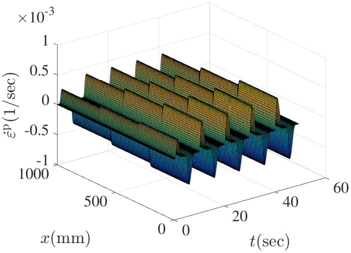
Large time increment approach for fatigue damage computations

S. Alameddin[†], M. Bhattacharyya[†], A. Fau[†], U. Nackenhorst[†], D. Néron[‡], P. Ladevèze[‡]

† IBNM, Leibniz Universität Hannover

‡ LMT, ENS Cachan, CNRS, Université Paris Saclay

15. March 2017



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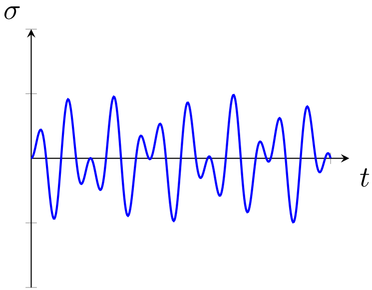
IRTG-1627

Fatigue damage

- Cyclic loading



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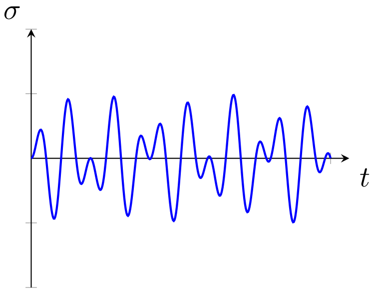

- Damage

Image by alegri / 4freephotos.com

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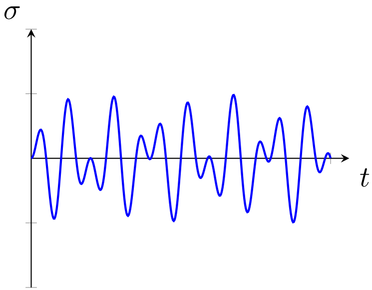

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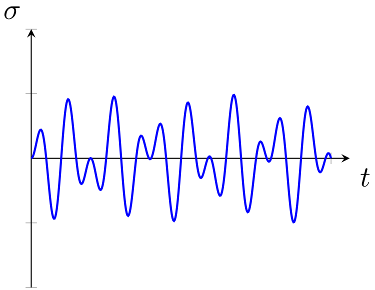

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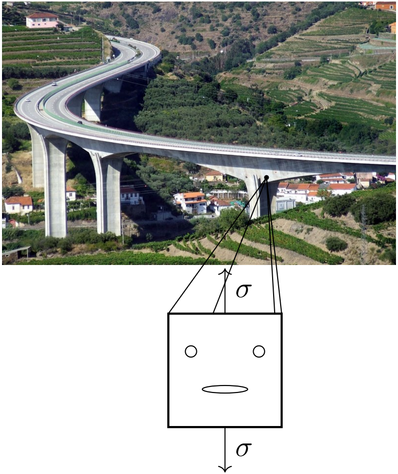

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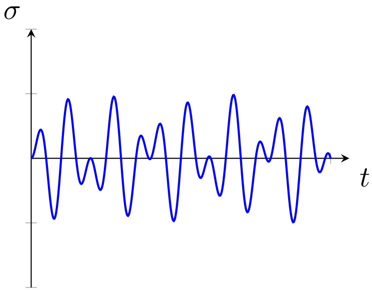

- Virtual experiments
- Continuum damage model
- Millions of cycles
- Computationally expensive

Fatigue damage

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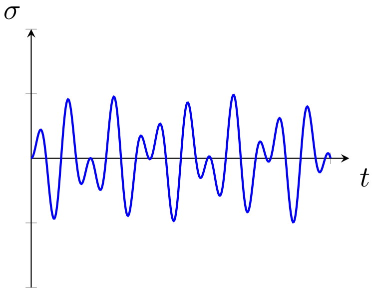

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Model order reduction (MOR) techniques

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Need for a convenient framework to utilise PGD

