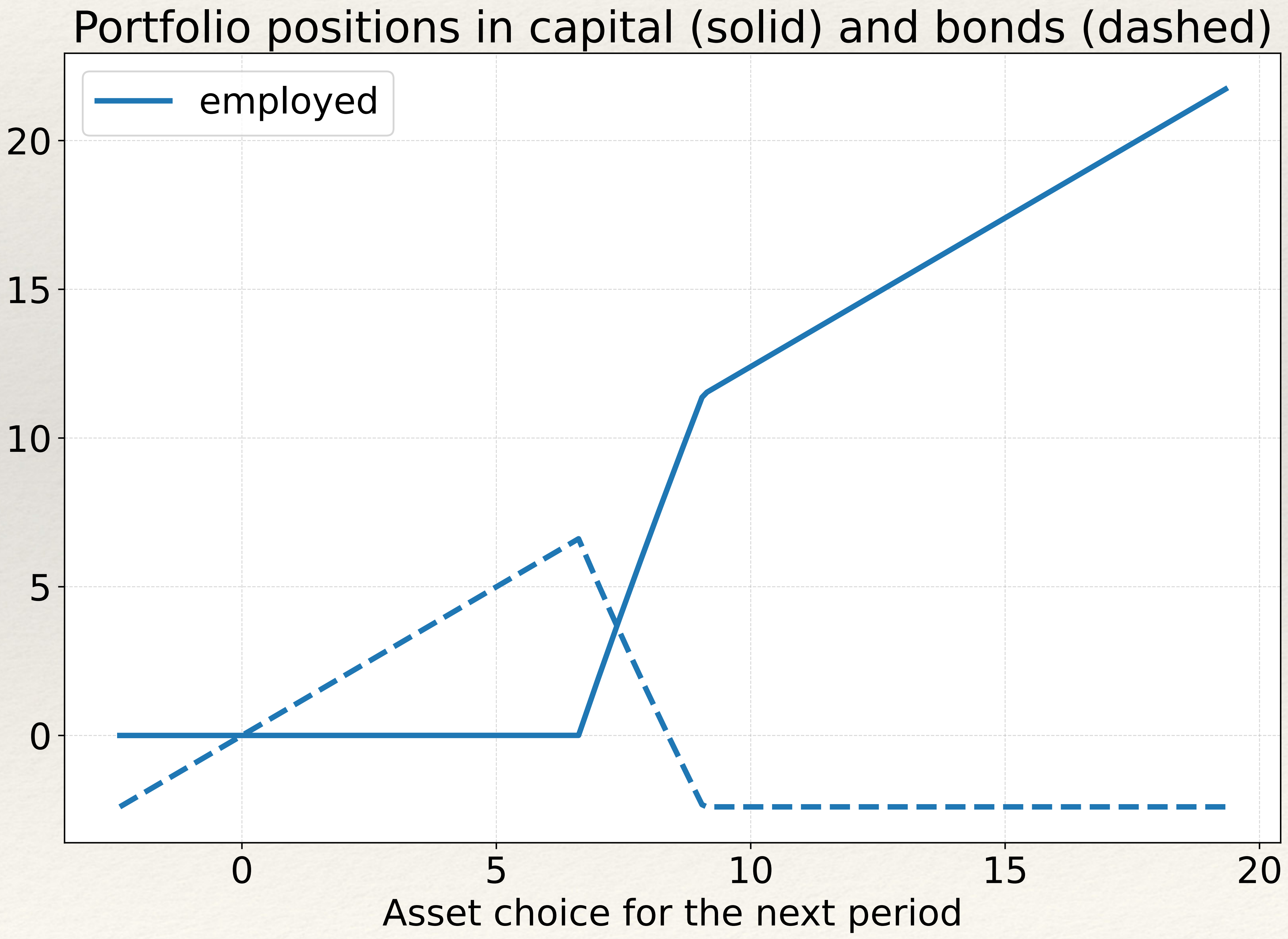

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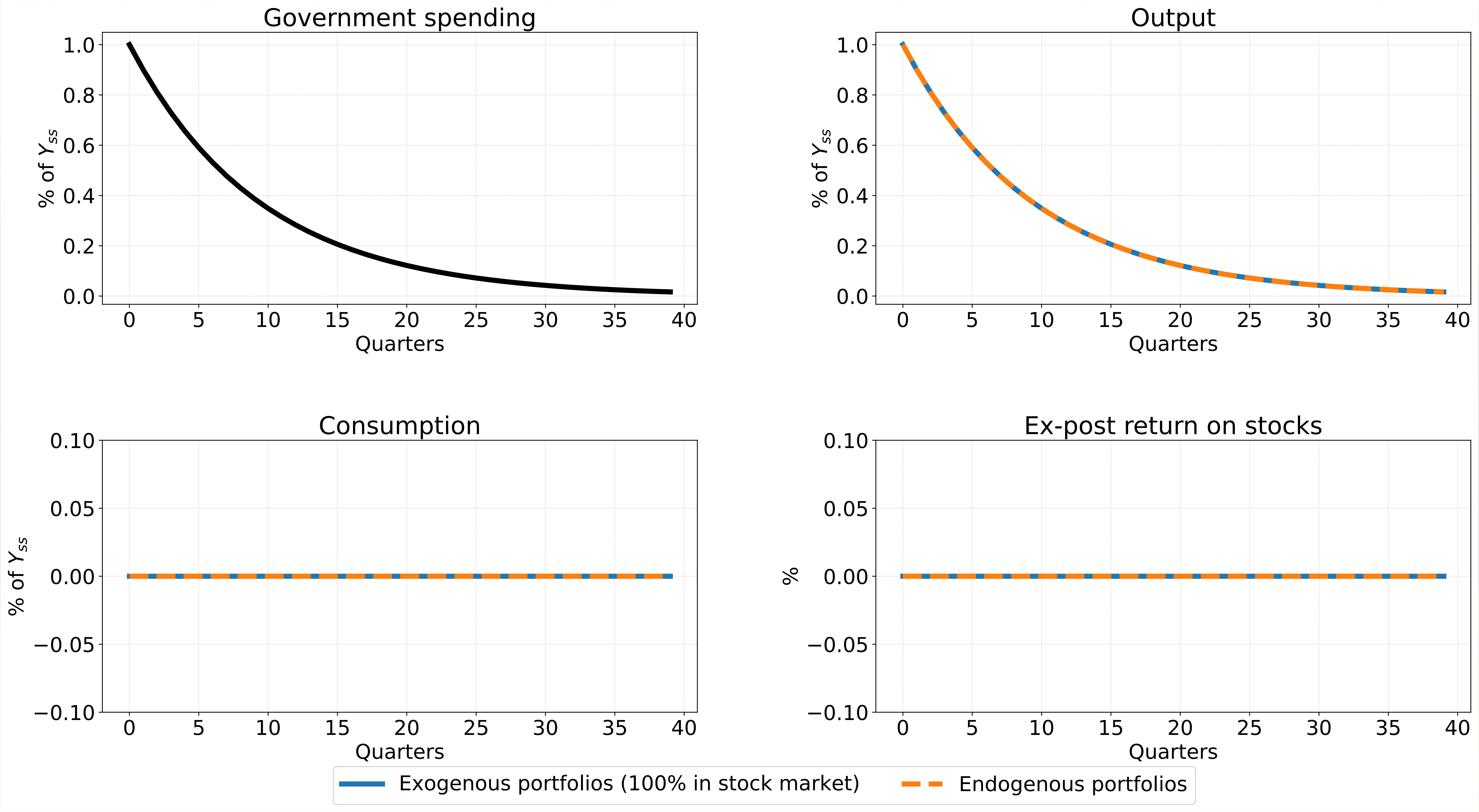
  - s stocks (price p_t, dividends d_t), b bonds, assume $\sigma = 1$
  - Rest of the model is as usual, calibrate so that $B_{ss} = 0$ (Werning case)

