Intro to HANK models: Fiscal Policy

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This session

- Just saw: How to solve steady states and simple transitional dynamics
- **Next:** Introducing “HANK” & how to use it for fiscal policy

1. Canonical HANK model
2. Fiscal policy in HANK
3. Fiscal policy simulations
4. Going beyond: Blocks & models

The canonical HANK model

Introducing the canonical HANK model

- Embed standard het. agent model into standard NK model
- Will allow for a **government:** bonds, taxes, gov. spending
- Build on "Intertemporal Keynesian cross" (IKC) and Annual Review papers
- Set up the model in the **sequence space**
  - assume economy in steady state, feed in **perfect foresight shock** at $t = 0$
  - keep in mind **certainty equivalence**

Household side

- Household $i \in [0,1]$ solves:

Discount factor shocks as before to avoid "asset-MPC trade-off"

Same hours worked for everyone
(details coming up)

$$\max_{c_{it}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_{it} \left(u(c_{it}) - v\left(N_t\right) \right)$$

$$c_{it} + a_{it} \leq (1 + r_t^p) a_{it-1} + \underline{(1 - \tau_t) \frac{W_t}{P_t} N_t} e_{it} \qquad a_{it} \geq \underline{a}$$

Total post-tax labor income $\quad Z_t = (1 - \tau_t) \dfrac{W_t}{P_t} N_t$

[Can also capture progressive taxation as in Heathcote-Storesletten-Violante]

First example of a "block": $\{r_t^p, Z_t\}_{t=0}^{\infty} \Rightarrow \{C_t, A_t\}_{t=0}^{\infty}$

Nominal rigidity

- Standard RANK model uses sticky prices with flexible wages:

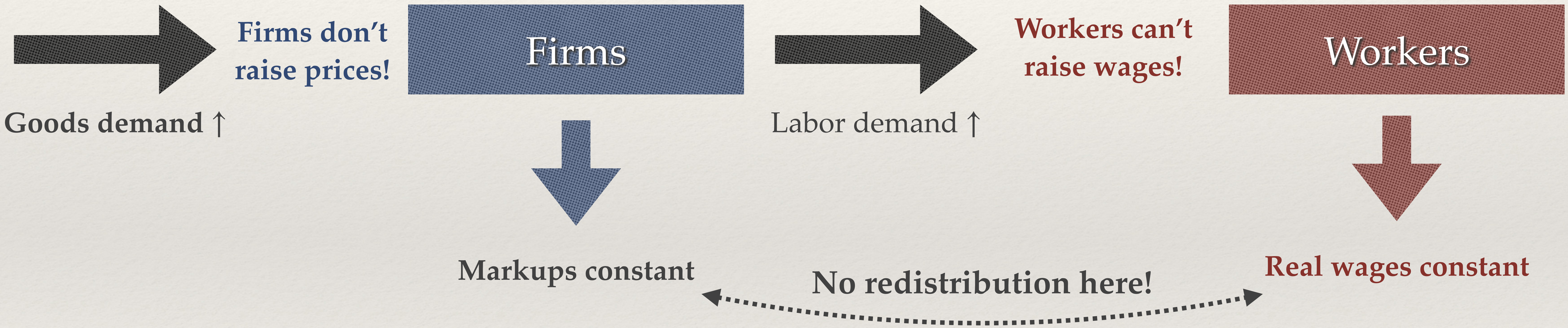
Sticky prices effectively redistribute resources from firm owners to workers

Not an issue in RANK. **Could be huge issue in HANK!**

[Bilbiie 2008, Broer et al. 2020]

Sticky wages

- Our canonical HANK model uses sticky wages as in Erceg et al (2000)



- How are wages adjusted?
  Unions set wages on behalf of workers…

$$\pi_t^w = \kappa\left(v'\left(N_t\right) - \frac{\epsilon - 1}{\epsilon}\left(1 - \tau_t\right)\frac{W_t}{P_t}u'\left(C_t\right)\right) + \beta\pi_{t+1}^w$$

Production

- Monopolistic competition, linear production in labor. In aggregate:

$$Y_t = N_t$$

- With flexible prices, price equals constant markup times marginal cost:

$$P_t = \mu W_t \qquad \Leftrightarrow \qquad \frac{W_t}{P_t} = \mu^{-1}$$

- Real wage exogenous. Goods inflation = wage inflation: $\pi_t = \pi_t^w$
- For this lecture: no markups, $\mu = 1$. Will revisit later.

Government: Fiscal policy

- Government sets fiscal policy, consisting of **three paths**:
  - G_t government spending
  - $T_t = \tau_t Y_t$ total tax revenue, governed via tax rate τ_t
  - B_t government bonds (uniformly bounded to avoid Ponzi schemes)
- … subject to **budget constraint:**

$$B_t = \left(1 + r_t\right) B_{t-1} + G_t - T_t$$

Government: Monetary policy

- Monetary authority follows an interest rate rule. Allow for two kinds of rules:
  - **standard Taylor rule rule:** $i_t = r + \phi_\pi \pi_t + \epsilon_t$ (linearized)

    steady state real rate

    Taylor rule coefficient

    Focus on this rule first

  - **real interest rate rule:** $i_t = r + \pi_{t+1} + \epsilon_t \longrightarrow r_{t+1} = r + \epsilon_t$

    Exogenous path of real rates!
    Will be hugely helpful for tractability…

- Two differences: *(i)* π_t vs π_{t+1} (not crucial); *(ii)* $\phi_\pi = 1$ (key)

Definition of equilibrium

- All agents optimize and markets clear

Bonds are only asset here
Later: add capitalized profits with $\mu > 1$

$$G_t + C_t = Y_t \qquad \text{equivalent by Walras' law} \qquad A_t = B_t$$

$$C_t = \int c_t^* (a_-, e)\, dD_t (a_-, e) \qquad\qquad A_t = \int a_t^*(a_-, e) dD_t (a_-, e)$$

Computing the steady state

- Simple to compute the steady state:

1. Normalize output $Y = 1$, pick r, B, G. Gov. budget: $T = rB + G$.

2. Can use **same code as before:**

   - use as income $Z\,e_{it}$ with $Z = Y - T$

   - choose β to match $A = B$

3. $G + C = Y$ holds by Walras' law! Done!

- **Next:** Solve for dynamic responses to fiscal policy shocks!

