49. Show that the sequence $(a_n)_{n\in\mathbb{N}}$ defined by $a_1 = 1$, $a_{n+1} = \frac{a_n+5}{2}$ is bounded by 5. Then, prove that it is increasing. Provide a justification for the existence of the limit of the sequence $(a_n)_{n\in\mathbb{N}}$, and then calculate it.

50. Let $B_1 = \begin{bmatrix} 4 & 5 \\ 1 & 2 \end{bmatrix}$, and let $B_{n+1} = B_1 B_n$ for $n \in \mathbb{N}$. Find pairs of real numbers (a, b) such that:
$$\lim_{n\to\infty} \frac{\det B_n}{5 + a^2 b^n} = \frac{1}{9}.$$

51. Determine all the accumulation points for the sequences $(a_n)_{n\in\mathbb{N}}$, $(b_n)_{n\in\mathbb{N}}$, $(c_n)_{n\in\mathbb{N}}$ defined by
    - $a_n := i^n$,
    - $b_n := (-1)^n + \frac{n^2+1}{n^2}$,
    - $c_n := \sin\left(n\frac{\pi}{6}\right)$.

52. Let $p > 1$, and let $a_n = \sqrt{n^p + n + 1} - \sqrt{n^p - n + 1}$. Determine for which $p > 1$ the sequence $(a_n)_{n\in\mathbb{N}}$ is bounded.

53. Determine limits of sequences $(a_n)_{n\in\mathbb{N}}$, $(b_n)_{n\in\mathbb{N}}$, $(c_n)_{n\in\mathbb{N}}$, $(d_n)_{n\in\mathbb{N}}$ defined by
$$a_n = \left(1 + \frac{1}{5n+1}\right)^{6n+1}, \quad b_n = \left(1 + \frac{15n+1}{7n^3+11n+5}\right)^{3n^2+1},$$
$$c_n = \left(\frac{2n^2-n}{2n^2+13}\right)^n, \quad d_n = \left(\frac{n^2+1}{n^2}\right)^n.$$

54. Determine the limit of the sequence $(a_n)_{n\in\mathbb{N}}$ defined by
$$a_n = (\sqrt{n^2+4} - n)\prod_{k=1}^{n}\left(1 + \frac{1}{k+1}\right),$$
where $n \in \mathbb{N}$.