Example 1: balanced-budget $\{G_t\}$ shock



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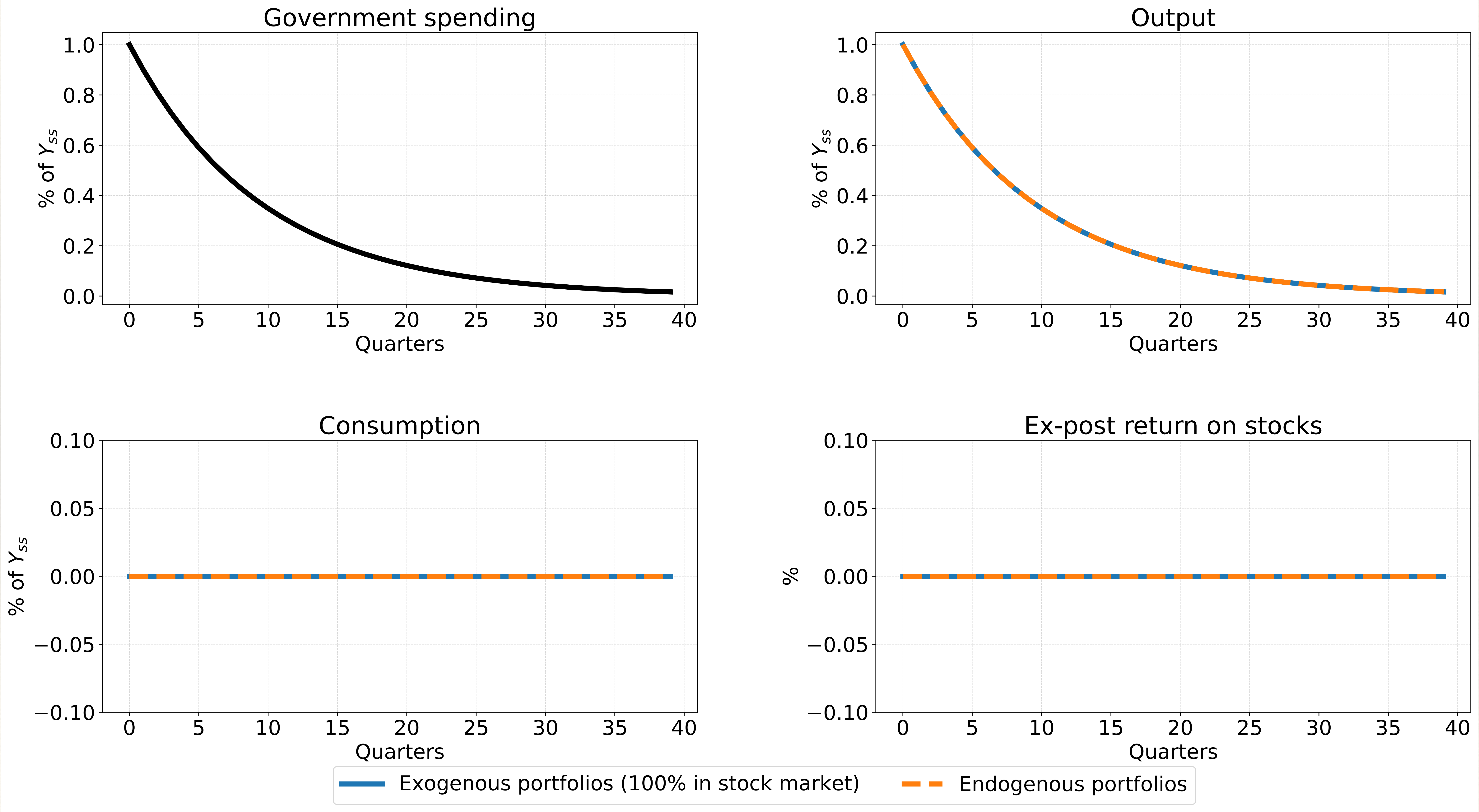


