COMP228-DD Computer Organization & Design Winter 2020
Assignment 2
Due: February 19th, 2020

Submit on Moodle before 17:45 and typed hardcopy in class.

1. [21 marks] Digital Logic.

Let 'X' be the ternary connective such that 'Xpqr' is logically equivalent to '(p --> q) /\ (q --> r)'. We have: p --> q |= =| ~p \/ q. Here, 'F' and 'T' denote the 0-place connectives 'false' and true, respectively. There are some constraints. In a), show a solution with one 'F'. In b), show a solution with the letters in alphabetical order. In c), show a solution with one 'p' and the letters in alphabetical order (ignore negation).

a) Using {'X', 'F'}, synthesize: ~p |= =| X ___ ___ ___

b) Using {'X', 'T'}, synthesize: p /\ q |= =| X ___ ___ ___

c) Using {'X', '~'}, synthesize: p \/ q |= =| X ___ ___ ___

2. [18 marks] Binary and Hexadecimal Numbers.

Convert each of the following six binary or hexadecimal natural numbers into decimal. Show work. The last number is large. Therefore, for it, show the last work step, and the number's value in scientific notation.

a) Binary numbers: 11101, 11101 11110, 11101 11110 11101 10111

b) Hexadecimal numbers: ae1, aeba1, aeba1 51de1

3. [25 marks] Fractional Numbers and Blackboard Notation.

Infinite binary expansions of rational numbers are either pure recurring or mixed recurring depending on whether the cycle starts immediately after the point.

a) [math] Show the infinite binary expansion of 1/9.

b) [math] Represent this infinite binary expansion in hexadecimal.

c) [math] Show the infinite binary expansion of 4/9. Don't change the cycle.

d) [math] Represent this infinite binary expansion in hexadecimal.

e) Show the normalized blackboard floating-point notation that best approximates 3 4/9. The fractional field is 16 bits. Show all 16 of them. Now, show just the 16-bit (4-hexit) fractional field, after normalization, in hexadecimal.

4. [16 marks] Integer Multiplication I.

a) Multiply the following two 10-bit binary natural numbers. The multiplicand is 10011 11100 (27c hex) and the multiplier is 11010 (1a hex).

Show, in hexadecimal, i) the initial value of the accumulator, ii) each term added to the accumulator _and_ the resulting partial sum. The last addition yields the final result.

b) Redo the multiplication steps exactly as in question 4 a), but initialize the accumulator to s = 11011 (1b hex) instead of 0. Show the same intermediate and final values. (This is called "fused multiply-add").

5. [20 marks] Integer Multiplication II.

a) Show that, regardless of the initial n-bit value of the accumulator, the fused multiply-add result of two n-bit natural-number operands is always representable in 2n bits. Now, suppose n = 16. Starting from the largest possible FMA result, what is the hexadecimal representation of the largest n = 16-bit number that can _still_ be added without producing overflow?

b) A modular-adder device 'M' operates with 16-bit registers. You give it two 16-bit natural numbers 'a' and 'b'. It adds them, divides by 2^16, keeps the quotient 'q' a secret, and publishes the remainder 'r'. Hint: Before answering, experiment with small addition tables.

i) If a = 23,979 and r = 63,400, what are 'b' and 'q'?

ii) If a = 33,472 and r = 8,047, what are 'b' and 'q'?