LOGIC EXERCISES

BY PANKAJ KAMTHAN

PROBLEM 1. [TIME ALLOWED = 5 MINUTES]

Is the following sentence a proposition? If it is a proposition, determine whether it is true or false.

“In the year 2000, Montreal was the capital of Quebec.”

PROBLEM 2. [TIME ALLOWED = 5 MINUTES]

Is the following sentence a proposition? If it is a proposition, determine whether it is true or false.

$1^{232} \neq 2^{232}$ and $\log(1) = 1$.

PROBLEM 3. [TIME ALLOWED = 5 MINUTES]

Is the following sentence a proposition? If it is a proposition, determine whether it is true or false.

“Enter your password.”

PROBLEM 4. [TIME ALLOWED = 5 MINUTES]

Give the negation of the following sentence:

“It is hot today.”

PROBLEM 5. [TIME ALLOWED = 5 MINUTES]

Give the negation of the following sentence:

“2 is negative.”

PROBLEM 6. [TIME ALLOWED = 5 MINUTES]

Give the negation of the following sentence:

"The number $\sqrt{2}$ is rational."

PROBLEM 7. [TIME ALLOWED = 5 MINUTES]

Give the negation of the following sentence:

"$2 + 3 = 6$."

PROBLEM 8. [TIME ALLOWED = 5 MINUTES]

Construct a truth table for $p \vee \neg(p \wedge q)$.

PROBLEM 9. [TIME ALLOWED = 5 MINUTES]

Let p and q be propositions. Give the truth value of $(p \vee q) \longrightarrow (p \wedge q)$ when both p and q are false.

PROBLEM 10. [TIME ALLOWED = 5 MINUTES]

Express the following propositions using logical connectives:

(a) I will go to the movie if I complete my assignment.
(b) I will go to the movie only if I complete my assignment.
(c) I will not go to the movie if I do not complete my assignment.

PROBLEM 11. [TIME ALLOWED = 5 MINUTES]

Express the contrapositive in English of the following sentence:

"I will buy the tickets only if you call."

PROBLEM 12. [TIME ALLOWED = 5 MINUTES]

Give the contrapositive and converse of the following proposition:

"If it is sunny, then I will go swimming."

PROBLEM 13. [TIME ALLOWED = 5 MINUTES]

Give the contrapositive, converse, and inverse of the following proposition:

"If the number is positive, then its square is positive."

PROBLEM 14. [TIME ALLOWED = 5 MINUTES]

Let p and q be the propositions:

p : I bought a lottery ticket this week.
q : I won the million dollar jackpot on Friday.

The proposition $\neg(\neg p \wedge \neg q)$ as an English sentence is:

(a) I did not buy a lottery ticket this week, and I did not win the million dollar jackpot on Friday.
(b) Either I bought a lottery ticket this week or I won the million dollar jackpot on Friday.
(c) Not better than good beer.
(d) None of the above.

PROBLEM 15. [TIME ALLOWED = 5 MINUTES]

Let p, q, and r be the propositions:

p : You have the flu.
q : You miss the final examination.
r : You pass the course.

The proposition $\neg q \leftrightarrow r$ as an English sentence is:

(a) You do not miss the final examination if and only if you pass the course.
(b) You do not pass the course if and only if you miss the final examination.
(c) I won the million dollar jackpot on Friday, and so I will not have flu and I want to miss the final examination.
(d) I hope the final examination will not be a lottery.

PROBLEM 16. [TIME ALLOWED = 5 MINUTES]

Show, (1) using a truth table, and (2) using a mathematical proof, that $\neg(p \vee \neg q)$ and $q \wedge \neg p$ are logically equivalent.

PROBLEM 17. [TIME ALLOWED = 5 MINUTES]

State whether "n is divisible by 9" is (a) necessary, (b) sufficient, or (c) neither necessary nor sufficient for "n is divisible by 6", where n is a natural number.

PROBLEM 18. [TIME ALLOWED = 5 MINUTES]

Using "laws" of logic, simplify $\neg(p \vee q) \vee (\neg p \wedge q)$.

PROBLEM 19. [TIME ALLOWED = 5 MINUTES]

Give a truth table of $\neg[(p \longrightarrow q) \wedge (q \longrightarrow p)]$. Explain.

PROBLEM 20. [TIME ALLOWED = 5 MINUTES]

This is about the truth value of the statement: "There exist positive integers x, y, and z such that $x^2 + y^2 = z^2$."

Select one of the following:

(a) $x = 0, y = 0, z = 0$.
(b) $x = 1, y = 2, z = 3$.
(c) $x = 2, y = 3, z = 4$.
(d) $x = 3, y = 4, z = 5$.
(e) None of the above.

PROBLEM 21. [TIME ALLOWED = 5 MINUTES]

This is about the truth value of the statement: "There exist positive integers x, y, and z such that $x^3 + y^3 = z^2$."

Select one of the following:

(a) $x = 1, y = 2, z = 3$.
(b) $x = 2, y = 2, z = 4$.
(c) $x = 3, y = 4, z = 5$.
(d) (a), but not (b).
(e) Both (a) and (b), but not (c).

PROBLEM 22. [TIME ALLOWED = 5 MINUTES FOR EACH PART]

Let P: $\{1, 2\} \times \{1, 2\} \longrightarrow \{T, F\}$. Express the following using conjunctions and disjunctions only.

(a) $\forall y\ \exists x\ P(x, y)$.
(b) $\exists y\ \forall x\ P(x, y)$.
(c) $\exists x\ \exists y\ P(x, y)$.
(d) $\forall x\ \forall y\ P(x, y)$.
(e) $\forall y\ \forall x\ P(x, y)$.
(f) $\exists x\ \forall y\ \neg P(x, y)$.

PROBLEM 23. [TIME ALLOWED = 5 MINUTES FOR EACH PART]

Let P: $\mathbf{Z} \times \mathbf{Z} \longrightarrow \{T, F\}$, where $P(x, y)$ denotes "$x + y^2 = 10$".

Give the truth value of the following propositions:

(1) $\forall y\ \exists x\ P(x, y)$.

(a) True.
(b) False.
(c) I do not know yet.
(d) I do not care.

(2) $\exists y\ \forall x\ P(x, y)$.

(a) True.
(b) False.
(c) I do not know.
(d) I do not care yet.

(3) $\exists x\ \exists y\ P(x, y)$.

(a) True.
(b) False.
(c) I know, but I am not going to tell you.
(d) Is this really a question?

PROBLEM 24. [TIME ALLOWED = 5 MINUTES]

Simplify $\neg[\exists x\ \exists y\ [P(x, y) \oplus P(y, x)]]$ so that there are no conjunction, disjunction, or negation symbols in the resulting logical expression.

PROBLEM 25. [TIME ALLOWED = 5 MINUTES]

Show, (1) using a truth table, and (2) using a mathematical proof, that $[(p \longrightarrow q) \wedge (q \longrightarrow r)] \longrightarrow (p \longrightarrow r)$ is a tautology.

PROBLEM 26. [TIME ALLOWED = 5 MINUTES]

Express the negation of the following statements in terms of quantifiers without using the negation operator:

(a) $\forall x\ ((x > -1) \vee (x < 1))$
(b) $\exists x\ (3 < x \leq 7)$