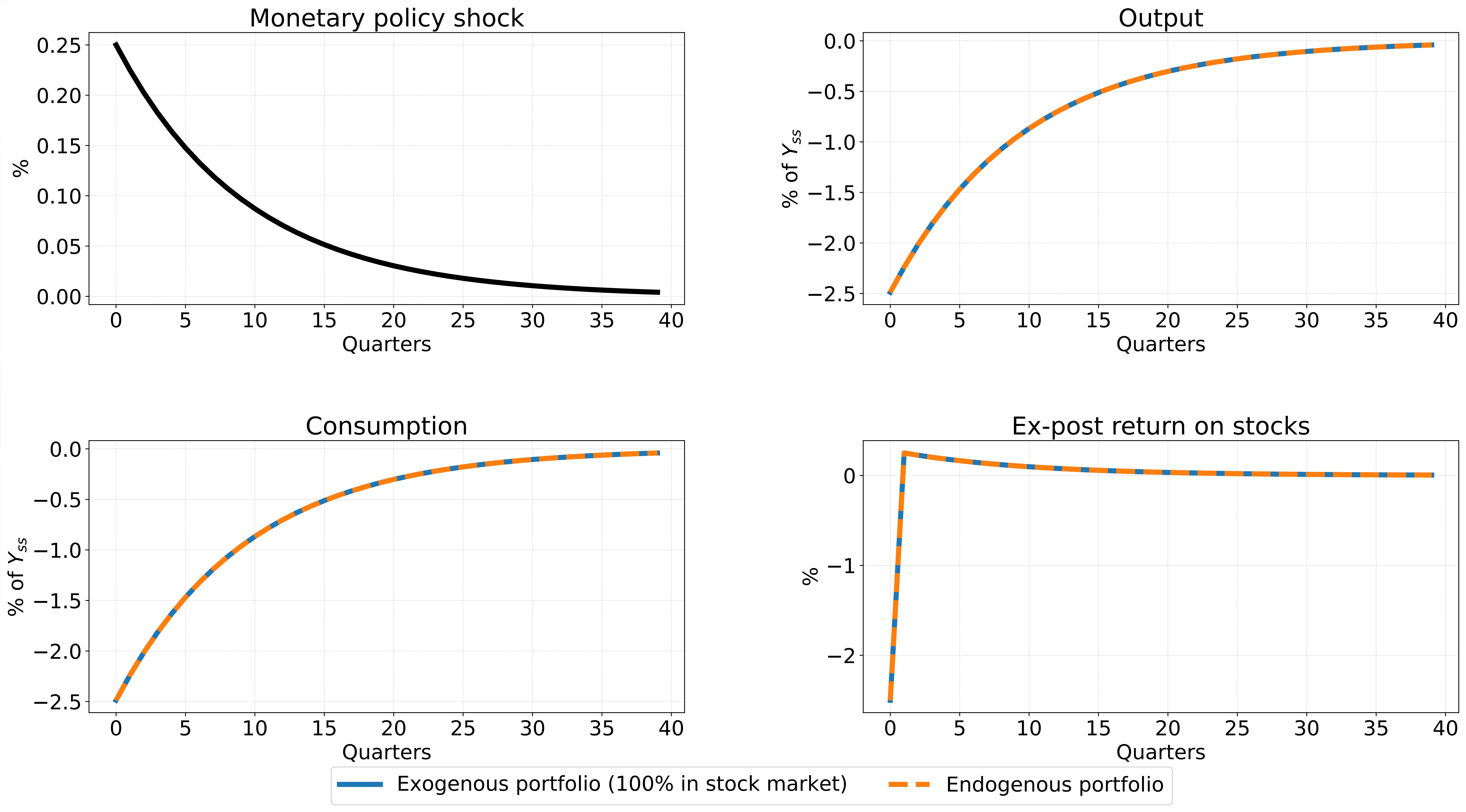
No effect from portfolio choice!

Why? Homogenous consumption effects at exogenous portfolios, $dc_{i0} = 0$

For endog. portfolios to matter, need unequal consumption effects of shocks

Also, here stock prices constant -> no difference between bonds and stocks

Example 2: monetary policy $\{r_t\}$ shock



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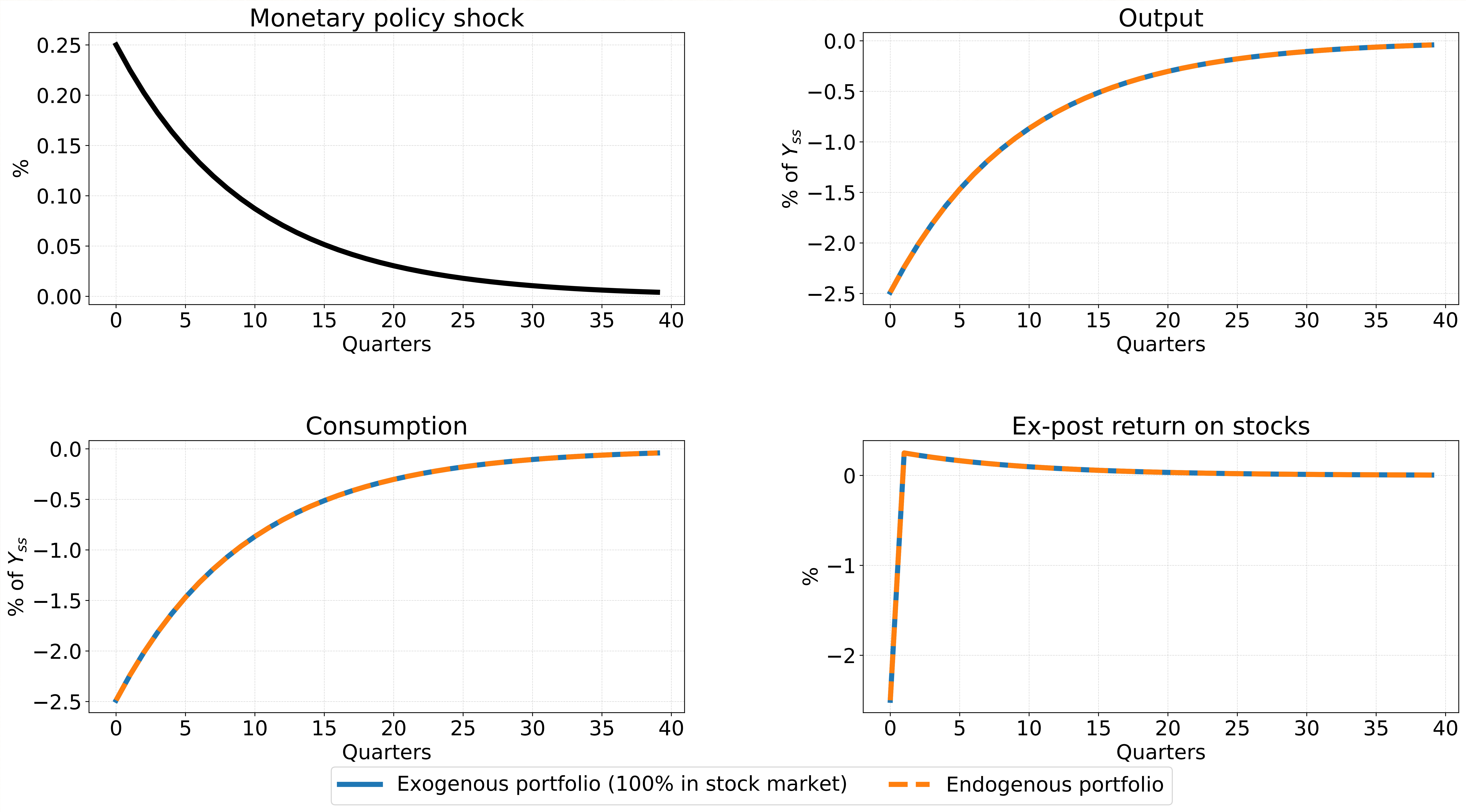
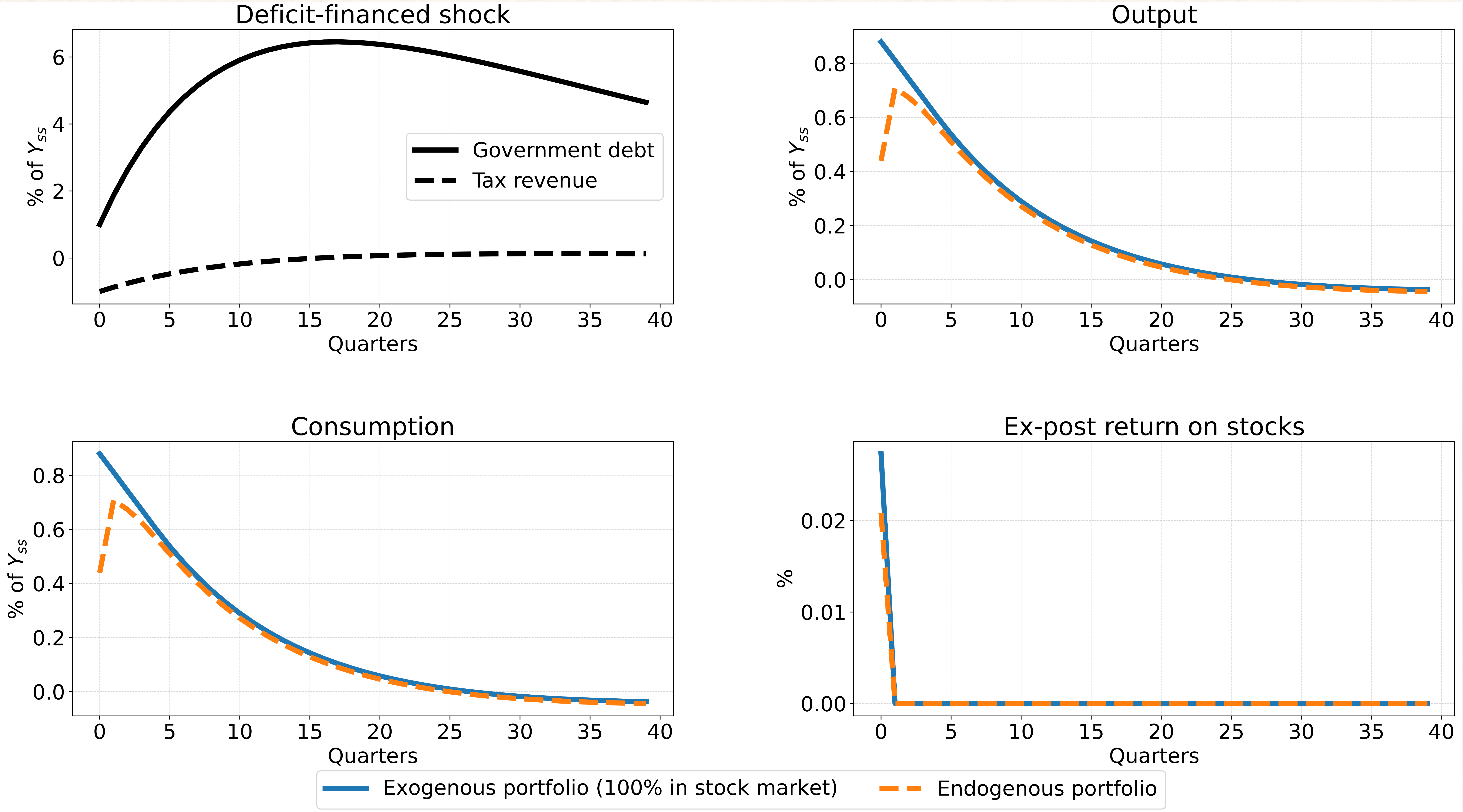


