Project 1
Quantum Walks and Monte Carlo

Introduction

From the paper *"Quantum Galton Boards"*, understand that the classical Galton board is a device where balls fall through a triangular array of pegs, making left or right decisions at each peg. The outcome is a binomial distribution of the number of steps taken in one direction, which for many layers approximates a Gaussian (Normal) distribution.

The paper explores a quantum analogue of this process *Quantum Galton Board (QGB)* where the "balls" are replaced by quantum states, and the "left/right" decisions are replaced by quantum operations. This allows superposition, interference, and entanglement to shape the distribution of outcomes in ways that have no direct classical counterpart.

Concept of a Quantum Galton Board

In the quantum version, each decision point is represented by a qubit.

- In the unbiased case, we apply a Hadamard gate to put the qubit in a superposition of $|0\rangle$ and $|1\rangle$, giving equal probabilities for both outcomes.
- In the biased case, we apply a rotation gate such as $R_y(\theta)$, which changes the relative probability of 0 and 1 according to the angle θ.

Instead of physically dropping a ball through pegs, the quantum circuit *evolves* through a series of these "coin flip" gates (one per layer). At the end, measuring all qubits gives a bitstring, and the number of 1s corresponds to the position where the ball would land in a classical board. Because this is quantum, amplitudes can interfere, meaning that some output positions can be enhanced depending on the structure of the circuit.

Models Described in the Paper

The paper had several ways to construct QGBs:

1. **Standard Bernoulli-sum model**

   Each qubit is independent and undergoes a single- or biased-coin operation.The probability distribution after measurement matches a classical binomial when unbiased.

2. **Biased QGB**

   Instead of a 50-50 Hadamard, the rotations bias the probabilities.With appropriate biasing, the output can approximate non-Gaussian shapes such as an exponential decay.

3. **Hadamard Quantum Walk model**

   Adds controlled-phase (CZ) interactions between neighboring qubits after the Hadamard step.This introduces quantum interference patterns similar to discrete-time quantum walks, creating distributions very different from classical ones.

Measurement and Output Analysis

After running the quantum circuit, the measurement results are collected as bitstrings. The important quantity is Hamming weight, how many 1s appear in the result. By counting how often each Hamming weight occurs, we obtain the position histogram analogous to the balls' final positions in the classical board.

For large numbers of layers, the unbiased QGB gives a distribution close to Gaussian, which can be checked by comparing the measured histogram to a normal distribution with the same mean and standard deviation. Metrics like Total Variation (TV) distance, Jensen–Shannon (JS) divergence, and Kullback–Leibler (KL) divergence quantify how close the observed distribution is to the target distribution.

Why QGBs are Interesting

Demonstrating quantum superposition and interference in an intuitive "falling ball" analogy. Studying biased quantum processes and how parameterized gates change statistical outcomes. Testing quantum walks and their statistical properties on real quantum hardware. Providing a platform for benchmarking quantum processors, since the target distributions are analytically known for certain cases.

Implementation Principles

1. From the paper and related resources, the essential steps for implementing a QGB are:
2. Assign one qubit per layer (decision point).
3. Apply the "coin" operation to each qubit:
   i) Hadamard for unbiased case.
   ii) $R_y(\theta)$ for bias.
4. Add entangling gates (CZ between neighbors) for interference-based models.
5. Measure all qubits and record the bitstrings.
6. Map results to Hamming weights to get the position histogram.
7. Compare to a theoretical target distribution to analyze behavior.

Takeaways

The quantum Galton board is essentially the quantum circuit representation of a random walk in a discrete space, with control over both bias and interference. The simplicity of its structure makes it suitable for running on actual quantum devices, yet rich enough to explore deeper quantum statistical effects.

The main insight is that even something as simple as the Galton board a classical physics toy becomes a powerful tool for exploring the boundary between quantum and classical behavior when implemented with qubits. By varying the gates and interactions, we can smoothly transition from classical-like binomial distributions to highly non-classical interference patterns, all within the same conceptual framework.

References

- Carney, M. & Varcoe, B., *Universal Statistical Simulator*, arXiv:2202.01735 (2022).
- The Galton Board: Randomness and the Gaussian Curve (https://conten.to/post/2024/12/19/)