A. Which of the following is a proposition?

(a) $(1+2)^2 = 1^2 + 2 \cdot 1 \cdot 2 + 2^2$.
(b) $\sqrt{42}$.
(c) $x - y$.
(d) $1 < 3$.
(e) $3 < 1$.
(f) Every positive even integer can be written as the sum of two primes.

B. Let A, B be propositions. Determine a truth table for the propositions

(a) $\neg(A \wedge \neg B)$,
(b) $\neg A \vee B$,

and compare them to the truth table for $A \Rightarrow B$.

C. Let A, B be any propositions and F a contradiction. Using truth tables, prove

$$A \Rightarrow B \text{ and } A \wedge \neg B \Rightarrow F \text{ are logically equivalent.}$$

D. Let A, B and C be any propositions and T a tautology. Using truth tables, prove

(a) $(A \Rightarrow B) \wedge (B \Rightarrow C) \Rightarrow (A \Rightarrow C)$ and T are logically equivalent,
(b) $A \Leftrightarrow B$ and $(A \Rightarrow B) \wedge (B \Rightarrow A)$ are logically equivalent.

E. Let A, B and C be any propositions. Prove

(a) $(A \wedge B) \wedge C$ and $A \wedge (B \wedge C)$ are logically equivalent,
(b) $(A \vee B) \wedge C$ and $(A \wedge C) \vee (B \wedge C)$ are logically equivalent.

F. For this exercise, the corresponding class of x is $\Omega :=$"all possible AI students". We define predicates

- $P(\cdot)$: "Student ... was in at least one Math lesson".
- $Q(\cdot)$: "Student ... was in all Math lessons".

Write the following propositions and their negations in words.

(a) $\forall x \in \Omega : \; P(x)$.
(b) $\exists x \in \Omega : \; P(x)$.
(c) $\forall x \in \Omega : \; Q(x)$.
(d) $\exists x \in \Omega : \; Q(x)$.