338.001, VL Logic, Martina Seidl / Wolfgang Schreiner / Wolfgang Windsteiger, 2022W

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State	Finished
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Time taken	15 mins
Grade	**0.4** out of 5.0 (**8.4**%)

Question **1**

Partially correct

Mark 0.1 out of 0.5

Flag question

In the following questions, you need to generate a formal proof (proof tree) of the statement $(K_1 \wedge K_2) \rightarrow G$, where K_1, K_2, and G abbreviate the following formulas:

$$K_1: \ \forall k\colon l(k) \rightarrow m(k, v(k))$$
$$K_2: \ \forall i, j\colon s(j, i) \rightarrow m(j, w(i))$$
$$G: \ \forall i\colon (l(i) \vee \exists j\colon s(i, j)) \rightarrow \exists j\colon m(i, j)$$

Note that, for reasons of space, we will sometimes use the abbreviations instead of the expanded formulas even in the proof tree.

In the first exercise, develop an "incomplete" proof tree until the step, where the proof divides into 2 branches called (1) and (2). These branches (1) and (2) have then to be completed in the subsequent exercises.

Like in the other examples, proof rule "GA" stands for "GoalAssum" and "CA" stands for "ContrAssum".

$$\forall k\colon \big(l(k) \rightarrow m(k, v(k))\big), \forall i, j\colon \big(s(j, i) \rightarrow m(j, w(i))\big) \vdash \forall i\colon \big((l(i) \vee \exists j\colon s(i, j)) \rightarrow \exists j\colon m(i, j)\big) \quad \downarrow$$

P-∨

$$\forall k\colon \big(l(k) \rightarrow m(k, v(k))\big), \ \boxed{l(i_0) \vee \exists j\colon s(i,j)} \ \vdash \ \boxed{(l(i_0) \vee \exists j\colon s(i_0,j)) \rightarrow \exists j\colon m(i_0,j)}$$

P-→

$$\forall k\colon \big(l(k) \rightarrow m(k, v(k))\big), \forall i, j\colon \big(s(j, i) \rightarrow m(j, w(i))\big), \ \boxed{\forall j\colon m(i_0,j)} \ \vdash \ \boxed{\forall j\colon m(i_0,j)}$$

A-∧

$$K_1, K_2, \ \boxed{m(i_0,j)} \ \vdash \ \boxed{\forall j\colon m(i_0,j)} \qquad K_1, K_2, \exists j\colon \boxed{\exists j\colon m(i_0,j)} \ \vdash \ \boxed{\exists j\colon s(i_0,j)}$$

$$(1) \qquad\qquad (2)$$

A-∀ | MP | P-∃ | CA | P-∀ | P-∧ | P- ¬ | A-∨ | P-→ | GA | A-∧ | A-∃ | P-∨ | A-→ | P-↔ | A- ¬

$\forall i\colon (l(i) \rightarrow m(i,v(i)))$ | $l(i_0) \wedge \exists j\colon s(i_0,j)$ | $(l(i_0) \vee \exists j\colon s(i_0,j)) \rightarrow \exists j\colon m(i_0,j)$ | $\exists j\colon m(i_0,j)$ | $\forall i\colon (l(i) \vee \exists j\colon s(i,j))$

$l(i_0) \vee \exists j\colon s(i,j)$ | $\forall i,j\colon (s(j,i) \rightarrow m(j,w(i)))$ | $\exists j\colon m(i,j)$ | $l(i_0) \vee \exists j\colon s(i_0,j)$ | $\exists j\colon s(i_0,j) \rightarrow \exists j\colon m(i_0,j)$

$l(i_0) \wedge \forall j\colon s(i_0,j)$ | $\forall j\colon m(i_0,j)$

$l(i_0)$ | $l(i)$ | $\exists j\colon m(i_0,j)$ | $\exists j\colon m(i,j)$ | $\forall j\colon m(i_0,j)$ | $s(j,i_0)$ | $m(i_0,j)$ | $\exists j\colon s(j,i_0)$ | $l(j)$ | $\exists j\colon s(i_0,j)$ | $m(j,i_0)$ | $s(i_0,j)$

Die Antwort ist teilweise richtig.

You have correctly selected 2.

