FOB3Q

Started on	Monday, 11 December 2023, 7:15 PM
State	Finished
Completed on	Monday, 11 December 2023, 7:30 PM
Time taken	15 mins
Grade	**3.25** out of 5.00 (**65**%)

Question 1

Partially correct

Mark 2.00 out of 2.50

Flag question

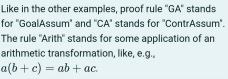
Generate a formal proof (proof tree) of the statement

$$\forall a, b \in \mathbb{N}: ((\exists c \in \mathbb{N}: a = 2c \wedge \exists c \in \mathbb{N}: b = 2c) \rightarrow$$

Intuitively, the statement says that the sum of two even numbers is always even.

The knowledge Γ contains all arithmetic rules and knowledge for addition and multiplication in the natural numbers. Note that, for reasons of space, we will sometimes apply more than one rule in one proof step (like it is often done in mathematical proofs)

Like in the other examples, proof rule "GA" stands for "GoalAssum" and "CA" stands for "ContrAssum". The rule "Arith" stands for some application of an arithmetic transformation, like, e.g., $a(b + c) = ab + ac$.



P-∧ | A-∧ | P-∨ | A-∨ | P-→ | P-¬

MP | A-→ | Arith | GA | A-∃ | P-=

A-∀ | CA | P-∃ | P-∀

$2c_0$ | $2c$ | $2c_1 + 1$ | $2c_0 + 1$ | $2c_1$ | $2c + 1$

$2(c_0 + c_1) + 1$ | $2(c_0 + c_1 + 1)$ | $2(c_0 + c_1)$ | $2c_0 + 1 + 2c_1 + 1$ | $2c + 1 + 2c$ | $2c + 2c$ | $2c + 1 + 2c + 1$ | $2c_0 + 1 + 2c_1$

$2 \cdot c_0 \cdot c_1$ | $c_0 \cdot c_1$ | $c_0 \cdot c_1$ | $c + c \in \mathbb{N}$ | $c.c \in \mathbb{N}$ | $2c.c \in \mathbb{N}$ | $c \cdot c \in \mathbb{N}$ | $c_0 \cdot c_1 \in \mathbb{N}$ | $c_0 \cdot c_1 \in \mathbb{N}$ | $2 \cdot c_0 \cdot c_1 \in \mathbb{N}$

Die Antwort ist teilweise richtig.

You have correctly selected 12.

Question 2

Partially correct

Mark 1.25 out of 2.50

Flag question

Prove by induction on n the goal formula

$$\forall n \in \mathbb{N}: n \geq 1 \rightarrow 9 \mid (10^n - 1).$$

Proof:

1. Induction base: we choose n=[1] ✔ and we have to prove 9 | [10^1 - 1] ✔, which is true by [induction base] ✘.
2. Induction hypothesis: we assume 9 | [10^(n+1) - 1] ✘ for [an arbitrary but fixed n∈ℕ with n≥1] ✔.
3. Induction step: we have to prove 9 | [10^(n+1) - 1] ✔, which is true because

[10^(n+1) - 1] ✔ = [10^10^n - 1] ✔ by [inductic] ✘

= [9*10^n + 10^n - 1] ✔ by [inductic] ✘

= [10^n - 9m] ✘ for some [inductio] ✘)

Hence, [10^1 - 1] ✘ is a multiple of 9, i.e. $9 \mid (10^{n+1} - 1)$, and the proof is finished. ✔

Die Antwort ist teilweise richtig.

You have correctly selected 7.

The correct answer is: Prove by induction on n the goal formula

$$\forall n \in \mathbb{N}: n \geq 1 \rightarrow 9 \mid (10^n - 1).$$

Proof:

1. Induction base: we choose n=[1] and we have to prove 9 | [10^1 - 1], which is true by [simple arithmetic].
2. Induction hypothesis: we assume 9 | [10^n - 1] for [an arbitrary but fixed n∈ℕ with n≥1].
3. Induction step: we have to prove 9 | [10^(n+1) - 1], which is true because

[10^(n+1) - 1] = [10*10^n - 1] by [simple arithmetic]

= [9*10^n + 10^n - 1] by [simple arithmetic]

= [9*10^n + 9m] for some integer m (by [induction hypothesis])

Hence, [10^(n+1) - 1] is a multiple of 9, i.e. $9 \mid (10^{n+1} - 1)$, and the proof is finished.

Finish review