Large time increment (LATIN) method

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  - An approximation on the **whole time-space domain** is obtained.
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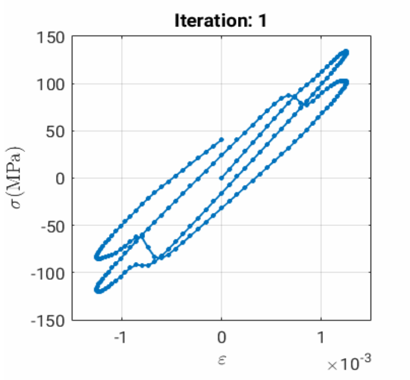
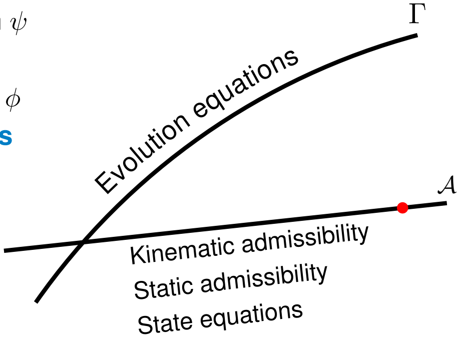


The LATIN framework

- Assumptions
  - Free energy function ψ
    State equations
  - Dissipation potential ϕ
    Evolution equations
  - No dynamic effects
- Linear initialisation

$$\varepsilon = \varepsilon^{\mathrm{e}}$$

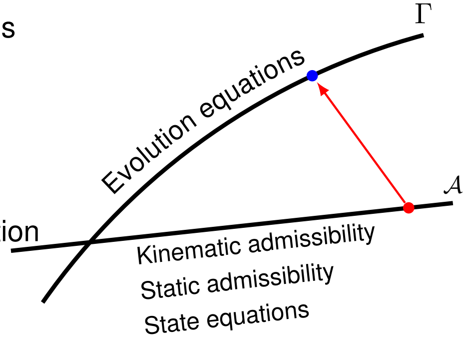


The LATIN framework

- Non-linear step
- The evolution equations

$$\text{e.g. } \dot{X} = -\frac{\partial \phi}{\partial Y}$$

- Local in space (cheap)
- Suitable for parallelisation

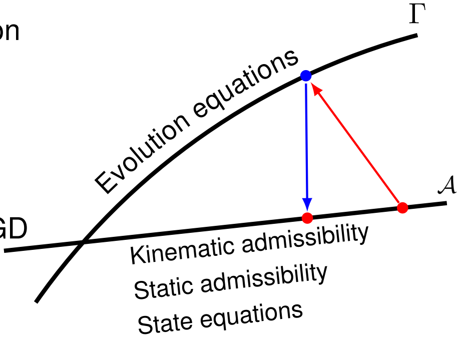


The LATIN framework

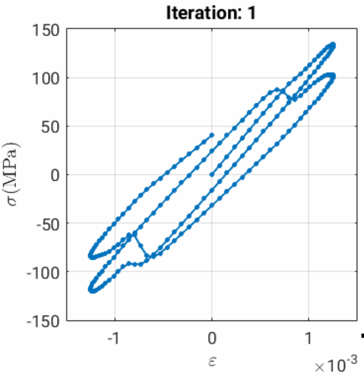
- Linear step
- The equilibrium equation

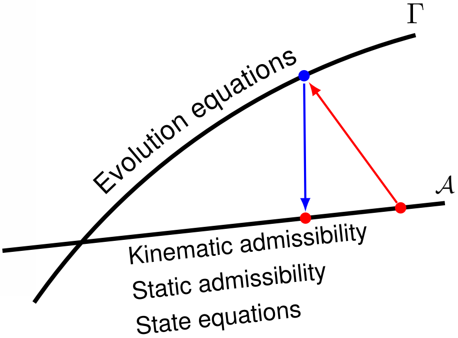
$$\mathrm{div}(\boldsymbol{\sigma}) = \mathbf{0}$$

- Global in space
- Convenient to apply PGD



The LATIN iterative scheme





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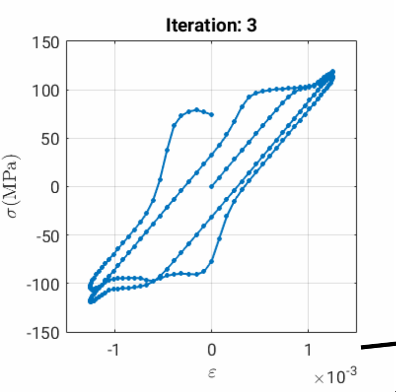

