COMP 352
Data Structures and Algorithms

BUCKET & RADIX SORT

Chapter 12

Bucket-Sort

- Let be S be a sequence of n (key, element) items with keys in the range $[0, N-1]$

- Bucket-sort uses the keys as indices into an auxiliary array B of sequences (buckets)
  - Phase 1: Empty sequence S by moving each entry (k, o) into its bucket $B[k]$
  - Phase 2: For $i = 0, \ldots, N-1$, move the entries of bucket $B[i]$ to the end of sequence S
- **Animation:** *https://www.cs.usfca.edu/~galles/visualization/BucketSort.html*

Bucket-Sort

```
Algorithm bucketSort(S):
Input: Sequence S of entries with integer keys in the range [0, N – 1]
Output: Sequence S sorted in non-decreasing order of the keys let B be an
            array of N sequences, each of which is initially empty
for each entry e in S do  //phase 1
   k = the key of e
   remove e from S
   insert e at the end of bucket B[k]
for i = 0 to N–1 do       //phase 2
   for each entry e in B[i] do
      remove e from B[i]
      insert e at the end of S
```

Bucket-Sort

Analysis:

- Phase 1 takes $O(n)$ time
- Phase 2 takes $O(n + N)$ time

Bucket-sort takes $O(n + N)$ time

Properties

Key-type Property

- The keys are used as indices into an array and cannot be arbitrary objects
- No external comparator

Stable Sort Property

- The relative order of any two items with the same key is preserved after the execution of the algorithm

Extensions

Extensions

- Integer keys in the range [***a***, ***b***]
  - Put entry (***k***, ***o***) into bucket ***B***[***k*** - ***a***]

- String keys from a set ***D*** of possible strings, where ***D*** has constant size (e.g., names of the 50 U.S. states)
  - Sort ***D*** and compute the rank ***r***(***k***) of each string ***k*** of ***D*** in the sorted sequence
  - Put entry (***k***, ***o***) into bucket ***B***[***r***(***k***)]

Lexicographic Order

A d-tuple is a sequence of d keys $(k_1, k_2, \ldots, k_d)$, where key k_i is said to be the i-th dimension of the tuple

Example:

- The Cartesian coordinates of a point in space are a 3-tuple

The lexicographic order of two d-tuples is recursively defined as follows

$$(x_1, x_2, \ldots, x_d) < (y_1, y_2, \ldots, y_d)$$
$$\Leftrightarrow$$
$$x_1 < y_1 \vee x_1 = y_1 \wedge (x_2, \ldots, x_d) < (y_2, \ldots, y_d)$$

I.e., the tuples are compared by the first dimension, then by the second dimension, etc.

Lexicographic-Sort

- Let C_i be the comparator that compares two tuples by their i-th dimension
- Let *stableSort*(S, C) be a stable sorting algorithm that uses comparator C
- Lexicographic-sort sorts a sequence of d-tuples in lexicographic order by executing d times algorithm *stableSort*, one per dimension
- Lexicographic-sort runs in $O(dT(n))$ time, where $T(n)$ is the running time of *stableSort*

Which value do we sort by?

1st, 2nd then 3rd?

Example:

(7, 4, 6) (5, 1, 5) (2, 4, 6) (2, 1, 4) (3, 2, 4)

(2, 4, 6) (2, 1, 4) (3, 2, 4) (5, 1, 5) (7, 4, 6)

(2, 1, 4) (5, 1, 5) (3, 2, 4) (2, 4, 6) (7, 4, 6)

(2, 1, 4) (3, 2, 4) (5, 1, 5) (2, 4, 6) (7, 4, 6)

Which value do we sort by?

3rd, 2nd, then 1st?

