We prove:

$$\underset{A,B,C,f}{\forall}((C \subseteq A) \wedge \mathbf{surjective}[\mathbf{restriction}[f, C], C, B]) \Rightarrow \mathbf{surjective}[f, A, B] \quad \text{(restr surj)}$$

under the assumptions:

$$\underset{A,B,f}{\forall} \mathbf{surjective}[f, A, B] :\Longleftrightarrow \underset{y \in B}{\forall} \underset{x \in A}{\exists} f[x] == y, \quad \text{(surj)}$$

$$\underset{C,f}{\forall} \underset{x \in C}{\forall} \mathbf{restriction}[f, C][x] == f[x], \quad \text{(restrict)}$$

$$\underset{A,B}{\forall} A \subseteq B :\Longleftrightarrow \underset{x \in A}{\forall} x \in B. \quad \text{(subset)}$$

For proving (restr surj) we choose $A$, $B$, $C$, and $f$ arbitrary but fixed and show

$$((C \subseteq A) \wedge \mathbf{surjective}[\mathbf{restriction}[f, C], C, B]) \Rightarrow \mathbf{surjective}[f, A, B]. \quad \text{(G\#1)}$$

In order to prove (G#1) we assume

$$(C \subseteq A) \wedge \mathbf{surjective}[\mathbf{restriction}[f, C], C, B] \quad \text{(A\#3)}$$

and then prove

$$\mathbf{surjective}[f, A, B]. \quad \text{(G\#4)}$$

We expand definitions:

In order to prove (G#4), using definition (surj), we now show

$$\underset{y \in B}{\forall} \underset{x \in A}{\exists} f[x] == y. \quad \text{(G\#577)}$$

From (A#3.1) we know, by definition (subset),

$$\underset{x \in C}{\forall} x \in A \quad \text{(A\#578)}$$

From (A#3.2) we know, by definition (surj),

$$\underset{y \in B}{\forall} \underset{x \in C}{\exists} \mathbf{restriction}[f, C][x] == y \quad \text{(A\#579)}$$

We apply substitutions:

From (A#579) we know, by (restrict),

$$\underset{y \in B}{\forall} \underset{x \in C}{\exists} f[x] == y \quad \text{(A\#785)}$$

For proving (G#577) we choose $y$ arbitrary but fixed and assume

$$y \in B. \quad \text{(A\#996)}$$

We have to show

$$\underset{x \in A}{\exists} f[x] == y. \quad \text{(G\#995)}$$

We augment the knowledge base:

From (A#996), using (A#785), we can deduce

$$\underset{x \in C}{\exists} f[x] == y. \quad \text{(A\#1201)}$$

From (A#1201) we know

$$x \in C, \quad \text{(A\#2207)}$$

$$f[x] == y \quad \text{(A\#2208)}$$

for some $x$.

We apply substitutions:

In order to prove (G#995), using (A#2208), we now show

$$\underset{x \in A}{\exists} f[x] == f[x]. \qquad \text{(G\#2701)}$$

We augment the knowledge base:

From (A#2207), using (A#578), we can deduce

$$x \in A. \qquad \text{(A\#3648)}$$

For proving (G#2701) we have to find an appropriate value for $x^*$, such that we can prove

$$(x^* \in A) \wedge (f[x^*] == f[x]). \qquad \text{(G\#5690)}$$

Let now $x^* \to x$. In order to prove (G#5690) it suffices to show

$$x \in A. \qquad \text{(G\#9480)}$$

The goal (G#9480) is identical to formula (A#3648) in the knowledge base. Thus, this part of the proof is finished.