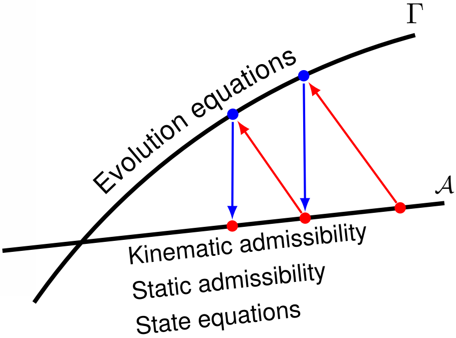

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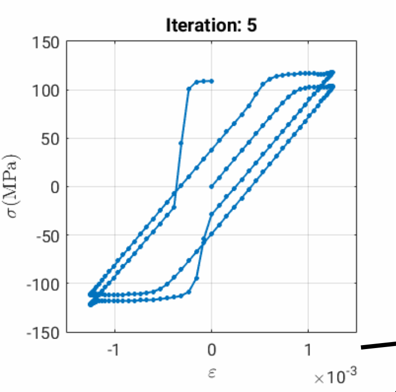

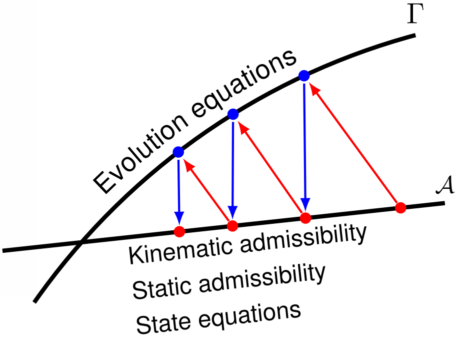

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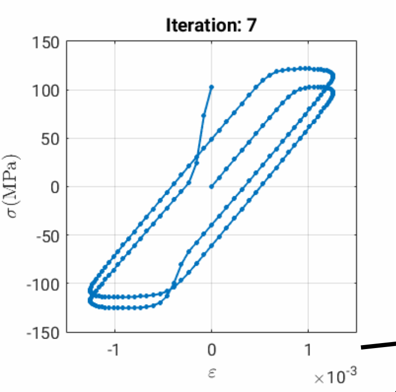

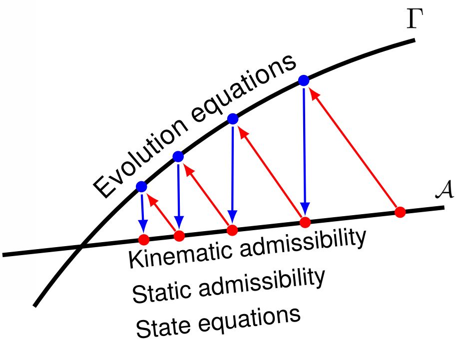

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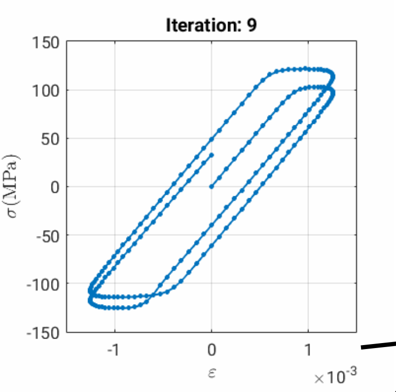

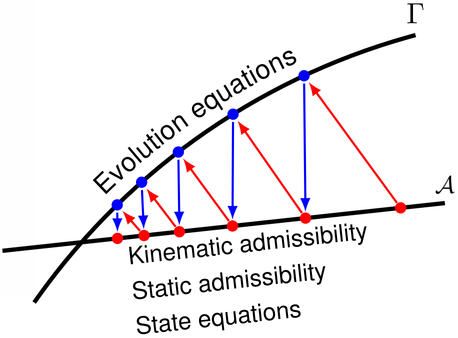

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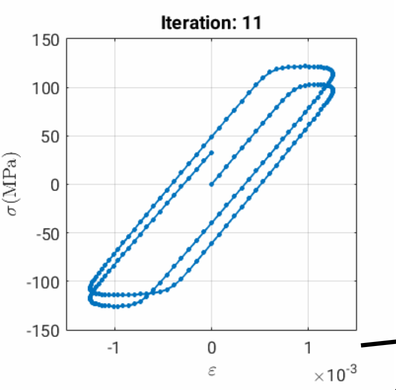

