COMP-232

MATHEMATICS FOR COMPUTER SCIENCE

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Assignment #1

Shadi Jiha

 # 40131284

1. For each of the following statements use a truth table to determine whether it is a tautology, a contra-diction, or a contingency.

a)

$\boldsymbol{p}$	$\boldsymbol{q}$	$\boldsymbol{r}$	$\boldsymbol{p \vee r}$	$\boldsymbol{p \vee r}$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	F

$(\boldsymbol{p} \vee \boldsymbol{r}) \wedge (\boldsymbol{q} \vee \boldsymbol{r})$	$\boldsymbol{p \wedge q}$	$(\boldsymbol{p} \wedge \boldsymbol{q}) \vee \boldsymbol{r}$	$((\boldsymbol{p} \vee \boldsymbol{r}) \wedge (\boldsymbol{q} \vee \boldsymbol{r})) \leftrightarrow (\boldsymbol{p} \wedge \boldsymbol{q}) \vee \boldsymbol{r}$
T	T	T	T
T	T	T	T
T	F	T	T
F	F	F	T
T	F	T	T
F	F	F	T
T	F	T	T
F	F	F	T

It is a tautology

b)

$\boldsymbol{p}$	$\boldsymbol{q}$	$\boldsymbol{p \oplus q}$	$\boldsymbol{p \oplus \neg q}$	$(\boldsymbol{p \oplus q}) \wedge (\boldsymbol{p \oplus \neg q})$
T	T	F	T	F
T	F	T	F	F
F	T	T	F	F
F	F	F	T	F

It is a contradiction

c)

p	q	r	$q \rightarrow r$	$p \rightarrow (q \rightarrow r)$
T	T	T	T	T
T	T	F	T	T
T	F	T	F	T
T	F	F	T	T
F	T	T	T	F
F	T	F	T	F
F	F	T	F	T
F	F	F	T	F

$(q \wedge r)$	$p \rightarrow (q \wedge r)$	$(p \rightarrow (q \rightarrow r)) \leftrightarrow (p \rightarrow (q \wedge r))$
T	T	F
F	T	F
F	T	F
F	T	F
T	F	F
F	T	T
F	T	F
F	T	T

It is a contingency

d)

p	q	$\neg q \rightarrow \neg p$	$p \wedge \neg q \rightarrow \neg p$	$(p \wedge \neg q \rightarrow \neg p) \rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

It is a tautology

2. For each of the following logical equivalences state whether it is valid or invalid. If invalid then give a counterexample (e.g., based on a truth assignment). If valid then give an algebraic proof using logical equivalences from Tables 6, 7, and 8 from Section 1.3 of textbook. (include logic laws).

a)

$(p \to r) \land (q \to r) \equiv (p \land q) \to r$

$(\neg p \lor r) \land (\neg q \lor r) \equiv \neg(p \land q) \lor r;\ (because\ p \to q \Leftrightarrow \neg p \lor q)$

$(r \lor \neg p) \land (r \lor \neg q) \equiv \neg p \lor \neg q \lor r; (Commutative\ law)$

$r \lor (\neg p \land \neg q) \equiv \neg p \lor \neg q \lor r; (Distributive\ law)$

$r \lor \neg(p \lor q) \equiv \neg(p \land q) \lor r; (De\ Morgan's\ law)$

$INVALID$

Proof:

Let's consider $p = T, q = F\ and\ r = T$

$(p \to r) \land (q \to r) \equiv (p \land q) \to r$

$T \land T \equiv F \to T$

$T = F, which\ is\ not\ true$

b)

$(p \to r) \lor (q \to r) \equiv (p \lor q) \to r$

$(\neg p \lor r) \lor (\neg q \lor r) \equiv \neg(p \lor q) \lor r; (because\ p \to q \Leftrightarrow \neg p \lor q)$

$\neg p \lor r \lor \neg q \lor r \equiv \neg(p \lor q) \lor r$

$(\neg p \lor \neg q) \lor (r \lor r) \equiv \neg(p \lor q) \lor r; (Associative\ law)$

$(\neg p \lor \neg q) \lor r \equiv \neg(p \lor q) \lor r; (Idempotent\ law)$

$\neg(p \land q) \lor r \equiv \neg(p \lor q) \lor r; (De\ Morgan's\ law)$

$INVALID$

Proof:

Let's consider $p = T,\ q = F,\ r = F$

$(p \to r) \lor (q \to r) \equiv (p \lor q) \to r$

$F \lor T \equiv T \to F$

$T = F$

$T = F, which\ is\ not\ true$

c)

$\left(\left((p \lor q) \land (p \to r) \land (q \to r)\right) \to r\right) \equiv T$