Fiscal Policy

Fiscal policy shocks

- We just introduced the canonical HANK model.
- **Next:** Focus on fiscal policy!
  - Switch off monetary policy shocks: $r_t = r = const$
  - Focus on **first-order** shocks to fiscal policy $d\mathbf{G} = \{dG_t\}, d\mathbf{T} = \{dT_t\}$ s.t.

$$\sum_{t=0}^{\infty} (1+r)^{-t}(dG_t - dT_t) = 0$$

Aggregate consumption function

- Recall household "block": mapping sequences $\{r_t^p, Z_t\}$ into $\{C_t, A_t\}$
- With constant r, this means that date-t consumption can be written as

$$C_t = \mathscr{C}_t\left(Z_0, Z_1, Z_2, \ldots\right) = \mathscr{C}_t\left(\{Z_s\}\right)$$

- We call this the *intertemporal consumption function.*
- With $\mathscr{C}_t$ we can write goods market clearing as

This exactly describes the equilibrium output response Y_t!

$$Y_t = G_t + \mathscr{C}_t\left(\{Y_s - T_s\}\right)$$

Intertemporal MPCs

$$Y_t = G_t + \mathscr{C}_t\left(\{Y_s - T_s\}\right)$$

- Feed in small shock $\{dG_t, dT_t\}$

$$dY_t = dG_t + \sum_{s=0}^{\infty} \frac{\partial \mathscr{C}_t}{\partial Z_s} \cdot (dY_s - dT_s)$$

- Response entirely characterized by Jacobian of $\mathscr{C}$, "**intertemporal MPCs**"

$$M_{t,s} \equiv \frac{\partial \mathscr{C}_t}{\partial Z_s}$$

- $M_{t,s} = \%$ of date-s income gain spent at date-t. Note: $\sum_{t=0}^{\infty} (1+r)^{s-t} M_{t,s} = 1$

Intertemporal MPCs

Response to income increase at $s = 0$

$$\mathbf{M} = \begin{pmatrix} M_{00} & M_{01} & M_{02} & M_{03} & \cdots \\ M_{10} & M_{11} & M_{12} & M_{13} & \cdots \\ M_{20} & M_{21} & M_{22} & M_{23} & \cdots \\ M_{30} & M_{31} & M_{32} & M_{33} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Intertemporal MPCs

Response to income increase at $s = 1$

$$\mathbf{M} = \begin{pmatrix} M_{00} & M_{01} & M_{02} & M_{03} & \cdots \\ M_{10} & M_{11} & M_{12} & M_{13} & \cdots \\ M_{20} & M_{21} & M_{22} & M_{23} & \cdots \\ M_{30} & M_{31} & M_{32} & M_{33} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Intertemporal MPCs

Response to income increase at $s = 2$

$$\mathbf{M} = \begin{pmatrix} M_{00} & M_{01} & M_{02} & M_{03} & \cdots \\ M_{10} & M_{11} & M_{12} & M_{13} & \cdots \\ M_{20} & M_{21} & M_{22} & M_{23} & \cdots \\ M_{30} & M_{31} & M_{32} & M_{33} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Note: **M** preserves present values: $\underbrace{\sum_{t=0}^{\infty} \frac{1}{(1+r)^t} M_{t,s}}_{\text{PV of spending response}} = \underbrace{\frac{1}{(1+r)^s}}_{\text{PV of date-}s\text{ income increase}} \quad \Leftrightarrow \quad \mathbf{q}'\mathbf{M} = \mathbf{q}'$

$\mathbf{q} = \left(1, (1+r)^{-1}, (1+r)^{-2}, \ldots\right)'$

PV of **spending** response

PV of date-s **income** increase

The intertemporal Keynesian cross

$$dY_t = dG_t + \sum_{s=0}^{\infty} \frac{\partial \mathscr{C}_t}{\partial Z_s} \cdot \left(dY_s - dT_s\right)$$

$$\boxed{d\mathbf{Y} = d\mathbf{G} - \mathbf{M}d\mathbf{T} + \mathbf{M}d\mathbf{Y}}$$

Intertemporal Keynesian cross

- Entire complexity of model is in $\mathbf{M}$
- "Sufficient statistic" — only $\mathbf{M}$ matters!

Comparison with old-Keynesian cross

- Very similar to the Old-Keynesian cross in IS-LM:

$$dY_t = dG_t - mpc \cdot dT_t + mpc \cdot dY_t$$

- Intertemporal Keynesian cross: microfounded, vector-valued, dynamic
- But: Many intuitions in HANK are similar to IS-LM intuitions
  - in some sense, HANK much more Keynesian than NK models!

Solving the intertemporal Keynesian cross

$$d\mathbf{Y} = d\mathbf{G} - \mathbf{M}d\mathbf{T} + \mathbf{M}d\mathbf{Y} \qquad \Rightarrow \qquad (\mathbf{I} - \mathbf{M})\, d\mathbf{Y} = d\mathbf{G} - \mathbf{M}d\mathbf{T}$$

- Can't we simply invert $\mathbf{I} - \mathbf{M}$?
- No! $\mathbf{q}$ is in (left) kernel: $\mathbf{q}'(\mathbf{I} - \mathbf{M}) = 0$. Also: $\mathbf{q}'\left(d\mathbf{G} - \mathbf{M}d\mathbf{T}\right) = 0$
- With some advanced math, can show that inverse still exists iff unique bounded $d\mathbf{Y}$ exists:

$$d\mathbf{Y} = \mathscr{M}\left(d\mathbf{G} - \mathbf{M}d\mathbf{T}\right)$$

where $\mathscr{M} \equiv \left(\mathbf{K}\left(\mathbf{I} - \mathbf{M}\right)\right)^{-1}\mathbf{K}$ with $\mathbf{K} = -\sum_{t=1}^{\infty}(1+r)^{-t}\mathbf{F}^{t}$

Solving in using asset market clearing

- An equivalent way to solve the model:

$$\mathscr{A}_t\left(\left\{Y_s - T_s\right\}\right) = B_t$$

- Linearized:

$$\mathbf{A}\left(d\mathbf{Y} - d\mathbf{T}\right) = d\mathbf{B} \qquad \Rightarrow \qquad d\mathbf{Y} = d\mathbf{T} + \mathbf{A}^{-1}d\mathbf{B}$$

- Same solution (can show):

$$d\mathbf{Y} = \mathscr{M}\left(d\mathbf{G} - \mathbf{M}d\mathbf{T}\right) = \underbrace{\mathscr{M}\left(\mathbf{I} - \mathbf{M}\right)d\mathbf{T}}_{= d\mathbf{T}} + \overbrace{\mathscr{M}}^{= \mathbf{A}^{-1}\mathbf{K}}\underbrace{\left(d\mathbf{G} - d\mathbf{T}\right)}_{\mathbf{K}\left(d\mathbf{G} - d\mathbf{T}\right) = d\mathbf{B}}$$