Example:

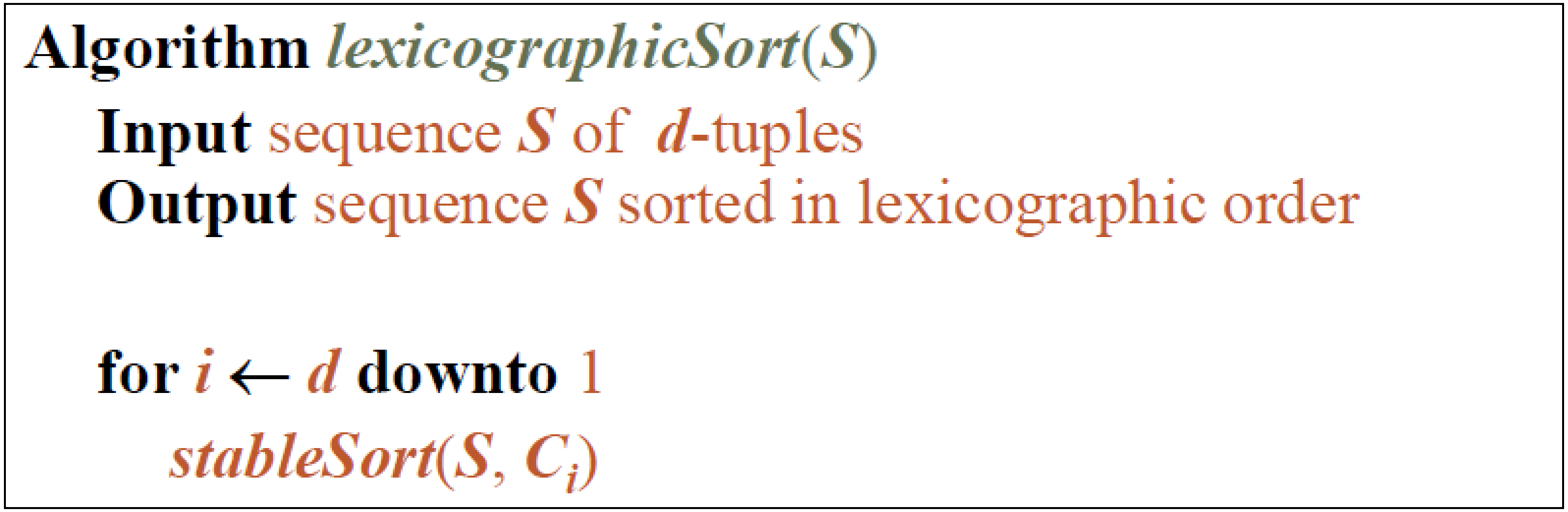
(7,4,6) (5,1,5) (2,4,6) (2, 1, 4) (3, 2, 4)

(2, 1, 4) (3, 2, 4) (5,1,5) (7,4,6) (2,4,6)

(2, 1, 4) (5,1,5) (3, 2, 4) (7,4,6) (2,4,6)

(2, 1, 4) (2,4,6) (3, 2, 4) (5,1,5) (7,4,6)

