Sheet 8

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43) a) $a_n = (-1)^n$

$[-1, 1]$

b) $b_n = \frac{1}{n}$

c) $c_n = 3.1 + \frac{1}{n}$

d) $d_n = \frac{1}{n}$

44) $a_{2k} = \frac{2k + \cos(2k \cdot \pi)}{4k} = \frac{2k+1}{4k} = \frac{1}{2} + \frac{1}{4k}$

as. $k \to \infty$, $a_{2k} \to \frac{1}{2}$

similarly, $a_{2k+1} = \frac{-(2k+1) + \cos((2k+1)\pi)}{2(2k+1)}$

$= \frac{-2k}{4k+2} = \frac{-1}{2} + \frac{1}{2k+1}$

Since even & odd subsequences of (a_n) approaches different limit

→ n goes to infinity

3

45a) $\lim_{n\to\infty} a_n = 3 \qquad \lim_{n\to\infty} (a_n b_n^2) = 0$

$b_n \neq 0$

$a_n = 3 + \frac{1}{n}$, $b_n = \frac{1}{\sqrt{n}}$

b) $\lim_{n\to\infty} a_n = 5$, $\lim_{n\to\infty} (a_n + (-1)^n b_n) = 0$; $(b_n) \neq (b_{n+1})$

$a_n = 5 + \frac{(-1)^n}{n}$, $b_n = \frac{1}{n}$

46a) $a_n = \sqrt{n^2+n} - n$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} (\sqrt{n^2+n} - n)$

$= \frac{n}{\sqrt{n^2+n} + n} = \frac{1}{2}$

b) $b_n = \frac{n^2+n+1}{n^2+n\sin n + 1}$

$\lim_{n\to\infty} \frac{n^2}{n^2} \to 1$

c)