61. Study the convergence of these series:

    (a) $\sum_{k=1}^{\infty} \frac{1}{\sqrt{(k+1)(k+2)(k+3)}}$

    (b) $\sum_{k=1}^{\infty} \frac{1}{((k+1)(k+2)(k+3))^{\frac{1}{4}}}$

    (c) $\sum_{k=1}^{\infty} \frac{k^3+k}{k^5+1}$

62. Study the convergence of these series:

    (a) $\sum_{k=1}^{\infty} \left(1-\frac{1}{k^2}\right)^{k^3}$

    (b) $\sum_{k=1}^{\infty} \frac{3^k}{k!}$

    (c) $\sum_{k=1}^{\infty} \frac{k+k^k}{k^{2k}}$

    (d) $\sum_{k=1}^{\infty} \frac{k!}{k^k}$

    Hint: Ratio or root tests might be useful...

63. Study the values of α for which $\sum_{k=2}^{\infty} \frac{1}{k \ln^{\alpha}(k)}$ converges.

    if and only if $\alpha > 1$. *Hint: you can try to use the condensation test.*

64. For each serie, find the values $b \in \mathbb{R}$ such as the following series converge:

    (a) $\sum_{k=1}^{\infty} \frac{(b^2+2b)^k}{k^2}$

    (b) $\sum_{k=1}^{\infty} \frac{(-1)^k}{k^b}$

65. Let $(F_n)_{n\in\mathbb{N}}$ be a Fibonacci sequence. Show that $\sum_{k=1}^{\infty} \frac{1}{F_k}$ is convergent.
    Hint: use the definition of the Fibonacci sequence

66. Let $(u_n)_{n\in\mathbb{N}}$ be a positive sequence such as the serie $\sum_{n=1}^{\infty} u_n$ is convergent. We want to study the convergence of $\sum_{n=1}^{\infty} \frac{\sqrt{u_n}}{n}$.

    (a) Prove that $\sum_{n=1}^{m} \frac{\sqrt{u_n}}{n} \leq \left(\sum_{n=1}^{m} u_n\right)^{\frac{1}{2}} \cdot \left(\sum_{n=1}^{m} \frac{1}{n^2}\right)^{\frac{1}{2}}$.
    Hint: Use a famous inequality you learnt at the beginning of the semester.

    (b) Conclude about the convergence of $\sum_{n=1}^{\infty} \frac{\sqrt{u_n}}{n}$.