Lexicographic-Sort



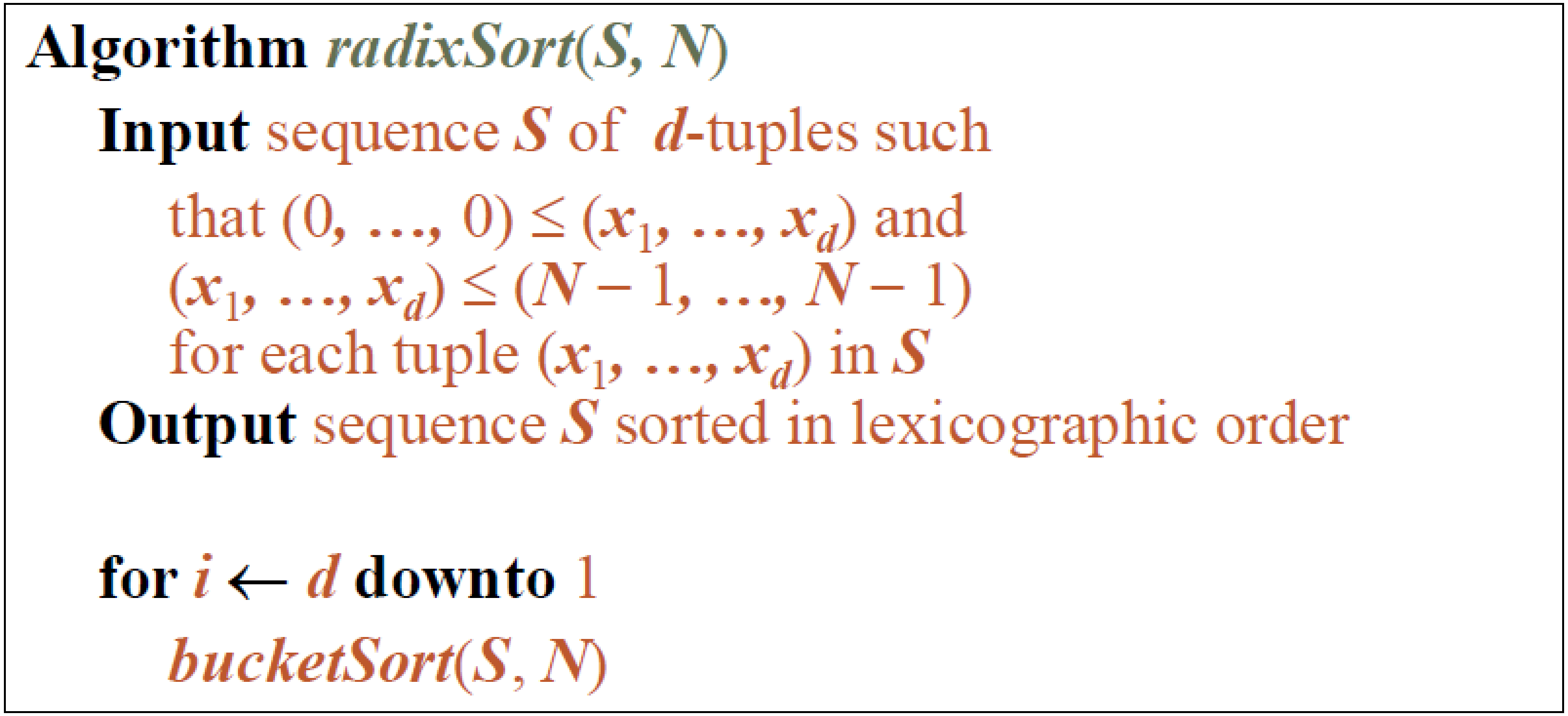
```
Algorithm lexicographicSort(S)
  Input sequence S of d-tuples
  Output sequence S sorted in lexicographic order

  for i ← d downto 1
    stableSort(S, $C_i$)
```

Radix-Sort

- Radix-sort is a specialization of lexicographic-sort that uses bucket-sort as the stable sorting algorithm in each dimension
- Radix-sort is applicable to tuples where the keys in each dimension i are integers in the range $[0, N - 1]$
- Radix-sort runs in time $O(d(n + N))$

Radix-Sort



```
Algorithm radixSort(S, N)
    Input sequence $S$ of $d$-tuples such
        that $(0, \ldots, 0) \leq (x_1, \ldots, x_d)$ and
        $(x_1, \ldots, x_d) \leq (N - 1, \ldots, N - 1)$
        for each tuple $(x_1, \ldots, x_d)$ in $S$
    Output sequence $S$ sorted in lexicographic order

    for $i \leftarrow d$ downto 1
        bucketSort(S, N)
```

