COMP-232

MATHEMATICS FOR COMPUTER SCIENCE

Fall 2019

Assignment #3

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PROBLEM 1: Let A and B be sets. Prove that A ⊆ B if and only if P(A) ⊆ P(B).

Let $A = \{a\}$ and $B = \{a, b\}$. Let's define $P(A)$ and $P(B)$:

- $P(A) = \{\emptyset, \{a\}\}$
- $P(B) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

We can see here that $P(A)$ is a subset of $P(B)$, because $P(B)$ contains $\emptyset, \{a\}$ which is equal to $P(A)$. Also, A is a subset of B because B contains a. Therefor it is a proof that $A \subseteq B \leftrightarrow P(A) \subseteq P(B)$

PROBLEM 2: Let A, B, C, and D be sets. Prove or disprove the following:

$$(A \cap B) \cup (C \cap D) = (A \cap D) \cup (C \cap B).$$

Let $A = \{a\}, B = \{b\}, C = \{c\}, D = \{?\}$

$$(\{a\} \cap \{b\}) \cup (\{c\} \cap D) = (\{a\} \cap D) \cup (\{c\} \cap \{b\})$$

$$\{\ \} \cup (\{c\} \cap D) = (\{a\} \cap D) \cup \{\ \}$$

$$(\emptyset \cup \{c\}) \cap (\emptyset \cup D) = (\{a\} \cup \emptyset) \cap \{D \cup \emptyset\}$$

$$\{c\} \cap D = \{a\} \cap D$$

So here we can see that the statement is only true if and only if $\{c\} \cap D = \emptyset$ and $\{a\} \cap D = \emptyset$. In other words, the set D has no common elements with neither C nor A. For this reason, we can conclude that the whole statement is not always true, because D can be something like $D = \{a, x, y, z\}$ so,

$$\{c\} \cap D = \{a\} \cap D$$

Will be,

$$\emptyset = a$$

Which is not true. Therefor the statement if false.

PROBLEM 3: Give an example of two uncountable sets A and B such that A – B is:

a) Countably Infinite.
b) Uncountable.

PROBLEM 4:

a) Let $x = 4m + n, 0 \leq n < 4$. Always possible since if $4 \mid x$ then $2 \mid x$.
Proof by cases:
   - Case n = 0:

$$x = 4m \to \frac{\frac{4m}{2}}{2} = \frac{4m}{4} \to \frac{2m}{2} = m \to m = m$$

- Case n = 2:

$$x = 4m + 2 \to \frac{\frac{4m+2}{2}}{2} = \left\lceil \frac{4m+2}{4} \right\rceil \to \left\lceil \frac{2m+1}{2} \right\rceil = \left\lceil m + \frac{1}{2} \right\rceil$$

$$\to \left\lceil m + \tfrac{1}{2} \right\rceil = \left\lceil m + \tfrac{1}{2} \right\rceil$$

- Case n = 4:

$$x = 4m + 4 \to \frac{\frac{4m+4}{2}}{2} = \left\lceil \frac{2m+2}{2} \right\rceil \to \lceil m + 1 \rceil = \lceil m + 1 \rceil$$

$$\to \lceil m + 1 \rceil = \lceil m + 1 \rceil$$

b) Let $x = n + \varepsilon, 0 \le \varepsilon < 1$. Proof by cases:

- Case $\varepsilon \in \left[0, \frac{1}{3}\right)$:

$$x = n + \frac{1}{4} \to \lfloor 3x \rfloor = \lfloor x \rfloor + \left\lfloor x + \frac{1}{3} \right\rfloor + \left\lfloor x + \frac{2}{3} \right\rfloor$$

$$\to \left\lfloor 3\left(n + \frac{1}{4}\right) \right\rfloor = \left\lfloor n + \frac{1}{4} \right\rfloor + \left\lfloor n + \frac{1}{4} + \frac{1}{3} \right\rfloor + \left\lfloor n + \frac{1}{4} + \frac{2}{3} \right\rfloor$$

$$\to \left\lfloor 3n + \frac{3}{4} \right\rfloor = \left\lfloor n + \frac{1}{4} \right\rfloor + \left\lfloor n + \frac{7}{12} \right\rfloor + \left\lfloor n + \frac{11}{12} \right\rfloor$$

$$\to \lfloor 3n \rfloor + \left\lfloor \frac{3}{4} \right\rfloor = \lfloor n \rfloor + \left\lfloor \frac{1}{4} \right\rfloor + \lfloor n \rfloor + \left\lfloor \frac{7}{12} \right\rfloor + \lfloor n \rfloor + \left\lfloor \frac{11}{12} \right\rfloor$$

