Sheet 7

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41. $A = \begin{pmatrix} 1 & 4 \\ 2 & 6 \end{pmatrix}$

$|A| = \begin{vmatrix} 1 & 4 \\ 2 & 6 \end{vmatrix} = 1 \times 6 - 4 \times 2 = -2$

$B = \begin{pmatrix} 1 & 3 & -2 \\ 2 & 6 & 0 \\ 4 & 11 & 8 \end{pmatrix}$

$|B| = \begin{vmatrix} 1 & 3 & -2 \\ 2 & 6 & 0 \\ 4 & 11 & 8 \end{vmatrix} = -2 \begin{vmatrix} 2 & 6 \\ 4 & 11 \end{vmatrix} + 8 \begin{vmatrix} 1 & 3 \\ 2 & 6 \end{vmatrix}$

$= -2(22-24) + 8(6-6)$

$= 4$

$C = \begin{pmatrix} C_1 & 0 \\ 0 & C_2 \end{pmatrix}$

A/c to question, let, $C_1 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $C_2 = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$, $0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

Then;

$C = \begin{pmatrix} a & b & 0 & 0 \\ c & d & 0 & 0 \\ 0 & 0 & e & f \\ 0 & 0 & g & h \end{pmatrix}$

Now,

$$|C| = \begin{vmatrix} a & b & 0 & 0 \\ c & d & 0 & 0 \\ 0 & 0 & e & f \\ 0 & 0 & g & h \end{vmatrix}$$

$$= a \cdot \det \begin{pmatrix} d & 0 & 0 \\ 0 & e & f \\ 0 & g & h \end{pmatrix} - c \cdot \det \begin{pmatrix} b & 0 & 0 \\ 0 & e & f \\ 0 & g & h \end{pmatrix}$$

$$= a \cdot d \cdot \det \begin{pmatrix} e & f \\ g & h \end{pmatrix} - c \cdot b \cdot \det \begin{pmatrix} e & f \\ g & h \end{pmatrix}$$

$$= (ad - cb) \cdot \det \begin{pmatrix} e & f \\ g & h \end{pmatrix}$$

Since, $\det(C_1) = ad - cb$ and $\det(C_2) = \det \begin{pmatrix} e & f \\ g & h \end{pmatrix}$

$$\underline{|C| = \det(C_1) \cdot \det(C_2)}$$

$$D = \begin{pmatrix} 0 & 0 & \cdots & d_n \\ 0 & 0 & \cdots d_{n-1} & 0 \\ \vdots & \vdots & & \vdots \\ 0 & d_2 & \cdots 0 & 0 \\ d_1 & 0 & \cdots 0 & 0 \end{pmatrix}$$

If n is odd, $(i+j)$ is always even for the position of $d_1, d_2, \ldots d_{n-1}, d_n$.

So,

$$|D| = d_1 \cdot d_2 \cdots \det\begin{pmatrix} 0 & d_n \\ d_{n-1} & 0 \end{pmatrix}$$

$$= d_1 \cdot d_2 \cdots (0 - d_n \cdot d_{n-1})$$

$$= -d_1 \cdot d_2 \cdots d_n \cdot d_{n-1}$$

If n is even, $(i+j)$ is always odd ~~for the~~ for the position of $d_1, d_2 \cdots d_{n-1}, d_n$.

So, $$|D| = (-d_1) \cdot (-d_2) \cdots (-1) \det\begin{pmatrix} 0 & d_n \\ d_{n-1} & 0 \end{pmatrix}$$

$$= \cancel{(-d_1) \cdot (-d_2) \cdot (-1)(d_{n-2})(0 - d_n \cdot d_{n-1})}$$

$$= (-1) \cdot (d_1) \cdot (-1) d_2 \cdots (-1) d_{n-2} \cdot (0 - d_n \cdot d_{n-1})$$

There are even number of (-1) till d_{n-2}, when all multiplied it becomes 1

$$\therefore |D| = d_1 \cdot d_2 \cdots (-d_n \cdot d_{n-1})$$

$$= -d_1 \cdot d_2 \cdots d_{n-1} \cdot d_n$$

Hence,

$$|D| = -d_1 \cdot d_2 \cdots d_{n-1} \cdot d_n$$

17. a) $A = \begin{pmatrix} -9 & 7 & 3 \\ -13 & 9 & 4 \\ -3 & 2 & 1 \end{pmatrix}$

By Gaussian elimination.

