1. Compute the finite sums below. Give the computational steps, not only the results. Apply the learned rules for Σ whenever appropriate.

$$A = \sum_{j=5}^{10} (3j),$$

$$B = \sum_{i=1}^{50} (i^3 - (i-1)^3),$$

C is the sum of the first 110 numbers in the list $3, 7, 11, 15, 19, 23, \ldots$.

2. Compute the finite products below. Give the computational steps, not only the results. Apply the learned rules for Π whenever appropriate.

$$A = \prod_{i=1}^{5} (2i),$$

$$B = \prod_{i=3}^{6} (i-2)!,$$

$$C = \prod_{i=1}^{3} \prod_{k=2}^{4} (2i + k).$$

3. Let $n \in \mathbb{N}$. Try to give the following expressions in a simple closed form. Perform some computations for particular, small values of n. Give the computational steps, not only the results.

$$A = \sum_{k=1}^{n} k \cdot k! \ (\textit{Hint: } A = \sum_{k=1}^{n} (k+1-1) \cdot k! = \cdots = (n+1)! - 1\,),$$

$$B = \prod_{i=2}^{n} \left(1 - \frac{1}{i}\right), \ (\textit{Hint: common denominator for the factors})$$

$$C = \sum_{k=1}^{n} \frac{1}{k(k+1)}$$ (*Hint: use that the term* $\frac{1}{k(k+1)}$ *can be rewritten as* $\frac{\alpha}{k} + \frac{\beta}{k+1}$ *for some* α *and* β *in* $\mathbb{R}$),

4. Think about how the following expressions can be written explicitly and try to calculate the sum of the product and the product of the sum:

   a) $\displaystyle\sum_{i=1}^{2} \prod_{k=1}^{3} \frac{i}{k},$

   b) $\displaystyle\prod_{k=1}^{3} \sum_{i=1}^{2} \frac{i}{k},$

   c) Are points a) and b) sufficient to say that, in general, in sum and products together, the order of the symbols sum and product is important and should not be changed?

5. Consider the following table.

x_i	-6	-3	-1	1	3	4
y_i	5	4	2	1	1	0

   (a) Compute $\mu_x := \frac{1}{6}\sum_{\ell=1}^{6} x_\ell$ and $\mu_y := \frac{1}{6}\sum_{\ell=1}^{6} y_\ell$.

   (b) Compute $\sigma_x^2 := \frac{1}{6}\sum_{\ell=1}^{6}(x_\ell - \mu_x)^2$ and $\sigma_{xy} := \frac{1}{6}\sum_{\ell=1}^{6}(x_\ell - \mu_x)(y_\ell - \mu_y)$.

   (c) Determine $k := \frac{\sigma_{xy}}{\sigma_x^2}$.

   (d) Solve for d: $\mu_y = k\mu_x + d$.

   (e) Plot the points (x_i, y_i) and $f\colon \mathbb{R} \to \mathbb{R}$, $f(x) = kx + d$.

   (f) Compute $f(-2)$ and $f(2)$.

6. Let $n \in \mathbb{N}$, and $x_0 < x_1 < x_2 < \cdots < x_n \in \mathbb{R}$ a sorted list of $n+1$ different real numbers. For $i = 0, 1, 2, \ldots, n$, we define the polynomials

$$\ell_i(x) := \prod_{\substack{j=0 \\ j\neq i}}^{n} \frac{x - x_j}{x_i - x_j}\,.$$

This notation for the lower and upper index of the product corresponds to the index set

$$\{0, 1, 2, \ldots, n\} \setminus \{i\} = \{0, 1, 2, \ldots, i-1, i+1, \ldots, n\}.$$

Furthermore, for $y_0, y_1, y_2, \ldots, y_n \in \mathbb{R}$, another list of $n+1$ real numbers (not necessarily sorted and different from each other), we define

$$p(x) := \sum_{i=0}^{n} y_i \cdot \ell_i(x)\,.$$

For this exercise, let $n = 3$ and take the following table:

i	0	1	2	3
x_i	-1	0	2	3
y_i	3	-3	6	5

   (a) Compute the four polynomial functions $\ell_0(x), \ell_1(x), \ell_2(x), \ell_3(x)$.

   (b) Plot the graph of $\ell_i(x)$ for $i = 0, \ldots, 3$ in the interval $[-1.5, 3.5] \subset \mathbb{R}$.
   - Look at the zeros of the graphs. What do you observe?
   - Also look at the points $(x_i, \ell_i(x_i)) \in \mathbb{R}^2$ for $i = 0, \ldots, 3$. What do you observe?

   Can you justify both observations by considering the definition of $\ell_i(x)$?
   (For plotting/visualizing, use a tool of your choice!)
   Hint: a zero of $\ell_i(x)$ is a value $\overline{x} \in \mathbb{R}$, such that $\ell_i(\overline{x}) = 0$.

(c) Compute $p(x)$ and plot the graph of $p(x)$ in the interval $[-1.5, 3.5] \subset \mathbb{R}$. In this plot, highlight the points $(x_i, y_i) \in \mathbb{R}^2$ for $i = 0, \dots, n$.

(For plotting/visualizing, use a tool of your choice!)

(d) Compute $p(1)$.