Lecture 1
Introduction to Differential Equations
1.1 Definitions and Terminology

The derivative $\frac{dy}{dx}$ of a function $y = f(x)$ is another function $f'(x)$ found by appropriate rules.

Definition 1. An equation containing the derivatives of dependent variables, with respect to independent variables, is said to be **a differential equation (DE)**.

Classification of DE

a) by type;
b) by order;
c) by linearity.

Classification by type

Definition 2. If a DE contains only ordinary derivatives of one or more dependent variables with respect to a single independent variable, it is said to be **ordinary differential equation (ODE)**.

Definition 3. If a DE contains the partial derivatives of one or more dependent variables with respect to two or more independent variables, it is said to be **partial differential equation (PDE)**.

Classification by order

Definition 4. The order of DE (ODE or PDE) is the order of the highest derivative in the equation.

Definition 5. The differential equation $\frac{d^n y}{dx^n} = f(x, y, y', \ldots, y^{(n-1)})$, where f is a real-valued continuous function, is said to be a differential equation in **normal form**.

Classification by linearity

Definition 6. An n-th order DE $F(x, y, y', \ldots, y^{(n)}) = 0$ is said to be **linear** if F is linear in $y, y', \ldots, y^{(n)}$

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1} y}{dx^{n-1}} + \ldots, a_1(x)\frac{dy}{dx} + a_0(x)y = g(x).$$

Definition 7. A nonlinear DE is simply one that is not linear.

Definition 8. A solution of n-th order DE $F(x, y, y', \ldots, y^{(n)}) = 0$ is a function $f(x)$ that possess at least n derivatives and

$$F(x, f(x), f'(x), \ldots, f^{(n)}(x)) = 0 \quad \text{for all} \quad x \in I.$$

Definition 9. The graph of a solution of ODE is called a **solution curve**.

Definition 10. A solution of DE that is identically zero on an interval I is said to be **trivial solution**.

Explicit and implicit solutions

Definition 11. If a solution of differential equation

$$F(x, y, y', \ldots, y^{(n)}) = 0$$

is an explicit function $y = f(x)$, then the solution of DE is said to be **explicit solution**.

Definition 12. If a solution of differential equation

$$F(x, y, y', \ldots, y^{(n)}) = 0$$

is an implicit function $G(x, y) = 0$, then the solution of DE is said to be **implicit solution**

Parametric families of solutions

Definition 13. A solution of DE $F(x, y, y') = 0$ containing an arbitrary constant represent a set $G(x, y, c) = 0$ of solutions called **a one-parameter family of solutions**.

Singular solution

Definition 14. A solution of DE that is not a member of family of solutions is called **a singular solution**.

Systems of DE

Definition 15. A system of ODE is two or more DE involving the derivatives of two or more unknown functions of a single variable.

Definition 16. A solution of a system is a set of differentiable functions defined on common interval, that satisfy each equation of the system on this interval.