Sheet 6

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32) $A = \begin{pmatrix} 1 & 1 & 0 \\ -7 & 1 & 0 \end{pmatrix}_{2\times3}$, $B = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}_{3\times3}$, $C = \begin{pmatrix} 1 & -3 \\ -2 & 1 \\ 0 & 0 \end{pmatrix}_{3\times2}$

a) $A \cdot B$ (Numbers of columns of A = Numbers of rows of B)
Hence, multiplication is possible

$$= \begin{pmatrix} 1 & 1 & 0 \\ -7 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$$

$$= \begin{pmatrix} 1\times1+1\times2+0\times3 & 1\times2+1\times4+0\times6 & 1\times3+1\times6+0\times9 \\ -7\times1+1\times2+0\times3 & -7\times2+1\times4+0\times6 & -7\times3+1\times6+0\times9 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 6 & 9 \\ -5 & -10 & -15 \end{pmatrix}$$

b) $B \cdot A$
(Numbers of colums of $B \neq$ Numbers of rows of A)
Hence, this is not possible

c) $A \cdot (B \cdot C)$
Firstly,

$$B \cdot C = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix} \begin{pmatrix} 1 & -3 \\ -2 & 1 \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1-4+0 & -3+2+0 \\ 2-8+0 & -6+4+0 \\ 3-12+0 & -9+6+0 \end{pmatrix} = \begin{pmatrix} -3 & -1 \\ -6 & -2 \\ -9 & -3 \end{pmatrix}$$

Now,

$A\cdot(B\cdot C)$

$$= \begin{pmatrix} 1 & 1 & 0 \\ -7 & 1 & 0 \end{pmatrix} \begin{pmatrix} -3 & -1 \\ -6 & -2 \\ -9 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} -3-6+0 & -1-2+0 \\ 21-6+0 & 7-2+0 \end{pmatrix}$$

$$= \begin{pmatrix} -9 & -3 \\ 15 & 5 \end{pmatrix}$$

d) $C\cdot(B\cdot A)$

As, $B\cdot A$ is not possible, $C\cdot(B\cdot A)$ is also not possible.

e) $A\cdot(B+C)$

B and C does not have same dimensions. So, addition of B and C is not possible and hence the operation $A\cdot(B+C)$ cannot be calculated.

f) $5\cdot(A^T+C)$

Dimension of A^T and C are equal. So, the addition of both can be calculated.

$$A^T = \begin{pmatrix} 1 & 1 & 0 \\ -7 & 1 & 0 \end{pmatrix}^T = \begin{pmatrix} 1 & -7 \\ 1 & 1 \\ 0 & 0 \end{pmatrix}$$

Now,

$$5\cdot(A^T+C) = 5\cdot\left(\begin{pmatrix} 1 & -7 \\ 1 & 1 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 1 & -3 \\ -2 & 1 \\ 0 & 0 \end{pmatrix}\right) = 5\begin{pmatrix} 2 & -10 \\ -1 & 2 \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 10 & -50 \\ -5 & 10 \\ 0 & 0 \end{pmatrix}$$

g) $B^T \cdot A^T$

$$A^T = \begin{pmatrix} 1 & 1 & 0 \\ -7 & 1 & 0 \end{pmatrix}^T = \begin{pmatrix} 1 & -7 \\ 1 & 1 \\ 0 & 0 \end{pmatrix}$$

$$B^T = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}^T = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$$

Now, $B^T \cdot A^T$

$$= \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix} \begin{pmatrix} 1 & -7 \\ 1 & 1 \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1+2+0 & -7+2+0 \\ 2+4+0 & -14+4+0 \\ 3+6+0 & -21+6+0 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & -5 \\ 6 & -10 \\ 9 & -15 \end{pmatrix}$$

33.a) Let, $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 0 & 0 \end{pmatrix} \in R^{3\times 3}$, $B = \begin{pmatrix} 1 & 4 & 5 \\ 6 & 1 & 0 \\ 0 & 5 & 2 \end{pmatrix} \in R^{3\times 3}$

$$A \cdot B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 4 & 5 \\ 6 & 1 & 0 \\ 0 & 5 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1+12+0 & 4+2+15 & 5+0+6 \\ 4+30+0 & 16+5+30 & 20+0+12 \\ 1+0+0 & 4+0+0 & 5+0+0 \end{pmatrix}$$

$$= \begin{pmatrix} 13 & 21 & 11 \\ 34 & 51 & 32 \\ 1 & 4 & 5 \end{pmatrix}$$

$$B \cdot A = \begin{pmatrix} 1 & 4 & 5 \\ 6 & 1 & 0 \\ 0 & 5 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1+16+5 & 2+20+0 & 3+24+0 \\ 6+4+0 & 12+5+0 & 18+6+0 \\ 0+20+2 & 0+25+0 & 0+30+0 \end{pmatrix}$$

$$= \begin{pmatrix} 22 & 22 & 27 \\ 10 & 17 & 24 \\ 22 & 25 & 30 \end{pmatrix}$$

Here, $A \cdot B \neq B \cdot A$, Hence vector multiplication in $R^{3\times 3}$ is not commutative.