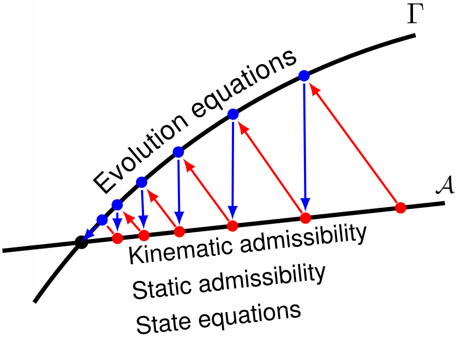

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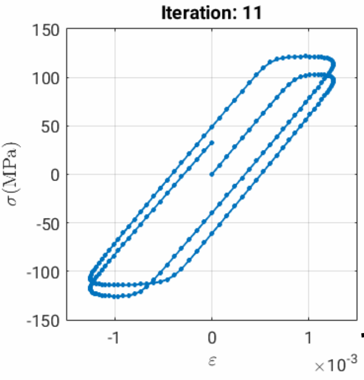
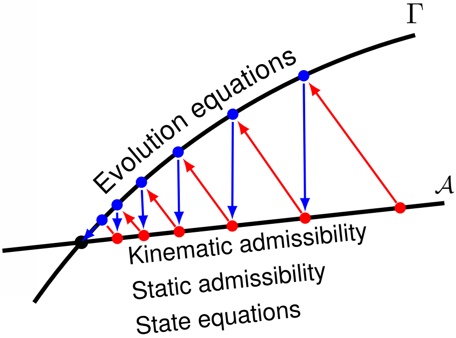


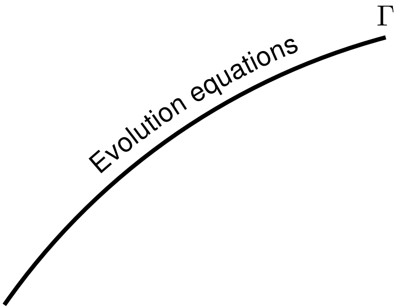


Established for viscoplasticity, contact and large deformations problems [Surveys: Ladevèze 1999, Chinesta 2014]

LATIN with isotropic damage

- State equation

  e.g. $\boldsymbol{\sigma} = E\,(1 - D)\,\boldsymbol{\varepsilon}^{\mathbf{e}}$



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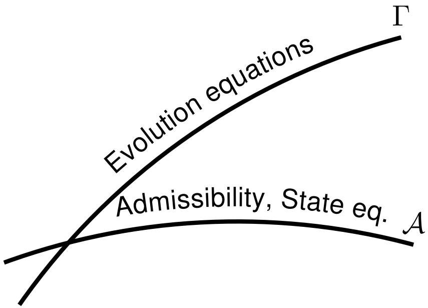

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Workaround

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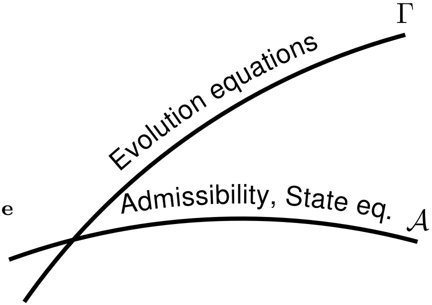

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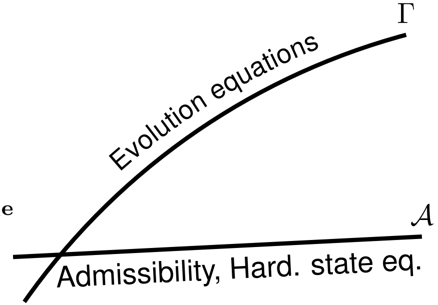
  in the local step

- Linear $\mathcal{A}$
- Use PGD for the global step



PGD

$$\varepsilon^{\mathrm{p}}(x,t) \approx \sum_{i=1}^{n} \lambda_i(t)\ \bar{\varepsilon}_i^{\mathrm{p}}(x)$$