The balanced budget multiplier

- In some cases, solution is simple!
- E.g. suppose $d\mathbf{G} = d\mathbf{T}$, i.e. gov. spending increase financed by tax hike.
- **Result:** $d\mathbf{Y} = d\mathbf{G}$ **!**
- Why? Simple to verify:

$$d\mathbf{Y} = d\mathbf{G} - \mathbf{M}d\mathbf{T} + \mathbf{M}d\mathbf{Y} \quad \Rightarrow \quad d\mathbf{G} = d\mathbf{G} - \mathbf{M}d\mathbf{G} + \mathbf{M}d\mathbf{G}$$

or using asset market: $d\mathbf{Y} = d\mathbf{T} + \mathbf{A}^{-1}d\mathbf{B} = d\mathbf{T} = d\mathbf{G}$

- IS-LM antecedents: Gelting (1941), Haavelmo (1945)

Deficit financed fiscal policy

- With deficit-financing $d\mathbf{G} \neq d\mathbf{T}$ we have

$$d\mathbf{Y} = d\mathbf{G} + \underbrace{\mathscr{M} \cdot \mathbf{M} \cdot \left(d\mathbf{G} - d\mathbf{T}\right)}_{d\mathbf{C}}$$

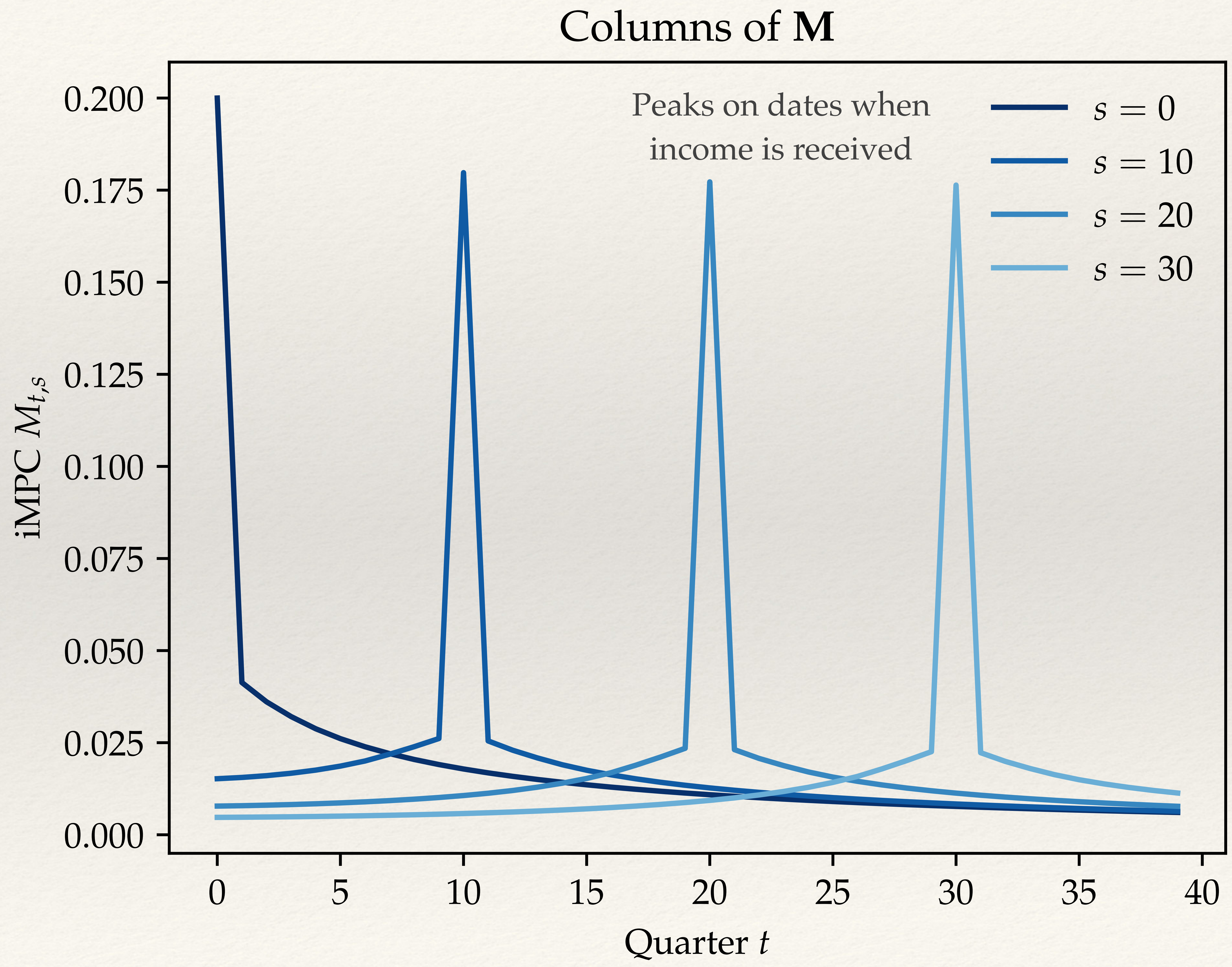
Interaction term:
large if $\mathbf{M}$ is large and primary
deficits $d\mathbf{G} - d\mathbf{T}$ large

- **Next:** Compute $\mathbf{M}$ and simulate this!

Fiscal policy simulations

Getting the intertemporal MPCs

- To solve the intertemporal Keynesian cross, all we need is $\mathbf{M}$
- Potentially costly!
- Quick to solve using "fake-news algorithm" (see later today)
- What does $\mathbf{M}$ look like in other models? (repr. agent? two-agent?)