$\left(\left((p \lor q) \land (\neg p \lor r) \land (\neg q \lor r)\right) \to r\right) \equiv T; (because\ p \to q = \neg p \lor q)$

$\left(\neg\left((p \lor q) \land (\neg p \lor r) \land (\neg q \lor r)\right) \lor r\right) \equiv T; (because\ p \to q = \neg p \lor q)$

$(\neg(p \lor q) \lor \neg(\neg p \lor r) \lor \neg(\neg q \lor r) \lor r) \equiv T; (De\ Morgan's\ law)$

$(\neg p \land \neg q) \lor (p \land \neg r) \lor (q \land \neg r) \lor r \equiv T; (De\ Morgan's\ law)$

$(\neg p \land \neg q) \lor \left(\neg r \land (p \lor q)\right) \lor r \equiv T;\ (Distributive\ law)$

$\big((\neg p \wedge \neg q) \vee r\big) \vee \big(\neg r \wedge (p \vee q)\big) \equiv T; (Associative\ law)$
$let\ P = (\neg(p \vee q) \vee r)$
$(\neg(p \vee q) \vee r) \vee \neg\big(r \vee \neg(p \vee q)\big) \equiv T$
$So{:}\ \ P \vee \neg P \equiv T$
They are the opposite of each other and since it is a OR in-between and they will never be equal, the statement will always be true

$VALID$

d)

$\Big(\big((p \to q) \wedge (q \to r)\big) \to (p \to r)\Big) \equiv T$
$\Big(\neg\big((p \to q) \wedge (q \to r)\big) \vee (p \to r)\Big) \equiv T; (because\ p \to q = \neg p \vee q)$
$\big(\neg(p \to q) \vee \neg(q \to r) \vee (p \to r)\big) \equiv T; (De\ Morgan's\ law)$
$\big(\neg(\neg p \vee q) \vee \neg(\neg q \vee r) \vee (\neg p \vee r)\big) \equiv T;\ (because\ p \to q = \neg p \vee q)$
$\big((p \wedge \neg q) \vee (q \wedge \neg r) \vee (\neg p \vee r)\big) \equiv T; (De\ Morgan's\ law)$

Here we can see that each one has a proposition that it the opposite of the other one. Example: one has q, the other has-q. And they are separated by OR so there is no combination of p, q and r that will give a FALSE statement.

$VALID$

3. Which of the following conditions are necessary, and which conditions are sufficient, for the natural number n to be divisible by 6? We say that integer a is divisible by integer b x 0 if there is an integer c such that a = bc. The natural numbers are N = {0, 1, 2,…}.
JUSTIFY
Necessary: 6|n => condition
Sufficient: condition => 6|n

a) Necessary, because 6 = 3 . 2 so if 6 | n then 3 | n also.
b) Neither, because if 6 | n ≠ 9 | n because n can be 6 so it is not necessary. Also 9 | n ≠ 6 | n, because n can be a number such as 30.
c) Sufficient, because if 6 | n ≠ 9 | n because n can be 6 or 18 so it is not necessary. However, 12 | n = 6 | n, 12 = 3 . 2 . 2 and 6 = 3 . 2. They have common factor so if a number if divisible by 12 it is divisible by 6.
d) Sufficient, because 24 is divisible by 6 but n doesn't need to be 24. n can be a lot of other numbers such as 6, 18, 36, …
e) Necessary, because if 3 | n then 3 | n^2 as proven in a)
f) Necessary, because all 6 = 3 . 2, which means that if 6 | n then 2 | n then is even. Also as proven above. If 6 | n then 3 | n. So naturally, n is even and divisible by 3.

ONE SHOULD BE NECESSARY AND SUFFICIENT

4. A set of propositions is consistent if there is an assignment of truth values to each of the variables in the propositions that makes each proposition true. Is the following set of propositions consistent?

Let p: "The file system is not locked"

q: "New messages will queue"

r: "The system is functioning normally"

s: "New messages will be sent to the message buffer"

So we have:

1) $p \rightarrow q$
2) $p \leftrightarrow r$
3) $\neg q \rightarrow s$
4) $p \rightarrow s$
5) $\neg s$

Because all sentences have to be true:

a) s = false
b) p = false
c) q = true
d) r = true
e) p = false, q = true

Therefor it is consistant

5. Suppose the domain of the propositional function $\boldsymbol{P(x;y)}$ consists of pairs x and y, where x = 1, 2, or 3, and y = 1, 2, or 3. Write out the propositions below using disjunctions and conjunctions only.

