COMP-228 Assignment 2

Shadi Jiha

ID: 40131284

<u>Question 1</u>

Truth table of $(p \to q) \land (q \to r)$:

p	q	r	$(p \to q) \land (q \to r)$
F	F	F	T
F	F	T	T
F	T	F	F
F	T	T	T
T	F	F	F
T	F	T	F
T	T	F	F
T	T	T	T

a) Using {'X', 'F'}

p	$\sim p$	$XpFF$
F	T	T
T	F	F

Thus, $\sim p| == |XpFF$

b) Using {'X', 'T'}

p	q	$p \land q$	$XpqT$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

Thus, $p \land q| == | XpqT$

c) Using {'X','~'}

p	q	$p \vee q$	$XpqT$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	F

Thus, $p \vee q$|==| $X{\sim}p{\sim}qF$

Question 2

Convert the following binary and hex to decimal numbers:

a) Binary to decimal:

a. 11101:

0	0	0	1	1	1	0	1
128	64	32	16	8	4	2	1

= 16+8+4+1=29

b. 11101 11110:

$(0 \cdot 2^0) + (1 \cdot 2^1) + (1 \cdot 2^2) + (1 \cdot 2^3) + (1 \cdot 2^4) + (1 \cdot 2^5) + (0 \cdot 2^6) + (1 \cdot 2^7) + (1 \cdot 2^8) + (1 \cdot 2^9) = 958$

c. 11101 11110 11101 10111:

$(1 \cdot 2^0) + (1 \cdot 2^1) + (1 \cdot 2^2) + (0 \cdot 2^3) + (1 \cdot 2^4) + (1 \cdot 2^5) + (0 \cdot 2^6) + (1 \cdot 2^7) + (1 \cdot 2^8) + (1 \cdot 2^9) + (0 \cdot 2^{10}) + (1 \cdot 2^{11}) + (1 \cdot 2^{12}) + (1 \cdot 2^{13}) + (1 \cdot 2^{14}) + (1 \cdot 2^{15}) + (0 \cdot 2^{16}) + (1 \cdot 2^{17}) + (1 \cdot 2^{18}) + (1 \cdot 2^{19}) = 981943$

b) Hex to decimal:

a. AE1:

$(1 \cdot 16^0) + (14 \cdot 16^1) + (10 \cdot 16^2) = 2785$

b. AEBA1:

$(1 \cdot 16^0) + (10 \cdot 16^1) + (11 \cdot 16^2) + (14 \cdot 16^3) + (10 \cdot 16^4) = 715681$

c. AEBA1 51DE1:

$(1 \cdot 16^0) + (14 \cdot 16^1) + (13 \cdot 16^2) + (1 \cdot 16^3) + (5 \cdot 16^4) + (1 \cdot 16^5) + (10 \cdot 16^6) + (11 \cdot 16^7) + (14 \cdot 16^8) + (10 \cdot 16^9) = 750446255585 = 7.50 \times 10^{11}$

Question 3:

a) $\frac{1}{9} = 0.\overline{11} = 00111101111000111000111000111001$

b) $3de38e39$

c) $\frac{4}{9} = 0.4\overline{4} = 00111110111000111000111000111001$

d) $3ee38e39$

Question 4

a) Multiply 10011 11100 (27c) by 11010 (1a)

a. $$\begin{array}{r} 27c \\ \times\ 1a \\ \hline 18D8 \\ +27c0 \\ \hline 4098 \end{array}$$

$$\frac{c \times a}{16} = \frac{120}{16} = 7 \text{ } and \text{ } reste \text{ } 8$$
$$\frac{7 \times a + 7}{16} = \frac{77}{16} = 4 \text{ } and \text{ } reste \text{ } 13 \text{ } (D)$$
$$\frac{2 \times a + 4}{16} = \frac{24}{16} = 1 \text{ } and \text{ } reste \text{ } 8$$

Same thing for the other half

b) $27c \times 1a$
   a. $c \times a + 1b = 93 \rightarrow 3 \text{ } and \text{ } rest \text{ } 9$
   b. $7 \times a + 9 = 79 \rightarrow 9 \text{ } and \text{ } rest \text{ } 7$
   c. $2 \times a + 7 = 27 \rightarrow 27$

   So, we have 2793 for the first half. Now we should calculate the second half

c) $27c \times 1 \text{ } with \text{ } (1b \text{ } accumulator)$
   a. $c \times 1 + 1b = 39 \rightarrow 9 \text{ } and \text{ } rest \text{ } 3$
   b. $7 \times 1 + 3 = 10 \rightarrow a \text{ } and \text{ } rest \text{ } 0$
   c. $2 \times 1 + 0 = 2$

   The second half is $2a9$. And thus, the final answer is $2793 + 2a9 = 2a3c$

Question 5

a) Proof:

$$(2^n - 1) \cdot (2^n - 1) + (2^n - 1) \leq 2^{2n} - 1$$
$$2^{2n} - 2 \cdot 2^n + 1 + (2^n - 1) \leq 2^{2n} - 1$$
$$2^{2n} - 2^n \leq 2^{2n} - 1$$

This statement is always true regardless of the value of n. We are subtracting 2^nform the left-hand-side while on the right-hand-side we are only subtracting -1.

Now find the maximum number we can add without producing overflow:

$$2^{2n} - 2^n + x = 2^{2n} - 1$$
$$x = 2^n - 1$$

Which means that for $n = 16$. The maximum number that can be added is $FFFF$

b) Using $\left(\frac{a+b}{2^{16}}\right) = \frac{p}{k} + r$

$$\frac{23979 + b}{2^{16}} = q + 63400$$
$$23979 + b = (q + 63400) \cdot 2^{16}$$