Intro to HANK models: Monetary Policy

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NBER Heterogeneous-Agent Macro Workshop, 2025

This lecture

This lecture

- So far: canonical HANK model, fiscal policy, Jacobians

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- **Next:** monetary policy! (in closed economy for now)
  1. Review of transmission in the RANK model
  2. Monetary policy in the canonical HANK model
  3. Direct and indirect effects of monetary policy

Monetary policy in the RANK model

Representative agent NK model

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- Separable preferences, sticky prices or wages, linearized + perfect foresight:
  - $c_t = c_{t+1} - \frac{1}{\sigma}\left(i_t - \pi_{t+1}\right)$
  - $\pi_t = \kappa c_t + \beta \pi_{t+1}$
  - $i_t = \pi_{t+1} + \epsilon_t$

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Can also use more standard Taylor rule:

$$i_t = \phi \pi_t + \epsilon_t$$

Monetary transmission in RANK

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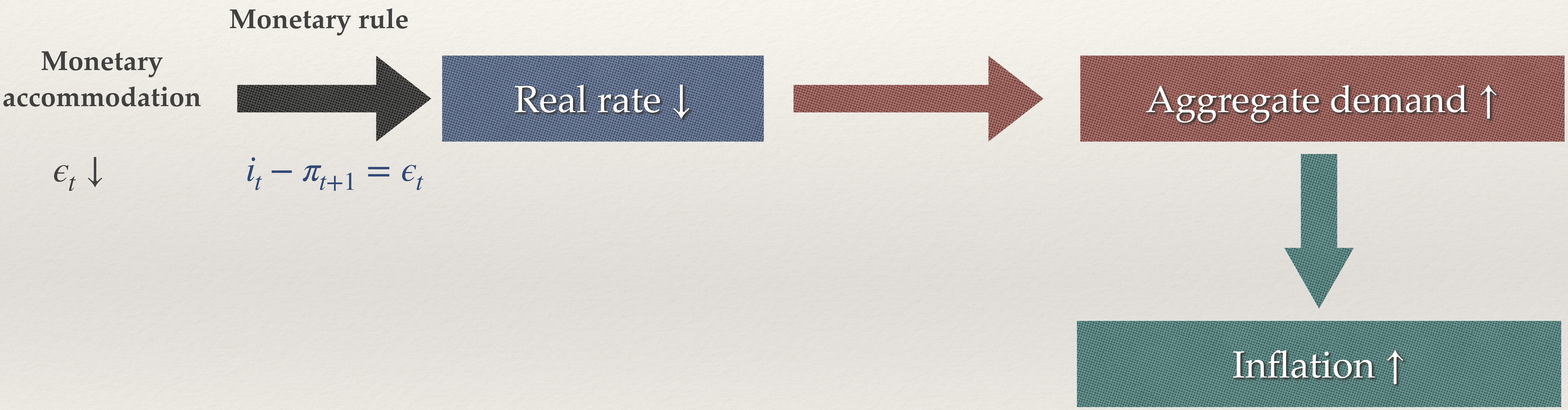

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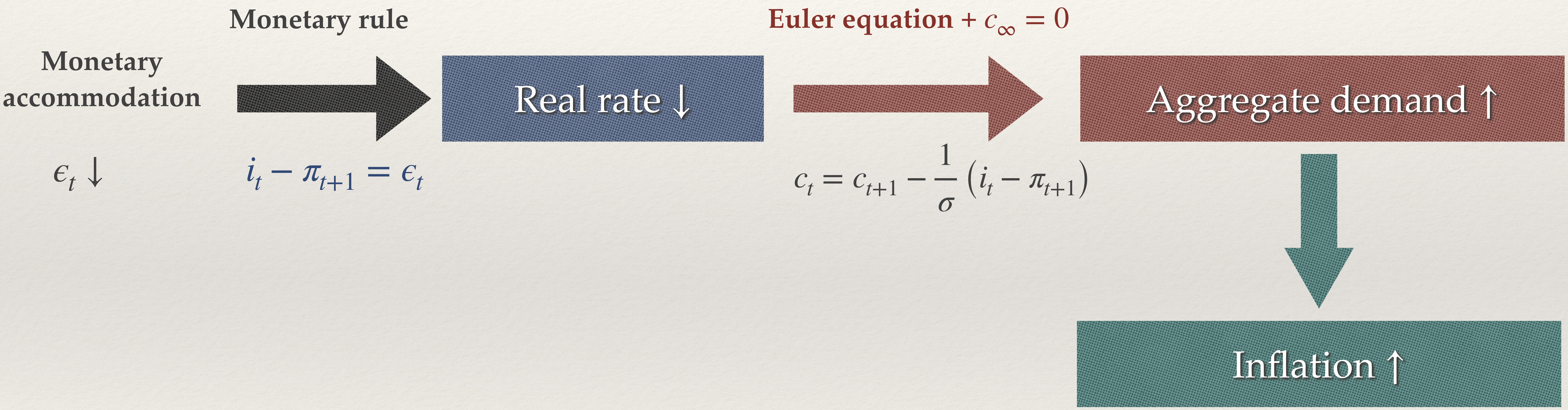

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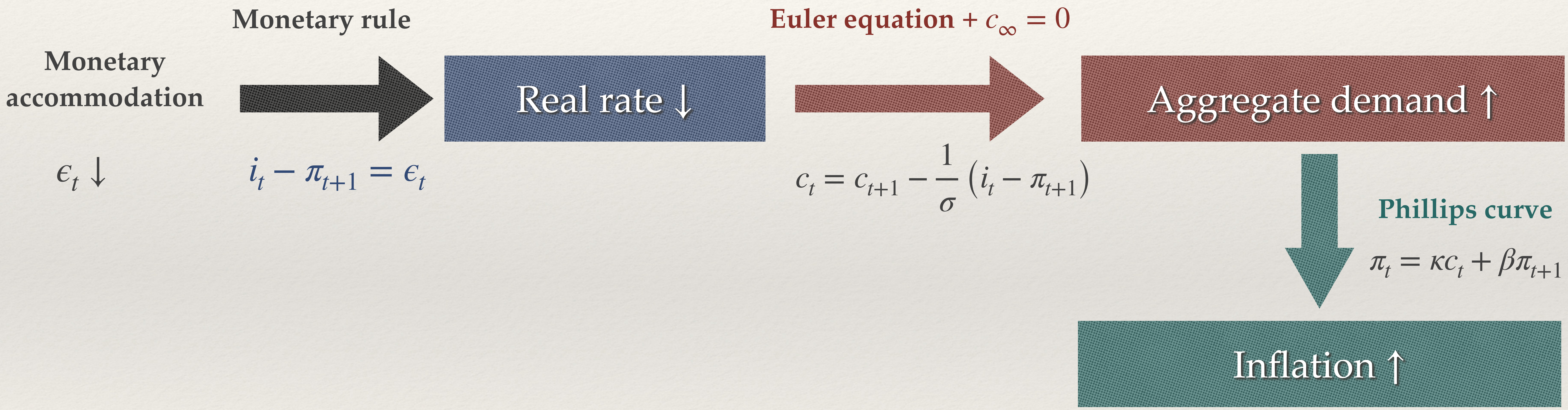

Monetary transmission in RANK

Monetary accommodation

$\epsilon_t \downarrow$

Monetary rule

$i_t - \pi_{t+1} = \epsilon_t$

Real rate $\downarrow$

Euler equation + $c_\infty = 0$

$$c_t = c_{t+1} - \frac{1}{\sigma}\left(i_t - \pi_{t+1}\right)$$

Aggregate demand $\uparrow$

Phillips curve

$$\pi_t = \kappa c_t + \beta \pi_{t+1}$$

Inflation $\uparrow$

Feedback with standard Taylor rule

$i_t = \phi \pi_t + \epsilon_t$

Monetary transmission in RANK

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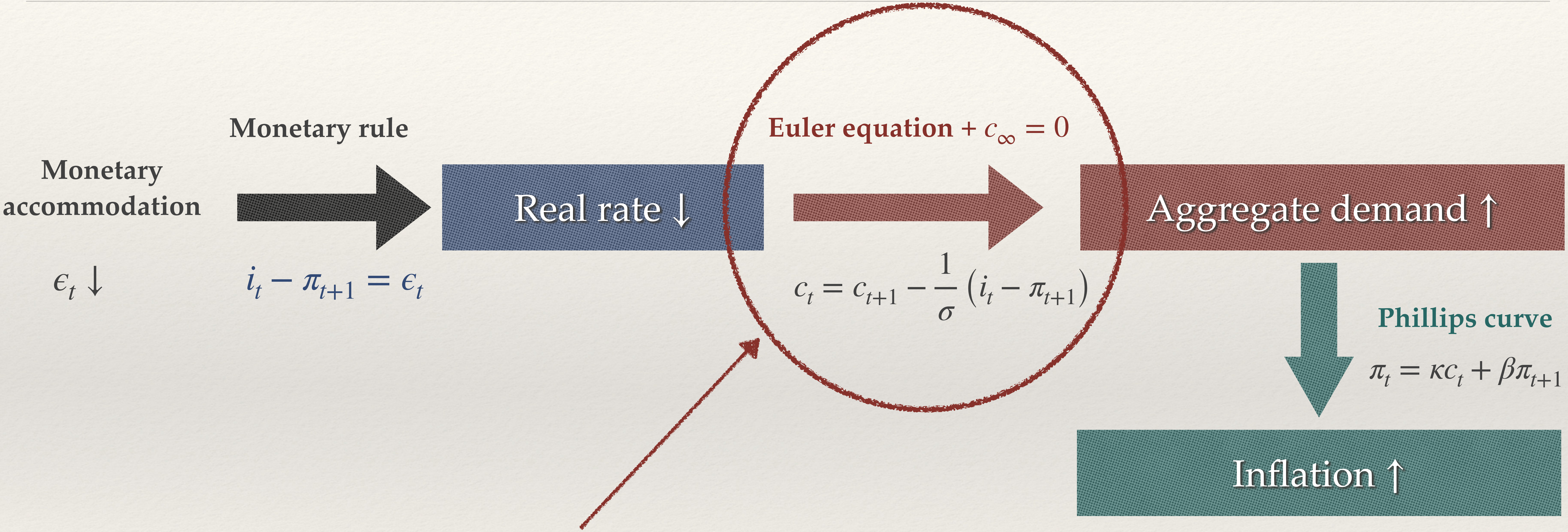

HANK revisits this link! Two key questions:

(1) Is transmission through Euler equation alone plausible?

