25. (a) Determine the set $\{x \in \mathbb{R} : 3|x+2| - 4x + 3 \leq 5|x-1|\}$. (Hint: Use case distinction.) Plot the solution set and the functions $f(x) = 3|x+2| - 4x + 3$ and $g(x) = 5|x-1|$. You may use any software of your choice for the plots.

    (b) Let $x_i \in \mathbb{R}, i \in \{1, \ldots, n\}$. Prove the generalized triangle inequality

$$\left|\sum_{i=1}^{n} x_i\right| \leq \sum_{i=1}^{n} |x_i| \,.$$

    Hint: Use induction on the predicate $P(n) : |\sum_{i=1}^{n} x_i| \leq \sum_{i=1}^{n} |x_i|$.

26. (a) Determine the following sets:
    - $\left\{x \in (4, \infty) : 4(\sqrt{3})^{2x-1} 5^{4x+3} = 3\pi^{x-4}\right\}$.
    - $\{x \in (0, \infty) : \log_3 x = 5\}$.
    - $\{x \in (-1, \infty) : 2\ln(x+3) - 3\ln(x+2) + \ln(x+1) = 0\}$.

    (b) Compute

$$\left\{x \in [-\pi, \pi] : \cos x = \frac{\sqrt{3}}{2}\right\} \quad \text{and} \quad \left\{x \in [-\pi, \pi] : \sin^2 x = \frac{1}{2}\right\}$$

27. Let $f : \mathbb{R} \to \mathbb{R}$ be the ReLU activation function defined as

$$f(x) := \begin{cases} x & \text{if } x \geq 0, \\ 0 & \text{if } x < 0. \end{cases}$$

Plot and describe in words, how the following transformations change the ReLU activation function f, that is, how the graph of the function $F(x)$ looks like compared to the graph of $f(x)$. You may use any software of your choice for the plots.

Let $k \in \{-1,\ 0.5,\ 2\}$

(a) $F(x) = f(x) + k$,

(b) $F(x) = f(x + k)$,

(c) $F(x) = k \cdot f(x)$,

(d) $F(x) = f(k \cdot x)$,

(e) $F(x) = |f(x)|$,

(f) $F(x) = f(|x|)$.

28. Let $z_1 = \dfrac{\sqrt{3}}{2} i + \dfrac{\sqrt{3} - i}{(\sqrt{2} + \sqrt{2}i)^2}$ and $z_2 = 2\left(\cos\left(\frac{\pi}{3}\right) + i \sin\left(\frac{\pi}{3}\right)\right)$.

    (a) Determine the real and imaginary part of z_1 and z_2.

    (b) Determine the absolute value and the argument of z_1 and z_2.

    (c) Determine the absolute value and the argument of $z_1 z_2$. Furthermore, give the real and imaginary part of $z_1 z_2$.

29. (a) Calculate $(\frac{1}{2}\sqrt{3} + \frac{1}{2}i)^{12}$ using de Moivre's formula.

    (b) How many real solutions do the following equations have? Write them down.

i. $x^2 = -1, \quad x \in \mathbb{R}$ ii. $x^3 = -1, \quad x \in \mathbb{R}$

(c) How many complex solutions do the following equations have? Write them down.

i. $x^2 = -1, \quad x \in \mathbb{C}$ ii. $x^3 = -1, \quad x \in \mathbb{C}$

(*Hint: For c) ii), use Euler's formula or/and de Moivre's formula.*)

30. Let z and w be two non-zero complex numbers. Show that $|z + w| = |z| + |w|$ if and only if z and w have the same argument.
(Hint: Use Theorem 1.50 from the lecture notes and use polar coordinates for z and w.)