Initialise $\lambda(t)$

while $err > tol$ **do**

$\int_t \bullet \, dt \quad \rightarrow \quad$ the space function $\bar{\varepsilon}^{\mathrm{p}}(x)$

$\int_\Omega \bullet \, d\Omega \quad \rightarrow \quad$ the time function $\lambda(t)$

end

Algorithm: PGD enrichment step

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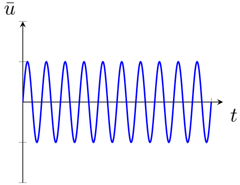
Algorithm: PGD enrichment step

- Auto. generation of the best pairs by a greedy algorithm
- No a priori assumption on the reduced order basis

Numerical example

- Chaboche Marquis constitutive model [Chaboche 1993; Cognard 1993]
  (Cr-Mo steel at $580^\circ\mathrm{C}$, Unilateral damage)



$$\psi = \frac{1}{2}\, E\, (\varepsilon^{\mathrm{e}})^2 + \underbrace{\frac{1}{2}\, C\, (\alpha)^2}_{\text{hardening}}$$

$$\phi = \phi^{\mathrm{p}}$$

$$\phi^{\mathrm{p}} = \frac{k}{n+1} \langle f^{\mathrm{p}} \rangle_+^{n+1} \qquad f^{\mathrm{p}} = |\sigma - \beta| + \frac{a}{C}\, \beta^2 - \sigma_{\mathrm{y}}$$

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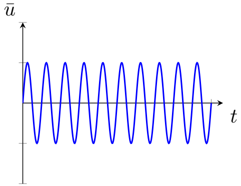

$$\psi = \frac{1}{2}\, E\, (\varepsilon^{\mathrm{e}})^2\, (1 - D) + \underbrace{\frac{1}{2}\, C\, (\alpha)^2}_{\text{hardening}}$$

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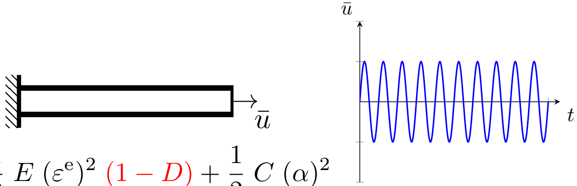

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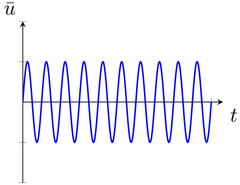

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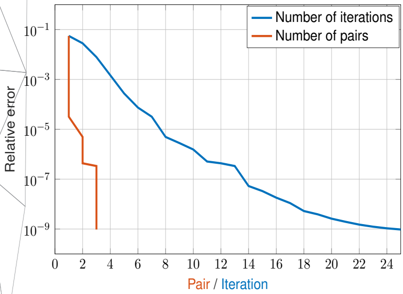
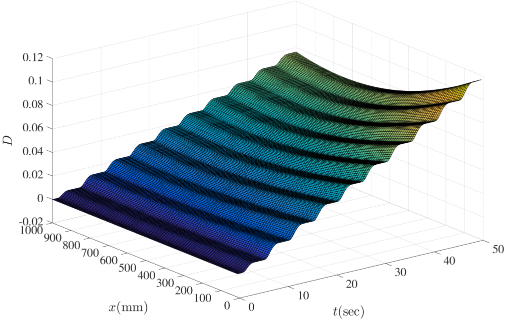
$$\phi^{\mathrm{d}} = \frac{k_{\mathrm{d}}}{n_{\mathrm{d}}+1} \langle f^{\mathrm{d}} \rangle_+^{n_{\mathrm{d}}+1} \qquad f^{\mathrm{d}} = Y - Y_0$$

