Hidden Opinions

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May 20, 2022

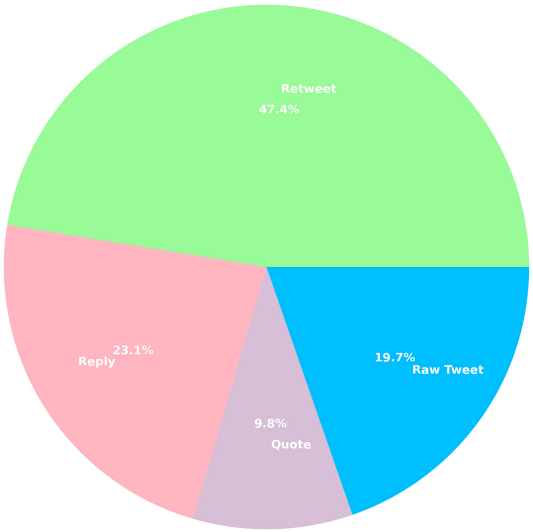
Motivation

- Group **Polarization**
  - ⋆ Empirical studies, regularity: high internal consensus and sharp disagreement across groups. Examples: climate catastrophe...
  - ⋆ Implications: economic, political, institutional... also can be a driver of radicalization, cultural shifts, etc.
  - ⋆ Role of Digital platforms? Online Social networks?
  - ⋆ Is it the **technology** (e.g. echo chambers, algorithms) ? Is it the **actors** themselves?
- Here, **focus of one possible driver** & model it:
  - ⋆ Inequality of attention towards expressed opinions (mix of the technology argument and the individual behavior argument)

Motivation: hidden opinions

Motivation: hidden opinions

- ⋆ Tweets of an "influential" set of actors with +10k followers, that speak about climate over 6 month in 2021
  (independent project, here just to illustrate)
  - Green: Retweets 47% of the corpus, and
  - Blue: only 19.7 of "raw" tweets
  - Take the set of Tweets who have used the hashtag COP26 during COP26: the whole pie becomes green...



Motivation & Idea of the model

- Here, focus on one possible driver & model it:
  - ⋆ Inequality of attention towards expressed opinions: Influence matters
  - ⋆ not all opinions have an impact on a given discourse: hidden opinions, but will matter for the fan base...
- How ? refine DeGroot...
  - ⋆ Introduce an **Expression heuristic**
  - ⋆ Express
    - ◦ Influential individuals **(high centrality)** take part of the discussion and can interact with other influential individuals: **like-minded** or **ideologically-opposed**.
  - ⋆ Hide
    - ◦ Normal individuals **(low centrality)** will have little impact on the conversation (update à la Degroot): e.g. **Retweet** count

Literature: models of opinion dynamics

- ⋆ Large literature about **consensus** or agreement
  - take the average of opinions expressed by all direct contacts, see DeGroot (1974), Harary (1959), French (1956)...
- ⋆ Strand about **disagreement**
  - **Stubborn agents**, see Friedkin(2015), Acemoglu et al (2013), Yildiz et al. (2013), Sadler (2019).
  - **Bounded confidence interval**, see for example, Hegselmann and Krauss (2002), Jager and Amblard (2005).
- ⋆ But ... (social psychology & sociology)
  - Not all individuals are equal... influence matters.
  - Agents may not always share their opinions with others.

Literature: social psychology

- Stasser and Titus (1985) : individuals in social contexts, do not always share their own opinion or the information they hold.
  - ⋆ Political set-up: elect Best, Okay or Ohum.
  - ⋆ The group as a whole had complete (but dispersed) information about Best, from each one's individual perspective Okay appeared more positive than Best.
  - ⋆ Before AND after discussion Best received 25% of endorsement.
- Larson et al. (1996): sharing a unique piece of information during a discussion, is more likely by higher status members rather than lower status members.
  - ⋆ Experiment: residents, interns & 3rd-year medical students.
  - ⋆ Main result: residents were more likely to repeat (unique) information when compared to interns and students.