Question **2**

Partially correct

Mark 0.2 out of 1.5

Flag question

Suppose now the proof situation in branch (1) is

$$\forall c\colon \big(d(c) \rightarrow h(c, s(c))\big), \forall a, b\colon \big(k(b, a) \rightarrow h(b, t(a))\big), d(a_0) \ \vdash \ \exists b\colon h(a_0, b).$$

Note that this might not be exactly what you derived in the first example, it is a "hypothetical" proof situation. Complete **this branch** of the proof.

Like in the other examples, proof rule "GA" stands for "GoalAssum" and "CA" stands for "ContrAssum".

$$(1)$$

$$\forall c\colon \big(d(c) \rightarrow h(c, s(c))\big), \forall a, b\colon \big(k(b, a) \rightarrow h(b, t(a))\big), d(a_0) \ \vdash \ \exists b\colon h(a_0, b) \quad \downarrow$$

P-→

$$\forall c\colon \big(d(c) \rightarrow h(c, s(c))\big), \forall a, b\colon \big(k(b, a) \rightarrow h(b, t(a))\big), d(a_0), \ \boxed{k(a_0,a_0) \rightarrow h(a_0,t(a_0))} \ \vdash \ \boxed{\forall b\colon h(a_0,b)}$$

A-∨

$$\forall c\colon \big(d(c) \rightarrow h(c, s(c))\big), \forall a, b\colon \big(k(b, a) \rightarrow h(b, t(a))\big), d(a_0), \ \boxed{k(b_0,a_0) \rightarrow h(b_0,t(a_0))}, h(\boxed{a_0}, \boxed{c_0}) \ \vdash \ \boxed{d(c_0) \rightarrow h(c_0,s(c_0))}$$

A-∀

$$\forall c\colon \big(d(c) \rightarrow h(c, s(c))\big), \forall a, b\colon \big(k(b, a) \rightarrow h(b, t(a))\big), d(a_0), \ \boxed{d(a_0) \rightarrow h(a_0,s(a_0))}, h(\boxed{t(c_0)}, \boxed{s(c_0)}) \ \vdash \ \boxed{h(a_0,t(a_0))}$$

P-∧

P-∧ | A-∃ | P-→ | A- ¬ | A-∀ | P-↔ | CA | GA | A-→ | A-∨ | P- ¬ | MP | P-∀ | A-∧ | P-∃ | P-∨

$k(a_0,a_0) \rightarrow h(a_0,t(a_0))$ | $k(b_0,a_0) \rightarrow h(b_0,t(a_0))$ | $\exists b\colon h(a_0,b)$ | $\forall b\colon h(a_0,b)$ | $d(a_0) \rightarrow h(a_0,s(a_0))$ | $d(c_0) \rightarrow h(c_0,s(c_0))$

$\perp$ | $k(b_0,b_0) \rightarrow h(b_0,t(b_0))$

$h(s(a_0),a_0)$ | $h(t(a_0),a_0)$ | $h(t(a_0),t(a_0))$ | $h(a_0,b_0)$ | $\perp$ | $h(s(a_0),s(a_0))$ | $h(a_0,s(a_0))$ | $h(a_0,a_0)$

a_0 | c_0 | $t(c_0)$ | $s(c_0)$ | $t(a_0)$ | $s(a_0)$

Die Antwort ist teilweise richtig.

You have correctly selected 2.

Question **3**

Partially correct

Mark 0.1 out of 1.5

Flag question

Suppose now the proof situation in branch (2) is

$$\forall c\colon (l(c) \to m(c,d(c))), \forall a,b\colon (s(b,a) \to m(b,h(a))), \exists b\colon s(a_0,b) \ \vdash\ \exists b\colon m(a_0,b).$$

Note that this might not be exactly what you derived in the first example, it is a "hypothetical" proof situation. Complete **this branch** of the proof.

Like in the other examples, proof rule "GA" stands for "GoalAssum" and "CA" stands for "ContrAssum".