(2) Is aggregate effect plausible?

Forward guidance puzzle in RANK

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- Solving RANK with this policy rule:

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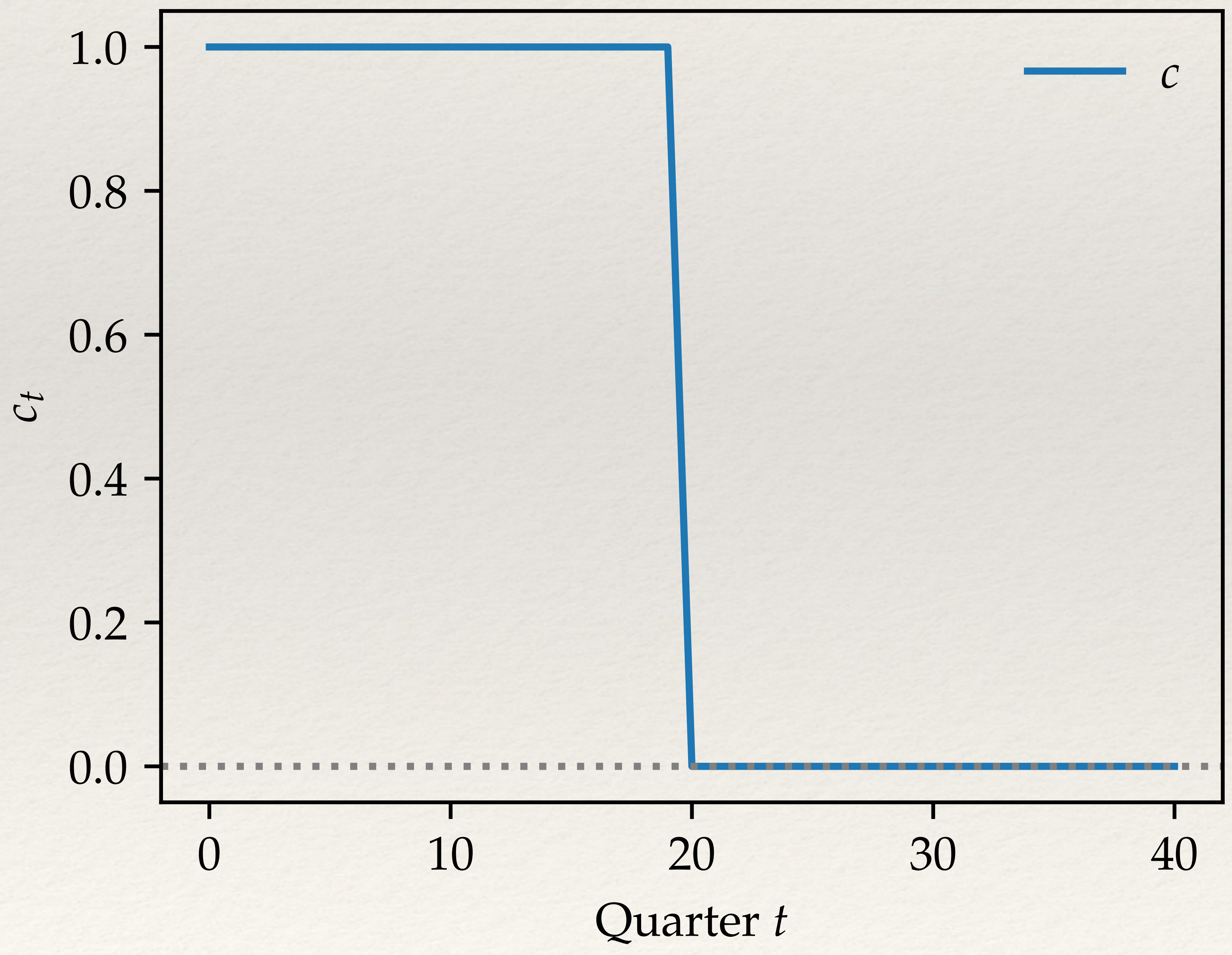

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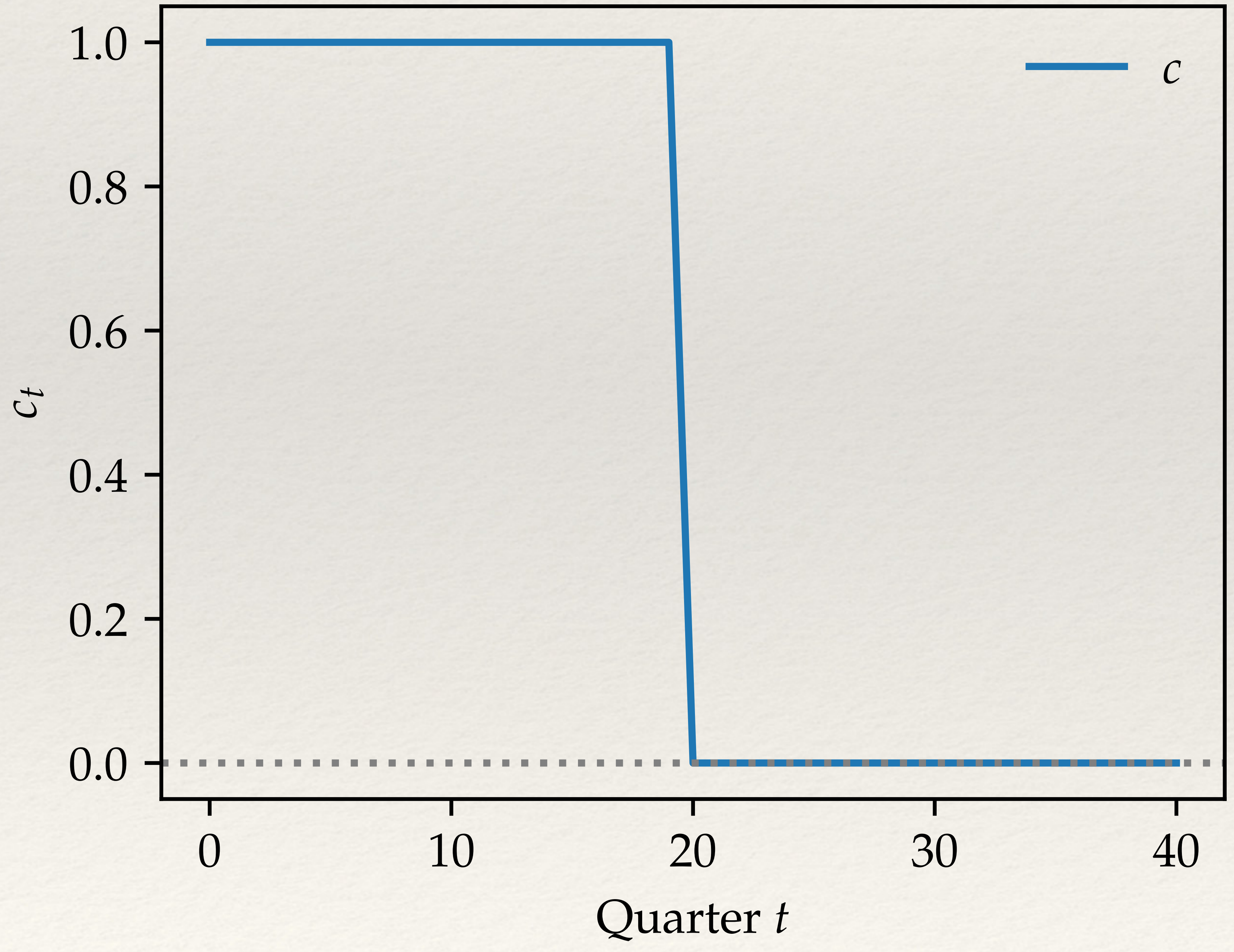

