① $\boxed{a}$ To detect the gender of the voice.

There are 2 classes: (i) Male and (ii) Female.
(ignoring trans).

The predictors / variables could be.

(i) Pitch of the voice – male have low pitched voice than female.

(ii) Loudness – male tend to talk louder than female (I know this is based on stereotype and not necessarily true).

(iii) The talking rate – Usually women talk faster. (again it is based on stereotypes

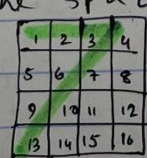
② $\boxed{b}$ To detect alphabets or number.

The number of classes include:

Alphabets (26) + Numbers (10) → since 0 – 9

= 36

- The variables / predictors as mentioned could account for the space occupied by each segment.



Since segment (1, 2, 3, 4, 6, 7, 9, 10, 13) are occupied it is probably 7.

- Also the time to write can also play critical role. like writing 'O' is faster than writing 'F'.

2. $P(y=1|x) = \frac{1}{1+e^{-z}}$ $z = \beta_0 + \beta_1 x_1 + \beta_2 x_2$

$\beta = [1, 2, 3]^T$

2 (a) $P(y=1|x) > P(y=0|x)$

$= 1 + 2x_1 + 3x_2$

Multiclass – logistic Regression

$z = Wx + W_0$

$$P(y=k|x) = \frac{e^{z_k}}{\sum_{l=1}^{k} e^{z_l}}$$

$\beta = [1, 2, 3]^T$

~~For~~

$\cancel{P(y=1/x) = \frac{e^1}{e^5}}$

$$\frac{1}{1+e^{-z}} > 0$$

$0 > 1+e^{-z}$

$1 < e^{-z}$

$1 < e^{-(\beta_0 - \beta_1 x_1 - \beta_2 x_2)}$

$1 < e^{-(1-2x_1-3x_2)}$

$1 - 2x - 3x = \ln(1)$

$1 - 2x - 3x = 0$

$1 - 5x = 0$

$x = 1/5$

$\boxed{2(b)}$ $P(y=1|x) > 0.8$

$$\frac{1}{1+e^{-z}} > 0.8$$

$$\frac{1}{1+e^{-(\beta_0-\beta_1 x_1-\beta_2 x_2)}} > 0.8$$

$$e^{-\beta_0-\beta_1 x_1-\beta_2 x_2} > \frac{1}{0.8}$$

$$e^{-(1-2x-3x)} > 1.25$$

$$\cancel{-5x > 1.25 - 1}$$

