43. Find an example for a sequence with the following properties, if possible:
    (a) a bounded sequence that is divergent.
    (b) a non-increasing bounded sequence that is divergent.
    (c) a strictly increasing sequence converging to π.
    (d) an unbounded null sequence.

44. Show that the sequence $(a_n)_{n\in\mathbb{N}}$ defined by $a_n := (-1)^n\left(\frac{n+\cos n\pi}{2n}\right)$ for $n \in \mathbb{N}$ is not convergent.

45. Give an example for non-constant sequences $(a_n)_{n\in\mathbb{N}}$, $(b_n)_{n\in\mathbb{N}}$ satisfying
    (a) $\lim_{n\to\infty} a_n = 3, b_n \neq 0$ and $\lim_{n\to\infty}(a_n b_n^2) = 0$.
    (b) $\lim_{n\to\infty} a_n = 5, |b_n| \neq |a_n|$ and $\lim_{n\to\infty}(a_n + (-1)^n b_n) = 0$.

46. Determine the limits of sequences

$$a_n = \sqrt{n^2+n} - n, \qquad b_n = \frac{n^2+n+1}{n^2+n\sin n+1},$$
$$c_n = \frac{n!(n+5)+2^n}{(n+1)!+3^n}, \qquad d_n = \sqrt[n]{3^n+4^n},$$
$$e_n = \frac{n^3-3n+7}{7n+1}, \qquad t_n = \sqrt{n^3-n^2+1} - n,$$

where $n \in \mathbb{N}$.

47. Let $q > 0$, determine the limit of sequences $(a_n)_{n\in\mathbb{N}}$, $(b_n)_{n\in\mathbb{N}}$, and $(c_n)_{n\in\mathbb{N}}$, defined by

$$a_n = (q^n + 5^n + 11^n)^{\frac{1}{n}},$$
$$b_n = \sqrt[n]{q^n + n^2},$$
$$c_n = \frac{\sin(q^n\pi)}{n}, \quad \text{where } n \in \mathbb{N}.$$

48. Let $\lim_{n\to\infty} a_n = a$. We define $b_n = \frac{1}{n}\sum_{j=1}^{n} a_j$ for $n \in \mathbb{N}$. Prove that

$$\lim_{n\to\infty} b_n = a$$

Hint: Use the boundedness of sequence $(a_n)_{n\in\mathbb{N}}$ and $a = \frac{\sum_{i=1}^{n} a}{n}$ for $a \in \mathbb{C}$, $n \in \mathbb{N}$.