Exercise Sheet 0

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Mathematics for AI I ⇒ Ex 0 (Exercise sheet 0)

A. Which of the following is a proposition?

⇒ d) $1<3$ e) $3<1$ f) Every positive integer can be written as the sum of two prime numbers.

g) $(1+2)^2 = 1^2 + 2\cdot1\cdot2 + 2^2$

* These four statements are proposition as they give truth value while rest doesnot give any truth value.

B. Let A, B be propositions. Determine a truth table for the propositions.

a) $\neg(A \wedge \neg B)$, b) $\neg A \vee B$

Truth table

A	B	$\neg B$	$A \wedge \neg B$	$\neg(A \wedge \neg B)$	$\neg A$	$\neg A \vee B$	$A \Rightarrow B$
T	T	F	F	T	F	T	T
T	F	T	T	F	F	F	F
F	T	F	F	T	T	T	T
F	F	T	F	T	T	T	T

The truth table of $\neg(A \wedge \neg B)$, $\neg A \vee B$ and $A \Rightarrow B$ are same or equivalent.

C. Let A, B be any propositions an F a contradiction. Using truth tables, prove: $A \Rightarrow B$ and $A \wedge \neg B \Rightarrow F$ are equivalent.

Truth table

A	B	$\neg B$	F	$A \Rightarrow B$	$A \wedge \neg B \Rightarrow F$
T	T	F	F	T	T
T	F	T	F	F	F
F	T	F	F	T	T
F	F	T	F	T	T

proved

Hence, $A \Rightarrow B$ and $A \wedge \neg B \Rightarrow F$ are logically equivalent.

D. Let A, B and C be any propositions and T a tautology. Using truth tables prove:

a) $(A \Rightarrow B) \wedge (B \Rightarrow C) \Rightarrow (A \Rightarrow C)$ and T are logically equivalent.

A	B	C	$A \Rightarrow B$	$B \Rightarrow C$	$A \Rightarrow C$	$(A \Rightarrow B) \wedge (B \Rightarrow C) \Rightarrow (A \Rightarrow C)$	T
T	T	T	T	T	T	T	T
T	T	F	T	F	F	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	F	T	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Hence, $(A \Rightarrow B) \wedge (B \Rightarrow C) \Rightarrow (A \Rightarrow C)$ and T are equivalent.

b) $A \Leftrightarrow B$ and $(A \Rightarrow B) \wedge (B \Rightarrow A)$ are logically equivalent.

A	B	$A \Leftrightarrow B$	$A \Rightarrow B$	$B \Rightarrow A$	$(A \Rightarrow B) \wedge (B \Rightarrow A)$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

Hence, $(A \Leftrightarrow B)$ and $(A \Rightarrow B) \wedge (B \Rightarrow A)$ are equivalent! proved

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E. Let A, B and C any propositions. Prove

a) $(A \land B) \land C$ and $A \land (B \land C)$

A	B	C	$(A \land B)$	$(B \land C)$	$(A \land B) \land C$	$A \land (B \land C)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	F	T	F	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

Approved

~~Hence~~, The truth table of $(A \land B) \land C$ and $A \land (B \land C)$ is same.
Hence, they are equivalent.

b) $(A \lor B) \land C$ and $(A \land C) \lor (B \land C)$ are logically equivalent

A	B	C	$(A \lor B) \land C$	$(A \land C)$	$(B \land C)$	$(A \land C) \lor (B \land C)$
T	T	T	T	T	T	T
T	T	F	F	F	F	F
T	F	T	T	T	F	T
T	F	F	F	F	F	F
F	T	T	T	F	T	T
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

The truth table of $(A \lor B) \land C$ and $(A \land C) \lor (B \land C)$ are equal.
Hence, they are equivalent.

F. For this exercise the corresponding class of x is Ω := "all possible AI students". We define predicates.

- $P(\cdot)$: "Student ... was in at least one Math lesson."
- $Q(\cdot)$: "student... was in all Math lessons."

Write the following propositions and their negations in words.

a) $\forall x \in \Omega : P(x)$

→ "For every student in the class of all possible AI students, it is true that ~~it is true that~~ the student was in at least one Math lesson."

<u>Negation</u>: → "There exists at least one student in the class of all possible AI students such that the student was not in at least one Math lesson."

b) $\exists x \in \Omega : P(x)$

→ "There exists at least one student in all possible AI students, ~~it is~~ such ~~not true~~ that the student was in at least one Math lesson."

<u>Negation</u>: → "For every students in the class of all possible AI students, it is not true that the students was in at least one Math lesson."

c) $\forall x \in \Omega : Q(x)$

→ "For every students in the class of all possible AI students, it is true that the student was in all Math lessons."

Negation: "There exists at least one student in the class of all possible AI students, such that the student was not in all Math lessons."

d) $\exists x \in \Omega : Q(x)$

→ "There exists at least one student in the class of all possible AI students, such that the student was in all Math lessons.

Negation : → "For every student in the class of all possible AI students, it is not true that the student was in all Math lessons."