$$\to \lfloor 3n \rfloor = \lfloor n \rfloor + \lfloor n \rfloor + \lfloor n \rfloor$$

$$\to \lfloor 3n \rfloor = \lfloor 3n \rfloor$$

- Case $\varepsilon \in \left[\frac{1}{3}, \frac{2}{3}\right)$:

$$x = n + \frac{1}{2} \to \lfloor 3x \rfloor = \lfloor x \rfloor + \left\lfloor x + \frac{1}{3} \right\rfloor + \left\lfloor x + \frac{2}{3} \right\rfloor$$

$$\to \left\lfloor 3\left(n + \frac{1}{2}\right) \right\rfloor = \left\lfloor n + \frac{1}{2} \right\rfloor + \left\lfloor n + \frac{1}{2} + \frac{1}{3} \right\rfloor + \left\lfloor n + \frac{1}{2} + \frac{2}{3} \right\rfloor$$

$$\to \left\lfloor 3n + \frac{3}{2} \right\rfloor = \left\lfloor n + \frac{1}{2} \right\rfloor + \left\lfloor n + \frac{5}{6} \right\rfloor + \left\lfloor n + \frac{7}{6} \right\rfloor$$

$$\to \lfloor 3n \rfloor + \left\lfloor \frac{3}{2} \right\rfloor = \lfloor n \rfloor + \left\lfloor \frac{1}{2} \right\rfloor + \lfloor n \rfloor + \left\lfloor \frac{5}{6} \right\rfloor + \lfloor n \rfloor + \left\lfloor \frac{7}{6} \right\rfloor$$

$$\to \lfloor 3n \rfloor + 1 = \lfloor n \rfloor + \lfloor n \rfloor + \lfloor n \rfloor + 1$$

$$\to \lfloor 3n \rfloor + 1 = \lfloor 3n \rfloor + 1$$

- Case $\varepsilon \in \left[\frac{2}{3}, 1\right)$:

$$x = n + \frac{3}{4} \to \lfloor 3x \rfloor = \lfloor x \rfloor + \left\lfloor x + \frac{1}{3} \right\rfloor + \left\lfloor x + \frac{2}{3} \right\rfloor$$

$$\rightarrow \left\lfloor 3\left(n+\frac{3}{4}\right)\right\rfloor = \left\lfloor n+\frac{3}{4}\right\rfloor + \left\lfloor n+\frac{3}{4}+\frac{1}{3}\right\rfloor + \left\lfloor n+\frac{3}{4}+\frac{2}{3}\right\rfloor$$

$$\rightarrow \left\lfloor 3n+\frac{9}{4}\right\rfloor = \left\lfloor n+\frac{3}{4}\right\rfloor + \left\lfloor n+\frac{13}{12}\right\rfloor + \left\lfloor n+\frac{17}{12}\right\rfloor$$

$$\rightarrow \lfloor 3n\rfloor + \left\lfloor\frac{9}{4}\right\rfloor = \lfloor n\rfloor + \left\lfloor\frac{3}{4}\right\rfloor + \lfloor n\rfloor + \left\lfloor\frac{13}{12}\right\rfloor + \lfloor n\rfloor + \left\lfloor\frac{17}{12}\right\rfloor$$

$$\rightarrow \lfloor 3n\rfloor + 2 = \lfloor n\rfloor + \lfloor n\rfloor + 1 + \lfloor n\rfloor + 1$$

