Department of Computer Science & Software Engineering

Comp232 Mathematics for Computer Science

Fall 2019

Assignment 2. Due date: October 13

1. Let $P(x, y, z)$ denote the statement "$x + y \leq z$, " where $x, y, z \in \mathbb{Z}^+$. What is the truth value of each of the following? Explain your answers.

   (a) $\forall x \exists y \exists z P(x, y, z)$.

   (b) $\forall y \exists x \forall z P(x, y, z)$.

   (c) $\exists z \exists y \forall x P(x, y, z)$.

2. For each of the premise-conclusion pairs below, give a valid step-by-step argument (proof) along with the name of the inference rule used in each step. For examples, see pages 73 and 74 in textbook.

   (a) Premise: $\{\neg p \vee q \to r,\ s \vee \neg q,\ \neg t,\ p \to t,\ \neg p \wedge r \to \neg s\}$, conclusion: $\neg q$.

   (b) Premise: $\{\neg p \to r \wedge \neg s,\ t \to s,\ u \to \neg p,\ \neg w,\ u \vee w\}$, conclusion: $\neg t \vee w$.

   (c) Premise: $\{p \vee q,\ q \to r,\ p \wedge s \to t,\ \neg r,\ \neg q \to u \wedge s\}$, conclusion: t.

3. For each of the following, determine whether the argument is valid. You may use a counterexample or equivalence transformations to justify your answer.

   (a)
   $$\begin{array}{l} p \to q \\ \underline{\neg p \quad\quad} \\ \therefore\ \neg q \end{array}$$

   (b)
   $$\begin{array}{l} \underline{\neg p \to \neg q \quad} \\ \therefore\ (\neg p \to q) \to p \end{array}$$

   (c)
   $$\begin{array}{l} p \to r \\ q \to r \\ \underline{\neg(p \vee q)} \\ \therefore\ \neg r \end{array}$$

   (d)
   $$\begin{array}{l} p \to q \\ \underline{p \to (q \to \neg p)} \\ \therefore\ \neg p \end{array}$$

4. For each of the arguments below, indicate whether it is valid or invalid.

   (a) All healthy people eat an apple a day.
   Helen eats an apple a day.
   $\therefore$ Helen is a healthy person.

   (b) All healthy people eat an apple a day.
   Herbert is not a healthy person.
   $\therefore$ Herbert does not eat an apple a day.

   (c) If a product of two real numbers is 0, then at least one of the numbers is 0.
   For a particular real number x, neither $(x-1)$ nor $(x+1)$ equals 0.
   $\therefore$ The product $(x-1)(x+1)$ is not 0.

5. Use rules of inference to show that if $\forall x\big(P(x) \to Q(x)\big)$, $\forall x\big(Q(x) \to R(x)\big)$, and $\exists x\big(\neg R(x)\big)$ are true, then $\exists x\big(\neg P(x)\big)$ is true.

6. (a) Give a direct proof of: "If x is an odd integer and y is an even integer, then $x+y$ is odd."

   (b) Give a proof by contradiction of: "If n is an odd integer, then n^2 is odd."

   (c) Give an indirect proof of: "If x is an odd integer, then $x+2$ is odd."

   (d) Use a proof by cases to show that there are no solutions in positive integers to the equation $x^4 + y^4 = 100$.

   (e) Prove that given a nonnegative integer n, there is a unique nonnegative integer m, such that $m^2 \leq n < (m+1)^2$.

7. For each of the statements below state whether it is True or False. If True then give a proof. If False then explain why, e.g., by giving a counterexample.

   (a) The difference of any two odd integers is odd.

   (b) Let a and b be integers. If $a+b$ is even, then either a or b is even.

   (c) For all positive integers n, it holds that n is even if and only if $3n^2+8$ is even.

   (d) For all positive $x, y \in \mathbb{R}$, if x is irrational and y is irrational then $x+y$ is irrational.

   (e) $\forall x, y \in \mathbb{R}$, if x is irrational and y is rational then $x \cdot y$ is irrational.