7. For the given functions (expressions)

   (a) $f(x) = (2-x)^4$,

   (b) $g(x) = \frac{1}{1+\sqrt{x}}$,

   (c) $h(x) = 3 - |x|$,

   (d) $u(x) = \frac{1}{1-\sqrt{x}}$,

   find the largest possible domain in $\mathbb{R}$ and the corresponding range.

8. Let $f\colon D \to C$, defined as $x \mapsto x^2$, with $D, C \subset \mathbb{R}$.

   (a) Give an example for $D, C \subset \mathbb{R}$, such that f is injective. Find another example for $D, C \subset \mathbb{R}$, such that f is not injective.

   (b) Give an examples for $D, C \subset \mathbb{R}$, such that f is surjective. Find another example for $D, C \subset \mathbb{R}$, such that f is not surjective.

   (c) Consider $D = \mathbb{R}$ and $C = \mathbb{R}$.

      i. For $S_1 := \{-1, 0, 1, 2, \pi\}$, $S_2 := \{x \in \mathbb{R} : \ -3 \leq x \leq 4\}$ and $S_3 := \{x \in \mathbb{R} : \ x < 0 \lor x > 16\}$, determine the image of S_i under f, i.e. compute $f(S_i)$, for $i = 1, 2, 3$.

      ii. For $T_1 := \{0, 1, 2, \pi\}$, $T_2 := \{x \in \mathbb{R} : \ x \geq 0\}$ and $T_3 := \{x \in \mathbb{R} : \ x < 0\}$, determine the pre-image of T_i under f, i.e. compute $f^{-1}(T_i)$, for $i = 1, 2, 3$.

9. (a) Let $f\colon \mathbb{R} \to \mathbb{R}$, with $f(x) = x + 3$ and $g\colon \mathbb{R} \to \mathbb{R}$, $g(x) = x - 5$. Determine $(g \circ f)(x)$ and $(f \circ g)(x)$.

   (b) Let $f\colon \mathbb{R} \to \mathbb{R}$, with $f(x) = 3x$ and $g\colon \mathbb{R} \to \mathbb{R}$, $g(x) = -5x$. Determine $(g \circ f)(x)$ and $(f \circ g)(x)$.

   (c) Is function composition commutative, i.e. does $(g \circ f)(x) = (f \circ g)(x)$ hold in general? Justify your answer!

10. Investigate if the following functions are injective, surjective or bijective. Sketch the graph of each function and calculate its inverse if possible.

    (a) $f : \mathbb{R} \to \mathbb{R} \quad \text{with } f(x) = x^4$,

    (b) $g : \mathbb{R}_0^+ \to \mathbb{R}_0^- \quad \text{with } g(x) = -x^2$,

    (c) $h : \mathbb{R}\backslash\{0\} \to \mathbb{R}\backslash\{0\} \quad \text{with } h(x) = -x^{-1}$.

11. (a) Show that the divisibility relation over $\mathbb{N}$, that is, $| \subset \mathbb{N} \times \mathbb{N}$, is a partial order. (Recall that the divisibility relation $|$ is defined in the lecture notes as follows: Let m and n be integers. We say that m divides n, and write $m|n$, if there exists $k \in \mathbb{N}$ such that $mk = n$.)

    (b) Restrict the relation above to the finite set $\{1, 2, 3, 4, 5, 6\}$ and give the pairs of elements which are in relation.

12. Let $f : M \to N$ be a function and $C, D \subset N$. By

$$f^{-1}(C) := \{x : f(x) \in C\} \subset M$$

we denote the preimage of C under f (Definition 1.8). Is the following statement true?

$$f^{-1}(C) \cap f^{-1}(D) = f^{-1}(C \cap D).$$

If true, provide a proof, and if false, a counterxample!