

CONCORDIA UNIVERSITY
DEPARTMENT OF COMPUTER SCIENCE AND SOFTWARE ENGINEERING
COMP 232: MATHEMATICS FOR COMPUTER SCIENCE
FALL 2019

ASSIGNMENT 3

PROBLEM 1.

Let A and B be sets. Prove that $A \subseteq B$ if and only if $P(A) \subseteq P(B)$.

PROBLEM 2.

Let A , B , C , and D be sets. Prove or disprove the following:

$$(A \cap B) \cup (C \cap D) = (A \cap D) \cup (C \cap B).$$

PROBLEM 3.

Give an example of two uncountable sets A and B such that $A - B$ is

- (a) Countably Infinite.
- (b) Uncountable.

PROBLEM 4.

Let x be a real number.

- (a) Prove that $\lceil \lceil x/2 \rceil / 2 \rceil = \lceil x/4 \rceil$.
- (b) Prove that $\lfloor 3x \rfloor = \lfloor x \rfloor + \lfloor x + 1/3 \rfloor + \lfloor x + 2/3 \rfloor$.

PROBLEM 5.

- (a) Give an example of a function from \mathbf{Z}^+ to \mathbf{Z}^+ that is neither one-to-one nor onto.
- (b) Let $g : A \rightarrow B$ and $f : B \rightarrow C$ be functions. Let $f \circ g$ be onto. Are both f and g necessarily onto?
- (c) Let f be a function from \mathbf{R} to \mathbf{R} defined by $f(x) = x^2$. Find $f^{-1}(\{x \mid 0 < x < 1\})$.

PROBLEM 6.

Draw the graph of $\lfloor 2 \lceil x/2 \rceil + 1/2 \rfloor$.

PROBLEM 7.

Let a , b , and m be integers, and $m \geq 2$. Prove that

$$ab \equiv [(a \bmod m) \cdot (b \bmod m)] \pmod{m}.$$

PROBLEM 8.

Prove that $a^3 \equiv a \pmod{3}$ for every positive integer a .

PROBLEM 9.

Prove that if p is a prime number greater than 3, then $p^2 = 6k + 1$, for some integer k .

PROBLEM 10.

Let a , b , and d be integers such that $d \geq 2$ and $a \equiv b \pmod{d}$. Prove that $\gcd(a, d) = \gcd(b, d)$.