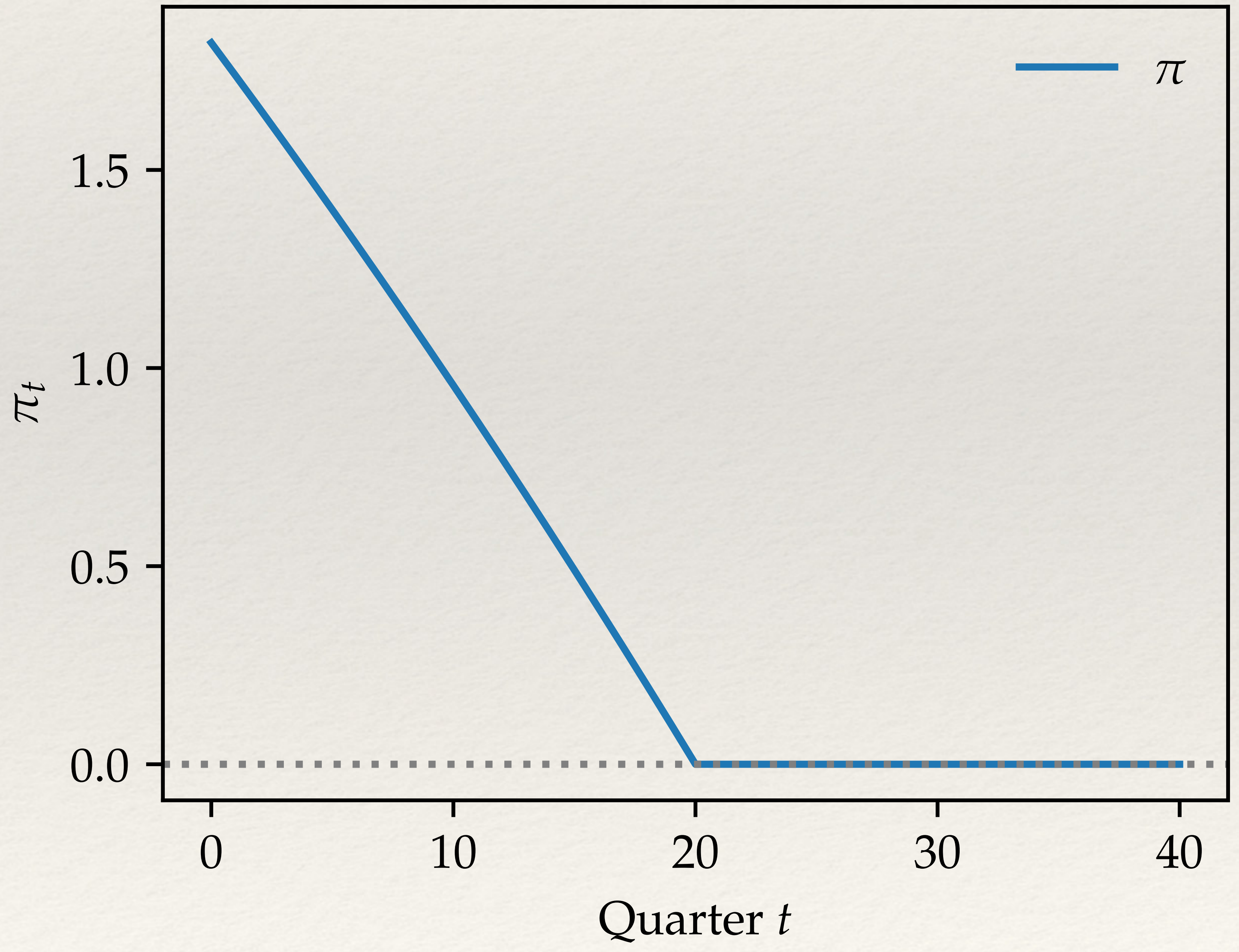

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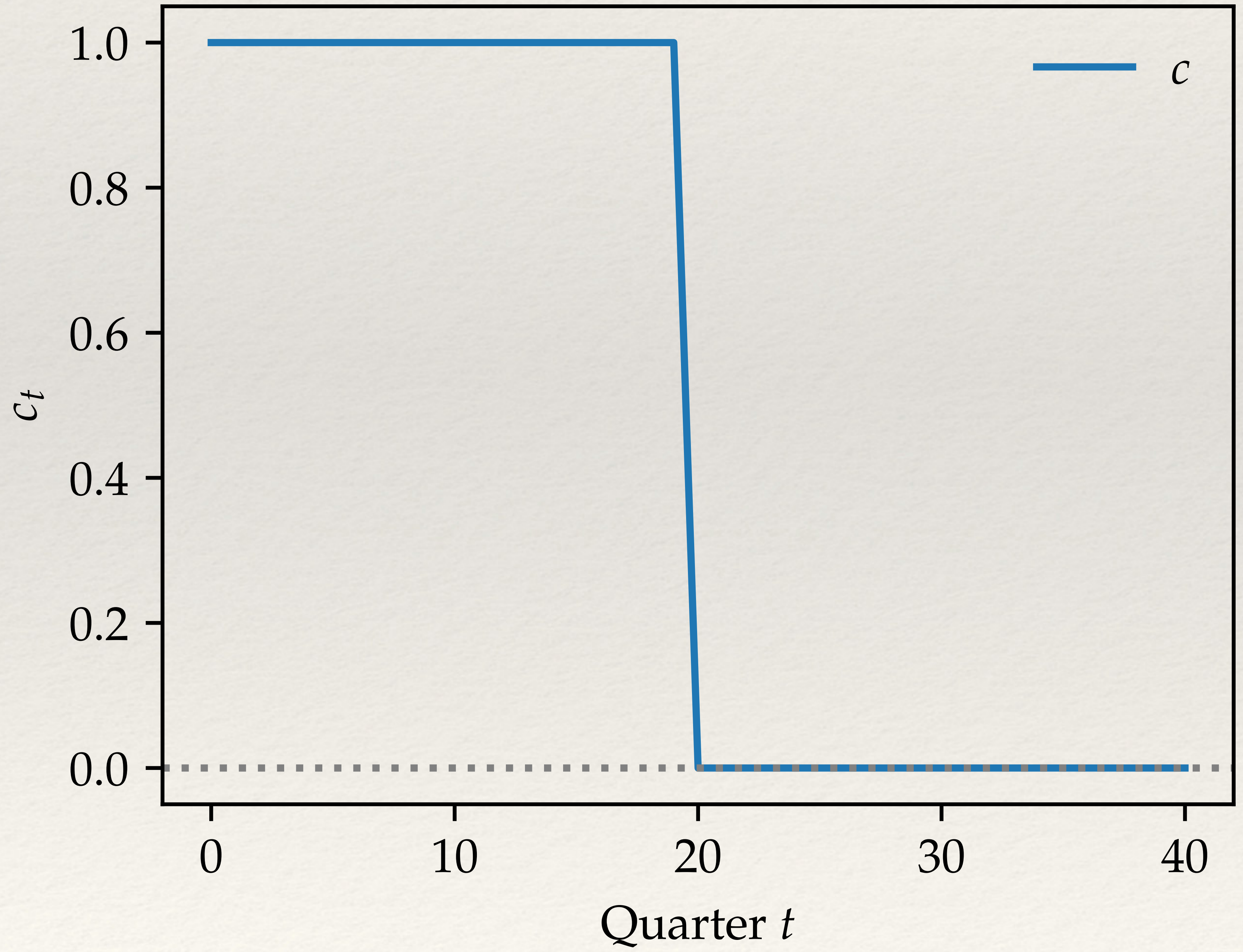

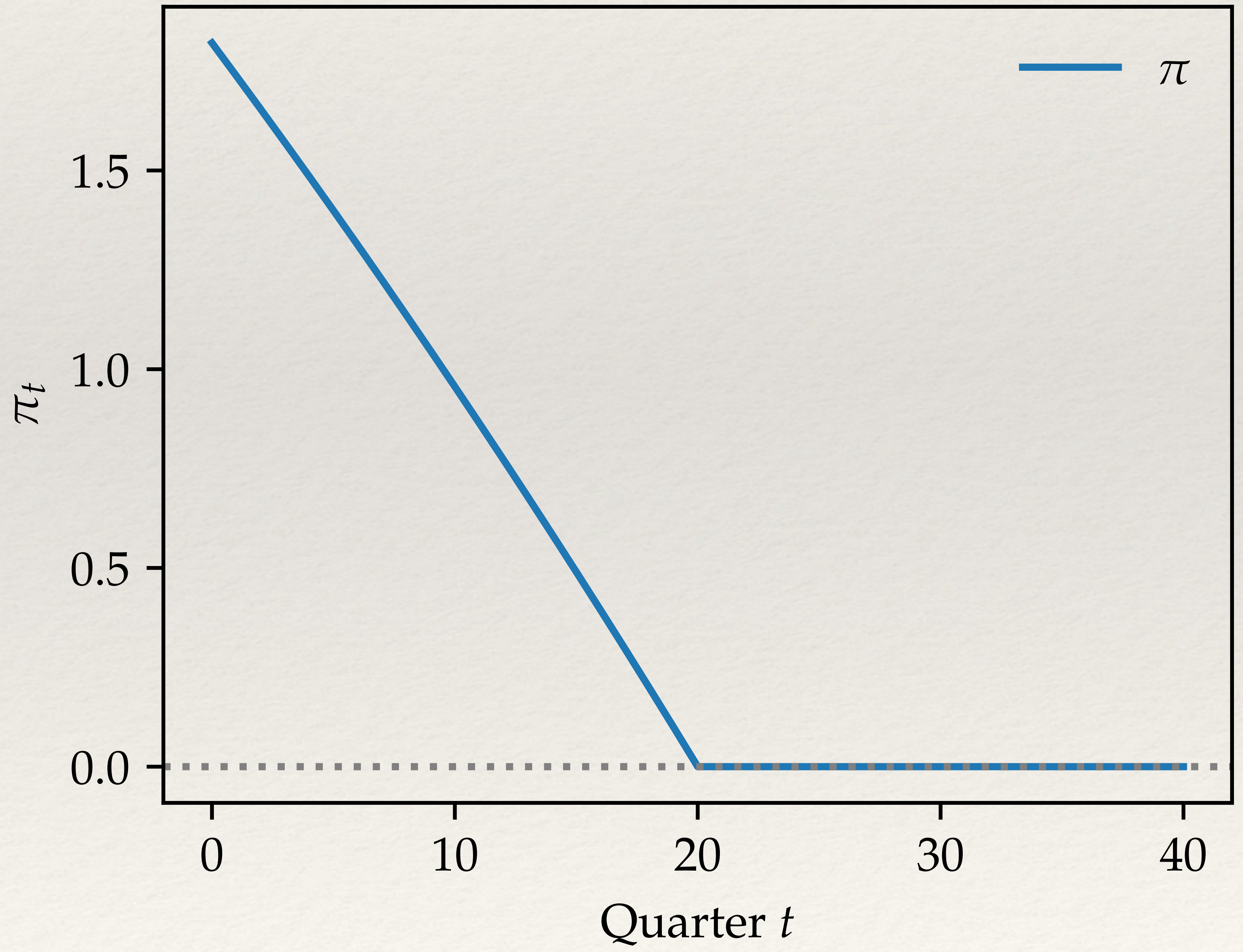

- Implausible? “Forward guidance puzzle”. Euler just too forward looking?

Monetary policy in the canonical HANK model

Recall canonical HANK model

- Household $i \in [0,1]$ solves:

$$\max_{c_{it}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_{it} \left(u(c_{it}) - v\left(N_t\right) \right)$$

$$c_{it} + a_{it} \leq (1 + r_t^p) a_{it-1} + Z_t e_{it}$$

$$a_{it} \geq 0$$

$$\boxed{\{r_t^p, Z_t\}_{t=0}^{\infty}} \longrightarrow \boxed{\{C_t, A_t\}_{t=0}^{\infty}}$$

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- With no markups $\mu = 0$, all assets are real government bonds, $A_{ss} = B_{ss}$
- **Q:** What happens after a monetary policy shock?