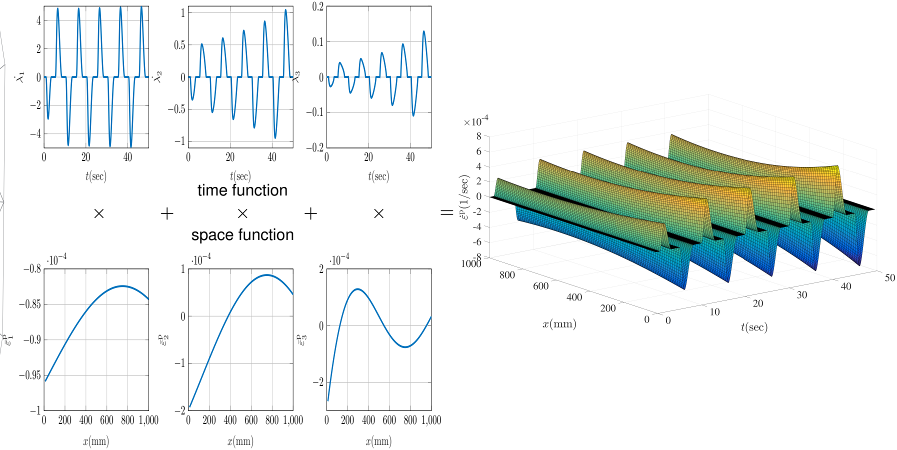
The convergence behaviour





Damage evolution at convergence

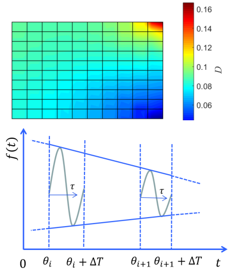
The plastic strain evolution using PGD



$$\dot{\varepsilon}^{\mathrm{p}}(x,t) \approx \sum_{i=1}^{3} \dot{\lambda}_i(t)\; \bar{\varepsilon}_i^{\mathrm{p}}(x)$$

Conclusion and current research

- A LATIN-based model reduction approach for the simulation of cycling damage [Mainak Bhattacharyya, IRTG 2nd cohort]
  - Crack closure effect
  - Non-proportional loading
- Two-time scale approach (*in progress*)



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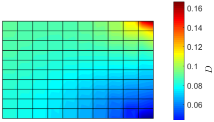
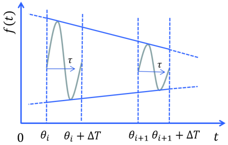




Current objectives

- Extend the two-time scale to consider

  Different amplitudes, frequencies and random loadings

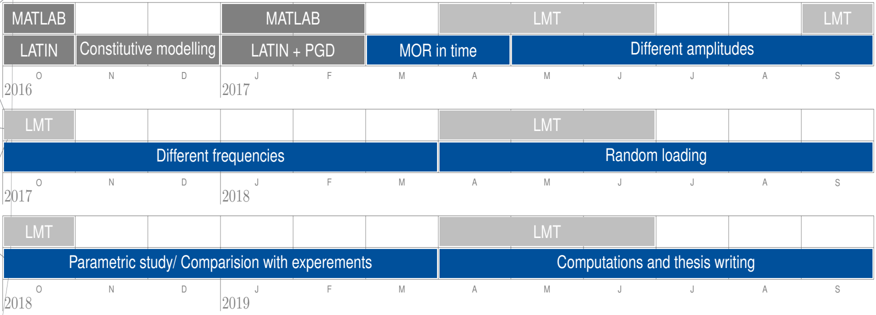
Milestone plan

O	N	D	J	F	M	A	M	J	J	A	S
MATLAB			MATLAB			LMT					LMT
LATIN	Constitutive modelling		LATIN + PGD		MOR in time		Different amplitudes				
2016			2017								

O	N	D	J	F	M	A	M	J	J	A	S
LMT						LMT					
Different frequencies						Random loading					
2017			2018								

O	N	D	J	F	M	A	M	J	J	A	S
LMT						LMT					
Parametric study/ Comparision with experements						Computations and thesis writing					
2018			2019								

Milestone plan



Thank you for your attention!

Time comparison

D. Néron et al, Time-space PGD for the rapid solution of 3D nonlinear parametrized problems in the many-query context, *IJNME*, 2015.

- LATIN convergence conditions

  Ladeveze 1999 [p84]

- PGD existence and convergence

  Ladeveze 1999 [p119]

- PGD for solving PDE

  A. Nouy. A priori model reduction through proper generalized decomposition for solving time-dependent partial differential equations. Computer Methods In Applied Mechanics and Engineering, 199(23- 24):1603–1626, 2010.

  Antonio Falco