Model

Model

- **Network**, **Influence** & **Initial Opinions**
  - ⋆ Group of individuals $N = \{1, \ldots, n\}$ embedded in
  - ⋆ ... a connected symmetric network: G, $g_{ij} = g_{ji} \in \{0, 1\}$.
  - ⋆ Set of friends of $i \in N$: $\{j \in N : g_{ij} = 1\}$ and $d_i = |N_i|$.
  - ⋆ The influence or **centrality** of $i \in N$: $\delta_i(G) \in \mathbb{R}_+^*$.
  - ⋆ Exogenous initial opinion for $i \in N$: $\alpha_{i,0} \in [-1, 1]$

Definition

At period $t \geq 0$, for $\tau \in (0, 1)$, $i \neq j \in N$ are:

- **like-minded**: $|\alpha_{i,t} - \alpha_{j,t}| < \tau$
- **ideologically-opposed**: $|\alpha_{i,t} - \alpha_{j,t}| \geq \tau$

Model

Opinion Updating

For $i \in N$, with centrality $\delta_i(G)$

$$\begin{cases} \text{Hide (rule 1): DeGroot} & \text{if } \delta_i(G) < \delta^*(G) \\ \text{Express (rule 2): see Below} & \text{if } \delta_i(G) \geq \delta^*(G) \end{cases}$$

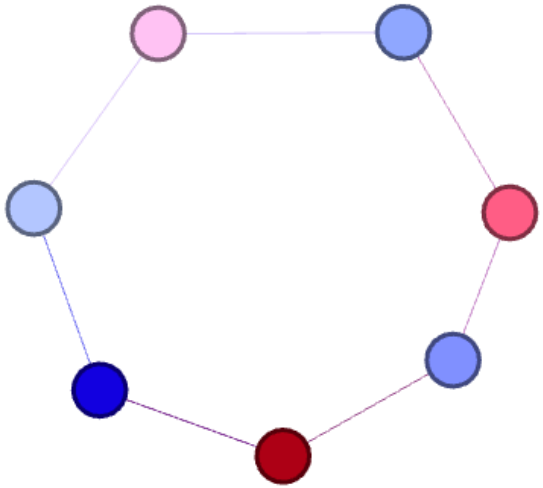
- Rule 2
  - ⋆ $\forall j \in N_i$, such that $\delta_j(G) \geq \delta^*(G)$
  - ⋆ $\mu \in (0, 1/2)$ and $\alpha_{i,t} \in [-1, 1]$

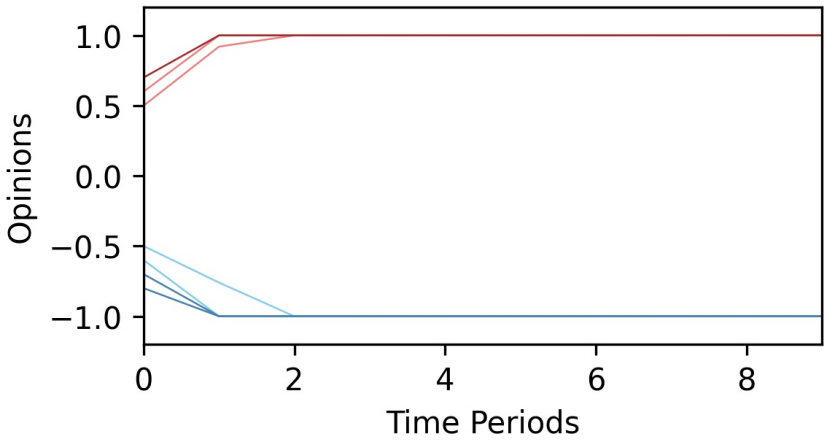
$$\alpha_{i,t} = \alpha_{i,t-1} + \mu\Big(\underbrace{\sum_{j \in \underline{N}_{i,t-1}} (\alpha_{j,t-1} - \alpha_{i,t-1})}_{\text{Attractive Effect}} - \underbrace{\sum_{j \in \overline{N}_{i,t-1}} (\alpha_{j,t-1} - \alpha_{i,t-1})}_{\text{Repulsive Effect}}\Big)$$