RA and TA

- Repr. agent (RA): $\quad C_t = (1-\beta)\sum_{s\geq 0}\beta^s Z_s + ra_{-1} \quad \Rightarrow \quad M_{t,s} = \frac{\partial C_t}{\partial Z_s} = (1-\beta)\beta^s$

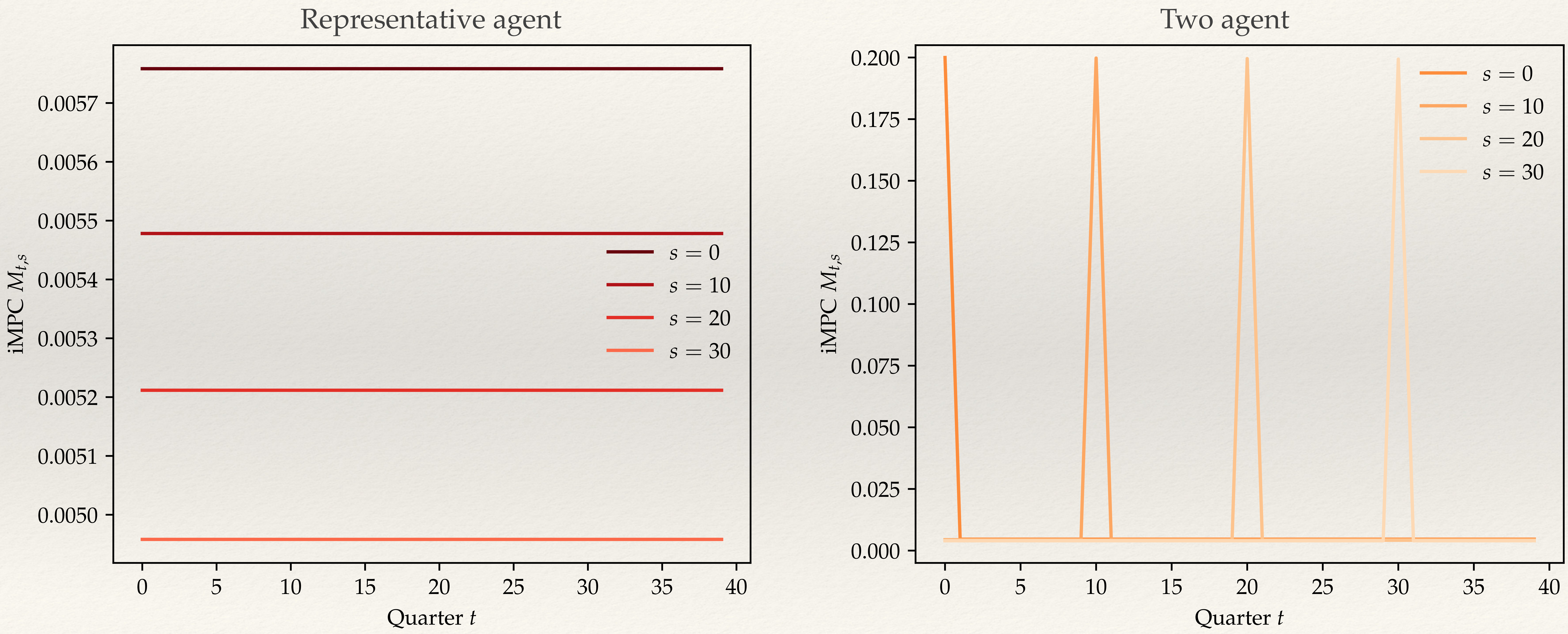
  $\beta = 1/(1+r)$

$$\mathbf{M}^{RA} = \begin{pmatrix} 1-\beta & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ 1-\beta & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ 1-\beta & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

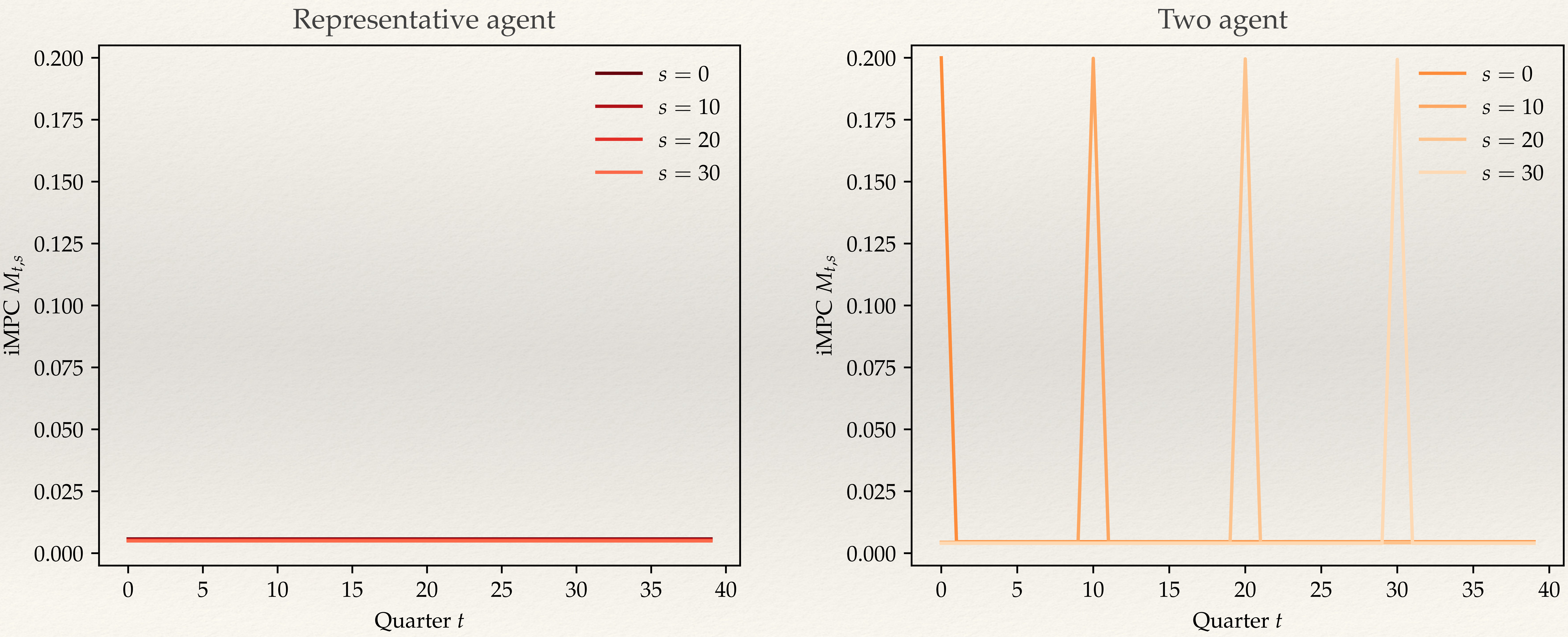
- Two-agent (TA): Like RA, except a fraction λ of households is hand-to-mouth

