COMP-232

MATHEMATICS FOR COMPUTER SCIENCE

Fall 2019

Assignment #2

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1. Let P(x; y; z) denote the statement \x + y B z, " where x; y; z > Z+. What is the truth value of each of the following? Explain your answers.

a) $\forall x \exists y \exists z P(x, y, z)$
True because as it says "Every x added to some y will be less than or equal to z. Suppose that $x = 5, y = 1, z = 6$ Then the statement is true because $6 \leq 6$.

b) $\forall y \exists x \forall z P(x, y, z)$
False, because x can be $5, y = 2, z = 3$ then $5 + 2 \leq 3$ is false

c) $\exists z \exists y \forall x P(x, y, z)$
True because as it says "Every x added to some y will be less than or equal to some z. Suppose that $x = 3, y = 1, z = 10$ Then the statement is true because $4 \leq 10$.

2. For each of the premise-conclusion pairs below, give a valid step-by-step argument (proof) along with the name of the inference rule used in each step. For examples, see pages 73 and 74 in textbook.

3. For each of the following, determine whether the argument is valid. You may use a counterexample or equivalence transformations to justify your answer.

a) $\big((p \rightarrow q) \wedge \neg p\big) \rightarrow \neg q \equiv T$
$\big((\neg p \vee q) \wedge \neg p\big) \rightarrow \neg q \equiv T$
$\neg\big((\neg p \vee q) \wedge \neg p\big) \vee \neg q \equiv T$
$\neg\big((\neg p \wedge \neg p) \vee (q \wedge \neg p)\big) \vee \neg q \equiv T$
$\neg(\neg p \vee (q \wedge \neg p)) \vee \neg q \equiv T$
$(p \wedge \neg q \vee p) \vee \neg q \equiv T$
$\neg p \vee \neg q \equiv T$
False if $p = F, q = T$, so it is not valid

b) $(\neg p \rightarrow \neg q) \rightarrow \big((\neg p \rightarrow q) \rightarrow p\big) \equiv T$
$(\neg p \rightarrow \neg q) \rightarrow (\neg(p \vee q) \vee p) \equiv T$
$\neg(p \vee \neg q) \vee (\neg(p \vee q) \vee p) \equiv T$
$(\neg p \wedge q) \vee ((\neg p \wedge \neg q) \vee p) \equiv T$

p	q	$(\neg p \wedge q) \vee ((\neg p \wedge \neg q) \vee p)$
T	T	T
T	F	T
F	T	T
F	F	T

So, it is valid

c) $((p \rightarrow r) \wedge (q \rightarrow r) \wedge \neg(p \vee q)) \rightarrow \neg r \equiv T$

$((\neg p \vee r) \wedge (\neg q \vee r) \wedge \neg(p \vee q)) \rightarrow \neg r \equiv T$
$((\neg p \vee r) \wedge (\neg q \vee r) \wedge \neg p \wedge \neg q) \rightarrow \neg r \equiv T$
$(\neg p \vee r) \wedge \neg p \wedge (\neg q \vee r) \wedge \neg q) \rightarrow \neg r \equiv T$
$(\neg p \wedge \neg q) \rightarrow \neg r \equiv T,\ \ Absorption\ law$
$\neg(\neg p \wedge \neg q) \vee \neg r \equiv T$
$p \vee q \vee \neg r \equiv T$
Now it is easy to do truth table and we see that if $p = F, q = F\ and\ r = T$ the result will be false. So, the argument is invalid.

d) $(p \rightarrow q) \wedge \big(p \rightarrow (q \rightarrow \neg p)\big) \rightarrow \neg p \equiv T$
$\Big((\neg p \vee q) \wedge \big(\neg p \vee (\neg q \vee \neg p)\big)\Big) \rightarrow \neg p \equiv T$
$\big((\neg p \vee q) \wedge (\neg q \vee \neg p)\big) \rightarrow \neg p \equiv T, Idempotent\ laws$
$\neg\big((\neg p \vee q) \wedge (\neg q \vee \neg p)\big) \vee \neg p \equiv T$
$\neg(\neg p \vee q) \vee \neg(\neg q \vee \neg p) \vee \neg p \equiv T$
$(p \wedge \neg q) \vee (q \wedge p) \vee \neg p \equiv T$

p	q	$(p \wedge \neg q) \vee (q \wedge p) \vee \neg p$
T	T	T
T	F	T
F	T	T
F	F	T

So, the argument is valid.

