12340306 - Mohammad Shadik Ansari

Exercise sheet # I

1. Sol^n

$$A = \sum_{j=5}^{10} (3j)$$

$$= 3 \sum_{j=5}^{10} j$$

$$= 3(5+6+7+8+9+10)$$

$$= 3 \times 45$$

$$= 135$$

$$B = \sum_{i=1}^{50} \left(i^3 - (i-1)^3\right)$$

$$= \sum_{i=1}^{50} \left(i^3 - (i^3 - 3i^2 + 3i - 1)\right)$$

$$= \sum_{i=1}^{50} (3i^2 - 3i + 1)$$

$$= 3 \cdot \sum_{i=1}^{50} i^2 - 3\sum_{i=1}^{50} i + \sum_{i=1}^{50} 1$$

[$\because$ Sum of n square numbers]
[$\because$ Sum of n natural num.]

$$= 3 \times \left(\frac{50(50+1)(2\times 50+1)}{6}\right) - 3\times\left(\frac{50\times(50+1)}{2}\right) + 50 \times 1$$

$$= 128775 - 3825 + 50$$

$$= 125,000$$

C is the sum of first 110 numbers in the list 3, 7, 11, 15, 19, 23, ...

then, $a_i = 3 + (i-1) \times 4 = 3 + 4i - 4 = 4i - 1$

$$\therefore \sum_{i=1}^{110} 4i - 1 = 4\sum_{i=1}^{110} i - \sum_{i=1}^{110} 1$$

$$= 4 \times \left(\frac{110 \cdot (110+1)}{2}\right) - 110$$

$$= 24310$$

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2. Sol^n

$$A = \prod_{i=1}^{5} (2i)$$

$$= (2\times1)(2\times2).(2\times3).(2\times4).(2\times5)$$

$$= 2^5 \times 5!$$

$$= 3840$$

$$B = \prod_{i=3}^{5} (i-2)!$$

$$= (3-2)! \cdot (4-2)! \cdot (5-2)!$$

$$= 1! \cdot 2! \cdot 3!$$

$$= 1 \times 2 \times 6$$

$$= 12$$

$$C = \prod_{i=1}^{3} \prod_{k=2}^{4} (2i+k).$$

$$= \prod_{k=2}^{4} (2\times1+k) \times \prod_{k=2}^{4} (2\times2+k) \times \prod_{k=2}^{4} (2\times3+k)$$

$$= (2+2) \cdot (2+3) \cdot (2+4) \times (4+2).(4+3).(4+4) \times (6+2).(6+3).(6+4)$$

$$= 4.5.6 \times 6.7.8 \times 8.9.10$$

$$= 29030400$$

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3. $A = \sum_{k=1}^{n} k \cdot k!$

$= \sum_{k=1}^{n} (k+1-1)\, k!$

$= \sum_{k=1}^{n} \left((k+1)\cdot k! - 1\cdot(k!)\right)$

$\therefore \boxed{A = \sum_{k=1}^{n} \left((k+1)! - k!\right)}$ $\left[\because (n+1)\, n! = (n+1)!\right]$

For, $n=1$,

$\sum_{k=1}^{n} (k+1)! - k! = (1+1)! - 1! = 1$

For $n=2$,

$\sum_{k=1}^{n} (k+1)! - k! = (2+1)! - 2! = 4$

$B = \prod_{i=2}^{n} \left(1 - \frac{1}{i}\right)$

$= \prod_{i=2}^{n} \left(\frac{i-1}{i}\right)$

$= \prod_{i=2}^{n} (i-1) \times \prod_{i=2}^{n} \frac{1}{i}$

$= (n-1)! \times \frac{1}{n!}$ $\left[\because \prod_{i=1}^{n} i \text{ or } \prod_{i=2}^{n} i = i!\right]$

$= \frac{\cancel{(n-1)!}}{n\cancel{(n-1)!}}$

$\therefore \boxed{B = \frac{1}{n}}$

For $n=2$,

$\prod_{i=2}^{n} \left(1 - \frac{1}{i}\right) = \frac{1}{n} = \frac{1}{2}$

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3. continue

C. $\sum_{k=1}^{n} \frac{1}{k(k+1)}$

$= \sum_{k=1}^{n} \left(\frac{1}{k} - \frac{1}{k+1} \right)$

$= \sum_{k=1}^{n} \frac{1}{k} - \sum_{k=1}^{n} \frac{1}{k+1}$

$= \cancel{\left(\frac{1}{1} - \frac{1}{2} \right)}$

$= \left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \dots + \frac{1}{n} \right) - \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \dots\dots + \frac{1}{n+1} \right)$

$= 1 - \frac{1}{n+1}$

$\therefore \boxed{C = 1 - \frac{1}{n+1}}$

For $n = 1$,

$C = 1 - \frac{1}{1+1} = \frac{1}{2}$

For $n = 2$,

$C = 1 - \frac{1}{2+1} = \frac{2}{3}$

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4. a) $\sum_{i=1}^{2} \prod_{k=1}^{3} \frac{i}{k}$

$= \prod_{k=1}^{3} \frac{1}{k} + \prod_{k=1}^{3} \frac{2}{k}$

$= \frac{1}{1} \times \frac{1}{2} \times \frac{1}{3} + \frac{2}{1} \times \frac{2}{2} \times \frac{2}{3}$

$= \frac{1}{6} + \frac{8}{6}$

$= \frac{9}{6} = \frac{3}{2}$

b) $\prod_{k=1}^{3} \sum_{i=1}^{2} \frac{i}{k}$

$= \sum_{i=1}^{2} \frac{i}{1} \times \sum_{i=1}^{2} \frac{i}{2} \times \sum_{i=1}^{2} \frac{i}{3}$

$= (1 + 2) \times \left(\frac{1}{2} + \frac{2}{2}\right) \times \left(\frac{1}{3} + \frac{2}{3}\right)$

$= 3 \times \frac{3}{2} \times \cancel{\frac{3}{3}}$

$= \frac{9}{2}$

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4. <u>continue</u>

c) No, these two instances are not enough to establish the general statement about the order of summation and product operation. But, indeed the order of both operation is very important and should not be changed.

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5. Sol^n

x_i	-6	-3	-1	1	3	4
y_i	5	4	2	1	1	0

a) $\mu_x = \frac{1}{6}\sum_{l=1}^{6} x_l$

$= \frac{1}{6}(x_1 + x_2 + x_3 + x_4 + x_5 + x_6)$

$= \frac{1}{6}(-6 - 3 - 1 + 1 + 3 + 4)$

$\therefore \boxed{\mu_x = \frac{-1}{3}}$

$\mu_y = \frac{1}{6}\sum_{l=1}^{6} y_l$

$= \frac{1}{6}(y_1 + y_2 + y_3 + y_4 + y_5 + y_6)$

$= \frac{1}{6}(5 + 4 + 2 + 1 + 1 + 0)$

$\therefore \boxed{\mu_y = \frac{13}{6}}$

b) $\sigma_x^2 = \frac{1}{6}\sum_{l=1}^{6}(x_l - \mu_x)^2$

$= \frac{1}{6}\left((x_1-\mu_x)^2 + (x_2-\mu_x)^2 + (x_3-\mu_x)^2 + (x_4-\mu_x)^2 + (x_5-\mu_x)^2 + (x_6-\mu_x)^2\right)$

$= \frac{1}{6}\left((-6+\frac{1}{3})^2 + (-3+\frac{1}{3})^2 + (-1+\frac{1}{3})^2 + (1+\frac{1}{3})^2 + (3+\frac{1}{3})^2 + (4+\frac{1}{3})^2\right)$

$= \frac{1}{6}\left(\frac{289}{9} + \frac{64}{9} + \frac{4}{9} + \frac{16}{9} + \frac{100}{9} + \frac{169}{9}\right)$

$= \frac{642}{54} = \frac{107}{9}$ $\quad \therefore \boxed{\sigma_x^2 = \frac{107}{9}}$

$\Rightarrow \sigma_{xy} = \frac{1}{6}\sum_{l=1}^{6}(x_l - \mu_x)(y_l - \mu_y)$

$= \frac{1}{6}\Big((x_1-\mu_x)(y_1-\mu_y) + (x_2-\mu_x)(y_2-\mu_y) + (x_3-\mu_x)(y_3-\mu_y) + (x_4-\mu_x)(y_4-\mu_y)$
$+ (x_5-\mu_x)(y_5-\mu_y) + (x_6-\mu_x)(y_6-\mu_y)\Big)$

$= \frac{1}{6}\Big((-6+\frac{1}{3})(5-\frac{13}{6}) + (-3+\frac{1}{3})(4-\frac{13}{6}) + (-1+\frac{1}{3})(2-\frac{13}{6}) + (1+\frac{1}{3})(1-\frac{13}{6}) + (3+\frac{1}{3})(1-\frac{13}{6})$
$+ (4+\frac{1}{3})(0-\frac{13}{6})\Big)$

$= \cancel{\ } \frac{-107}{18}$

$\therefore \boxed{\sigma_{xy} = \frac{-107}{18}}$

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c) $k = \dfrac{\sigma_{xy}}{\sigma_x^2}$

$= \dfrac{\frac{-107}{182}}{\frac{107}{91}}$

$\therefore \boxed{k = -\frac{1}{2}}$

d) $\mu_y = k\mu_x + d$

or, $d = k\mu_x - \mu_y$

$= -\frac{1}{2} \times \left(-\frac{1}{3}\right) - \frac{13}{6}$

$= \frac{1}{6} - \frac{13}{6}$

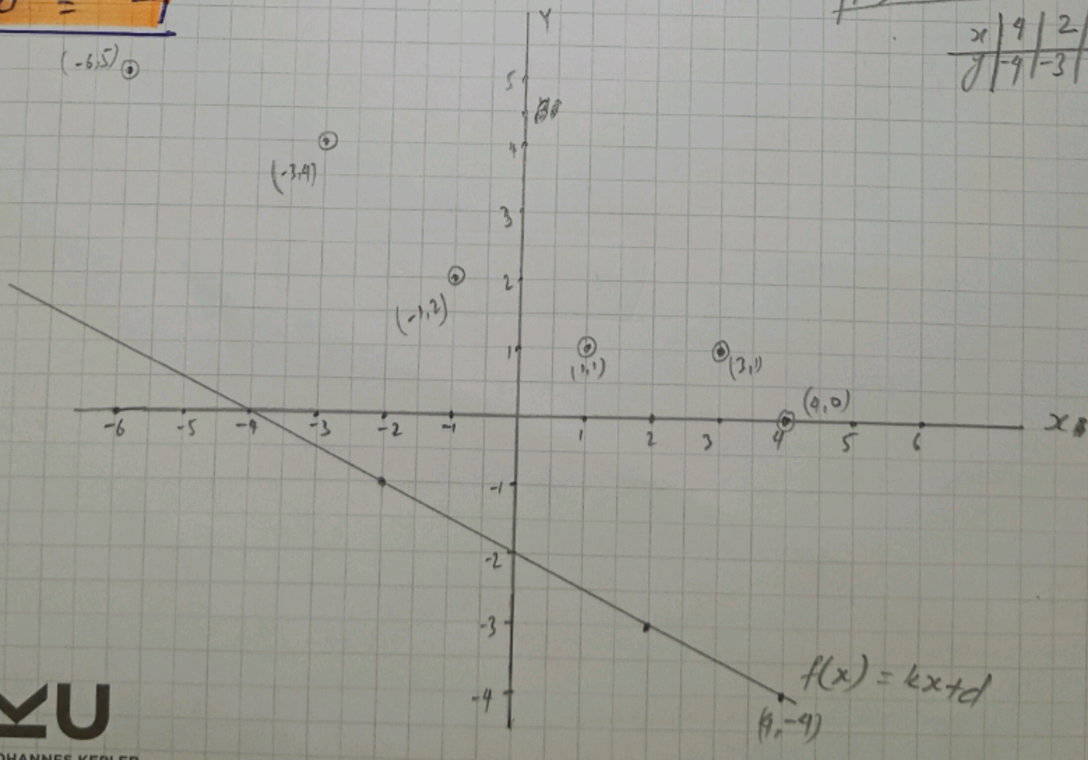
$= \frac{-12}{6}$

$\therefore \boxed{d = -2}$

For: $\boxed{f(x) = kx + d = -\frac{1}{2}x - 2}$

x	4	2	-2
y	-4	-3	-1

e)



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5.f) $\cancel{f(-2) = k}$

Here,

$$f(x) = kx + d = -\frac{1}{2}x - 2$$

$$\therefore f(2) = -\frac{1}{2} \times 2 - 2 = -3$$

$$\therefore f(-2) = -\frac{1}{2} \times (-2) - 2 = -1$$

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