338.001, VL Logic, Martina Seidl / Wolfgang Schreiner / Wolfgang Windsteiger, 2022W

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Time taken	14 mins 18 secs
Grade	**2.5** out of 5.0 (**50.8**%)

Question **1**
Partially correct
Mark 0.7 out of 2.5
Flag question

Generate a formal proof (proof tree) of the statement

$$\forall x, y \in \mathbb{N}\colon ((\exists z \in \mathbb{N}\colon x = 2z+1 \wedge \exists z \in \mathbb{N}\colon y = 2z+1) \rightarrow \exists z \in \mathbb{N}\colon x \cdot y = 2z+1).$$

Intuitively, the statement says that the product of two odd numbers is always odd.

The knowledge Γ contains all arithmetic rules and knowledge for addition and multiplication in the natural numbers. Note that, for reasons of space, we will sometimes apply more than one rule in one proof step (like it is often done in mathematical proofs)

Like in the other examples, proof rule "GA" stands for "GoalAssum" and "CA" stands for "ContrAssum". The rule "Arith" stands for some application of an arithmetic transformation, like, e.g., $a(b+c) = ab + ac$.

$\Gamma \vdash \forall x, y \in \mathbb{N}\colon ((\exists z \in \mathbb{N}\colon x = 2z+1 \wedge \exists z \in \mathbb{N}\colon y = 2z+1) \rightarrow \exists z \in \mathbb{N}\colon x \cdot y = 2z+1)$

P-∀, P-→, A-∧ ↓

$\Gamma, x_0 \in \mathbb{N}, y_0 \in \mathbb{N}, \exists z \in \mathbb{N}\colon x_0 =$ [2z] $, \exists z \in \mathbb{N}\colon y_0 =$ [2z₀ + 1] $\vdash \exists z \in \mathbb{N}\colon x_0 \cdot y_0 =$ [2z₀ + 1]

[P-∨] , A-∧

$\Gamma, x_0 \in \mathbb{N}, y_0 \in \mathbb{N}, z_0 \in \mathbb{N}, x_0 =$ [2z₁] $, z_1 \in \mathbb{N}, y_0 =$ [2z + 1] $\vdash \exists z \in \mathbb{N}\colon x_0 \cdot y_0 =$ [2z₁ + 1]

A-=

$\Gamma, x_0 \in \mathbb{N}, y_0 \in \mathbb{N}, z_0 \in \mathbb{N}, x_0 = 2z_0+1, z_1 \in \mathbb{N}, y_0 = 2z_1+1 \vdash \exists z \in \mathbb{N}\colon$ [2(2 z₀ . z₁ + z₁)] $= 2z+1$

Arith

$\Gamma, x_0 \in \mathbb{N}, y_0 \in \mathbb{N}, z_0 \in \mathbb{N}, x_0 = 2z_0+1, z_1 \in \mathbb{N}, y_0 = 2z_1+1 \vdash \exists z \in \mathbb{N}\colon$ [2(2 z₀ . z₁ + z₀ + z₁) + 1] $= 2z+1$

P-∃, [P-∧], Drop

$\Gamma, z_0 \in \mathbb{N}, z_1 \in \mathbb{N} \vdash$ [z₀ - z₁] $\in \mathbb{N}$ $\Gamma \vdash 2(2z_0z_1 + z_0 + z_1) + 1 = 2($ [2 z₀ . z₁ + z₀ + z₁] $) + 1$ [CA]

MP, Drop

$\Gamma,$ [z₀ . z₁ ∈ ℕ] $\vdash 2z_0z_1 + z_0 + z_1 \in \mathbb{N}$

[GA]

A-∨ | A-∀ | Arith | P-∀ | P-= | P-∨ | GA | CA | MP | P-¬ | P-∃ | P-∧ | A-∃ | P-→ | A-∧ | A-→

2z₀ | 2z₁ | 2z₀ + 1 | 2z | 2z + 1 | 2z₁ + 1

2z . 2z | 2z₀ . 2z₁ | 2(2 z₀ . z₁ + z₀ + z₁) + 1 | (2z + 1) 2z | 2(2 z₀ . z₁ + z₁) | (2z₀ + 1) . 2z₁

(2z + 1) . (2z + 1) | (2z₀ + 1) . (2z₁ + 1) | 2(z₀ + z₁)

z₀ . z₁ | z₀ - z₁ | z₀ + z₁ | 2 z₀ . z₁ + z₀ + z₁

z - z ∈ ℕ | z.z ∈ ℕ | z₀ - z₁ ∈ ℕ | 2z.z + z ∈ ℕ | 2 z₀ . z₁ ∈ ℕ

z + z ∈ ℕ | 2 z₀ . z₁ + z₀ + z₁ ∈ ℕ

Die Antwort ist teilweise richtig.

You have correctly selected 4.

Question **2**
Partially correct
Mark 1.9 out of 2.5
Flag question

The knowledge base consists of the formulas

$S(0) = 1$ (1)

$\forall n \in \mathbb{N}\colon S(n+1) = 2 \cdot S(n) + 1$ (2)

Prove by induction on *n* the goal formula

$$\forall n \in \mathbb{N}\colon S(n) = 2^{n+1} - 1.$$

Proof:

1. Induction base: we choose *n*= [0] ✔ and we have to prove [S(0)] ✔ = [2^0-1] ✘ , which is true because [S(0)=0] ✘ by [knowledge formula (1)] ✔ and [2^0-1=0] ✘ by simple arithmetic.
2. Induction hypothesis: we assume [S(n)=2^(n+1)-1] ✔ for [an arbitrary but fixed n∈ℕ] ✔ .
3. Induction step: we have to prove [S(n+1)=2^(n+2)-1] ✔ , which is true because

[S(n+1)] ✔	=	[2*S(n)+1] ✔	by	[knowledge formula (2)] ✔
	=	[2*(2^(n+1)-1)+1] ✔	by	[induction hypothesis] ✔
	=	[2^(n+1)-1] ✘	by	[simple arithmetic] ✔

Hence, the proof is finished.

Die Antwort ist teilweise richtig.

You have correctly selected 12.
The correct answer is:
The knowledge base consists of the formulas

$$S(0) = 1 \qquad (1)$$
$$\forall n \in \mathbb{N}\colon S(n+1) = 2 \cdot S(n) + 1 \qquad (2)$$

Prove by induction on *n* the goal formula

$$\forall n \in \mathbb{N}\colon S(n) = 2^{n+1} - 1.$$

Proof:

1. Induction base: we choose *n*=[0] and we have to prove [S(0)]=[2^1-1], which is true because [S(0)=1] by [knowledge formula (1)] and [2^1-1=1] by simple arithmetic.
2. Induction hypothesis: we assume [S =2^(n+1)-1] for [an arbitrary but fixed n∈ℕ].
3. Induction step: we have to prove [S(n+1)=2^(n+2)-1], which is true because

[S(n+1)]	=	[2*S +1]	by [knowledge formula (2)]
	=	[2*(2^(n+1)-1)+1]	by [induction hypothesis]
	=	[2^(n+2)-1]	by [simple arithmetic]

Hence, the proof is finished.