34.) Let the price ~~th~~ of each drink be 'x' EURO,
Then,

price of pizza $= (x+8)$ EURO

The total I paid is 50.

So, A/c to question,

$$5x + 4(x+8) = 50$$

or, $5x + 4x + 32 = 50$

or, $9x = 18$

$\therefore \boxed{x = 2}$

$\therefore$ Price of each drink $(x) = 2$ EURO

Price of each pizza $(x+8) = 2+8 = 10$ EURO

35) a) $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ 0 & 3 \\ 0 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 3 \\ 0 & 0 & 0 \end{pmatrix}$,

$D = \begin{pmatrix} 1 & 2 & 3 & 1 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

A, D is in row echelon form

A is in reduced row echelon form

$\Rightarrow$ D is not in reduced row echelon form because the leading element of R_2 is 2 ~~and~~ ~~the~~ all the elements, ~~in the~~ ~~except 2 should~~ in the corresponding column except 2 should be 0. but it is not. Hence, it is not in (reduced) row echelon form.

35.b) $A = \begin{pmatrix} 2 & 3 \\ 1 & 5 \end{pmatrix}$.

making ~~R~~ Reduced echelon form.

$R_2 \to R_2 - \frac{1}{2} R_1$

$A = \begin{pmatrix} 2 & 3 \\ 0 & \frac{7}{2} \end{pmatrix}$

$R_2 \to \frac{2}{7} R_7$

$A = \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix}$

$R_1 \to R_1 - 3R_2$

$A = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$

$R_1 \to \frac{1}{2} R_1$

$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Rank of A is 2. i.e. numbes of non- zero rows.

33b

$$B = \begin{pmatrix} 1 & 2 & -2 & -4 \\ 2 & 4 & 1 & 2 \\ 1 & 3 & -3 & -3 \end{pmatrix}$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - R_1$

$$B = \begin{pmatrix} 1 & 2 & -2 & -4 \\ 0 & 0 & 5 & 10 \\ 0 & 1 & -1 & 1 \end{pmatrix}$$

$R_2 \to \frac{1}{5} R_2$

$$B = \begin{pmatrix} 1 & 2 & -2 & -4 \\ 0 & 0 & 1 & 2 \\ 0 & 1 & -1 & 1 \end{pmatrix}$$

$R_3 \to R_3 - \frac{1}{2} R_1$

$$B = \begin{pmatrix} 1 & 2 & -2 & -4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 3 \end{pmatrix}$$

$R_3 \to \frac{1}{3} R_3$, $R_1 \to R_1 + 2R_2$

$$B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$R_2 \to R_2 - 2R_3$

$$B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \Longrightarrow$$ Reduced Row echelon form

Rank = 3

36.a) $x_1 - x_3 = 2$

$x_2 + 2x_3 = 5$

$x_1 + x_2 + x_3 = 7$

~~Represent~~ ~~Do~~ Gauss elimination method.

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 1 & 2 & 5 \\ 1 & 1 & 1 & 7 \end{array}\right]$$

$R_1 \leftrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & 1 & 2 & 5 \\ 1 & 0 & -1 & 2 \end{array}\right]$$

$R_3 \to R_3 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & 1 & 2 & 5 \\ 0 & -1 & -2 & -5 \end{array}\right]$$

$R_3 \to R_3 + R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 + x_2 + x_3 = 7$

$x_2 + 2x_3 = 5$

It has infinitely many solutions.

36.3)
$$x_1 + 2x_2 - 2x_3 = -4$$
$$2x_1 + 4x_2 + x_3 = 2$$
$$x_2 - x_3 = 1$$

Gauss Elimination

$$\left[\begin{array}{ccc|c} 1 & 2 & -2 & -4 \\ 2 & 4 & 1 & 2 \\ 0 & 1 & -1 & 1 \end{array}\right]$$

$R_3 \leftrightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & -2 & -4 \\ 0 & 1 & -1 & 1 \\ 2 & 4 & 1 & 2 \end{array}\right]$$

$R_3 \rightarrow R_3 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & -2 & -4 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 5 & 10 \end{array}\right]$$

Now,

$5x_3 = 10$

$\therefore \boxed{x_3 = 2}$

substituting x_3 and x_2

$-x_1 + 2x_2 - 2x_3 = -4$

$-x_1 + 2\times 3 - 2\times 2 = -4$

or, $\therefore \boxed{x_1 = 6}$

Substituting the value of x_3

$x_2 - x_3 = 1$

$x_2 - 2 = 1$

$\therefore \boxed{x_2 = 3}$