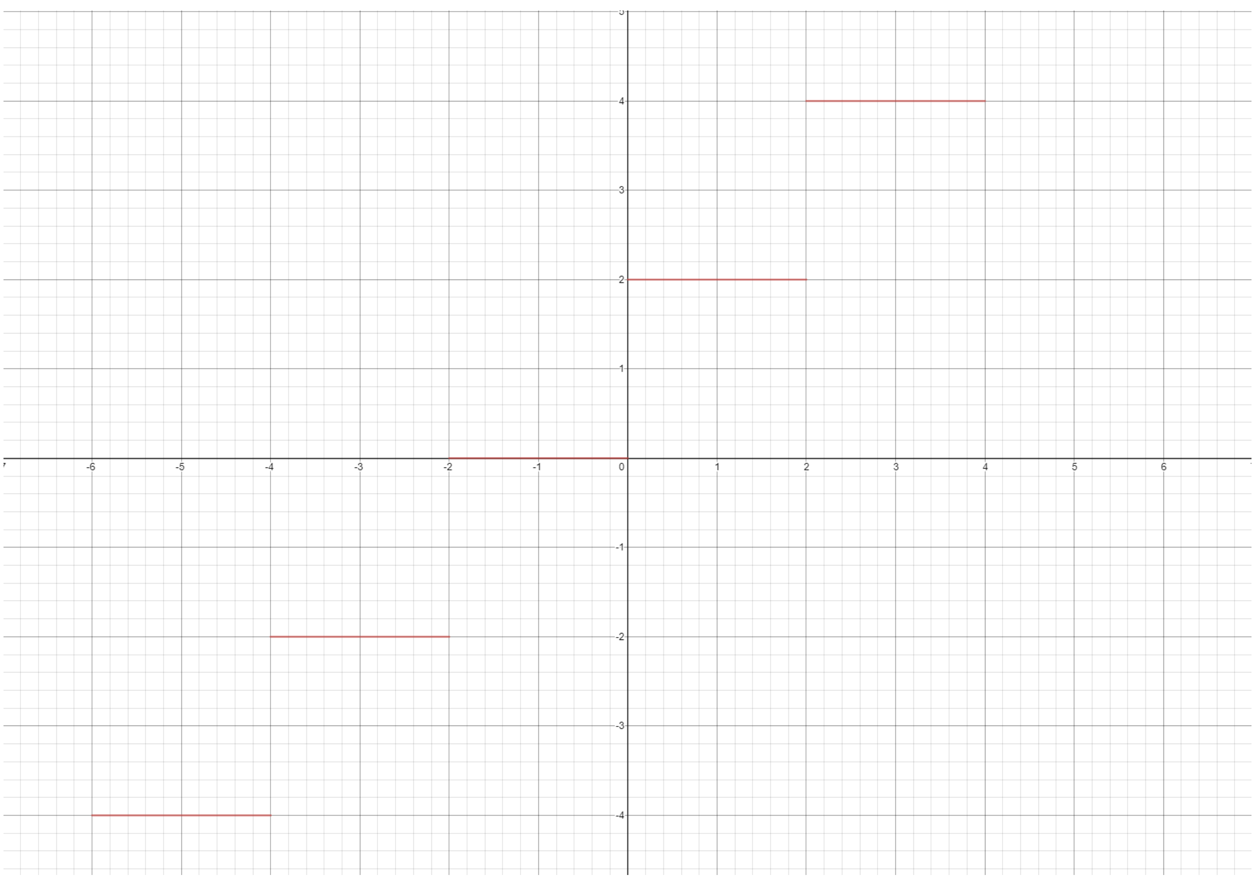
$$\rightarrow \lfloor 3n\rfloor + 2 = \lfloor 3n\rfloor + 2$$

PROBLEM 5:

Floor(Ceiling)…

PAGE 45 solution to a problem

PROBLEM 6:



PROBLEM 7:

A and B are integers: $ab = [(a \ \% \ m) \cdot (b \ \% \ m)](\% \ m)$

- Case $a > b$ and m = 3:

  Let a = 4, b = 2: $8 = [(4 \ \% \ 3) \cdot (2 \ \% \ 3)](\% \ 3)$

  $$8 = [1 \cdot 2](\% \ 3)$$

  $$8 - 2 = 6, \qquad 6\%3 = 0$$

  So, $8 = [(4 \ \% \ 3) \cdot (2 \ \% \ 3)](\% \ 3)$ is true

- Case $a < b$ and m = 3:

  Let a = 2, b = 4: $8 = [(2 \ \% \ 3) \cdot (4 \ \% \ 3)](\% \ 3)$

  $$8 = [2 \cdot 1](\% \ 3)$$

  $$8 - 2 = 6, \qquad 6\%3 = 0$$

  So, $8 = [(2 \ \% \ 3) \cdot (4 \ \% \ 3)](\% \ 3)$ is true

- Case $a = b$ and m = 3:

  Let a = 4, b = 4: $8 = [(4 \ \% \ 3) \cdot (4 \ \% \ 3)](\% \ 3)$

  $$8 = [1 \cdot 1](\% \ 3)$$

  $$8 - 1 = 6, \qquad 7\%3 = 1$$

  So, $8 = [(4 \ \% \ 3) \cdot (4 \ \% \ 3)](\% \ 3)$ is false

PROBLEM 8:

$$a^3 \equiv a(mod \ 3)$$

$$a^3 - a = mod \ 3$$

$$if \ 3 \mid (a^3 - a), then \ a^3 = 3q + a \ for \ an \ integer \ q$$

PROBLEM 9: