Lecture 2
1.2 Initial-value problems

Definition 1. The problem

$$\frac{d^n y}{dx^n} = f(x, y, y', \dots, y^{(n-1)}), \quad x \in I$$

subject to

$$y(x_0) = y_0, \;\; y'(x_0) = y_1, \;\; \dots \;\; , y^{(n-1)}(x_0) = y_{n-1}$$

where $x_0 \in I$, $y_0, y_1, \dots, y_{n-1}$ are some specified real constants, is said to be **an initial-value problem (IVP)**.

Definition 2. Conditions

$$y(x_0) = y_0, \;\; y'(x_0) = y_1, \;\; \dots \;\; , y^{(n-1)}(x_0) = y_{n-1}$$

are said to be **initial conditions**.

Existence-uniqueness of a solution of IVP

Theorem 1. Let **R** be a rectangular region in the xy-plane defined by $a \leq x \leq b$, $c \leq y \leq d$ that contains the point (x_0, y_0) interior. If $f(x, y)$ and $\partial f/\partial y$ are continuous on **R**, then there exists a unique solution $y = y(x)$ of IVP $\frac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$.

2. First order differential equations
2.2 Separable equations

Definition 3. A first order differential equation of the form

$$\frac{dy}{dx} = g(x) \cdot h(y)$$

or

$$M(x) \cdot N(y)dx + P(x) \cdot Q(y)dy = 0$$

is said to be **a separable** or to have separable variables.

Method of solution

1. Divide the equation by $N(y) \cdot P(x)$

$$\frac{M(x)}{P(x)}dx + \frac{Q(y)}{N(y)}dy = 0.$$

2. Integrate the last equation directly.