$$\mathbf{M}^{TA} = (1-\lambda)\mathbf{M}^{RA} + \lambda\mathbf{I}$$

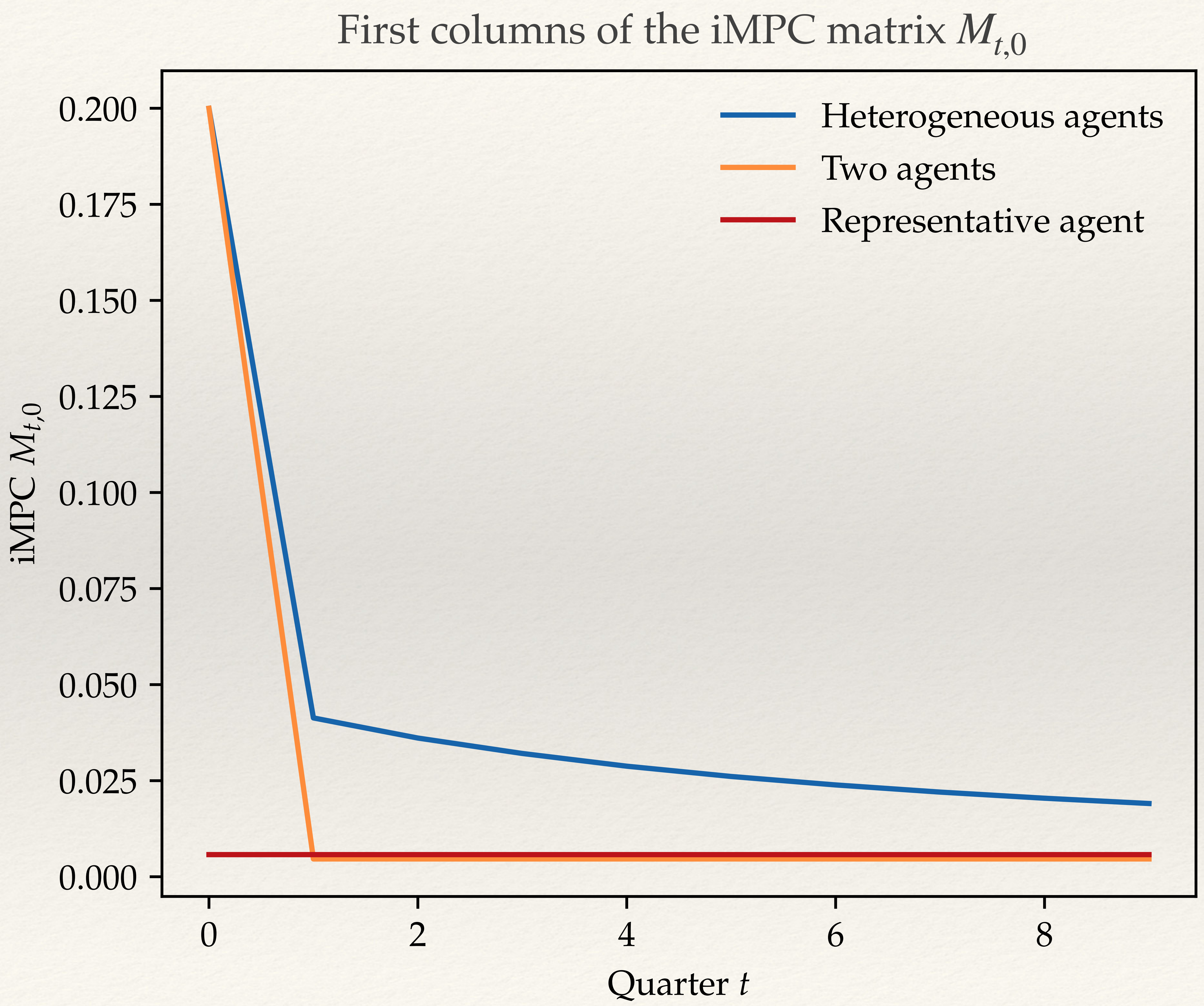
RA and TA iMPCs



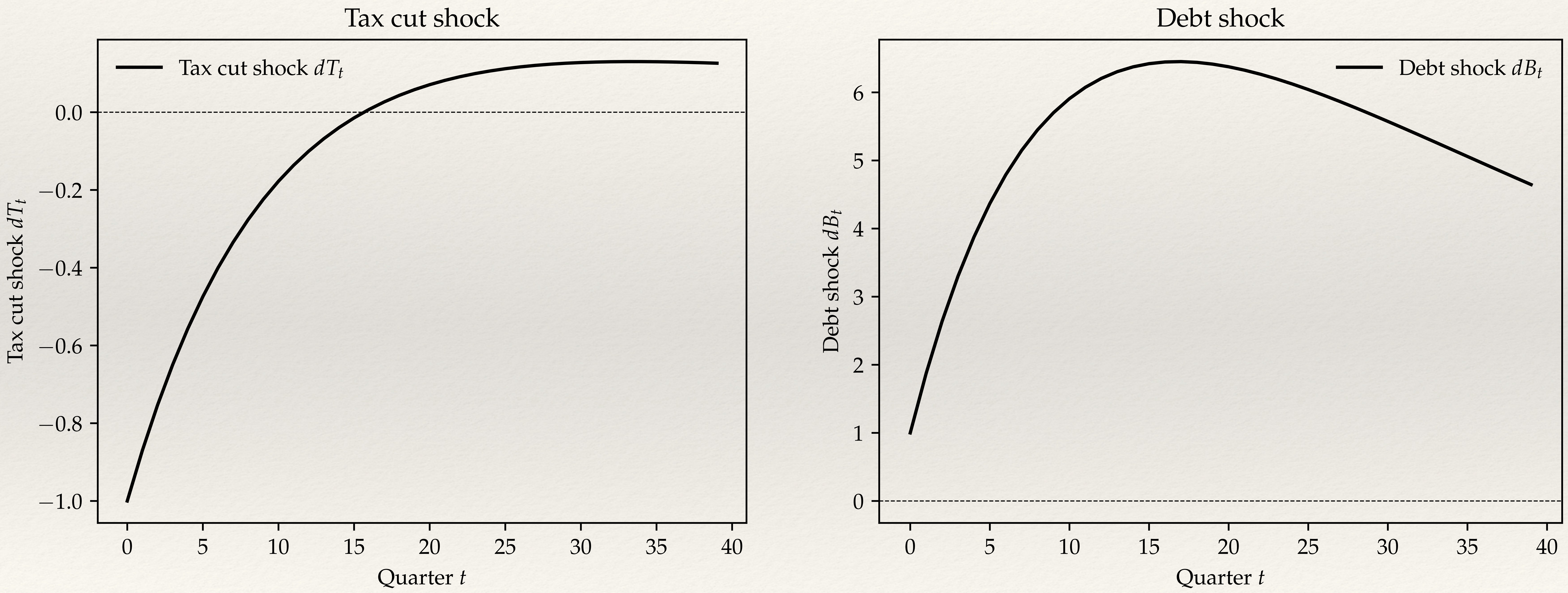
RA and TA iMPCs



Comparing the first columns



Deficit financed tax cut

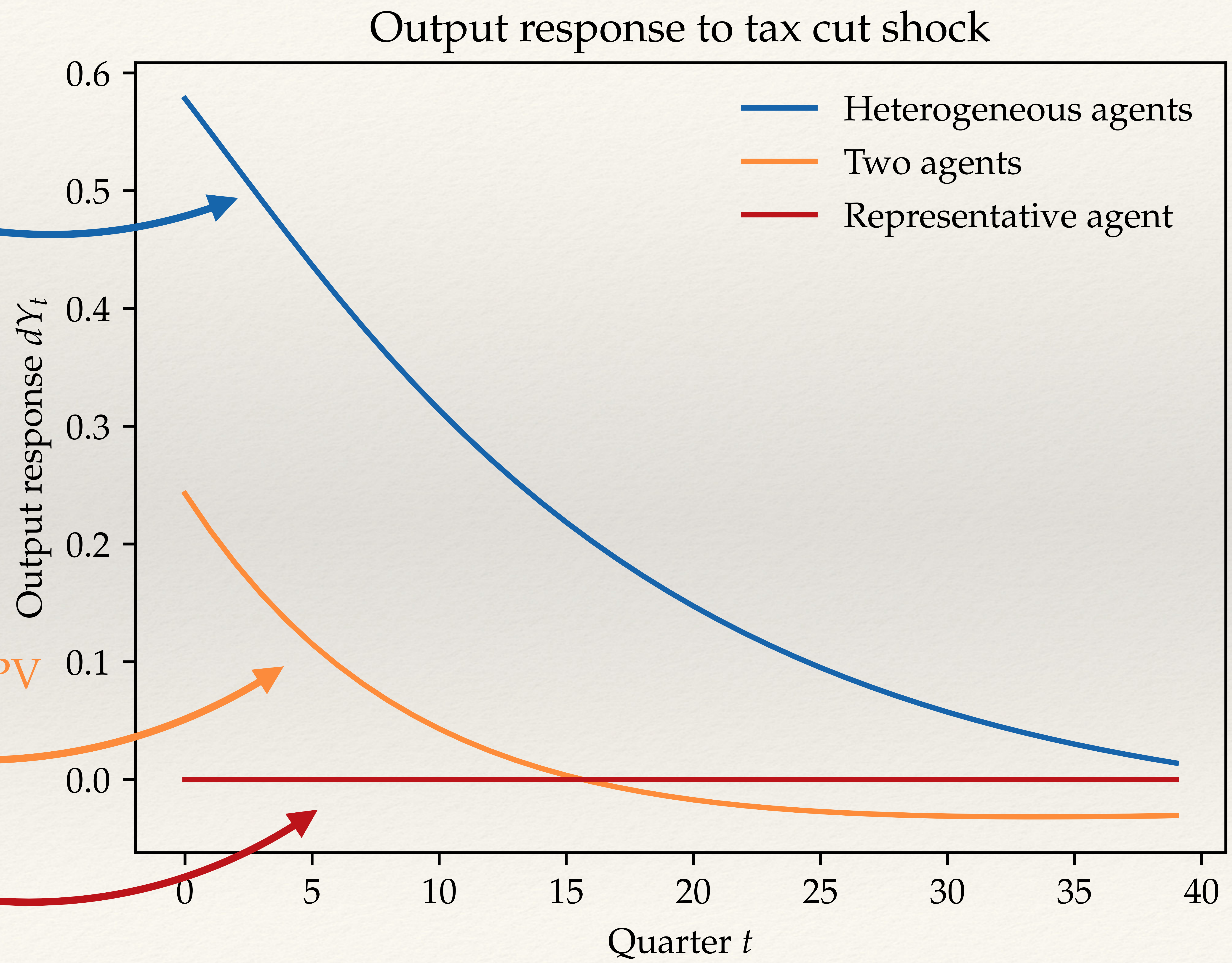


Deficit financed tax cut in RA, TA, HA

HA: sustained boom!

TA gets boom, but zero NPV

RA is Ricardian!



Going beyond: Blocks & models

Beyond IKC

- We solved everything here by solving an IKC-like equation
- What if the model is richer?
- A useful approach we will be following is one that thinks of models as consisting of many individual **blocks:**

Input sequences $\longrightarrow$ Output sequences

- In the following: Write sequences, e.g. $\{r_t\}$, as bold vectors: $\mathbf{r} = (r_0, r_1, \ldots)'$

Example: Household block

- Our households solved

$$V_t(e, a\_) = \max_{c,a} u(c) + \beta \mathbb{E}_t V_{t+1}(e', a)$$

$$c + a = (1 + r_t^p) a\_ + e\, Z_t \qquad a \geq \underline{a}$$

only aggregate sequences that matter!

- Can regard household **block** as (complex) mapping: $\mathbf{r}^p, \mathbf{Z} \to \mathbf{C}, \mathbf{A}$

1. backward iteration to get policies: $a_t(e, a\_), c_t(e, a\_)$
2. forward iteration to get distribution: $D_t(e, a\_)$
3. aggregation: $A_t = \int a_t(e, a\_) dD_t(e, a\_)$ and $C_t = \int c_t(e, a\_) dD_t(e, a\_)$

Will call such a block: **HetBlock**

- Can define Jacobians $\mathbf{J}^{C,r}, \mathbf{J}^{A,r}, \mathbf{J}^{C,Z}, \mathbf{J}^{A,Z}$ for all pairs of outputs and inputs!