4. For each of the arguments below, indicate whether it is valid or invalid.

a) let $P(x) = x\ eats\ an\ apple\ a\ day$
$Q(x) = x\ is\ healthy$
Then, $\forall x\big(P(x) \rightarrow Q(x)\big)$
So:
$P(Helen)$: Helen eats an apple a day
$Q(Helen)$: Helen is healthy

$P(Helen) \rightarrow Q(Helen)$

Which satisfies the initial equations

b) let $P(x) = x\ eats\ an\ apple\ a\ day$
$Q(x) = x\ is\ healthy$
Then, $\forall x\big(P(x) \rightarrow Q(x)\big)$
So:

$P(Herbert)$: Herbert eats an apple a day
$Q(Herbert)$: Helen is healthy

$!Q(Herbert) \rightarrow P(Herbert)$. It seams okay on paper. However, for this statement to be true the initial condition should be an "if and only if" as follows $\forall x\big(P(x) \leftrightarrow Q(x)\big)$. Because otherwise, Herbert can be unhealthy for other reasons, not necessarily not eating applies. So, the argument isn't valid

c) let $P(a,\ b) = the\ product\ of\ a\ and\ b$
$\forall a \forall b\big((a = 0\ \vee b = 0) \rightarrow (P(a, b) = 0)\big)$

So, let $a = (x - 1)$ and $b = (x + 1)$. If neither of those are equal to 0 then the quadratic equation $(x - 1)(x + 1) = 0$ cannot be satisfied and thus the argument is valid

5. Use rules of inference to show that if $\forall \boldsymbol{x}\big(\boldsymbol{P}(\boldsymbol{x}) \rightarrow \boldsymbol{Q}(\boldsymbol{x})\big), \forall \boldsymbol{x}\big(\boldsymbol{Q}(\boldsymbol{x}) \rightarrow \boldsymbol{R}(\boldsymbol{x})\big)$, and $\exists \boldsymbol{x}\big(\neg \boldsymbol{R}(\boldsymbol{x})\big)$ are true, then $\exists \boldsymbol{x}(\neg \boldsymbol{P}(\boldsymbol{x}))$ is true

$$\Big(\forall x\big(P(x) \rightarrow Q(x)\big)\Big) \wedge \Big(\forall x\big(Q(x) \rightarrow R(x)\big)\Big) \wedge \Big(\exists x\big(\neg R(x)\big)\Big) \rightarrow \exists x(\neg P(x)) \equiv T$$

6.
a) Give a direct proof of: \If x is an odd integer and y is an even integer, then x + y is odd."

let x be an odd number so $x = 2k + 1$ where $k \in \mathbb{Z}$
let y be an even number so $y = 2k$ where $k \in \mathbb{Z}$

So, we have:
$x + y = even$. In other words, if we can proof that $x + y = 2 \cdot (something \cdot k)$ then we proof that even + odd = even. Otherwise, even + odd = odd.

$$x + y = 2k + 1 + 2k$$
$$x + y = 4k + 1$$

We can see here that k is multiplied by 4 then added to 1. So, if k is odd then $4k$ is even and then $4k + 1$ is odd. Similarly, if k is even then $2k$ is even and $2k + 1$ is odd.

b) Give a proof by contradiction of: "If n is an odd integer, then n^2 is odd."