Still no effect from portfolio choice!

100% stock portfolios + log utility: homogeneous effects of monetary policy (Werning result!)

100% stocks are only optimal portfolios here

Example 3: deficit-financed transfer $\{B_t\}$ shock



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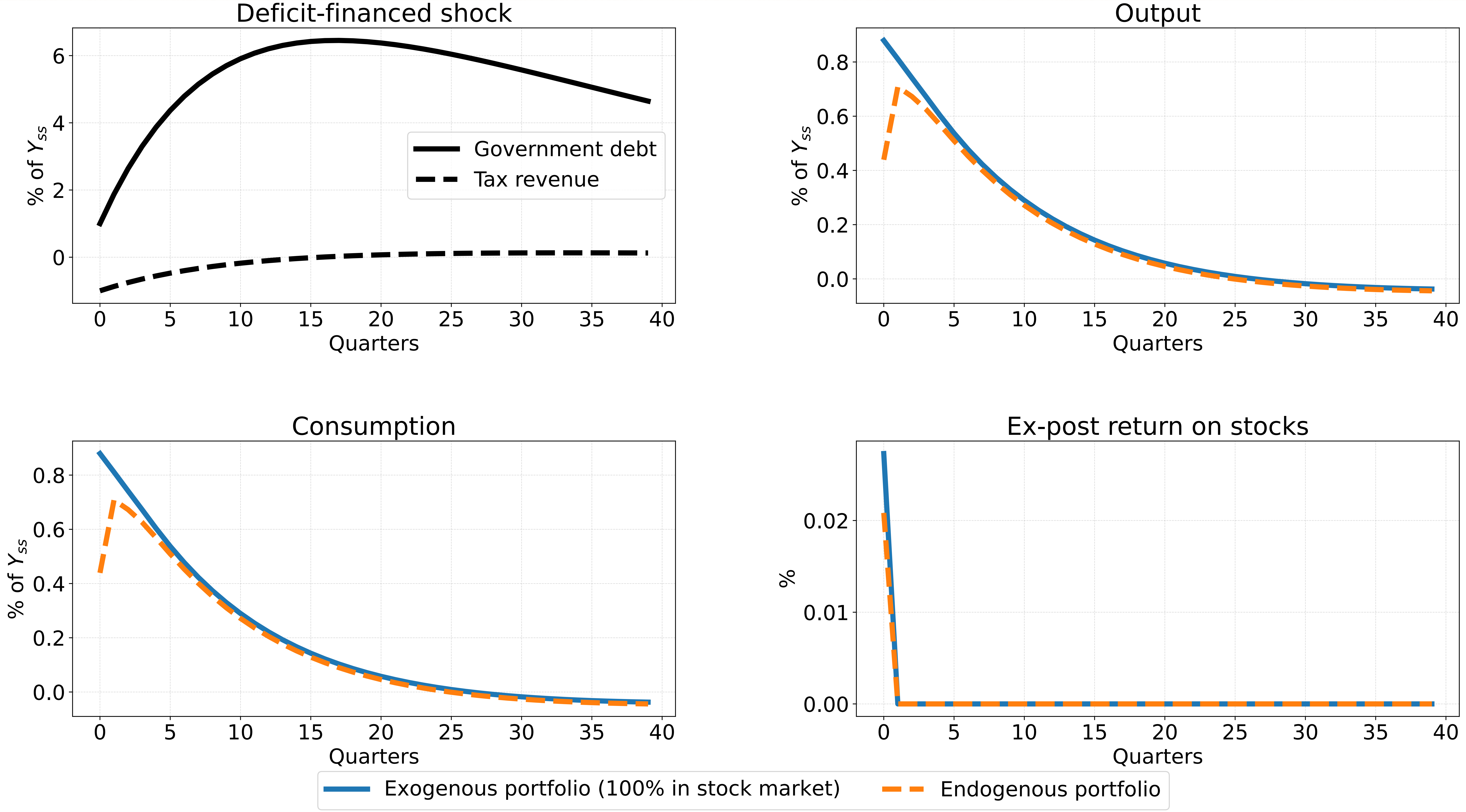