a) $P(x,y);\ \ x \in \{1,2,3\}$. So $\exists x\ P(x,3) \equiv P(1,3) \vee P(2,3) \vee P(3,3)$

b) $\forall y \neg(2,y); y \in \{1,2,3\}$
$\equiv \neg\exists y\ P(2,y) \equiv \neg[P(2,1) \vee P(2,2) \vee P(2,3)]$
$\equiv \neg P(2,1) \wedge \neg P(2,2) \wedge \neg P(2,3)$

c) $\forall x\ \exists y\ P(x,y)\ ;\ x,y \in \{1,2,3\}$. So,
$\forall x\ \exists y\ P(x,y)\ \equiv\ \forall x[P(x,1) \vee P(x,2) \vee P(x,3)]$
$\equiv [P(1,1) \vee P(1,2) \vee P(1,3)] \wedge [P(2,1) \vee P(2,2) \vee P(2,3)] \wedge [P(3,1) \vee P(3,2) \vee P(3,3)]$

d) $\exists x \forall y \neg P(x,y); x,y \in \{1,2,3\}. So$
$\equiv \exists x \neg\exists y P(x,y)$
$\equiv \exists x \neg[P(x,1) \vee P(x,2) \vee P(x,3)]$
$\equiv \neg[P(1,1) \vee P(1,2) \vee P(1,3)] \vee \neg[P(2,1) \vee P(2,2) \vee P(2,3)] \vee$

$\neg[P(3,1) \vee P(3,2) \vee P(3,3)]$

6. Let the domain for x be the set of all students in this class and the domain for y be the set of all countries in the world. Let P(x; y) denote student x that has visited country y and Q(x; y) denote student x that has a friend in country y. Express each of the following using logical operations and quantifiers, and the propositional functions P(x; y) and Q(x; y).

a) $P(Carlos, Bulgaria)$

b) $\forall x\ P(x, US)$

c) $\exists y\ \forall x\ P(x, y)$

d) $\forall x \neg \exists y P(x, y)$

e) $[(x_1 \neq x_2) \wedge \forall y(Q(x_1, y) \vee Q(x_2, y))]$

f) $(y_1 \neq y_2)\ \forall x \exists y\ P(x, y_1)\ Q(x, y_2)$

7. For each part in the previous question, form the negation of the statement so that all negation symbols occur immediately in front of predicates. For example:

a) $\neg P(Carlos, Bulgaria)$

b) $\forall x \neg P(x, US)$

c) $\forall y\ \forall x \neg\ P(x, y)$

d) $\forall x \neg \exists y P(x, y)$

$\equiv \neg(\forall x \neg \exists y P(x, y)) \equiv \forall x \neg \exists y \neg P(x, y) \equiv \forall x \forall y P(x, y)$

e) $\neg[(x_1 \neq x_2) \wedge \forall y(Q(x_1, y) \vee Q(x_2, y))]$

$\equiv \neg(x_1 \neq x_2) \vee \neg \forall y(Q(x_1, y) \vee Q(x_2, y))$

$\equiv \neg(x_1 \neq x_2) \vee \exists \neg y(Q(x_1, y) \vee Q(x_2, y))$

f) $\neg[(y_1 \neq y_2)\ \forall x \exists y\ P(x, y_1)\ Q(x, y_2)]$

$\equiv (x_1 = x_2) \wedge \exists \neg \neg y(Q(x_1, y) \vee Q(x_2, y))$

$\equiv (x_1 = x_2) \wedge \exists y(Q(x_1, y) \vee Q(x_2, y))$

8. Negate the following statements and transform the negation so that negation symbols immediately precede predicates. (See example in Question 7.)

a) $\exists x \exists y P(x, y) \vee (\forall x \forall y\ Q(x, y))$

$\equiv \neg[\exists x \exists y P(x, y) \vee (\forall x \forall y\ Q(x, y))]$

$\equiv \neg \exists x \neg \exists y \neg P(x, y) \wedge \neg \forall x \neg \forall y\ \neg Q(x, y)$

$\equiv (\ \forall x \forall y \neg\ P(x, y)) \wedge (\exists x \exists y \neg Q(x, y))$

b) $\forall x \forall y(\ Q(x, y) \leftrightarrow Q(y, x))$

$\equiv \neg\forall x \neg\forall y \neg(Q(x,y) \leftrightarrow Q(y,x))$

$\equiv \exists x \exists y(Q(x,y) \leftrightarrow \neg Q(y,x))$

c) $\forall y \exists x \exists z(T(x,y,z) \land Q(x,y))$

$\equiv \neg\forall y \neg\exists x \neg\exists z \neg(T(x,y,z) \land Q(x,y))$

$\equiv \exists y \forall x \forall z \neg(T(x,y,z) \land Q(x,y))$

$\equiv \exists y \forall x \forall z(\neg T(x,y,z) \lor \neg Q(x,y))$