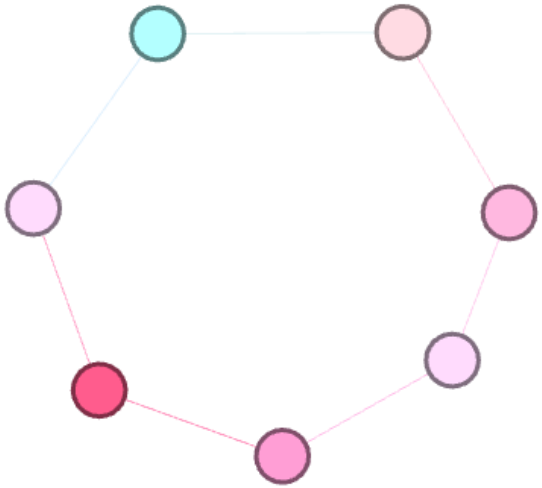
Like-minded neighbors $\underline{N}_{i,t}$ **Ideologically-opposed** neighbors $\overline{N}_{i,t}$

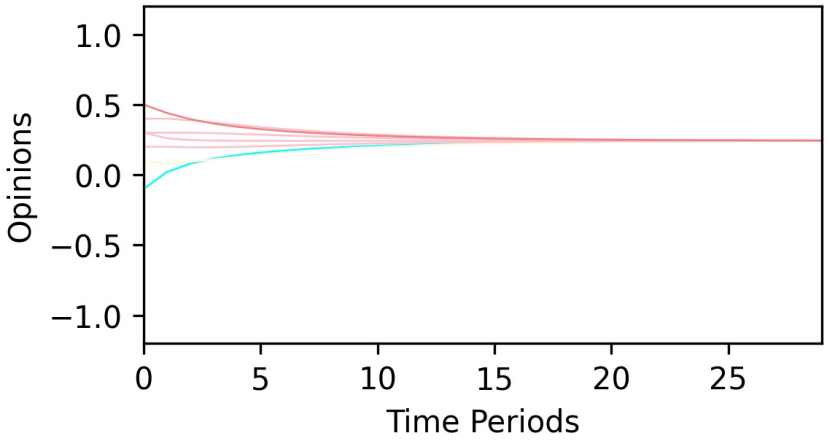
How does Rule 2 work? some examples!