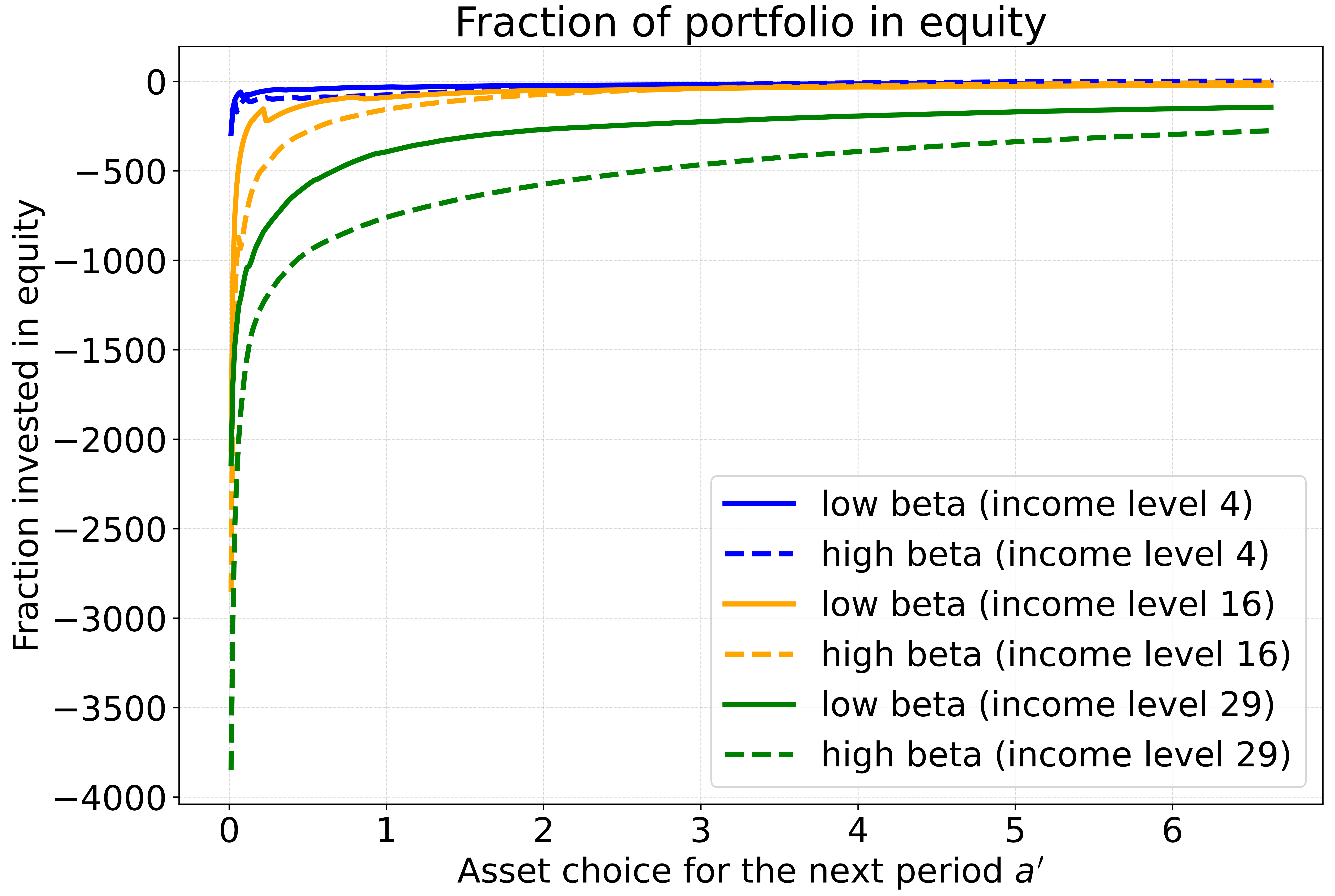
Large baseline output effects of deficit financed fiscal policy….

Largely undone by endogenous portfolios!

Poor disproportionately affected by shock, reduce stock exposure

But… how?

Under the hood: implausible portfolio shares!