Timing of interest rate changes

- Monetary policy rule: $i_t = \pi_{t+1} + \epsilon_t$
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- Government budget constraint with this notation:

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- Implies $T_t = T_{ss} + \dfrac{r_t - r_{ss}}{1 + r_t} B_{ss}$ (cut in interest rate leads to contemp. tax cut)

IKC revisited: monetary policy

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$$Y_t = G_{ss} + \mathscr{C}_t\left(\left\{r_t, Y_s - T_{ss} - \frac{r_t - r_{ss}}{1 + r_t} B_{ss}\right\}\right)$$

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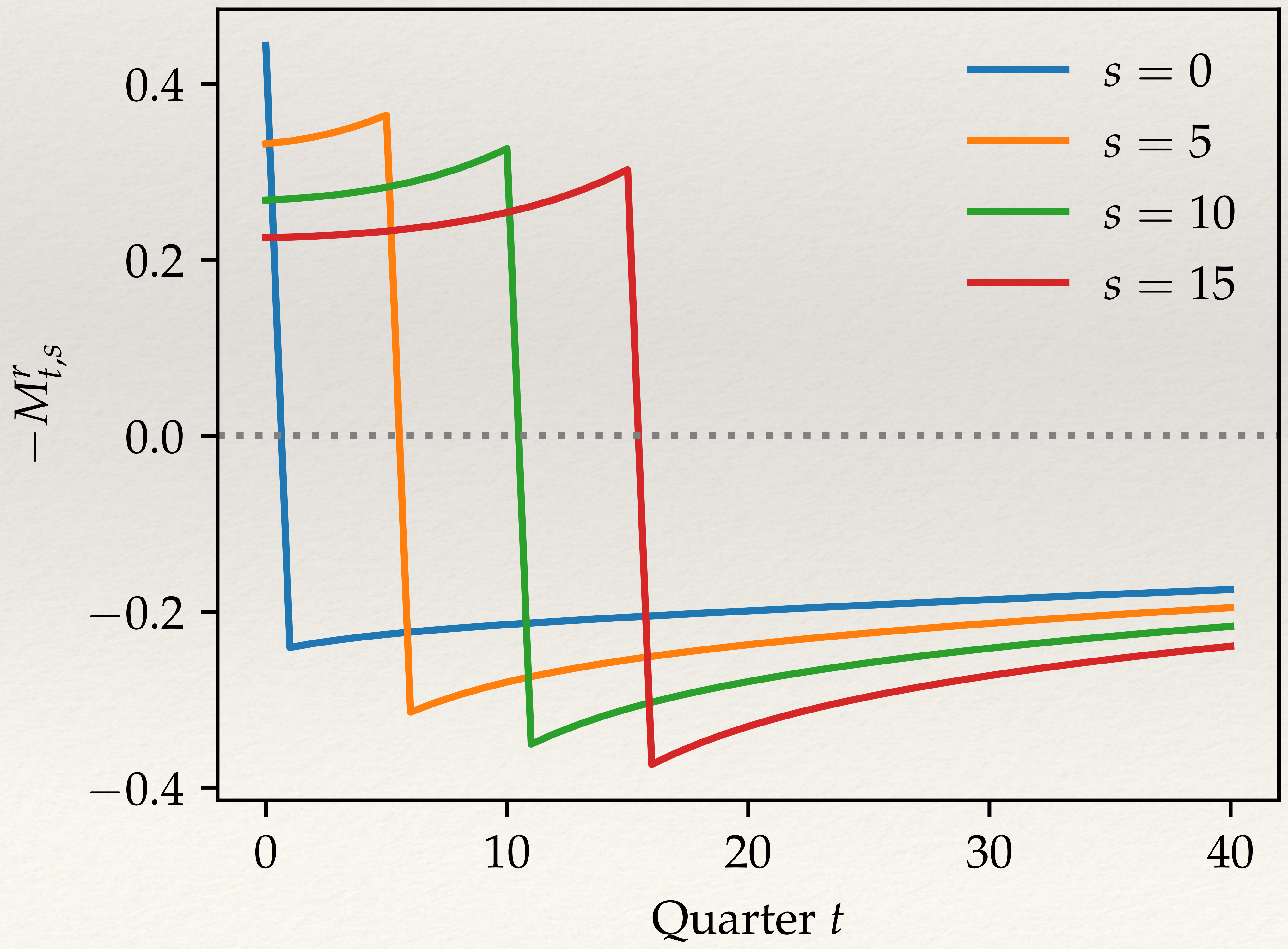
- New sequence space Jacobian: $M^r_{t,s} \equiv \frac{\partial \mathscr{C}_t}{\partial \log r_s}$, notation $d\mathbf{r} \equiv \frac{1}{1+r}(dr_0, dr_1, \ldots)$

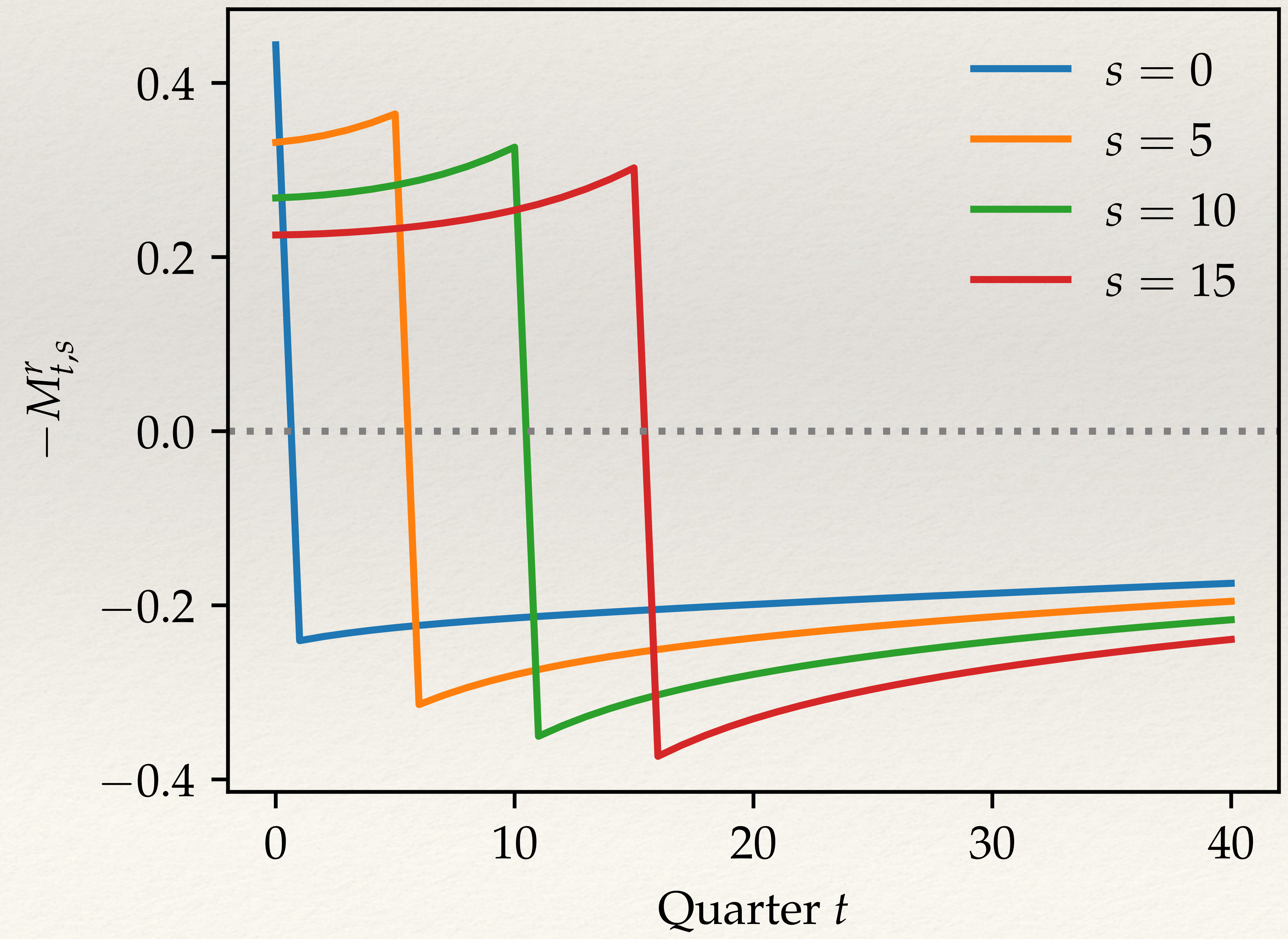
Visualizing $\mathbf{M}^r$ and $\mathbf{M}^r d\mathbf{r}$