(2)

$\forall c\colon (l(c) \to m(c,d(c))), \forall a,b\colon (s(b,a) \to m(b,h(a))), \exists b\colon s(a_0,b) \ \vdash\ \exists b\colon m(a_0,b)$

P-→ $\forall c\colon (l(c) \to m(c,d(c))), \forall a,b\colon (s(b,a) \to m(b,h(a))),$ ⬚ $\vdash$ ∀b: m(a₀,b)

P-∨ $\forall c\colon (l(c) \to m(c,d(c))), \forall a,b\colon (s(b,a) \to m(b,h(a))), s(⬚,⬚) \to$ ⬚ , ⬚ $\vdash$ ⬚

P-∀ $\forall c\colon (l(c) \to m(c,d(c))), \forall a,b\colon (s(b,a) \to m(b,h(a))), s(⬚,⬚) \to$ ⬚ , $m(a_0, h(b_0))$, ⬚ $\vdash$ ⬚

A-∨ $\forall c\colon (l(c) \to m(c,d(c))), \forall a,b\colon (s(b,a) \to m(b,h(a))), s(⬚,⬚) \to$ ⬚ , $m(b_0, d(c_0))$, ⬚ $\vdash$ ⬚

A-∧

P-∀ | P-↔ | A-∨ | MP | A-∧ | A-→ | P-∨ | A-∃ | P-¬ | P-∧ | P-→ | CA | P-∃ | GA | A-¬ | A-∀

$s(b_0,b_0) \to m(b_0,h(b_0))$ | $\perp$ | $\forall b\colon m(a_0,b)$ | $l(a_0) \to m(a_0,d(a_0))$ | $l(c_0) \to m(c_0,d(c_0))$ | $\exists b\colon m(a_0,b)$

$s(b_0,a_0) \to m(b_0,h(a_0))$ | $s(a_0,a_0) \to m(a_0,h(a_0))$

$m(a_0,h(b_0))$ | $m(d(a_0),a_0)$ | $\perp$ | $s(a_0,a_0)$ | $m(d(a_0),d(a_0))$ | $s(b_0,a_0)$ | $m(a_0,a_0)$ | $s(a_0,b_0)$ | $m(a_0,b_0)$

$m(h(a_0),h(a_0))$ | $m(a_0,d(a_0))$ | $m(h(b_0),a_0)$

a_0 | b_0

$h(a_0)$ | $d(a_0)$ | $d(c_0)$ | $h(b_0)$

Die Antwort ist teilweise richtig.

You have correctly selected 2.

Question **4**

Incorrect

Mark 0.0 out of 1.5

Flag question

Does the following proof tree represent a correct proof or is there a "bug" somewhere? If it is not correct, mark **all steps** that contain errors, since the proof may contain more than one error. As usual, "GA" and "CA" abbreviate "GoalAssum" and "ContrAssum", respectively.

$$\vdash\ (\forall m,n\colon (T(m) \lor S(n))) \to (\forall m\colon T(m) \lor \forall n\colon S(n)) \quad \downarrow$$
1
$$\forall m,n\colon T(m) \lor S(n)\ \vdash\ \forall m\colon T(m) \lor \forall n\colon S(n)$$
2
$$\forall m,n\colon T(m) \lor S(n),\ \neg\forall m\colon T(m)\ \vdash\ \forall n\colon S(n)$$
3
$$\forall m,n\colon T(m) \lor S(n),\ \exists m\colon \neg T(m)\ \vdash\ \forall n\colon S(n)$$
4
$$\forall m,n\colon T(m) \lor S(n),\ \neg T(m_0)\ \vdash\ S(n_0)$$
5
$$T(m_0) \lor S(n_0),\ \neg T(m_0)\ \vdash\ S(n_0)$$
6
$$\text{CA}\ \ T(m_0),\ \neg T(m_0)\ \vdash\ S(n_0) \qquad\qquad S(n_0),\ \neg T(m_0)\ \vdash\ S(n_0)\ \ \text{GA}$$

Select one or more:

- [x] In step 5 the universal quantifier is instantiated incorrectly. ✖
- [ ] There is no bug in the proof. The proof is correct.
- [ ] In step 1 we must not split the implication.
- [ ] In step 3 the de'Morgan-rule is applied incorrectly.
- [ ] In step 2 the disjunction is handled incorrectly.
- [ ] In step 4 we must not introduce different constants.
- [ ] In step 6 we must not split the "or" into two branches.

Die Antwort ist falsch.

The correct answer is:
There is no bug in the proof. The proof is correct.

Finish review

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