- ⋆ Hypo: threshold opinion difference $\tau = 0.5$ & same influence



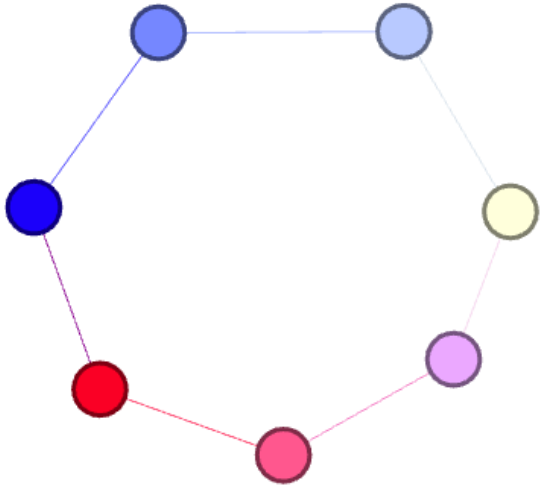


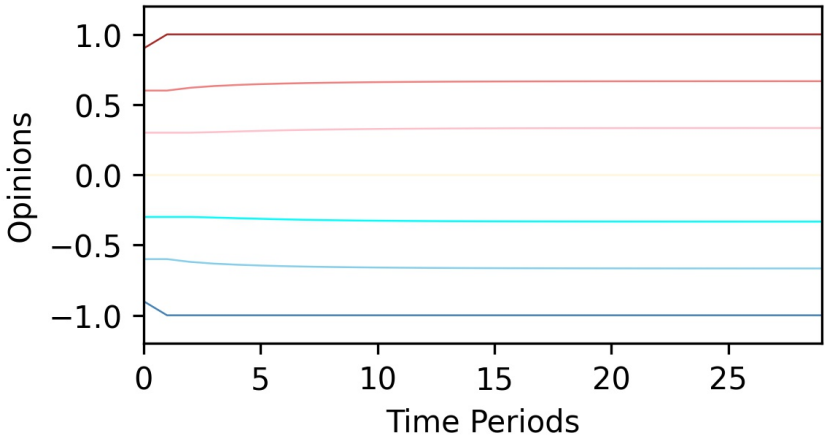


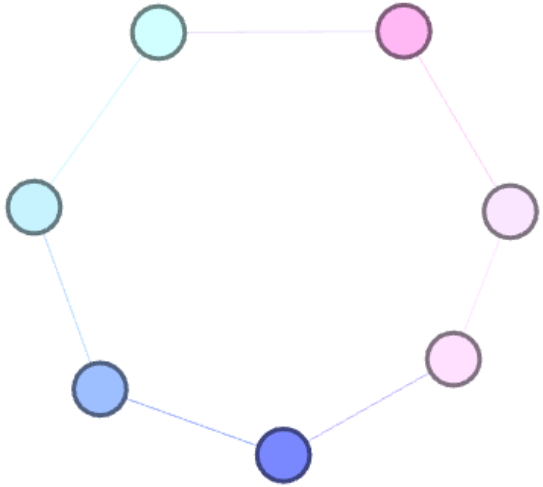


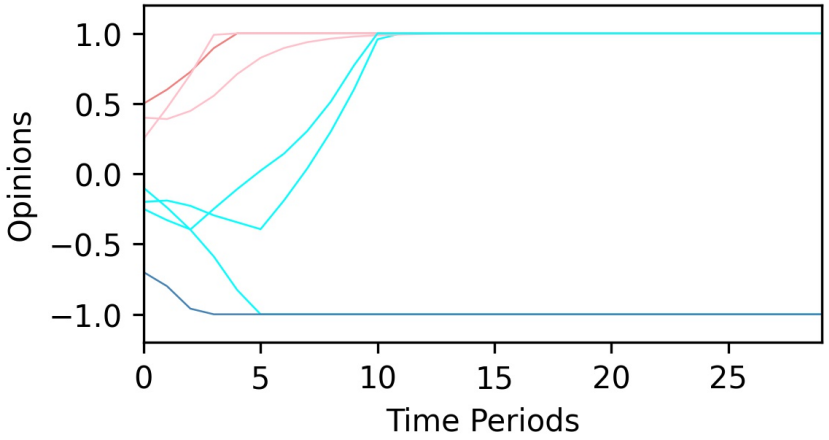
How does Rule 2 work? more examples!

- ⋆ Hypo: threshold opinion difference $\tau = 0.5$ & same influence









Long-run opinions? Hearing matrix

- Opinions of "expressers" (influential players) and the way they are connected governs the dynamics of opinions...

Proposition

(*i*) If i has exactly k neighbors who *express* and they are all **like-minded**, then $\exists t^* \geq t,\ \forall t' \geq t^*$: $a_{i,t'} = (\alpha_{i,0} + \alpha_{k_1,0} + \ldots + \alpha_{k_k,0})/k$

(*ii*) If player i has, at every period, **at least one neighbor** j who expresses and **ideologically opposed**, then $\exists t^* \geq t,\ \forall t' \geq t^*,\ \ a_{i,t'} = -1,\ a_{j,t'} = 1$

(*iii*) If player i has, at every period, **like-minded neighbors** who express and is **linked by a path** to a pair of ideologically opposed expressers, then $a_{i,t} = \alpha_{i,0}$

Long-run opinions? Hearing matrix

- Lemma : when a pair of expressers are initially **like-minded** they take a **longer time** to reach **consensus**
- but when the pair is ideologically-opposed they **disagree at a much faster rate**

Lemma

Let $i \neq j \in \mathcal{E} \subset E$ s.t. $|\mathcal{E}| = 2$ & $t_{\alpha_\infty} = \min\{t : |\alpha_t - \alpha_\infty| < \epsilon\}$.
If α^a_∞ is the long-run opinion vector when $|\alpha_{i,0} - \alpha_{j,0}| < \tau$ and α^r_∞ is the long-run opinion vector when $|\alpha_{i,0} - \alpha_{j,0}| \geq \tau$ then $t_{\alpha^r_\infty} < t_{\alpha^a_\infty}$.