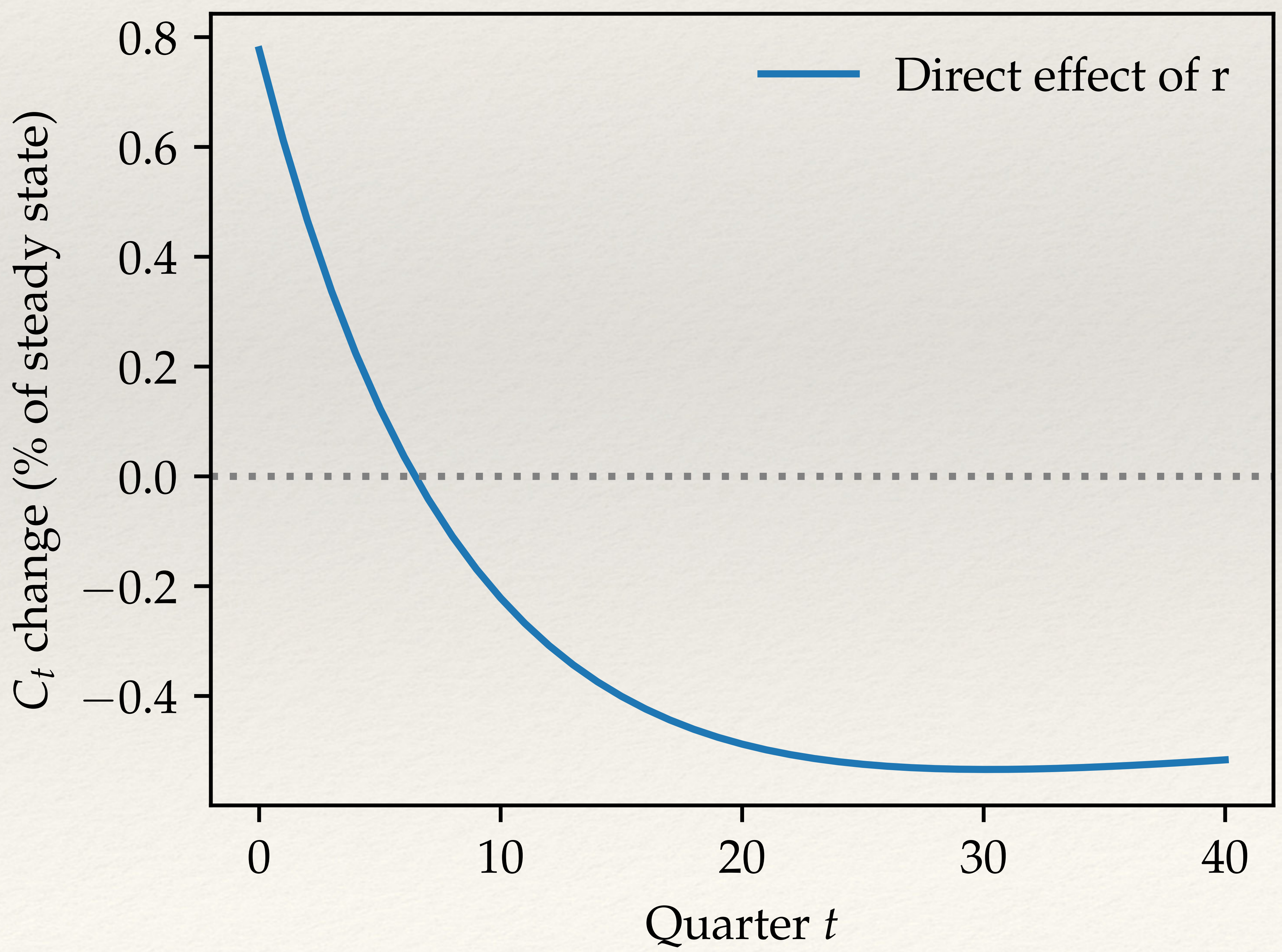
$$-M^r_{t,s} \equiv -\frac{\partial \mathscr{C}_t}{\partial \log r_s}$$



Visualizing $\mathbf{M}^r$ and $\mathbf{M}^r d\mathbf{r}$

$$-M^r_{t,s} \equiv -\frac{\partial \mathscr{C}_t}{\partial \log r_s} \xrightarrow{d\mathbf{r} = -0.25(1,\rho,\rho^2,\ldots)} (\mathbf{M}^r d\mathbf{r})_t = \sum_{s=0}^{\infty} \frac{\partial \mathscr{C}_t}{\partial \log r_s} \frac{dr_s}{1+r}$$





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$$\mathscr{A}_t\left(\left\{r_t, Y_s - T_{ss} - \frac{r_t - r_{ss}}{1+r_t}B_{ss}\right\}\right) = \frac{1+r_{ss}}{1+r_t}B_{ss}$$

Impulse response to AR(1) monetary shock

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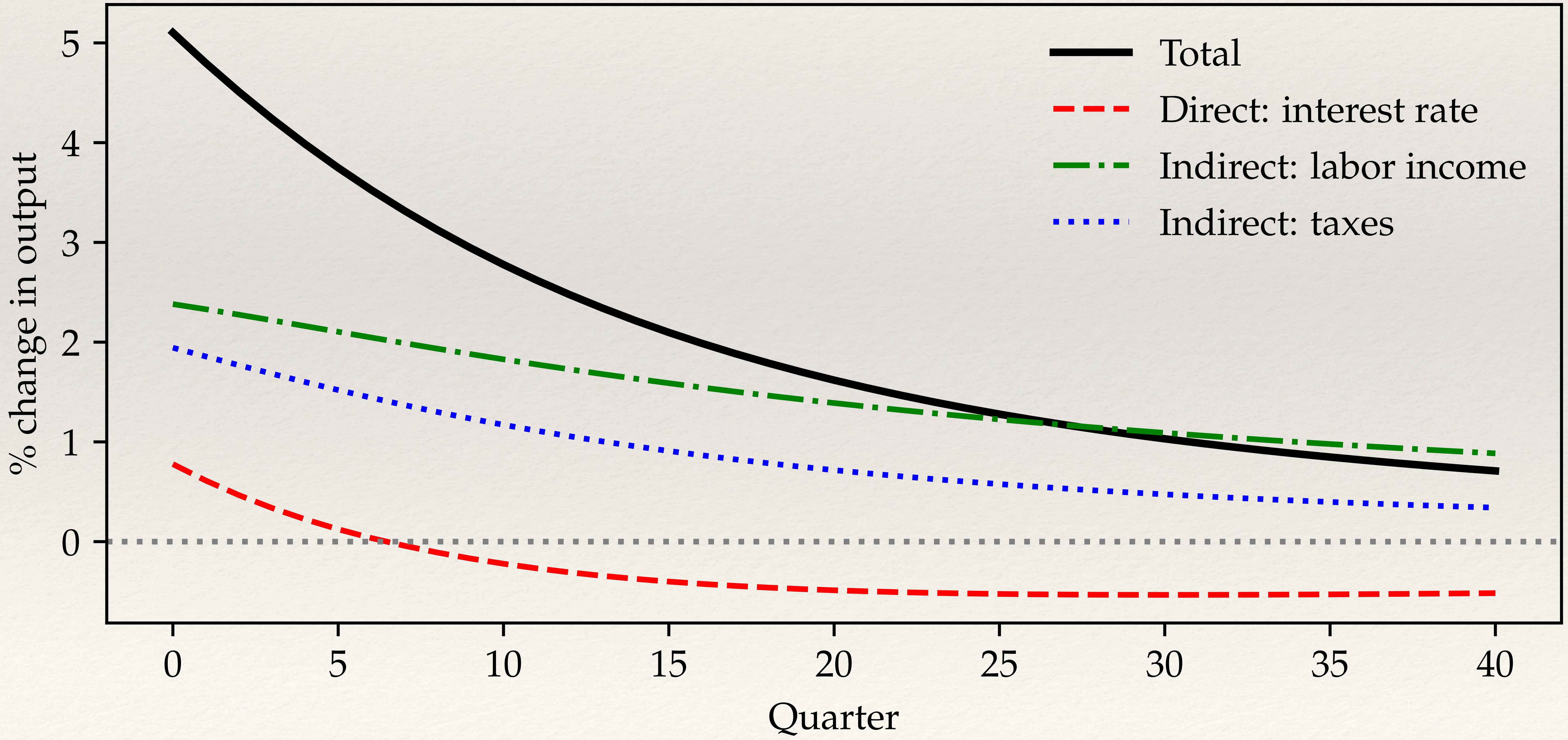

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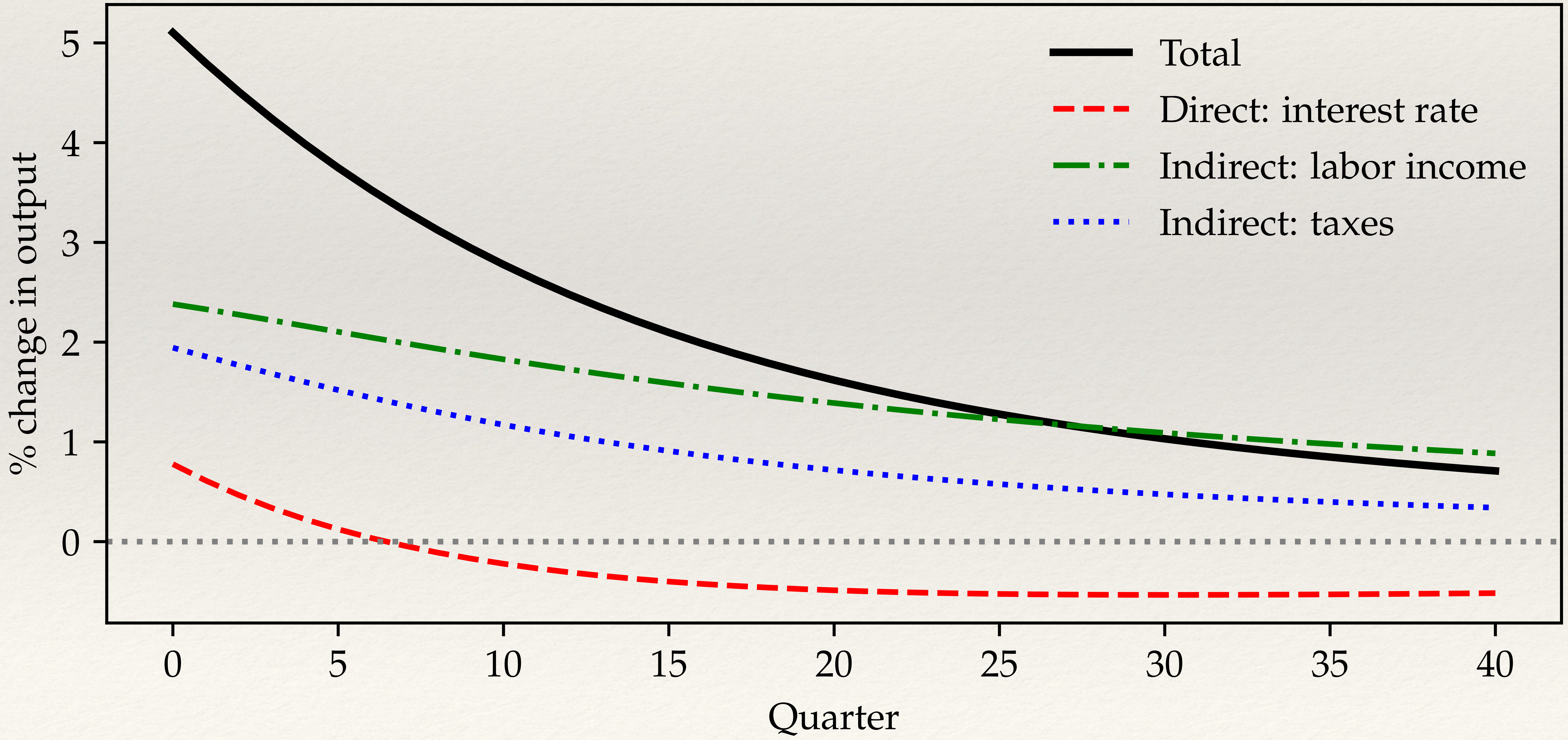

Very large indirect effects!!