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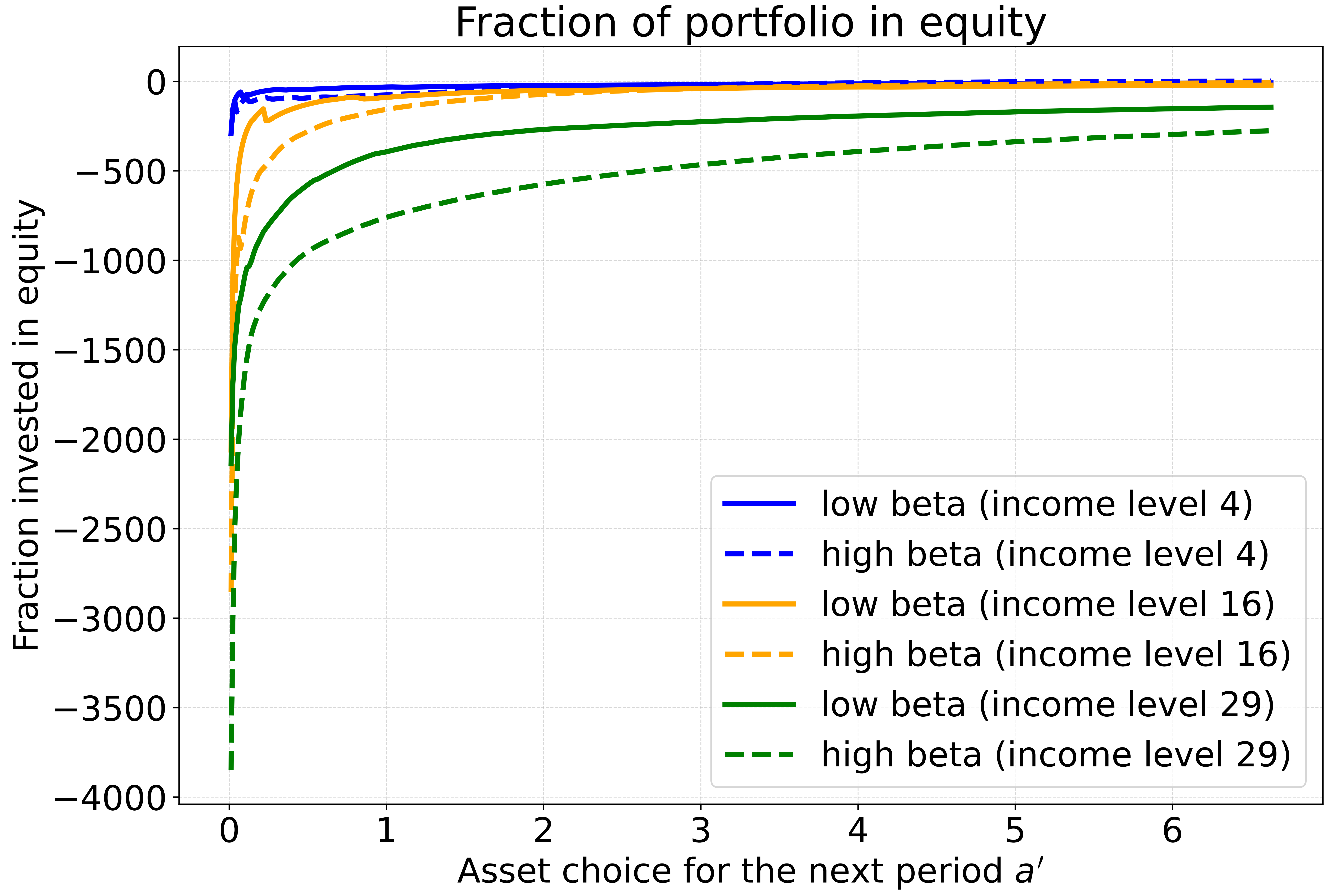
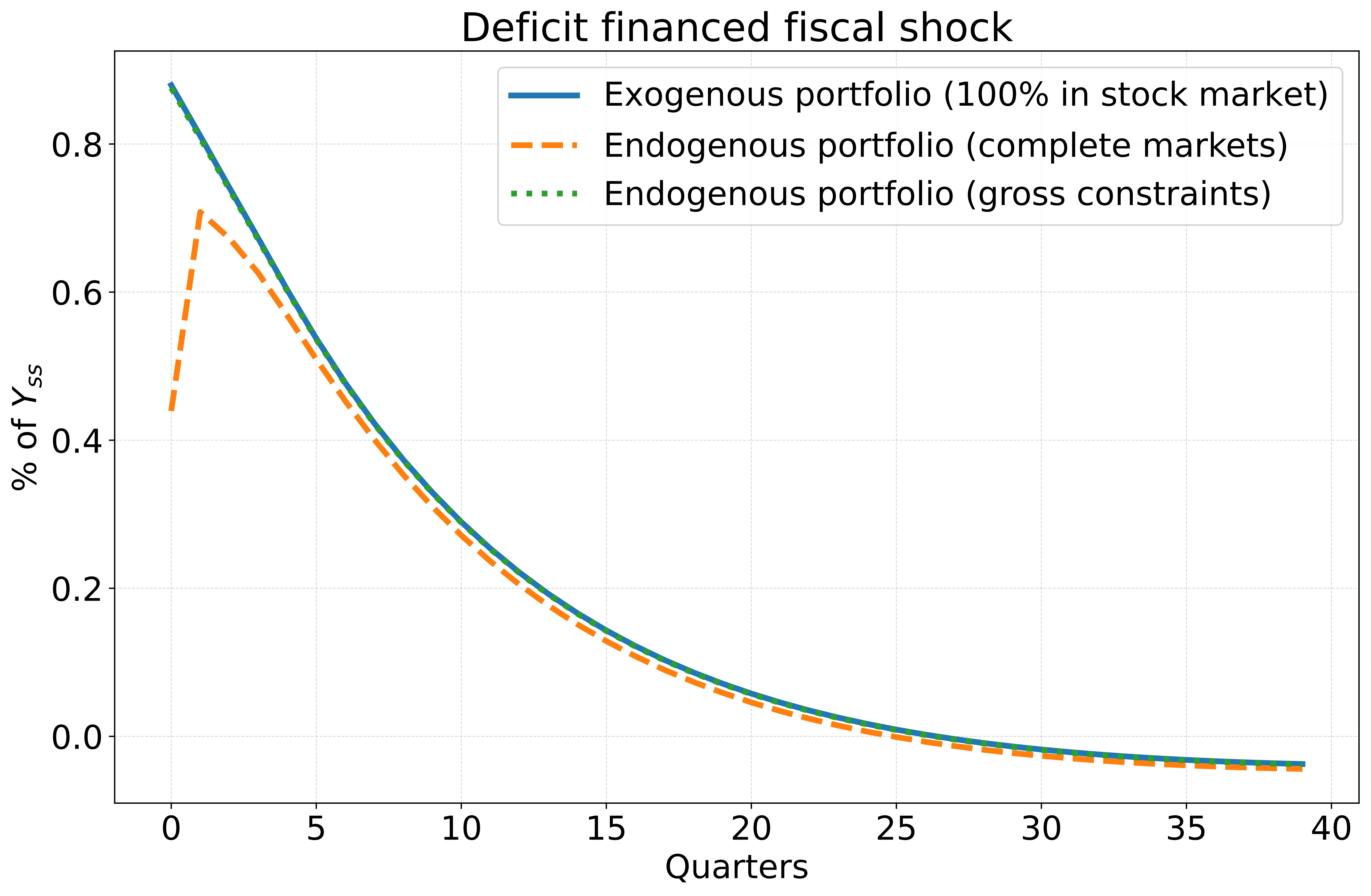


We didn't restrict gross positions in assets: borrowing constraint applied only to net position!

So "complete markets" transfers achieved with ultra-levered short-selling by the poor.

With portfolio constraints... (no short sales, 1.5x leverage limit)



Constrained endogenous portfolio now ~same as exogenous portfolio: no effect!

For endog. portfolios to matter, need high-MPC agents to be able to take large gross positions