Long-run opinions? Hearing matrix

- t^*: time period where repulsion is exhausted.
  - ⋆ by which the least ideologically-opposed pair of directly connected expressers, have repulsed each other to reach opinions at the upper and lower bound of the opinion interval.
- For any $t \geq t^*$, build a hearing matrix $\tilde{G}$ which takes into account both updating rules :
  - ⋆ Hide (rule 1): so for low influence players $\tilde{g}_{ij} = g_{ij}/d_i$
  - ⋆ Express (rule 2): three cases, either stubborn so $\tilde{g}_{ij} = 0$ for all j, or k like minded friends who express $\tilde{g}_{ij} = 1 - k\mu$, or at least one ideologically-opposed friend $\tilde{g}_{ii} = 1$ and $\tilde{g}_{ij} = 0$

Long-run opinions? Hearing matrix

Given the vector $\mathbf{a}_t = (a_{1,t}, \ldots, a_{n,t})$ for $t \geq t^*$, the evolution of opinions is now given by:

$$\mathbf{a}'_{t+1} = \tilde{G}\mathbf{a}'_t \tag{1}$$

Long-run opinions $\mathbf{a}_\infty$?

- Long-run opinion is simply: $a_\infty = (\lim_{t\to\infty} \tilde{G}^t) a_{t^*}$
- Entries of $(\lim_{t\to\infty} \tilde{G}^t)_{ij}$? how much eventual weight in the final belief of player i that each player j's initial belief has.
- $\tilde{G}$ is row-stochastic with strictly positive diagonal entries. I will use this to compute the limit...

Convergence

Theorem

Let a_{t^*} be the vector of opinions at period t^* and $\tilde{G}$ a row stochastic hearing matrix with strictly positive diagonal entries. $(\lim_{t\to\infty} \tilde{G}^t)$ exists and

$$\mathbf{a}'_\infty = (\lim_{t\to\infty} \tilde{G}^t) a_{t^*} = \mathcal{G} a_{t^*}$$

where $\mathcal{G}$ is the spectral projector associated with the eigenvalue $\lambda = 1$.

Proof

- ⋆ Intuition? expressers are like "absorbing states" (form an essential class, Seneta 1981).
- ⋆ $(\lim_{t\to\infty} \tilde{G}_\theta^t)_{ij} = \mathcal{G}_{ij}$ is "the probability that a random walk initiated at i is absorbed by node j".

Spectral influence

- ⋆ Entries of $\mathcal{G}$? how much eventual weight in the final beliefs of the society that each expresser's initial belief has.
- ⋆ Total influence of an expresser given by the following statistic, which I call spectral influence:

$$s_i = \frac{1}{n}(\mathcal{G}'\mathbf{1})_i$$

- ⋆ For any consensual player i (Rule 1), $s_i = 0$, while for *expressers* (Rule 2) $s_i > 0$.

Simulations

- ⋆ see: github.com/shadenshabayek/hidden_opinions_2022

Simulations

- Centrality measure ? Local popularity or expertise

$$\delta_i = \frac{d_i - 1}{\sum_{j \in N_i} d_j - 1}$$

  - ⋆ The lower the neighbors' influence sources, the higher the local popularity of i.
  - ⋆ **Dynamic Social Impact Theory** (Latané 96): marginally decreasing impact of increasing numbers of sources
  - ⋆ **Persuasiveness strength** : a very simple way to model probability of persuasiveness.
- Polarization ? Keep It Simple Stupid ...
  - ⋆ For now simply **variance in long-run opinions** (many measures of course: Esteban and Rey, appealing but α parameter?)

Simulations...

- **Scale free network**: Barabasi Albert Graph, 600 nodes
  - ⋆ randomly generate initial opinions
  - ⋆ here 2 runs with 6 initial hubs
  - ⋆ play with the **local centrality threshold**...
  - ⋆ play with the **number of hubs**...