Example: Fiscal block

- Can describe fiscal policy in a block, too, e.g.: $\mathbf{r}^p, \mathbf{B}, \mathbf{G}, \mathbf{Y} \rightarrow \mathbf{Z}, \mathbf{T}$

$$T_t = G_t + \left(1 + r_t^p\right) B_{t-1} - B_t \qquad\qquad Z_t = Y_t - T_t$$

- This block is totally analytical!
- Likewise, asset market clearing block: $\mathbf{A}, \mathbf{B} \rightarrow \mathbf{H}$ where $H_t = A_t - B_t$
- Jacobians are simple, e.g. $\mathbf{J}^{Z,Y} = \mathbf{I}$, $\mathbf{J}^{H,A} = \mathbf{I}$

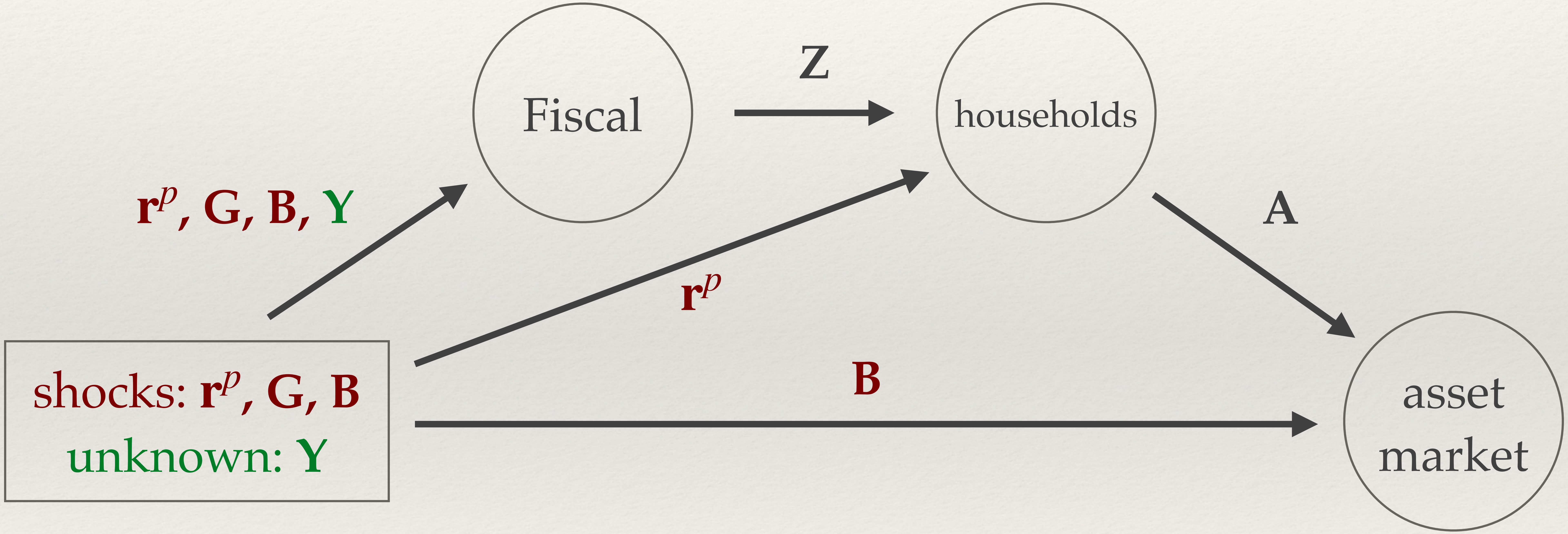
Will call such blocks: **SimpleBlock**

Model

- A **model** is then a **collection of blocks**, such that:
    - Some inputs are *exogenous shocks*, e.g. $\mathbf{r}^p, \mathbf{G}, \mathbf{B}$
    - Some inputs are *endogenous unknowns*, e.g. $\mathbf{Y}$
    - Some outputs are *targets* that must be zero in GE, e.g. $\mathbf{H}$
- Need to have that number of unknowns = number of targets
- Most models can be written this way!

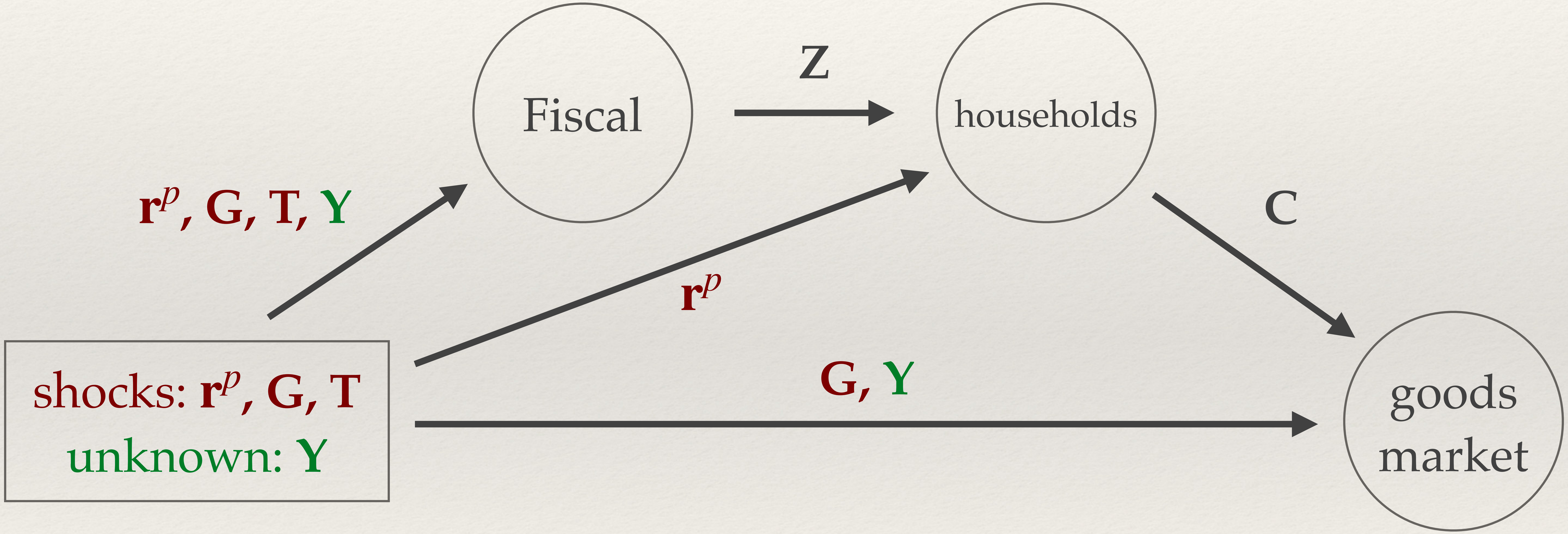
Directed Acyclic Graphs (DAGs)

- Can draw our model:



