1)

a) Let $T: \mathbb{R}^n \to \mathbb{R}^n$ be a function what does it mean if we say that T is a linear transformation (2 marks)

b) what does it mean if we say T is invertable? (2 marks)

c) Suppose that T is an invertable linear transformation with an inverse T^{-1}. Prove that T^{-1} is also a linear transformation. (6 marks)

2)

$$A = \begin{pmatrix} 2 & 1 & -2 \\ 0 & 3 & 6 \\ 2 & 0 & -4 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 3 & 1 \\ 1 & 2 \\ 0 & 1 \end{pmatrix}.$$

a) calculate row echelon form (4 marks)

b) what is the rank of A? (2 marks)

c) write down the determinant of A. (2 marks)

d) calculate matrix products AB and BA, if they exist. (4 marks)

3.)

a) let $X \subset \mathbb{R}$ what does it mean if we say that X is bounded from above? (2 marks)

b) let $X \subset \mathbb{R}$ and $t \in \mathbb{R}$. what does it mean if we say that t is the maximal element of X? (2 marks)

c) Suppose that $X \subset \mathbb{R}$ is bounded from above. what does it mean to say that t is the sup of X (2 marks)

d) let $X_1 = (0, 1]$

$\sup X_1 = ?$

$\max X_1 = ?$ (2 marks)

e) $X_2 = \left\{ 2 - \frac{1}{n} : n \in \mathbb{N} \right\}$.

$\sup X_2 = ?$

$\max X_2 = ?$ (3 marks)

f) $X_3 = \left\{ x \in \mathbb{Q} : x < \sqrt{2} \right\}$

$\sup X_3 = ?$

$\max X_3 = ?$ (3 marks)

4) Prove by ind.

a) $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$. (6 marks)

b) $\sum_{k=1}^{n} (-1)^k k^2 = (-1)^n \frac{n(n+1)}{2}$ (8 marks)