More shocks than assets:
the incomplete markets case

Incomplete markets case: the projection principle

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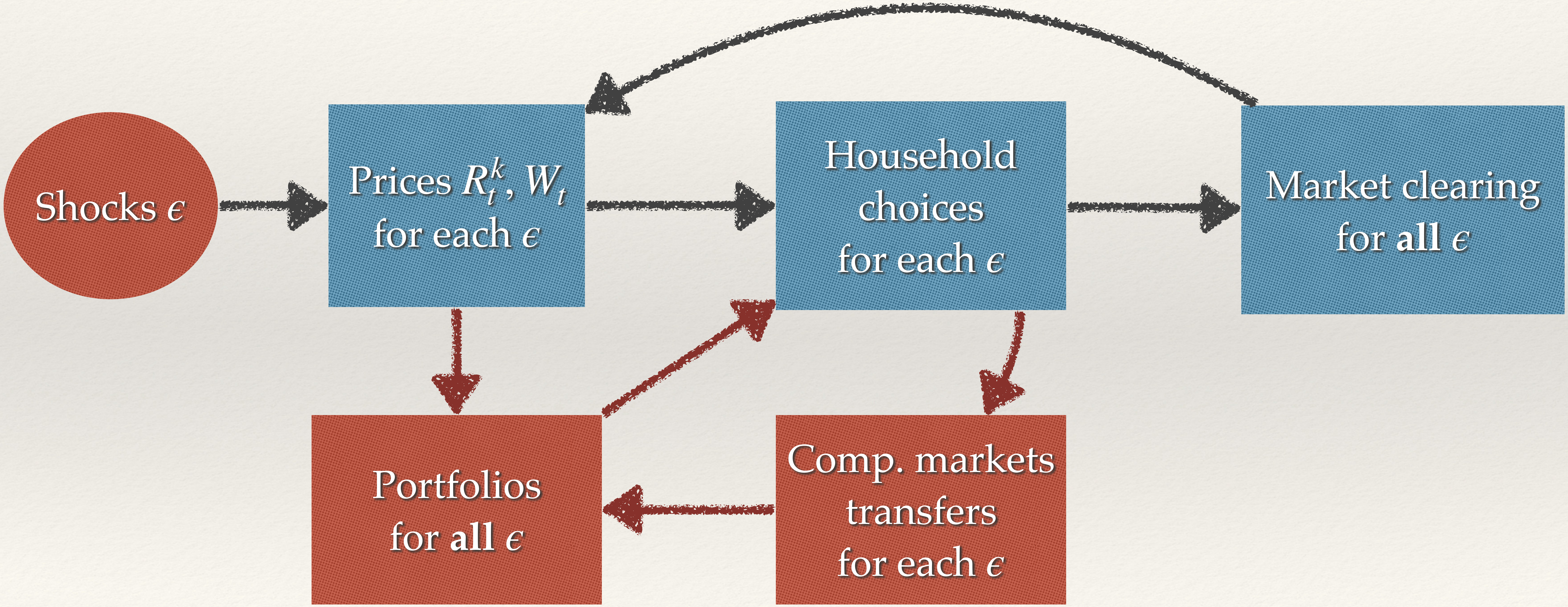
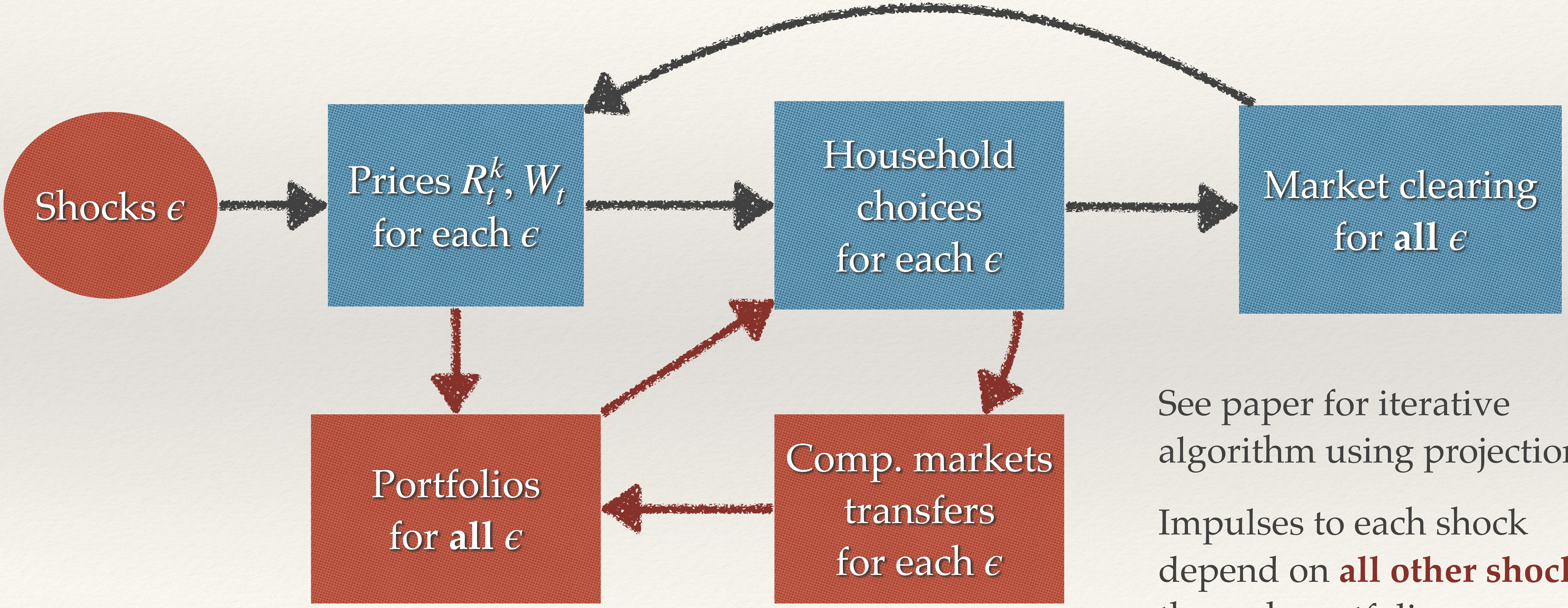
- Risk premia are the same as with complete-markets
- But: return matrix is endogenous, so nonlinear fixed point