Let's assume that n is even, then $n = 2k$, where $k \in \mathbb{Z}$. Then $n^2 = (2k)^2 = 4k^2 = 2(2k^2)$ so the result is a multiplier of 2 and thus the result will be even. However, if n is odd, so that $n = 2k + 1$ then

$$n^2 = (2k + 1)^2 = (2k + 1)(2k + 1) = 4k^2 + 2k + 2k + 1$$
$$= 4k^2 + 4k + 1$$

Thus, $4k^2 + 4k$ will be an even number because each k is multiplied by 4 which is a multiplier of 2 and add to 1 so the total result will be odd.

c) Give an indirect proof of: "If x is an odd integer, then x + 2 is odd."

If $x$is an odd integer, then we can write it as follows $x = 2k + 1$ (where $k \in \mathbb{Z}$). Thus, we have:

$$x + 2 = \text{odd}$$
$$x + 2 = 2k + 1 + 2$$
$$x + 2 = 2k + 3$$

We can see here that $2k$ is an even number because it is a multiplier of 2 and then we add it to an odd number. Thus, the result is odd because even + odd = odd as proven in a).

d) Use a proof by cases to show that there are no solutions in positive integers to the equation $\boldsymbol{x^4 + y^4 = 100}$.

$x, y = \{0,1,2,3\}$ because if they are larger than 3 $x^4 + y^4$ will go above 100.
So here are the combinations:

- $x = 0, y = 0 \rightarrow x^4 + y^4 = 0$
- $x = 1, y = 0 \rightarrow x^4 + y^4 = 1$
- $x = 2, y = 0 \rightarrow x^4 + y^4 = 16$
- $x = 3, y = 0 \rightarrow x^4 + y^4 = 81$
- $x = 0, y = 1 \rightarrow x^4 + y^4 = 1$
- $x = 1, y = 1 \rightarrow x^4 + y^4 = 2$
- $x = 2, y = 1 \rightarrow x^4 + y^4 = 17$
- $x = 3, y = 1 \rightarrow x^4 + y^4 = 82$
- $x = 0, y = 2 \rightarrow x^4 + y^4 = 16$
- $x = 1, y = 2 \rightarrow x^4 + y^4 = 17$
- $x = 2, y = 2 \rightarrow x^4 + y^4 = 32$
- $x = 3, y = 2 \rightarrow x^4 + y^4 = 97$
- $x = 0, y = 3 \rightarrow x^4 + y^4 = 81$
- $x = 1, y = 3 \rightarrow x^4 + y^4 = 82$
- $x = 2, y = 3 \rightarrow x^4 + y^4 = 97$

So we can see that there is no positive integer that can satisfy this equation.

e) Prove that given a nonnegative integer n, there is a unique nonnegative integer m, such that m2 B n < (m+ 1)2.

$$m \leq \sqrt{n} < m + 1$$

Case 1: n is a perfect square so $n = k^2$

Take $m = k$

For example, $k = 2,\quad 2 \leq \sqrt{4} < 3$

Case 2: n is not a perfect square so $n = k$

Take $m = [\sqrt{k}]$

For example, $k = 3$, $\quad [\sqrt{3}] \le \sqrt{3} < [\sqrt{3}] + 1$

$$= 1 \le 1.73205 < 2.73205, \textit{which is true}$$

7. For each of the statements below state whether it is True or False. If True, then give a proof. If False then explain why, e.g., by giving a counterexample.

a) False, because $9 - 3 = 6$ which is an even number

b) False, because $4 + 2 = 8$ and they are all even numbers

c) True

let $n = 2k$ then: $3(2k)^2 + 8 = 3n^2 + 8$

$$12k^2 + 8 = 3n^2 + 8$$

$$2(6k^2) + 8 = 3n^2 + 8$$

So, we see that $k^2$is multiplied by 2 and then added to an even number so the total result will be even

d) True, because an irrational number has an infinite number of decimals. So, when add 2 numbers with infinite number of decimals the result will also be with infinite number of decimals. Thus, it will be irrational.

e) True, because for example π is irrational and when we double π it is still an irrational.