☑ **FOB2Q**

Started on	Monday, 4 December 2023, 7:15 PM
State	Finished
Completed on	Monday, 4 December 2023, 7:30 PM
Time taken	15 mins 2 secs
Grade	**1.91** out of 5.00 (**38.15%**)

Question 1

Partially correct

Mark 0.41 out of 0.50

⚑ Flag question

In the following questions, you need to generate a formal proof (proof tree) of the statement $(K_1 \wedge K_2) \to G$, where K_1, K_2, and G abbreviate the following formulas:

$K_1 : \forall k : l(k) \to m(k, v(k))$
$K_2 : \forall i, j : s(j, i) \to m(j, w(i))$
$G : \forall i : (l(i) \vee \exists j : s(i, j)) \to \exists j : m(i, j)$

Note that, for reasons of space, we will sometimes use the abbreviations instead of the expanded formulas even in the proof tree.

In the first exercise, develop an "incomplete" proof tree until the step, where the proof divides into 2 branches called (1) and (2). These branches (1) and (2) have to be completed in the subsequent exercises.

In either of the examples, proof rule "GA" stands for "GoalAssum" and "CA" stands for "ContrAssum".

A∨	MP	P-3	P→	P-∨	P→	CA
GA	A-3	A-∧	P-∀	P-∧	P-∧	A-→

A-∀

$\forall i : (l(i) \vee \exists j : s(i,j))$

$l(i_0) \vee \exists j : s(i_0, j)$

$\forall j : m(i_0, j)$

$\exists m(i_0, j)$

$l(i_0) \wedge \forall j : s(i_0, j)$

$\exists \; s(i_0, j) \to \exists \; m(i_0, j)$

$l(i_0) \vee \exists \; s(i_0, j)$

$\forall i : l(i) \to m(i, v(i))$

$(l(i_0) \vee \exists \; s(i_0, j) \to \exists \; m(i_0, j))$

$l(i_0) \vee \exists \; s(i_0, j)$

$\forall \; (s(j, i) \to m(j, w(i)))$

$\exists \; m(i_0, j)$	$l(i)$	$s(i_0, j)$	$\exists \; s(i_0, j)$
$\exists \; m(i_0, j)$	$l(j)$	$m(i_0, j)$	$s(i_0, j)$
$l(i_0)$	$m(i_0, j)$	$\exists \; s(i_0, j)$	$\forall j : m(i_0, j)$

Die Antwort ist teilweise richtig.

You have correctly selected 9.

Question 2

Not answered

Marked out of 1.50

⚑ Flag question

Suppose now the proof situation in branch (1) is
$\forall x : (p(x) \to r(x, f(x))), \forall x, y : (q(y, x) \to r(y, g(x)))$

Note that this might not be exactly what you derived in the first example, it is a "hypothetical" proof situation. Complete **this branch** of the proof.

Like in the other examples, proof rule "GA" stands for "GoalAssum" and "CA" stands for "ContrAssum".

P-∨	A-...	A-∨	MP	P-∨	P→	GA
P-...	A-∀	P→	P-∧	CA	A-...	A-∧

P-3

A-3

$p(x_0) \to r(x_0, f(x_0))$	$q(y_0, x_0) \to r(y_0, g(x_0))$

$r(y_0, x_0) \to r(y_0, g(x_0))$	$p(x_0) \to r(x_0, f(x_0))$

$\forall y : r(x_0, y)$	$q(y_0, x_0) \to r(y_0, g(x_0))$

⊥

$\exists r : r(x_0, y)$

$r(p(x_0), q(x_0))$	$r(x_0, f(x_0))$	$r(x_0, g(x_0))$

$r(f(x_0), x_0)$	$r(x_0, y_0)$	$r(f(x_0), f(x_0))$

$r(x_0, x_0)$	⊥	$r(g(x_0), x_0)$

$q(x_0)$	x_0	$f(x_0)$	$f(x_0)$	$g(x_0)$	z_0

Die Antwort ist falsch.

Question 3

Correct

Mark 1.50 out of 1.50

⚑ Flag question

Suppose now the proof situation in branch (2) is
$\forall c : (l(c) \to m(c, d(c))), \forall a, b : (a(b, a) \to m(b, h(a)))$

Note that this might not be exactly what you derived in the first example, it is a "hypothetical" proof situation. Complete **this branch** of the proof.

Like in the other examples, proof rule "GA" stands for "GoalAssum" and "CA" stands for "ContrAssum".

A-...	P-∨	A-...	P-3	CA	P-∧	A-∀
GA	P-∀	A-∧	P-...	P→	A-∨	A-∀

A-3   MP

$n(b_0, b_0) \to m(b_0, h(b_0))$	

$l(c_0) \to m(c_0, d(c_0))$	$s(b_0, a_0) \to m(b_0, h(a_0))$

$(a_0, \dots) \to m(a_0, h(a_0))$	$\forall b : m(a_0, b)$

$s(a_0, a_0) \to m(b_0, h(a_0))$	⊥

$m(b_0, a_0)$	$m(b_0, b_0)$	$s(b_0, a_0)$

$m(d(a_0), a_0)$	$m(b_0, h(a_0))$	$m(a_0, h(a_0))$

⊥	$m(h(a_0), b_0)$	$s(a_0, b_0)$

$m(h(b_0), b_0)$	$s(a_0, b_0)$	$m(a_0, d(a_0))$

b_0	a_0	

$h(a_0)$	$d(a_0)$	$d(a_0)$

Die Antwort ist richtig.

Question 4

Incorrect

Mark 0.00 out of 1.50

⚑ Flag question

Does the following proof tree represent a correct proof or is there a "bug" somewhere? If it is not correct, mark **all steps** that contain errors, since the proof may contain more than one error. As usual, "GA" and "CA" abbreviate "GoalAssum" and "ContrAssum", respectively.

Select one or more:

☐ In step 1 we must not split the implication.

☒ In step 4 we must instantiate the two variables with different terms.   ✗

☐ In step 3 we must not introduce a new symbol.

☐ In step 6 we must not instantiate the universal quantifier, because it has a negation in front.

☐ There is no bug in the proof. The proof is correct.

☐ In step 2 the disjunction is handled incorrectly.

☐ In step 5 we must split the "or" into two branches.

Die Antwort ist falsch.

The correct answer is:
In step 6 we must not instantiate the universal quantifier, because it has a negation in front.