Fixed point with incomplete markets



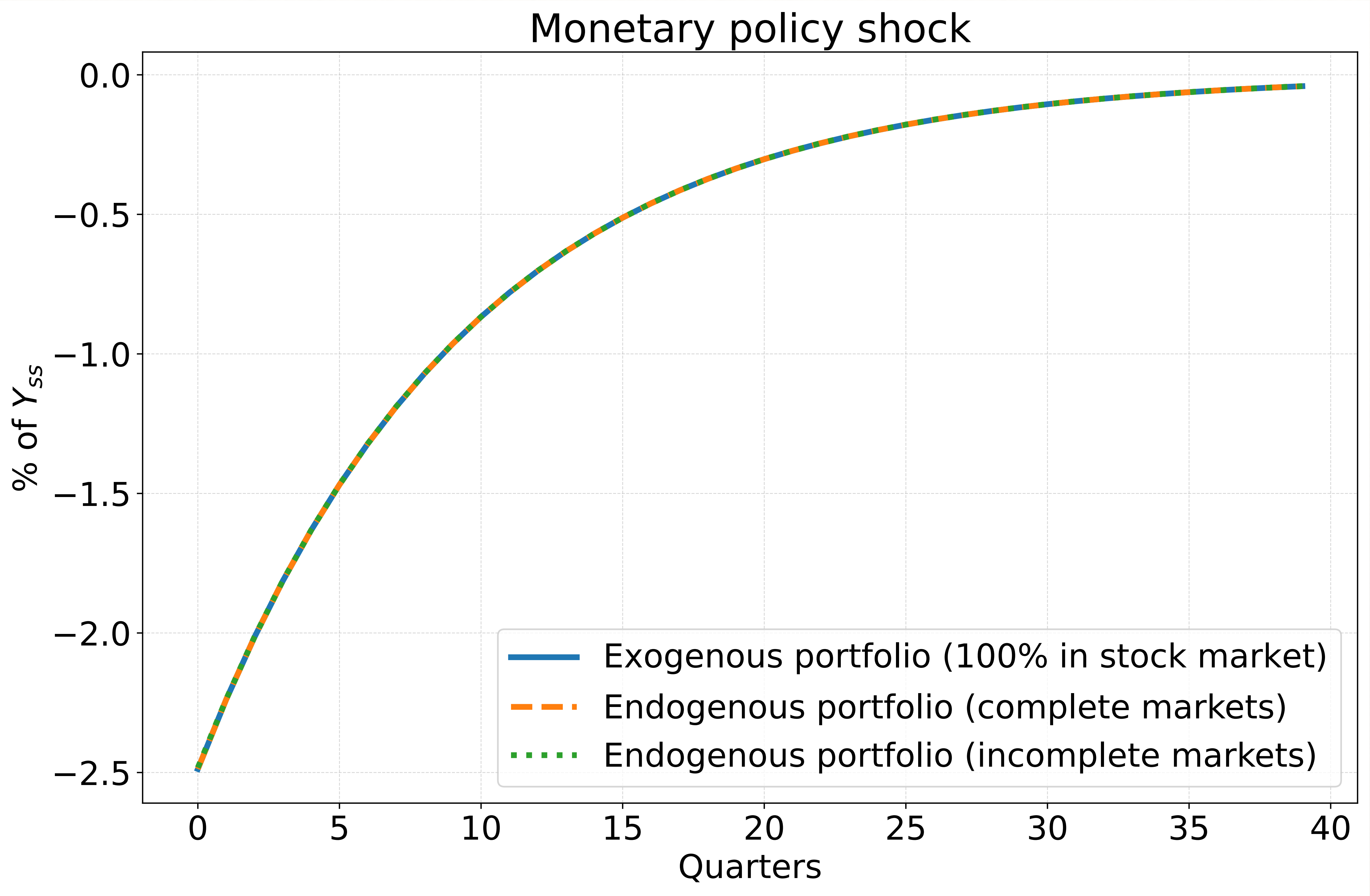
Fixed point with incomplete markets

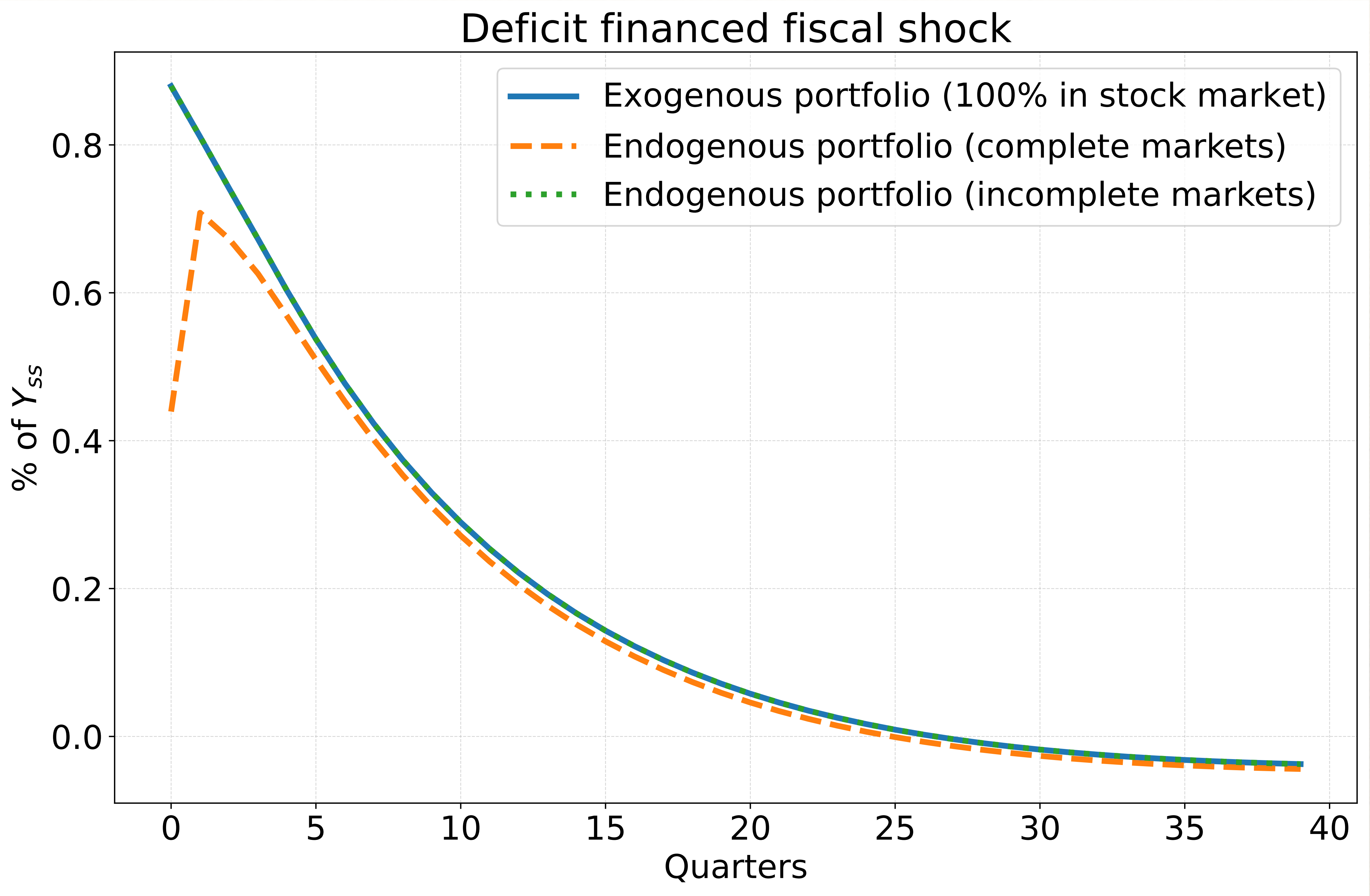


See paper for iterative algorithm using projection

Impulses to each shock depend on **all other shocks** through portfolios

Example 5: both monetary & deficit-financed fiscal shocks





With incomplete markets, the response to deficit-financed fiscal shock returns to ~ exogenous-portfolio case.

Why? Hedge better against shocks that drive asset prices more and are more volatile (here, m.p. shocks)

Conclusion

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- Simple modification of sequence-space Jacobian algorithm gives us:
  - impulses with endogenous portfolios and second-order risk premia
  - can add portfolio constraints, incomplete markets
  - new directions for asset pricing + heterogeneous agent macro
  - new directions for sequence-space macro
- When do endogenous portfolios matter in canonical HANK model?
  - When shocks have heterogeneous consumption effects
  - When high MPC agents are able to take highly leveraged positions