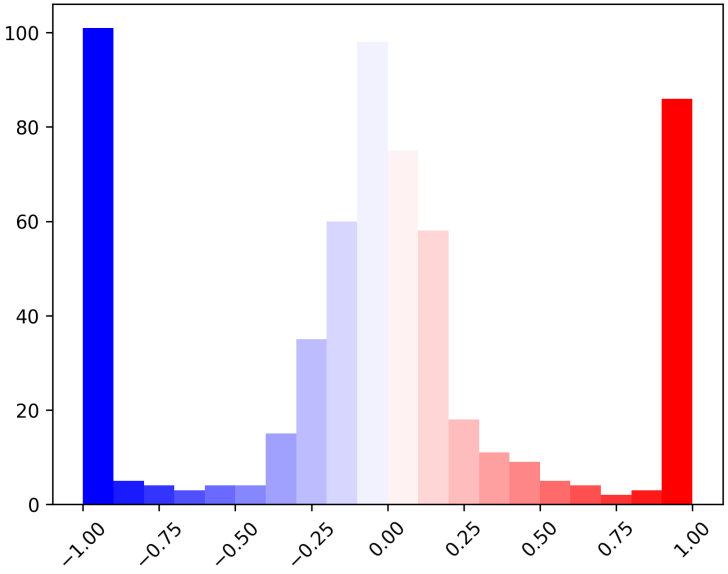


Figure: 240 expressers, ($v = 0.36$)

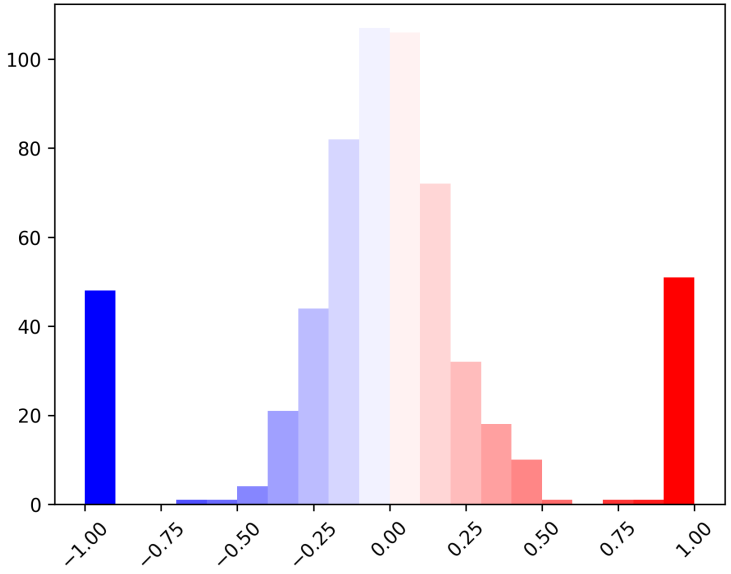


Figure: 122 expressers, ($v = 0.19$)

