COMP 233 Probability and Statistics for Computer Science
Winter 2020, Assignment 2
Due: February 14, 2020

Question 1 If $E[X] = 2$ and $E[X^2] = 8$, calculate the following.
  a) $E[(2+4X)^2]$.
  b) $E[X^2+(X+1)^2]$.

Question 2 A product is classified according to the number of defects it contains and the factory that produces it. Let X_1 and X_2 be the random variables that represent the number of defects per unit (taking on possible values of 0, 1, 2, or 3) and the factory number (taking on possible values 1 or 2), respectively. The entries in the table represent the *joint* possibility mass function of a randomly chosen product.

X_1 \ X_2	1	2
0	1/8	1/16
1	1/16	1/16
2	3/16	1/8
3	1/8	1/4

  a) Find the marginal probability distributions of X_1 and X_2, i.e., $P_{X_1}(i)$ and $P_{X_2}(i)$.
  b) Find $E[X_1]$, $E[X_2]$, $Var(X_1)$, $Var(X_2)$, and $Cov(X_1, X_2)$.

Question 3 From past experience, a professor knows that the test score of a student taking her final examination is a random variable with mean 75.
  a) Give an upper bound to the probability that a student's test score will exceed 85.
  b) Suppose in addition the professor knows that the variance of a student's test score is equal to 25. What can be said about the probability that a student will score between 65 and 85?
  c) How many students would have to take the examination so as to ensure, with probability at least 0.9, that the class average would be within 5 of 75?

Question 4 Let X be a binomial random variable with
$$E[X] = 7 \text{ and } Var(X) = 2.1$$
Find
  a) $P\{X = 4\}$
  b) $P\{X > 12\}$.

Question 5 The number of times that an individual contracts a cold in a given year is a Poisson random variable with parameter $\lambda = 3$. Suppose a new wonder drug (based on large quantities of vitamin C) has just been marketed that reduces the Poisson parameter to $\lambda = 2$ for 75 percent of the population. For the other 25 percent of the population, the drug has no appreciable effect on colds. If an individual tries the drug for a year and has 0 colds in that time, how likely is it that the drug is beneficial for him or her?

Question 6 The weekly demand for a product approximately has a normal distribution with mean 1,000 and standard deviation 200. The current on hand inventory is 2,200 and no deliveries will be occurring in the next two weeks. Assuming that the demands in different weeks are independent,

    a) What is the probability that the demand in each of the next 2 weeks is less than 1,100?

    b) What is the probability that the total of the demands in the next 2 weeks exceeds 2,200?

Question 7 A manufacturer produces bolts that are specified to be between 1.19 and 1.21 inches in diameter. If its production process results in a bolt's diameter being normally distributed with mean 1.20 inches and standard deviation .005, what percentage of bolts will not meet specifications?

Question 8 The number of years a radio functions is exponentially distributed with parameter $\lambda = 1/8$. If Jones buys a used radio, what is the probability that it will be working after an additional 10 years?