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Winter 2020 Assignment 2

COMP 233 Assignment 2 Solution

Question 1 if $E(X) = 2$ and $E(X^2) = 8$. Calculate the following:

a) $E((2+4X)^2)$

$$E(2^2 + 16X^2) = 4 + 16E(X^2) = 4 + 16 \cdot 8 = 132$$

b) $E(X^2 + (X+1)^2)$

$$E(X^2) + E((X+1)^2) = 8 + E(X^2 + 1) = 8 + 8 = 16$$

Question 2:

a) Find the marginal probability:

$X_1 \backslash X_2$	1	2	Total
0	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{3}{16}$
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{2}{16}$
2	$\frac{3}{16}$	$\frac{1}{8}$	$\frac{5}{16}$
3	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$
Total	$\frac{1}{2}$	$\frac{1}{2}$	1

b) Find:

a. $E(X_1)$

$$= \sum_{x \in X} x f(x) = 0 \cdot \frac{3}{16} + 1 \cdot \frac{2}{16} + 2 \cdot \frac{5}{16} + 3 \cdot \frac{6}{16} = \frac{15}{8}$$

b. $E(X_2)$

$$= \sum_{x_2 \in X_2} x_2 f(x_2) = 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{2} = \frac{3}{2}$$

c. $Var(X_1)$

$$Var(X_1) = E(X_1^2) - \big(E(X_1)\big)^2 = \left(\sum_{x \in X} x^2 f(x)\right) - \left(\frac{15}{8}\right)^2 =$$

$$\left(0^2 \cdot \frac{3}{16} + 1^2 \cdot \frac{2}{16} + 2^2 \cdot \frac{5}{16} + 3^2 \cdot \frac{6}{16}\right) - \left(\frac{15}{8}\right)^2 = \frac{79}{64}$$

d. $Var(X_2)$

$$= Var(X_2) = E(X_2^2) - \big(E(X_2)\big)^2 = \left(\sum_{x_2 \in X_2} x_2^2 f(x_2)\right) - \left(\frac{3}{2}\right)^2 =$$

$$= \left(1^2 \cdot \frac{1}{2} + 2^2 \cdot \frac{1}{2}\right) - \frac{9}{4} = \frac{1}{4}$$

e. $Cov(X_1, X_2)$

$$Cov(X_1, X_2) = E(X_1 X_2) = E(X_1)E(X_2) = \frac{15}{8} \cdot \frac{3}{2} = \frac{45}{16}$$

Question 3:

a) $P(X \geq a) \leq \frac{E(X)}{a} = P(X \geq 85) \leq \frac{75}{85} \rightarrow 0.8823$
b) $P(|X - \mu| \geq k) \leq \frac{\sigma^2}{k^2} = \frac{25}{(85-65)^2} = 0.0625$
c) $P(|X - 75| \leq 5) \geq 0.9 \rightarrow X \leq 80\ Students$

Question 4:

a) $P\{X = 4\} = C_4^n p^4 (1-p)^{n-4}$, However, we know that $E(X) = np = 7$ and $Var(X) = np(1-p) = 2.1$. Hence:

$$n = \frac{7}{p}$$

$$2.1 = \frac{7}{p} p(1-p) \rightarrow 2.1 = 7 - 7p$$

$$p = \frac{7}{10}$$

$$n = \frac{7}{\frac{7}{10}} = 10$$

$$P\{X = 4\} = \frac{10!}{(10-4)!\,4!} \cdot \left(\frac{7}{10}\right)^4 \cdot \left(\frac{3}{10}\right)^{10-4}$$

$$P\{X = 4\} = 210 \cdot 0.2401 \cdot 0.00073 = 0.03681$$

b) $P\{X > 12\} = 1 - P(X \leq 12)$:

$$P(X \leq 12) = \sum_{x \leq 12} C_x^n p^x (1-p)^{n-x} = 1.00716$$

$$P\{X > 12\} = 1 - 1.00716 = 0$$

Question 5:

$$P\{N_\lambda = k\} = e^{-\lambda} \frac{\lambda^k}{k!}$$

The chance to get 0 cold per year with $\lambda = 3$:

$$e^{-3}$$

The chance to get 0 cold per year with $\lambda = 2$:

$$e^{-2}$$

Let D denote "Drug is beneficial" and N "has 0 cold in the year":

$$P(D|N) = \frac{P(DN)}{P(N)} = \frac{e^{-3}}{e^{-2}} = 0.36787$$

Question 6:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, E(X) = \mu, \ \ Var(X) = \sigma^2$$

We have $\mu = 1000, \sigma = 200$

a) $P(X < 1100) = \int_{-\infty}^{1100} \frac{1}{\sqrt{2\pi}200} e^{-\frac{(x-1000)^2}{2(200)^2}}$

$$z = \frac{1100 - 1000}{200} = 0.5 \rightarrow Implies\ \phi(0.5)$$
$$= 0.6915\ for\ 1\ week$$

To get the probability for 2 weeks:

$$0.6915^2 = 0.4782$$

b) $P(X > 2200)$, Where $X_{total} = X_{Week1} + X_{week2}$

$$\mu = 2(1000) = 2000$$

$$\sigma = \sqrt{n\sigma_i^2} = \sqrt{2 \cdot (200)^2} = 200\sqrt{2}$$

Now we have:

$$P(X > 2200) = 1 - P(X \leq 2200)$$

$P(X \leq 2200)$ follows a $\phi(z)$ where $z = \frac{2200-2000}{200\sqrt{2}} = 0.70710$

$$\phi(0.70710) = 0.7611$$

Which means that $P(X > 2200) = 1 - 0.7611 = 0.2389$

Question 7:

Percentage of bolt no meeting requirement: $1 - P\{1.19 \leq X \leq \ 1.21\}$

$$z_a = \frac{a - 1.20}{0.005} = \frac{1.19 - 1.20}{0.005} = -2$$

$$z_b = \frac{b - 1.20}{0.005} = \frac{1.21 - 1.20}{0.005} = 2$$

$$P\{-2 \leq Z \leq \ 2\} = P\{Z \leq 2\} - P\{Z \leq -2\}$$

$$\phi(2) - [1 - \phi(2)]$$

$$P\{-2 \leq Z \leq \ 2\} = 0.9772 - (1 - 0.9772) = 0.9544$$

Now we can find $1 - P\{-2 \leq Z \leq \ 2\}$ which is equal to 0.0456

Question 8:

$$P(L \leq t) = 1 - e^{-\lambda t}$$

$$P(L > 10) = 1 - P(L \leq 10)$$

$$P(L \leq 10) = 1 - e^{-\frac{1}{8} \cdot 10} = 0.7135$$

$$P(L > 10) = 1 - 0.7135 = 0.2865$$