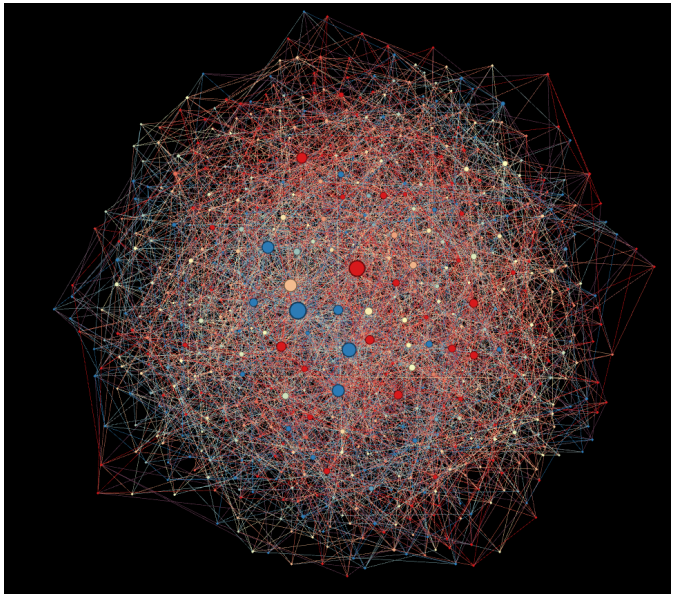


Figure: viz with 240 expressers

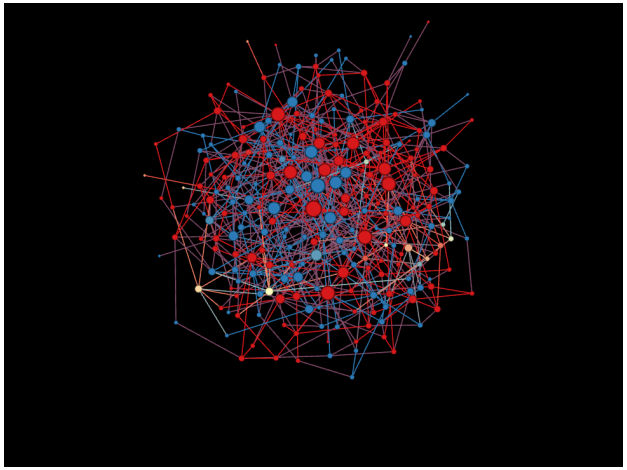
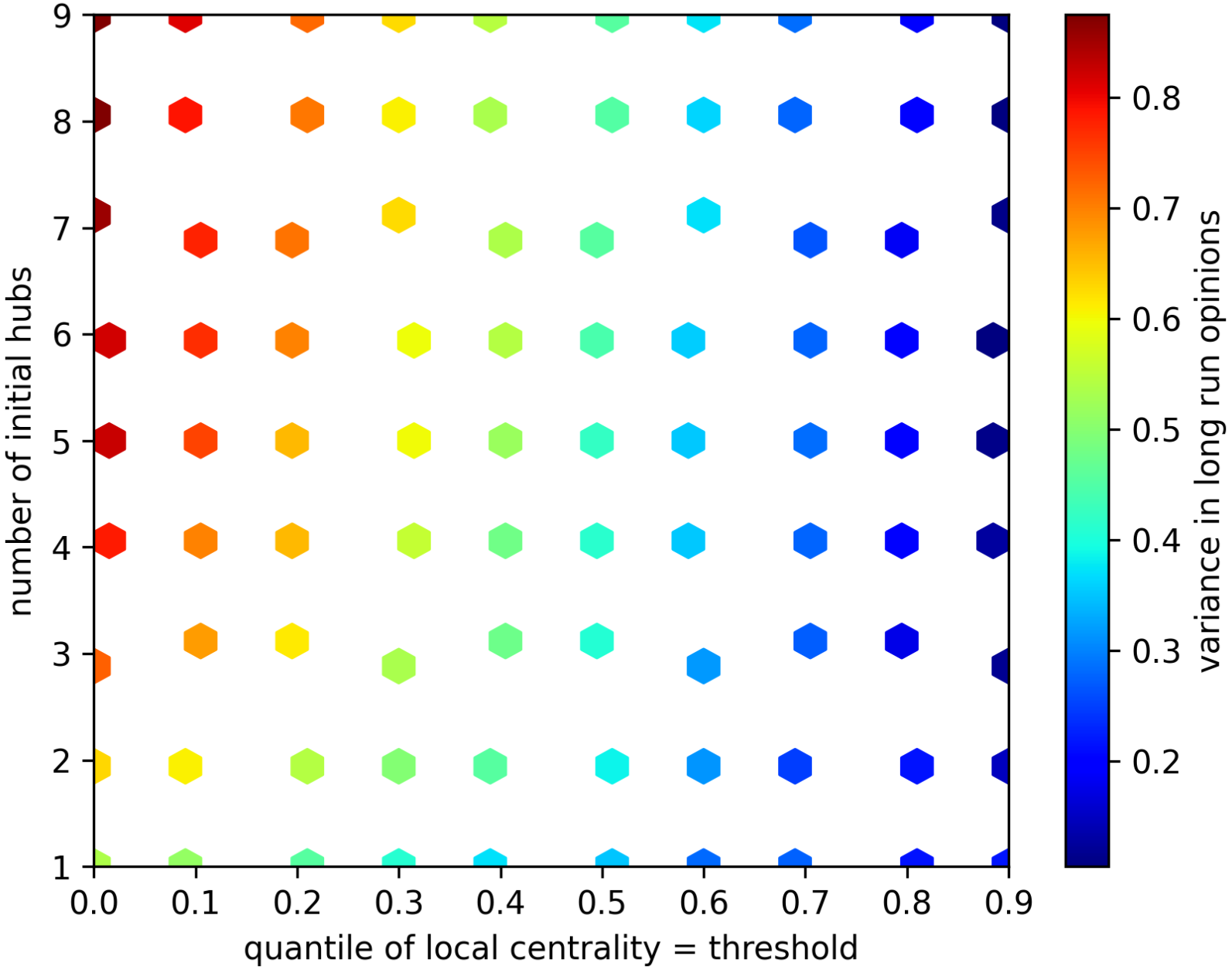