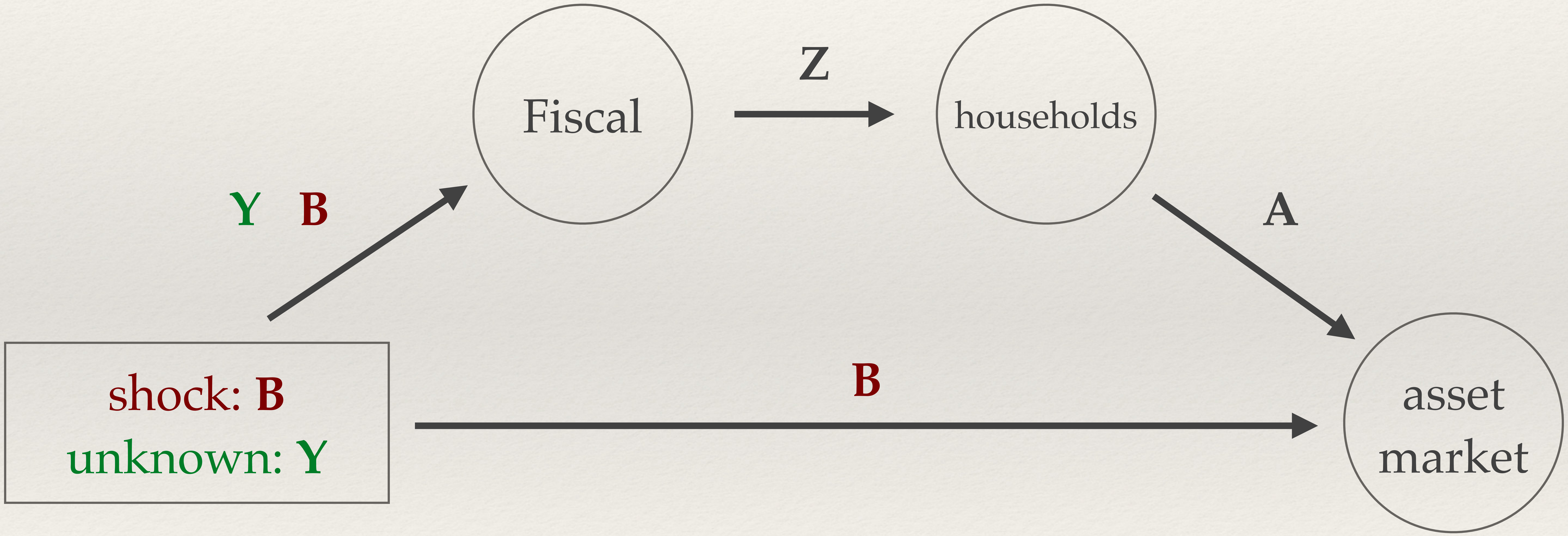
- Will use this later in our tutorial

DAGs are not unique!



Solving for output response of shocks

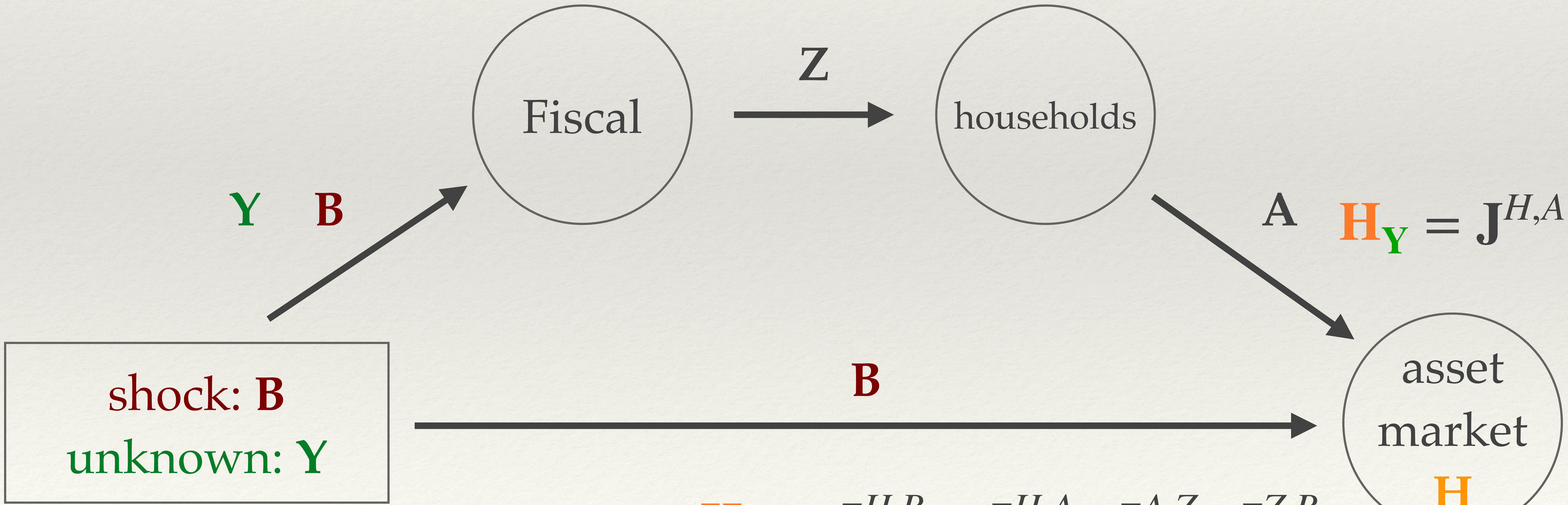
- Imagine we just shock $\mathbf{B}$ and want to solve for $\mathbf{Y}$, holding r, G constant.



- Want: asset market error $\mathbf{H}$ as function of shock and unknown $\mathbf{H}(\mathbf{Y}, \mathbf{B}) = 0$

Solving for output response of shocks

- Main idea: Use **implicit function theorem**: $d\mathbf{Y} = -\left(\mathbf{H_Y}\right)^{-1} \cdot \mathbf{H_B} \cdot d\mathbf{B}$
- But how do we compute the **H** Jacobians $\mathbf{H_Y}$, $\mathbf{H_B}$? $\rightarrow$ Walk along the DAG:



$$\mathbf{H_Y} = \mathbf{J}^{H,A} \cdot \mathbf{J}^{A,Z} \cdot \mathbf{J}^{Z,Y}$$

$$\mathbf{H_B} = \mathbf{J}^{H,B} + \mathbf{J}^{H,A} \cdot \mathbf{J}^{A,Z} \cdot \mathbf{J}^{Z,B}$$

Summary

Summary

- We introduced a **canonical HANK model**:
  - standard heterogeneous-agent household side
  - standard New-Keynesian supply side, but sticky wages + flexible prices
  - real rate rule for now, later Taylor rule
- Matters for **fiscal policy**!
  - deficit financing & front loading amplifies initial + cumulative multipliers
  - not the case in RA, and not even in TA