$$\cancel{-5x > 0.25}$$

$$\cancel{x > 0.05}$$

$$1-2x-3x = \ln(1.25)$$

$$1-5x = 0.223$$

$$1-0.223 = 5x.$$

$$\boxed{\therefore x = 0.155}$$

$\boxed{2(c)}$ $\frac{1}{1+e^{-z}} > 0.8$

Put $x_2 = 0.5$

$$e^{-\beta_0-\beta_1 x-\beta_2 x_2} > \frac{1}{0.8}$$

$$e^{-1-2x_1+3\times0.5} > \ln\left(\frac{1}{0.8}\right)$$

$$-1-2x_1+1.5 > 0.223$$

$$1-2x_1 > 1.277$$

$$2x_1 < 0.277$$

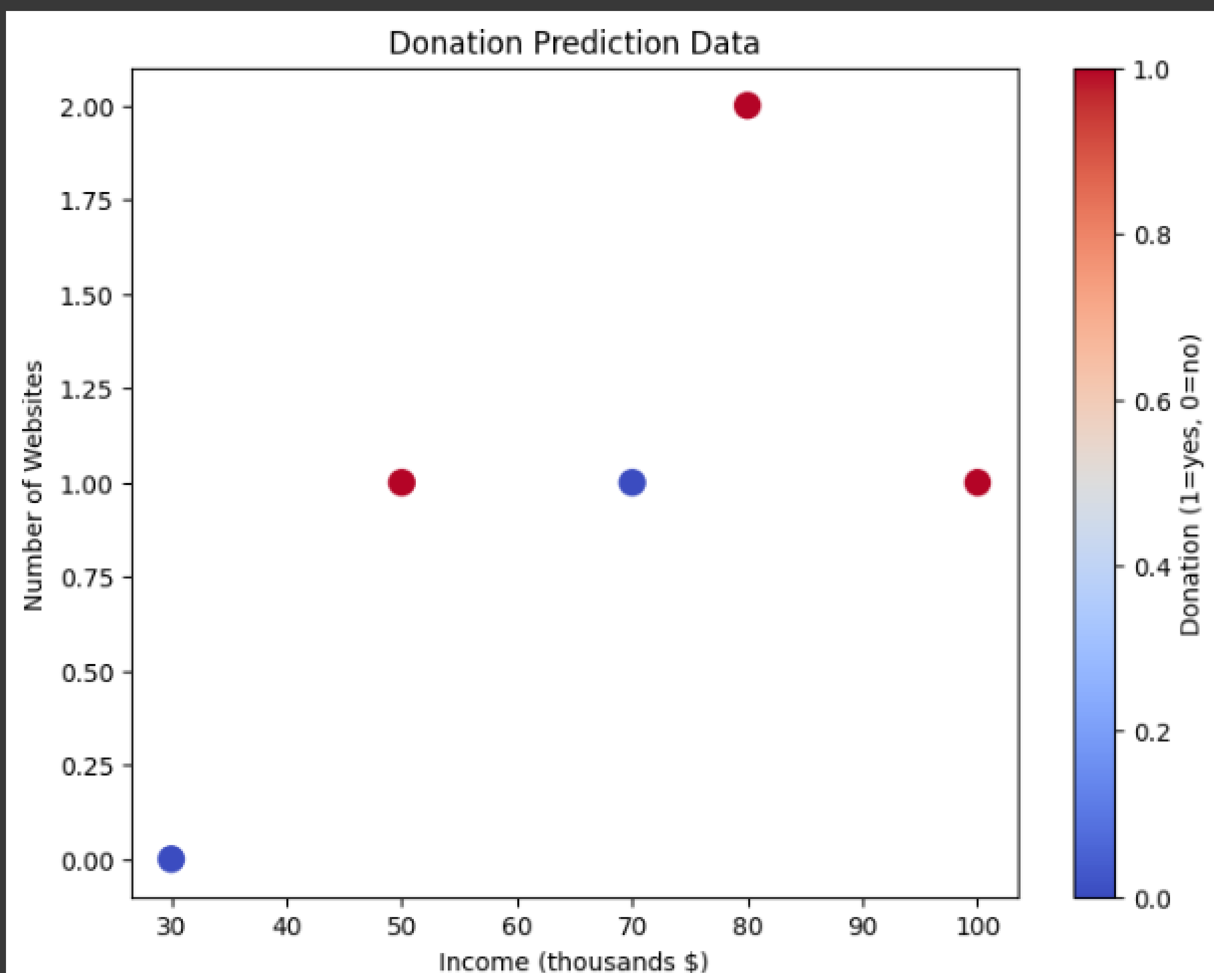
$$\boxed{x_1 < 0.1385}$$

Number -3

```
import matplotlib.pyplot as plt
import numpy as np

income = [30, 50, 70, 80, 100]
websites = [0, 1, 1, 2, 1]
donation = [0, 1, 0, 1, 1]


plt.figure(figsize=(8, 6))
plt.scatter(income, websites, c=donation, cmap='coolwarm', s=100)
plt.xlabel("Income (thousands $)")
plt.ylabel("Number of Websites")
plt.title("Donation Prediction Data")
plt.colorbar(label="Donation (1=yes, 0=no)")
plt.show()
```

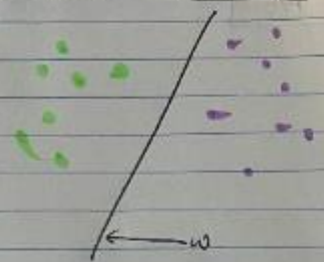


3(b) Linear classifier

$$y_i \begin{cases} 1 & \text{if } z_i > 0 \\ 0 & \text{if } z_i < 0 \end{cases} \qquad z_i = w^T x_i + b$$

Perfect Linear Separability.

Perfectly linearly separable.
It makes no error on the training data.



$$\begin{cases} W = (w_0, w_1, \ldots W_d) \\ \cdot\ w_0 + w_1 x_{i1} + \cdots w_d x_{id} > 0 \quad \text{when } y_i = 1 \\ \cdot\ w_0 + w_1 x_{i1} + \cdots w_d x_{id} < 0 \quad \text{when } y_i = 0 \end{cases}$$

$$\boxed{z_i = w x_i + bias}$$

3(c) $P(y_i = 1 | x_i) = \frac{1}{1 + e^{-z_i}}$

$$\boxed{z_i = w^T x_i + b}$$

$$P(y = 1 | x_i) = \frac{1}{1 + e^{-z_i}}$$

$$0 = \frac{1}{1 + e^{-z_i}}$$

$$1 + e^{-z_i} = 0$$

$$e^{-z_i} = 0$$

$$e^{-z_i} = 0 + 1$$

$e^{-z_i} =$ [redacted]