(Kaplan, Moll, Violante 2018)

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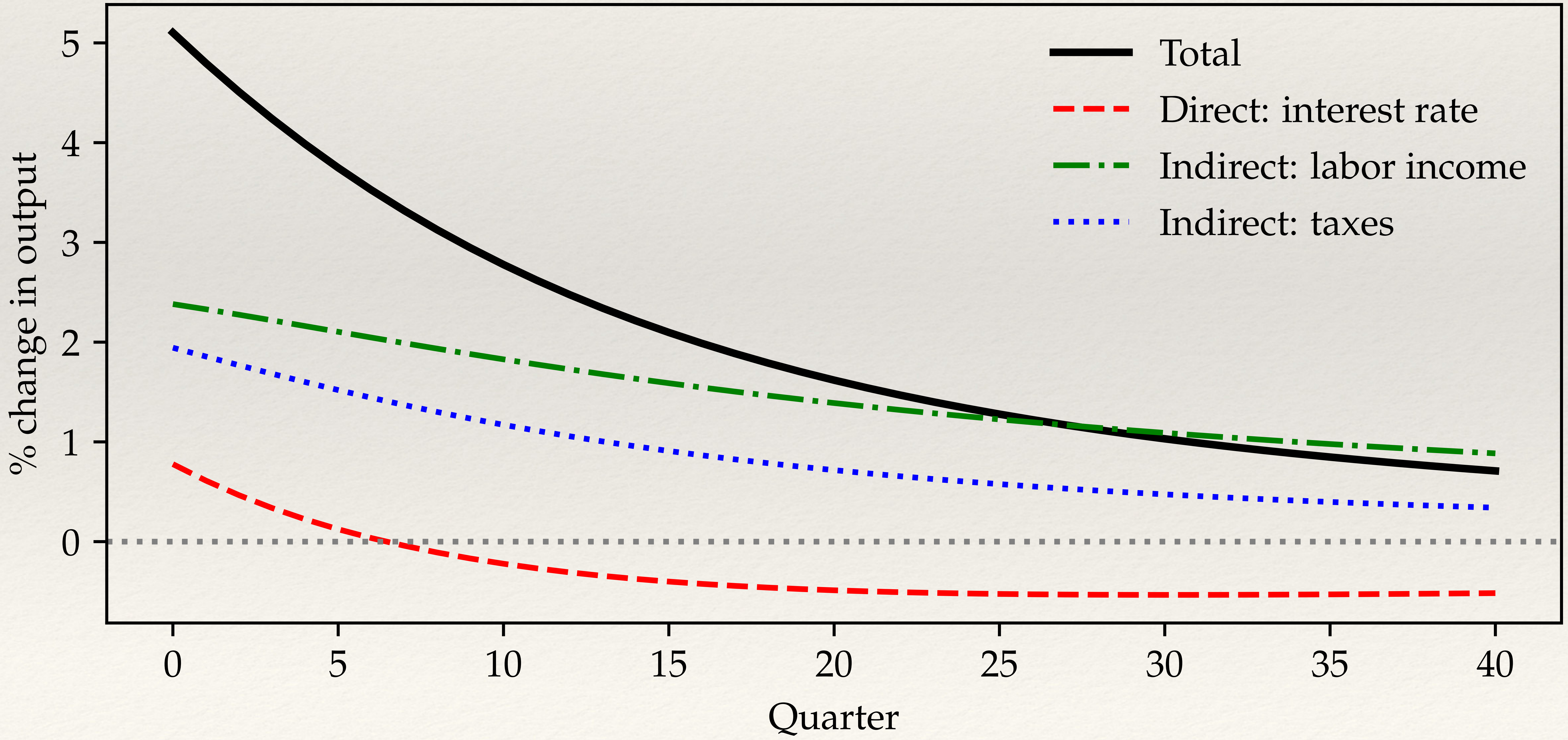

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Tax effect implausible here…

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- A mutual fund owns bonds and stocks, sells shares to hh's with return r_t^p

Introducing a stock market

- Calibration with $A_{ss} = B_{ss} = 5Y$ unrealistic
  - Income effect on the government is much too large! [1% r -> 5% of GDP]
- More realistic: households assets also invested in stock market
- Change calibration to positive markups $\mu > 0$
- Firm problem now implies $\frac{W_t}{P_t} = \frac{1}{\mu}$ and dividends $d_t = (1 - \tau_t)\left(1 - \frac{1}{\mu}\right) Y_t$
- Assume that stocks trade on claim to dividends with price p_t
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[Households owning stocks and bonds directly similar, see portfolio lecture]

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      in ss, just $p_{ss} = \dfrac{d_{ss}}{r}$

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Equations in model with markup

- Equations for equilibrium in model with markups:
  - $Y_t = \mathscr{C}_t\left(r_0^p, \{r_t\}, \{Z_t\}\right) + G_{ss}$
  - $Z_t = \dfrac{Y_t - T_t}{\mu}$
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Linearizing these equations

- Linearize in the sequence space:
    - $d\mathbf{Y} = \frac{\partial \mathbf{C}}{\partial r_0^p} dr_0^p + \mathbf{M}^r d\mathbf{r} + \mathbf{M} d\mathbf{Z}$
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    - $dr_0^p = \frac{1}{A_{ss}} \left(1 - \frac{1}{\mu}\right) \mathbf{q}'(d\mathbf{Y} - d\mathbf{T}) - \frac{p_{ss}}{A_{ss}} \mathbf{q}' d\mathbf{r}$
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IKC including asset price effects

- Rearranging the above, we find:

$$d\mathbf{Y} = \underbrace{\left(\mathbf{M}^r - p_{ss}\mathbf{m}\mathbf{q}'\right)}_{\equiv\overline{\mathbf{M}}^r} d\mathbf{r} + \underbrace{\left(\frac{1}{\mu}\mathbf{M} + \left(1 - \frac{1}{\mu}\right)\mathbf{m}\mathbf{q}'\right)}_{\equiv\overline{\mathbf{M}}} \left(d\mathbf{Y} - B_{ss} d\mathbf{r}\right)$$

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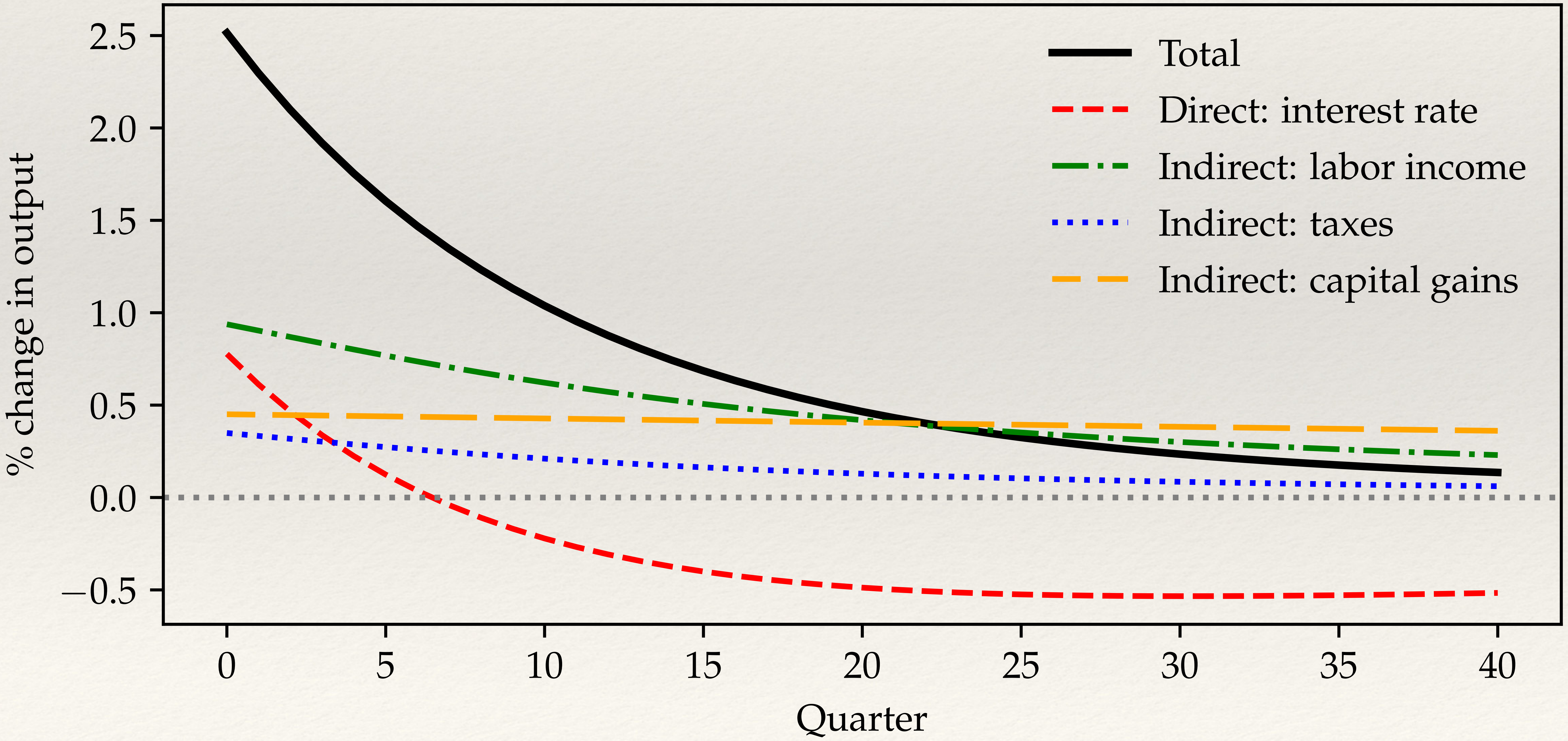
    As in our calibration!
- Same IKC as before, with bars instead: $d\mathbf{Y} = (\overline{\mathbf{M}}^r - \overline{\mathbf{M}}B_{ss})d\mathbf{r} + \overline{\mathbf{M}}d\mathbf{Y}$