$$\left(\begin{array}{ccc|ccc} -9 & 7 & 3 & 1 & 0 & 0 \\ -13 & 9 & 4 & 0 & 1 & 0 \\ -3 & 2 & 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow{R_1 \to -\frac{1}{9}R_1} \left(\begin{array}{ccc|ccc} 1 & -\frac{7}{9} & -\frac{3}{9} & -\frac{1}{9} & 0 & 0 \\ -13 & 9 & 4 & 0 & 1 & 0 \\ -3 & 2 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_2 \to R_2 - 13R_1} \left(\begin{array}{ccc|ccc} 1 & -\frac{7}{9} & -\frac{1}{3} & -\frac{1}{9} & 0 & 0 \\ 0 & -\frac{10}{9} & -\frac{1}{3} & -\frac{13}{9} & 1 & 0 \\ -3 & 2 & 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow{R_3 \to R_3 + 3R_1} \left(\begin{array}{ccc|ccc} 1 & -\frac{7}{9} & -\frac{1}{3} & -\frac{1}{9} & 0 & 0 \\ 0 & -\frac{10}{9} & -\frac{1}{3} & -\frac{13}{9} & 1 & 0 \\ 0 & -\frac{1}{3} & 0 & -\frac{1}{3} & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_2 \to -\frac{9}{10}R_2} \left(\begin{array}{ccc|ccc} 1 & -\frac{7}{9} & -\frac{1}{3} & -\frac{1}{9} & 0 & 0 \\ 0 & 1 & \frac{3}{10} & \frac{13}{10} & -\frac{9}{10} & 0 \\ 0 & -\frac{1}{3} & 0 & -\frac{1}{3} & 0 & 1 \end{array}\right) \xrightarrow{R_3 \to R_3 + \frac{1}{3}R_2} \left(\begin{array}{ccc|ccc} 1 & -\frac{7}{9} & -\frac{1}{3} & -\frac{1}{9} & 0 & 0 \\ 0 & 1 & \frac{3}{10} & \frac{13}{10} & -\frac{9}{10} & 0 \\ 0 & 0 & \frac{1}{10} & \frac{1}{10} & -\frac{3}{10} & 1 \end{array}\right)$$

$$\xrightarrow{R_3 \to 10R_3} \left(\begin{array}{ccc|ccc} 1 & -\frac{7}{9} & -\frac{1}{3} & -\frac{1}{9} & 0 & 0 \\ 0 & 0 & \frac{3}{10} & \frac{13}{10} & -\frac{9}{10} & 0 \\ 0 & 0 & 1 & 1 & -3 & 10 \end{array}\right) \xrightarrow{R_2 \to R_2 - \frac{3}{10}R_3} \left(\begin{array}{ccc|ccc} 1 & -\frac{7}{9} & -\frac{1}{3} & -\frac{1}{9} & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 & -3 & 10 \end{array}\right)$$

$$\xrightarrow{R_1 \to R_1 + \frac{1}{3}R_3} \left(\begin{array}{ccc|ccc} 1 & -\frac{7}{9} & 0 & \frac{2}{9} & -1 & \frac{10}{3} \\ 0 & 1 & 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 & -3 & 10 \end{array}\right) \xrightarrow{R_1 \to R_1 + \frac{7}{9}R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 & -3 & 10 \end{array}\right)$$

$$\therefore A^{-1} = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 0 & -3 \\ 1 & -3 & 10 \end{pmatrix}$$

b) $B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

For the matrix to be invertible: $\det(B) \neq 0$

or, $\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} \neq 0$

$\boxed{\text{or, } ad - bc \neq 0}$

39. We know,

$\det(E) \neq 0$, hence, it's invertible is possible.

Let, $E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, obtained by operation, $R_2 \longrightarrow 3R_2$ on I.

$\det E = 1 \begin{vmatrix} 3 & 0 \\ 0 & 1 \end{vmatrix} = 3 - 0 = 3 \neq 0$, hence invertible.

Calculating E^{-1},

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow{R_2 \to \frac{1}{3}R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$$

$E^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 \end{pmatrix}$; E^{-1} can be obtained by operation. $R_2 \to \frac{1}{3}R_2$ on I.

Hence, E^{-1} is also an elementary matrix.

39.b) $E_1 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

Applying: $R_1 \to R_1 + R_2$ on $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$E_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Applying: $R_2 \to -2R_2$ on $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$E_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$

Applying: $R_3 \longrightarrow R_3 + 2R_1$ on $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$