$z_i = \ln($[redacted]$)$

3(d) $\boxed{w' = \alpha w}$

$\boxed{b' = \alpha b}$

α is used in adjacent with weight

$$\boxed{z_i = \overset{1}{\alpha} W x_i + \overset{2}{\alpha}\, bias}$$

Because of α which is a +VE value, the magnitude of Z will increase but the increase is proportional throughout

$$P(y=1|x_i) = \frac{1}{1+e^{-z_i}}$$

Similarly $= \frac{1}{1+e^{-z(\alpha w + \alpha\, bias)}}$

~~Sim~~ However, with the $\boxed{\text{probability}}$ there might be

Number-4

$\boxed{\text{Number - 4}}$

(a)

x_1 = hours studied

x_2 = undergrad GPA

Y = receives an A

$\beta_0 = -6, \quad \beta_1 = 0.05, \quad \beta_2 = 1.$

i $\boxed{x_1 = 40}$ $\quad x_2 = 3.5$

We have to use logistic Regression

$$P(Y=1/x) = \frac{1}{1+e^{-z}}$$

$$z = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$

$$= -6 + 0.05\, x_1 + 1 \times x_2$$

$$= -6 + 0.05 \times 40 + 1 \times 3.5$$

$$= 0.5$$

$$P(Y=1/x) = \frac{1}{1+e^{+0.5}} = 0.377 = \boxed{37.7\% \text{ Ans}}$$

$$= \cancel{0.622 = 62.2\% \text{ Ans}}$$

(b) $P(Y=1|X) = 0.5$

$$0.5 = \frac{1}{1+e^{-z}}$$

$$0.5(1+e^{-z}) = 1$$

$$(1+e^{-z}) = 2$$

$$e^{-z} = 1$$

$$e^{-\beta_0 + \beta_1 x_1 - \beta_2 x_2} = 1$$

$$\boxed{e^{-6-0.05x_1-3.5\times 1} = 1}$$

$$-6-0.05x_1-3.5 = \ln(1)$$

$$-6-0.05x_1-3.5 = 0$$

$$+0.05x_1 = +9.5$$

$$\boxed{x_1 = 190}$$

Number 5

5 $J(\beta) = \sum_{i=1}^{N} \ln(1+e^{z_i}) - y_i z_i$

$z_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i}$

for x_{1i} & x_{2i}

(a) $\frac{\partial z}{\partial \beta_0} = \frac{\partial}{\partial \beta_0}[\beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i})$

$= 1 + 0 + 0$

$\boxed{= 1}$ Ans

$\frac{\partial z}{\partial \beta_1} = \frac{\partial}{\partial \beta_1}[\beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i}]$

$= 0 + x_{1i} + 0$

$\boxed{= x_{1i}}$ Ans

$\frac{\partial z}{\partial \beta_2} = \frac{\partial}{\partial \beta_2}[\beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i}]$

$\boxed{= x_{2i}}$ Ans

(b) $\frac{\partial J}{\partial \beta_0} = \frac{\partial}{\partial \beta_0}\left[\sum_{i=1}^{N} \ln(1+e^{z_i}) - y_i z_i\right.$

$= \frac{1}{N}\sum_{i=1}^{N} \frac{\partial}{\partial z_i}(\ln(1+e^{z_i}) - y_i z_i) \cdot \frac{\partial z_i}{\partial \beta_0}$

$= \frac{1}{N}\sum_{i=1}^{N} \left[\frac{e^{z_i}}{1+e^{z_i}} - y_i\right] \cdot 1$

$\boxed{\frac{\partial J}{d\beta_0} = \frac{1}{N}\sum_{i=1}^{N} \frac{e^{z_i}}{1+e^{z_i}} - y_i}$

$$\frac{\partial J(\beta)}{\partial \beta_1} = \frac{1}{N} \sum_{i=1}^{N} \frac{\partial}{\partial z_i} \left[\ln(1+e^{z_i}) - y_i z_i \right] \frac{\partial z_i}{\partial \beta_1}$$

$$\boxed{\frac{\partial J(\beta)}{\partial \beta_1} = \frac{1}{N} \sum_{i=1}^{N} \left[\frac{e^{z_i}}{1+e^{z_i}} - y_i \right] \cdot x_{1,i}}$$

$$\frac{\partial J(\beta)}{\partial \beta_2} = \frac{1}{N} \sum_{i=1}^{N} \frac{\partial}{\partial z_i} \left[\ln(1+e^{z_i}) - y_i z_i \right] \cdot \frac{\partial z_i}{\partial \beta_2}$$

$$\boxed{\frac{\partial J(\beta)}{\partial \beta_2} = \frac{1}{N} \sum_{i=1}^{N} \left[\frac{e^{z_i}}{1+e^{z_i}} - y_i \right] \cdot x_{2i}}$$

(c) Putting $\frac{\partial J}{\partial \beta_0} = 0$

$$0 = \frac{1}{N} \sum_{i=1}^{N} \frac{e^{z_i}}{1+e^{z_i}} - y_i$$

$$y_i = \frac{1}{N} \sum_{i=1}^{N} \frac{e^{z_i}}{1+e^{z_i}}$$

$$\boxed{Ny = \sum_{i=1}^{N} \frac{e^{z_i}}{1+e^{z_i}}}$$