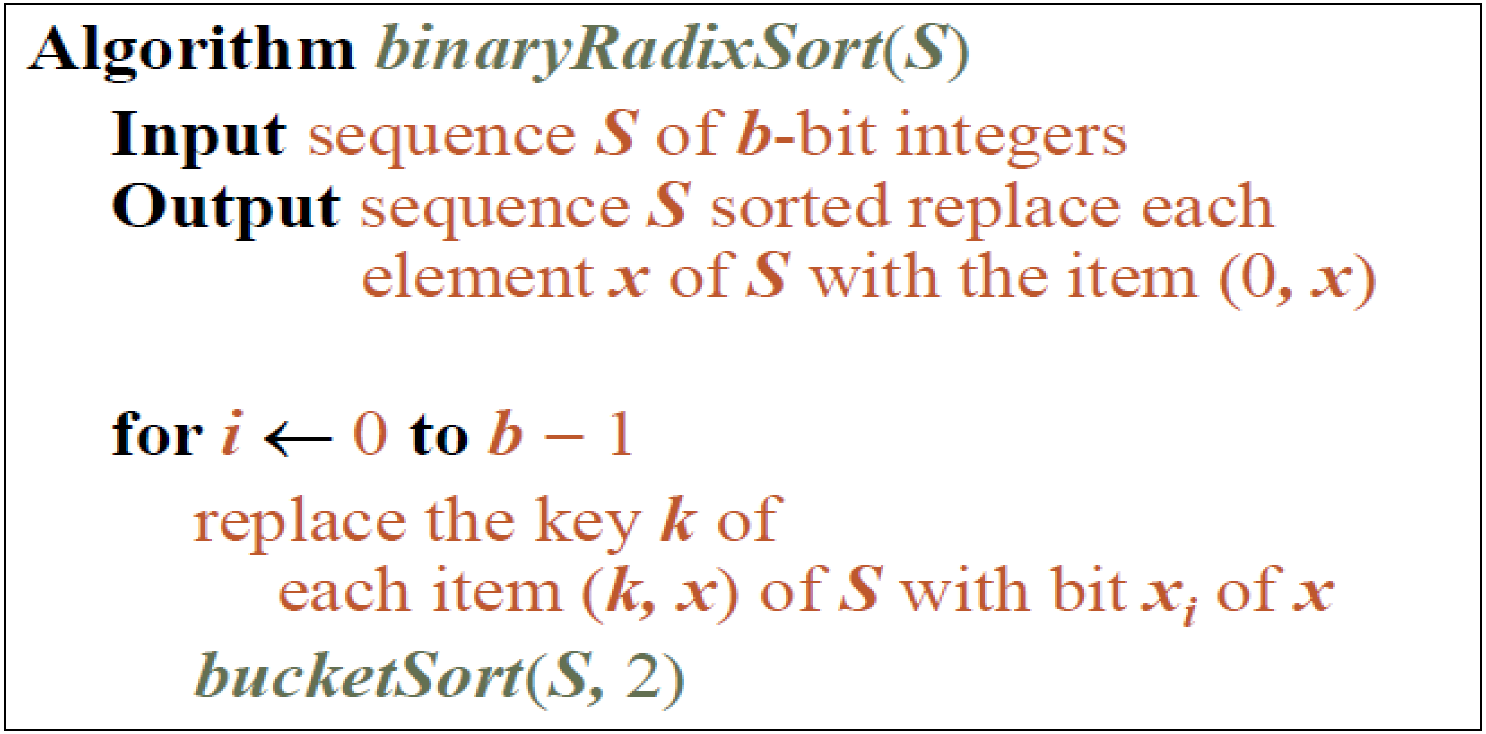
Radix-Sort for Binary Numbers

- Consider a sequence of n b-bit integers

  $$x = x_{b-1} \ldots x_1 x_0$$

- We represent each element as a b-tuple of integers in the range $[0, 1]$ and apply radix-sort with $N = 2$
- This application of the radix-sort algorithm runs in $O(bn)$ time
- For example, we can sort a sequence of 32-bit integers in linear time