Figure: subgraphs Expressers

	Run 1	**Run 2**
Diameter	4	4
Modularity	0.24	0.24
Deg assortativity	-0.073	-0.036
subgraph of expressers: number of components	**4**	**19**

Simulations



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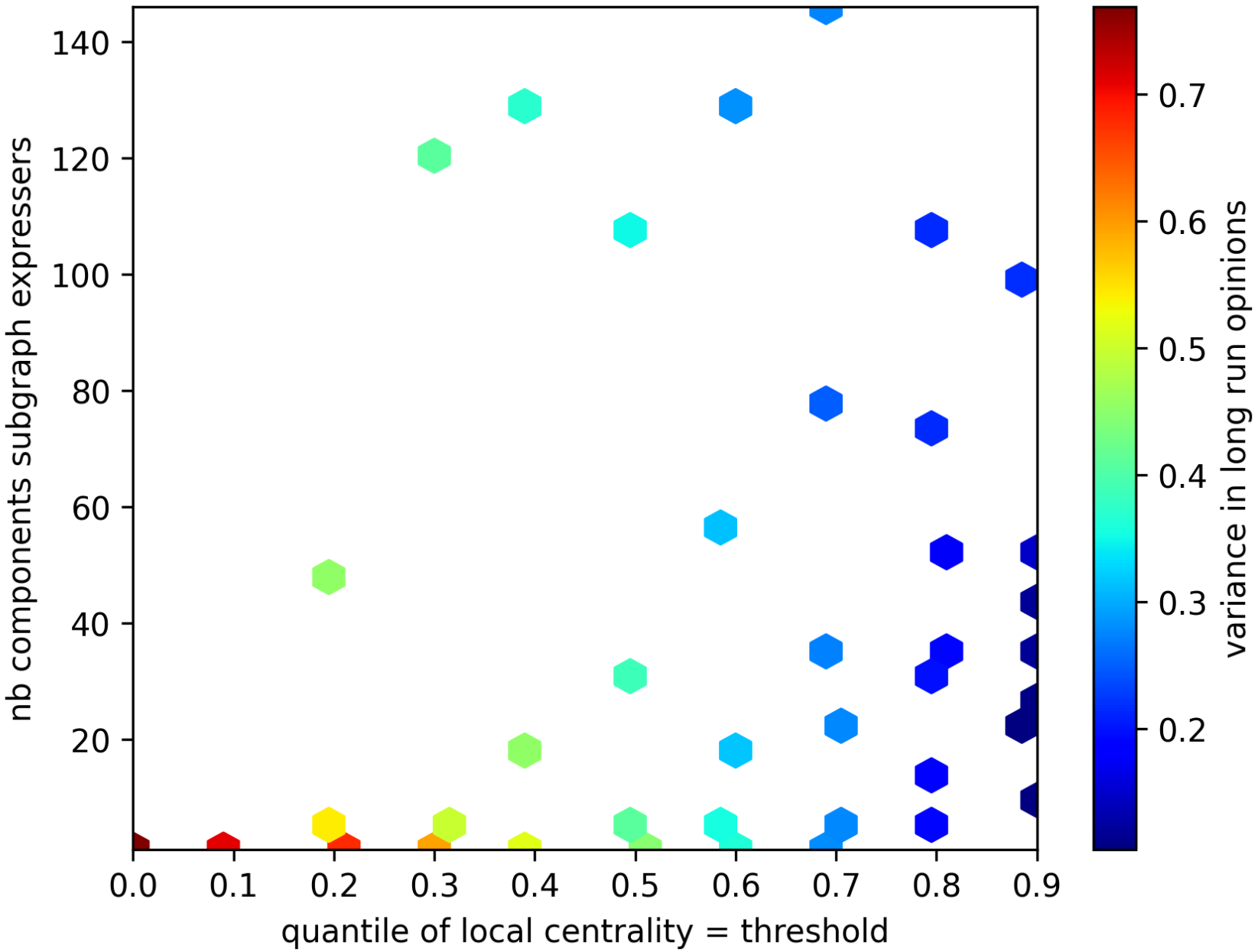

Way forward? Implications?

That's all Folks!