Concordia University, Individual project, Winter 2020

Course: Applied Advanced Calculus, ENGR233
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1. Answer:
   a. $AB = (0,1,0), AC = (0,0,1), BC = (0,-1,1)$
   $$Area = \frac{1}{2}|AB| \cdot |BC| = \frac{1}{2} \cdot 1 \cdot \sqrt{2} = \frac{\sqrt{2}}{2}$$
   b. Intersection between 2 planes:
   $$x + y + z = 1\,, x - y + 0z = 0$$
   - $x = 1 - y - z\ AND\ x = y$
   $$1 - y - z = y$$
   $$z = 1 - 2y$$
   $$y = y$$
   $$x = 1 - y - (1 - 2y) = 0 + y = y$$
   $$l = x = y = \frac{-z + 1}{2}$$
   - So, intersection:
     1. $l = \langle 1, 1, -2 \rangle \cdot t + (0,\ 0, 1\ )$

   - Now Calculate the distance:
   $$\overrightarrow{PP'} = (t, t, -2t + 1) - (1,0,0) = \langle t - 1, t, -2t + 1 \rangle$$
   $$\overrightarrow{PP'} \perp l = \overrightarrow{PP'} \cdot u = 0$$
   $$\begin{pmatrix} t - 1 \\ t \\ -2t + 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = 0$$
   $$t - 1 + t - 4t + 2 = 0$$
   $$-2t + 1 = 0$$
   $$t = \frac{1}{2}$$
   Coordinates of $\overrightarrow{PP'} = \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix}$. Hence, **Distance =** $\left|\overrightarrow{PP'}\right| =$
   $\sqrt{0.25 + 0.25 + 0} = \frac{\sqrt{2}}{2}$
2. Answer:
   a.
   b. $T(t) = \frac{r'(t)}{|\boldsymbol{r}'(\boldsymbol{t})|} = \frac{\langle 2t+1, 2t-1, 1 \rangle}{\sqrt{8t^2+3}} = \langle \frac{2t+1}{\sqrt{8t^2+3}}, \frac{2t-1}{\sqrt{8t^2+3}}, \frac{1}{\sqrt{8t^2+3}} \rangle$

- Now find $L_1 and\ L_2$:
  1. $L_1 = \langle \frac{3\sqrt{11}}{11}, \frac{\sqrt{11}}{11}, \frac{\sqrt{11}}{11} \rangle$
  2. $L_2 = \langle \frac{-\sqrt{11}}{11}, \frac{-3\sqrt{11}}{11}, \frac{\sqrt{11}}{11} \rangle$

3. a) $\nabla f = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} i + \frac{\partial f}{\partial z} i$

$$\nabla f = \langle 0, 0,0 \rangle$$
$$directional\ dir = \nabla f(1, -2,1) \cdot u$$
$$u = \frac{\langle -1, 0,2 \rangle}{\|\langle -1, 0,2 \rangle\|} = \langle -\frac{\sqrt{5}}{5}, 0, \frac{2\sqrt{5}}{5} \rangle$$
$$directional\ dir = 0$$

b) $\nabla f = \left(2xe^{x^2-1} + 2(x^2 + y^2 + 1)e^{x^2-1}xln(e)\right)i + 2ye^{x^2-1}j + 0k$

$$\nabla f(-1, 1, 3) = (-2 - 6\ln(e))i + 2j + 0k$$

Thus, the equation is: $\left(x - (-2 - 6)\right) - (y - 2) - (z - 0) = \boldsymbol{x - y - z = 6}$

4. Answer:

a) $W = \int_C ydx + xdy,$

$$W = \int_1^e ydx + \int_0^1 xdy = y(e - 1) + x|_{(e-1,1)} = e\ Watt$$

b) An integral is path-independent if $\int_C F \cdot dr = 0$

$F(x, y, z) = 8(2\cos t)(2\sin t)^3(t) + 12(2\cos t)^2(2\sin t)^2 t + 4(2\cos t)^4(2\sin t)^3$

$$dr = \left(-2sin(t)\right)i + 2cos(t)j + 1k$$
$$\int_C F \cdot r = \int_0^\pi F \cdot r = \frac{512}{21}$$

So, the integral in not path-independent

5. Answer:

a. $\iint_R \left(\frac{\partial(xy+xy^2)}{\partial x} - \frac{\partial\left(\frac{y^3}{3}\right)}{\partial y} \right) dxdy$

Upper y limit: $y^2 = 1 - y^2 \to y = \frac{\sqrt{2}}{2}$

$$\int_0^{\frac{\sqrt{2}}{2}} \int_{y^2}^{1-y^2} ydxdy = \frac{1}{8}$$

b. $\int_0^4 \int_{\sqrt{y}}^2 \sqrt{x^3 + 1}dxdy = \int_0^2 \int_{x^2}^2 \sqrt{x^3 + 1}dydx = 5.7778$

6. Age
7. Answer:

$$\int_{x-\pi}^{x} \int_{\frac{y-3}{-3}}^{\frac{y-6}{-3}} \frac{\cos\left(\frac{1}{2}(x-y)\right)}{3x+y}\, dxdy = -0.117\, cos(0.667x) + \ 0.58.\, sin(0.667x)$$