Radix-Sort for Binary Numbers

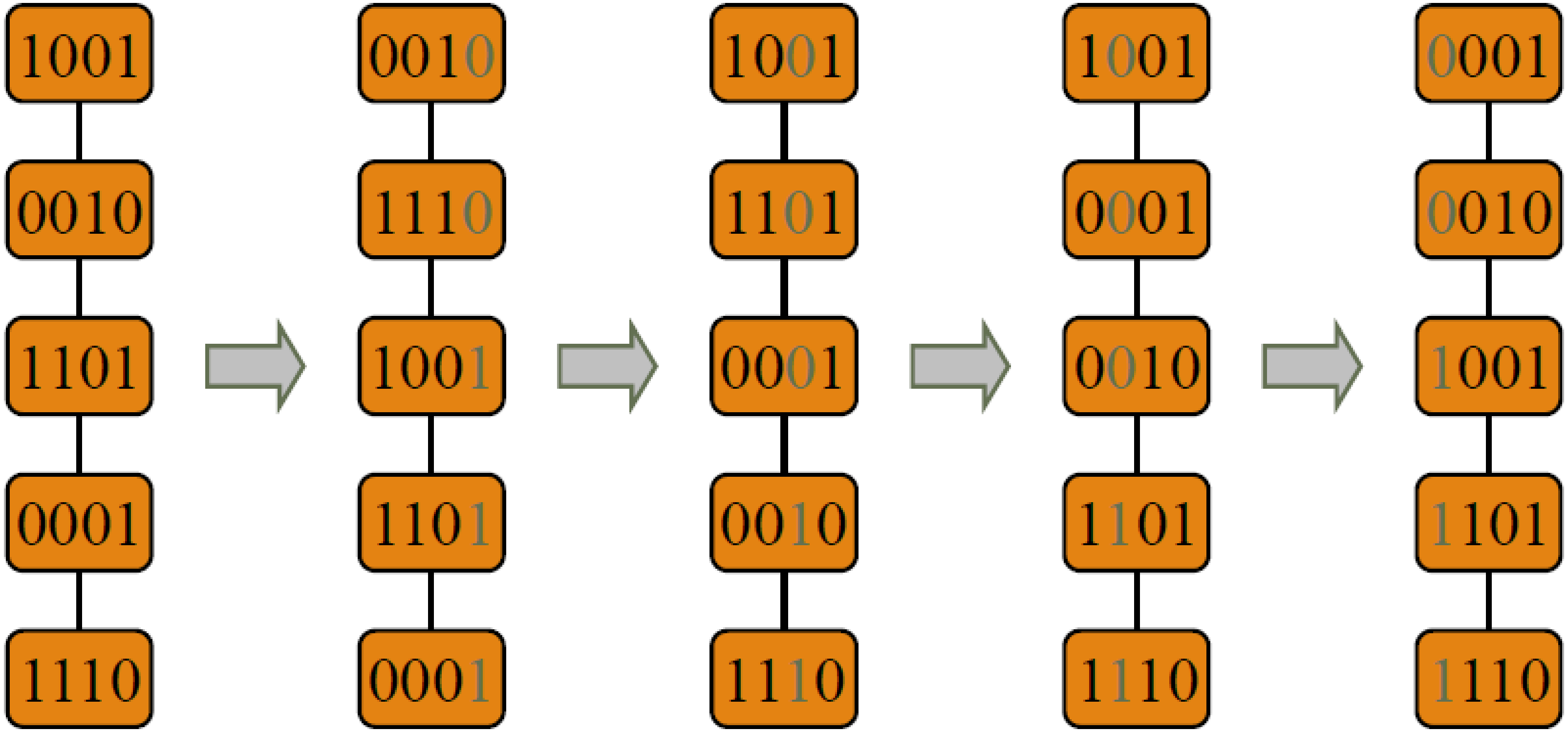


```
Algorithm binaryRadixSort(S)
    Input sequence $S$ of $b$-bit integers
    Output sequence $S$ sorted replace each
              element $x$ of $S$ with the item $(0, x)$

    for $i \leftarrow 0$ to $b - 1$
        replace the key $k$ of
            each item $(k, x)$ of $S$ with bit $x_i$ of $x$
        bucketSort(S, 2)
```

Example

Sorting a sequence of 4-bit integers



1001	0010	1001	1001	0001
0010	1110	1101	0001	0010
1101	1001	0001	0010	1001
0001	1101	0010	1101	1101
1110	0001	1110	1110	1110

References

These slides has been extracted, modified and updated from original slides of :

1. Data Structures and Algorithms in Java, 6th edition. John Wiley& Sons,
2. Introduction to Algorithms, 3rd Edition. Thomas H. Cormen and Charles E. Leiserson