Impulse response to AR(1) monetary shock

- Calibration: $B_{ss}/Y_{ss} = 1$

$$d\mathbf{Y} = \underbrace{\mathbf{M}^r d\mathbf{r}}_{\text{Direct interest rate effect}} - \underbrace{\mathbf{M}/\mu B_{ss} d\mathbf{r}}_{\text{Tax effect}} + \underbrace{\mathbf{M}/\mu d\mathbf{Y}}_{\text{Labor income effect}} + \underbrace{\mathbf{m} A_{ss} dr_0^p}_{\text{Cap. gain effect}}$$



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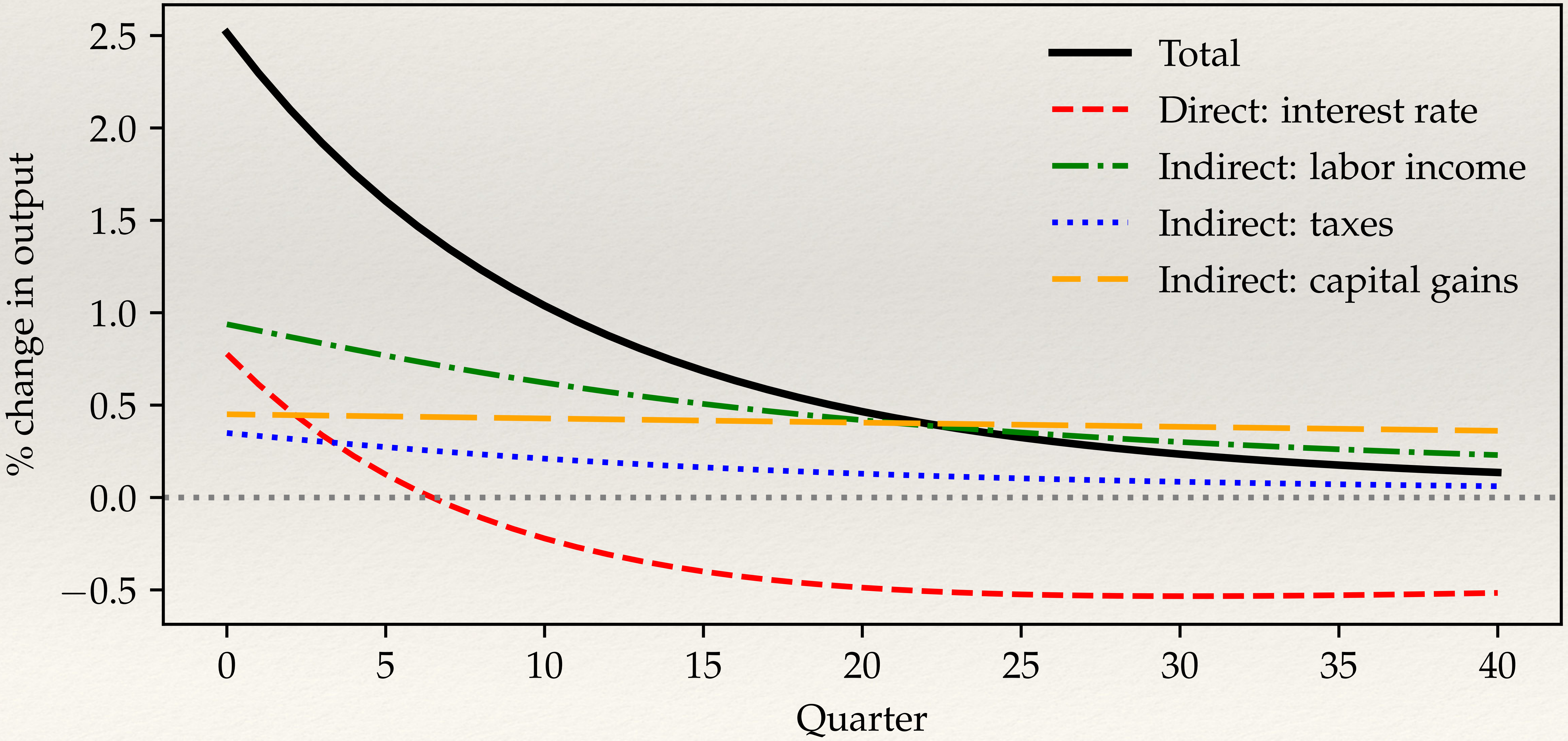

More plausible tax effect

Indirect effects still large…
a little over 2/3 of total

(similar to KMV paper)

Impulse response with no bonds in s.s.

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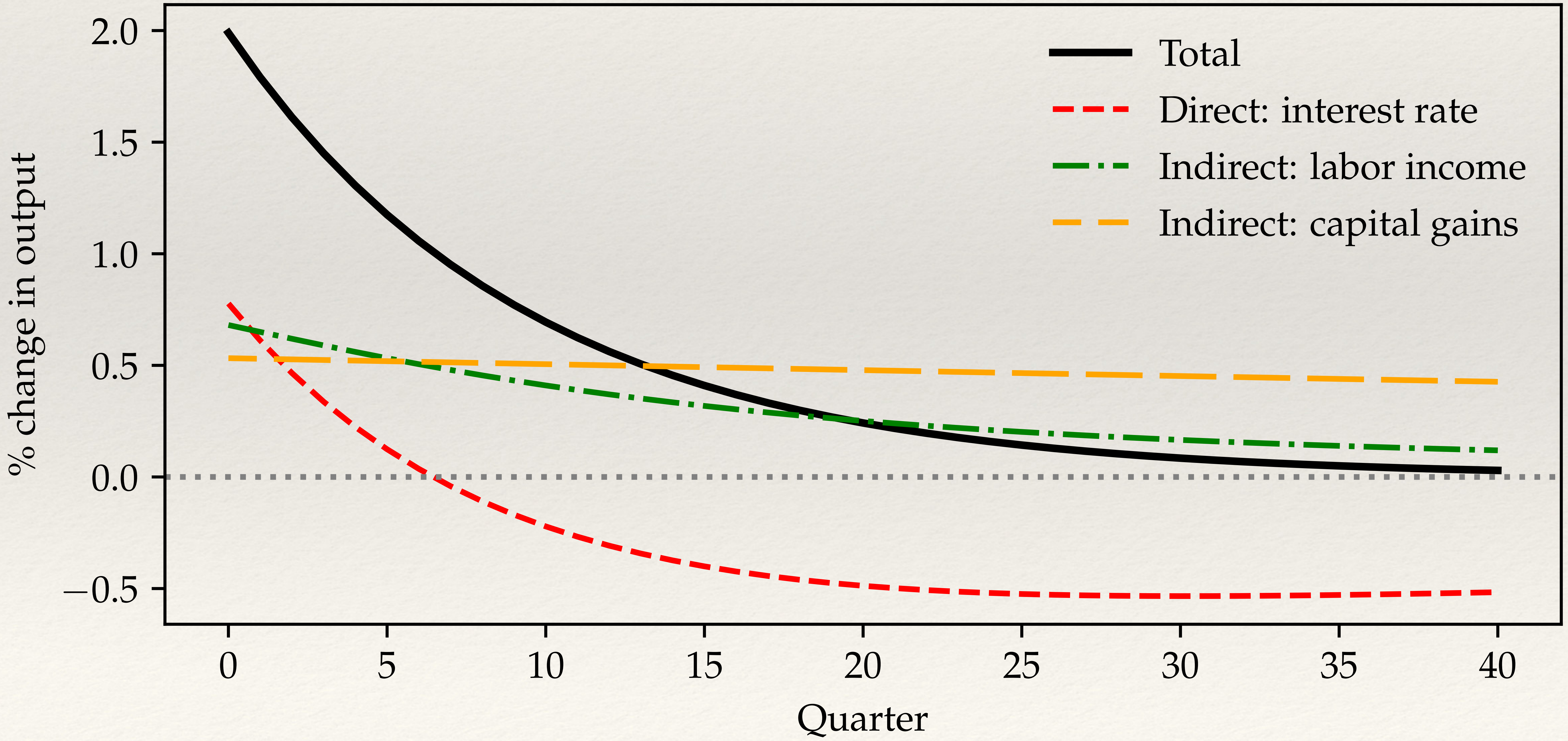

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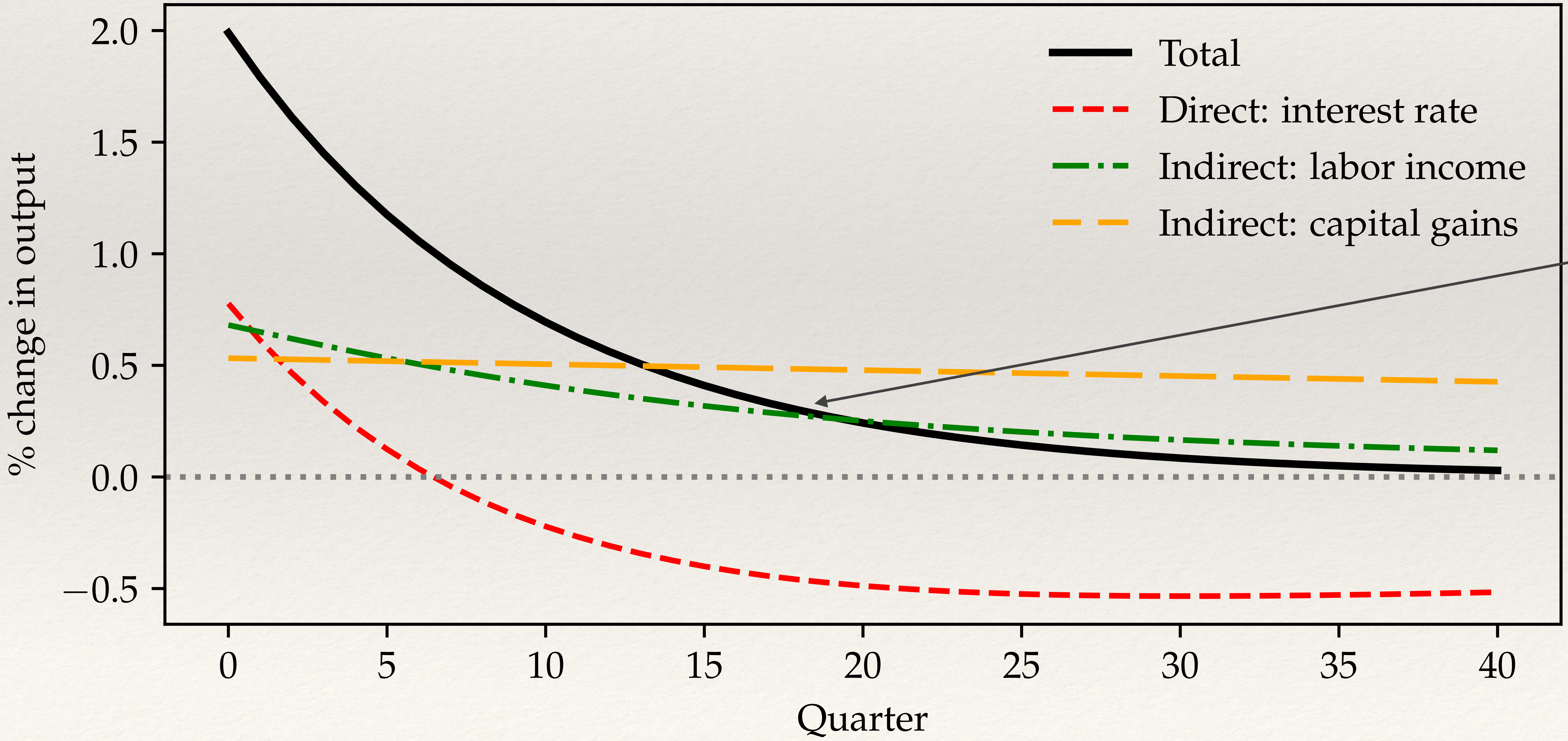

Total effect is exactly the same as in RANK!

$$dC_t = 0.25 \cdot C \cdot \frac{\rho^t}{1-\rho}$$

Coincidence?

Neutrality result

- No! For the calibration with $B_{ss} = 0$ and $u(c) = \log c$, we prove (see ARE):

$$d\mathbf{Y} = -C\mathbf{U}d\mathbf{r} \qquad \mathbf{U} \equiv \begin{pmatrix} 1 & 1 & 1 & 1 & \cdots \\ 0 & 1 & 1 & 1 & \cdots \\ 0 & 0 & 1 & 1 & \cdots \\ 0 & 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

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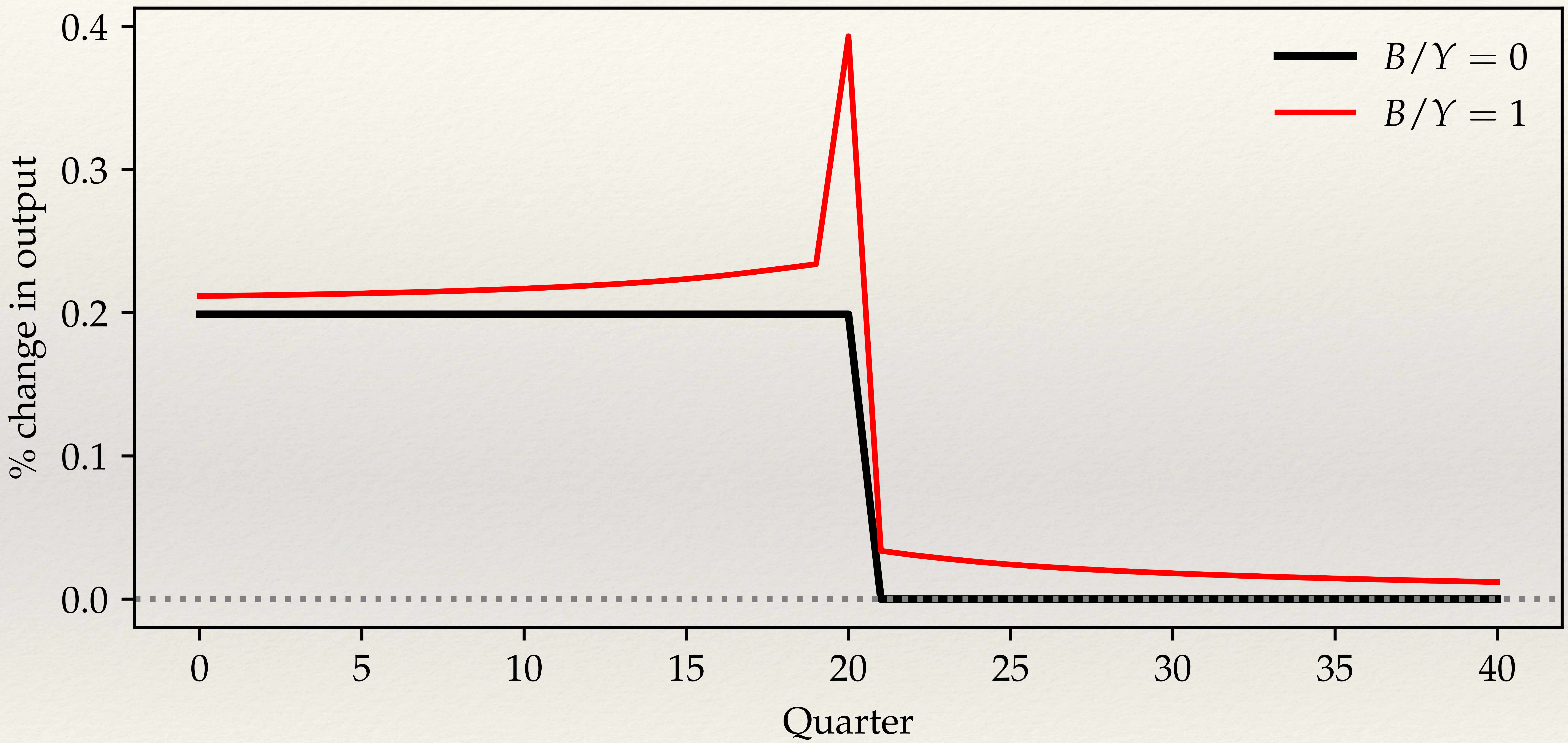
- Monetary policy effects in HANK and RANK are the same for **any** $d\mathbf{r}$!
- Werning (2015) result: need "acyclical income risk" and "acyclical liquidity"

HANK doesn't solve the forward guidance puzzle!



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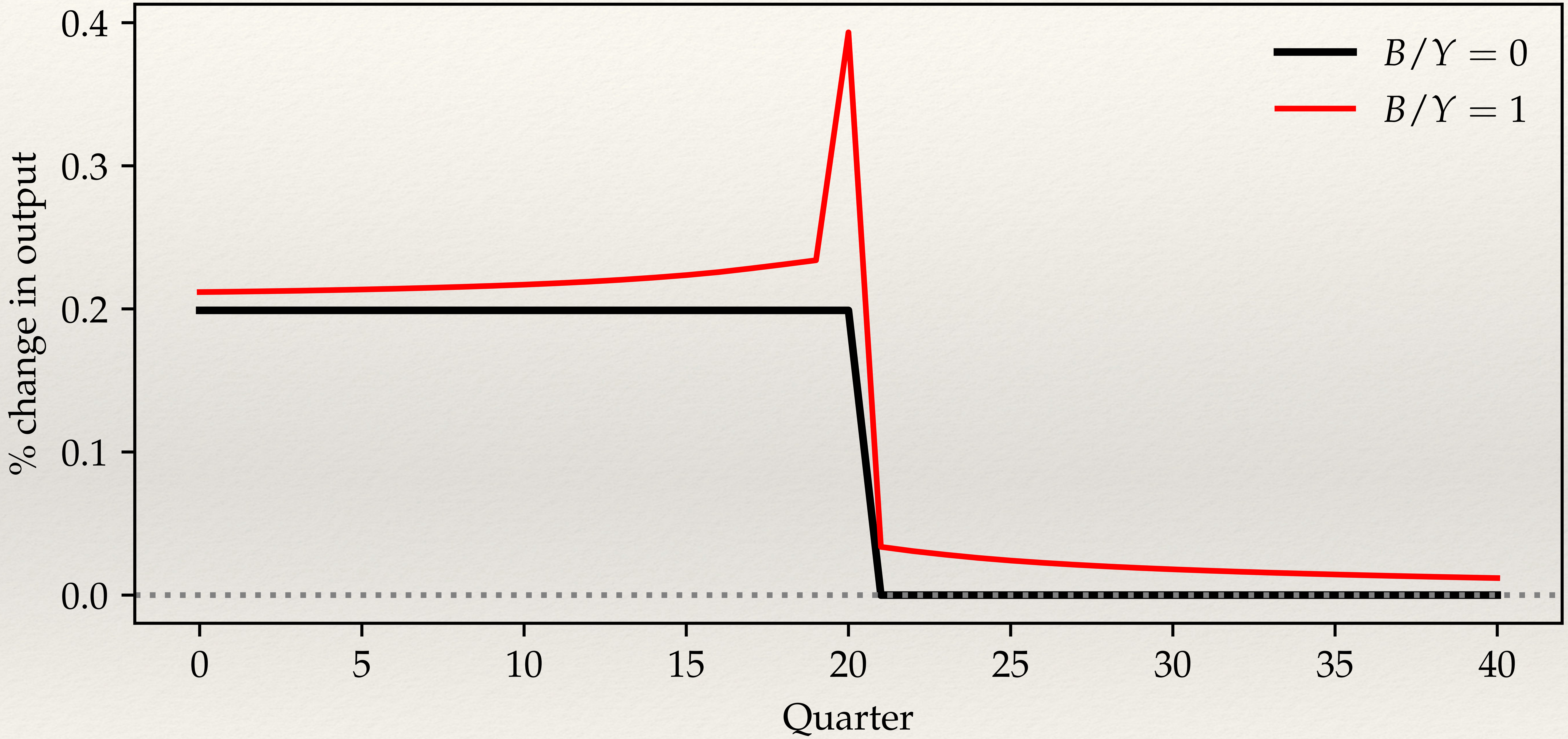

- The direct effect of d**r** is lower on impact… but not the GE response to d**r** !

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- Does HANK change the transmission of monetary policy?
  - Yes: indirect effects much more important than in RANK!