

ViVaCE "Virtual Materials and their
Validation: German-French School of
Computational Engineering" - IRTG 1627

Model Order Reduction Techniques for Fatigue Damage Assessment

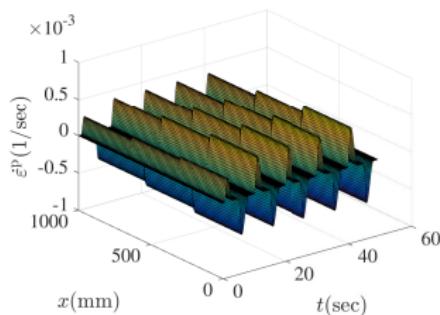
Shadi Alameddin,

Dr. Amelie Fau, Prof. Udo Nackenhorst

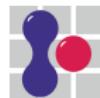
Institute of Mechanics and Computational Mechanics

Leibniz Universität Hannover

21. February 2017

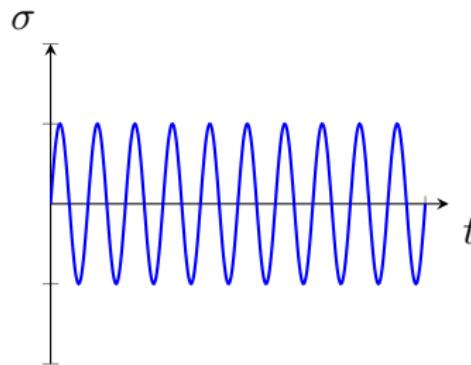


Deutsche
Forschungsgemeinschaft



Motivation

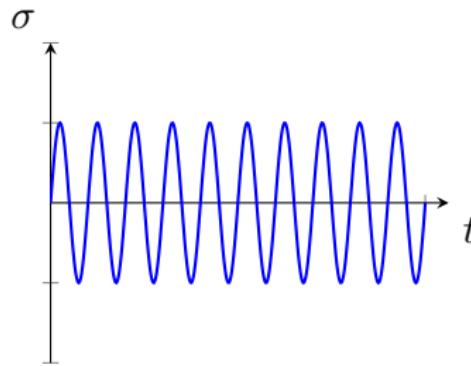
- Cyclic loading



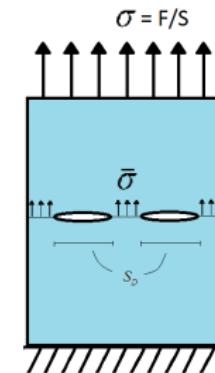


Motivation

- Cyclic loading



- Damage

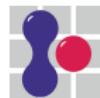




Motivation



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Motivation

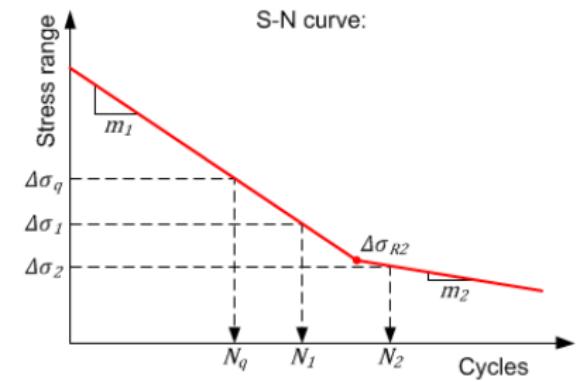
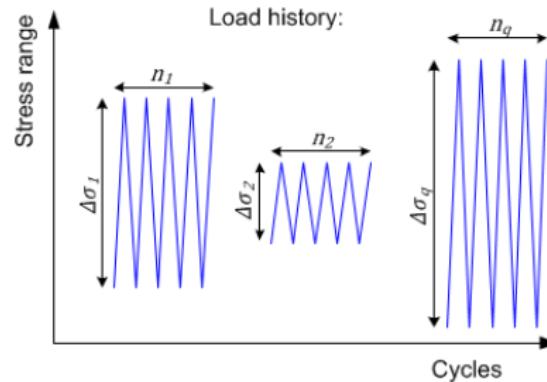


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Life prediction due to cyclic loading $\sim 10^9$ cycles



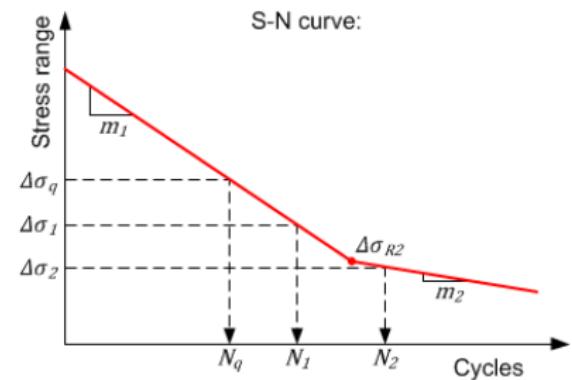
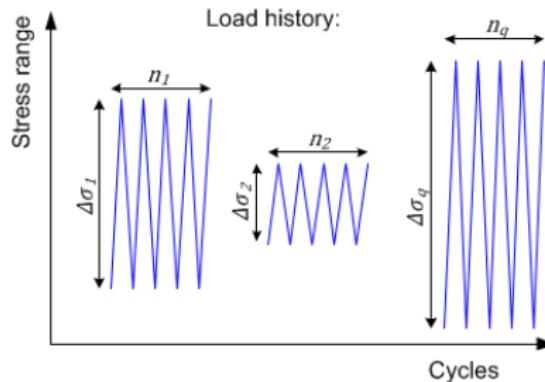
S-N curves



<http://fatiguetoolbox.org/documentation/theory-reference>



S-N curves



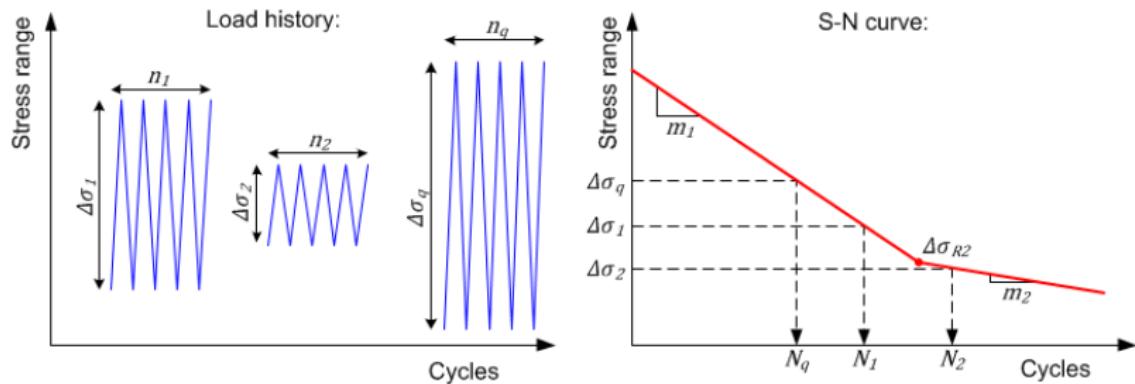
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Pros

- Simple and easy to use



S-N curves



<http://fatiguetoolbox.org/documentation/theory-reference>

Pros

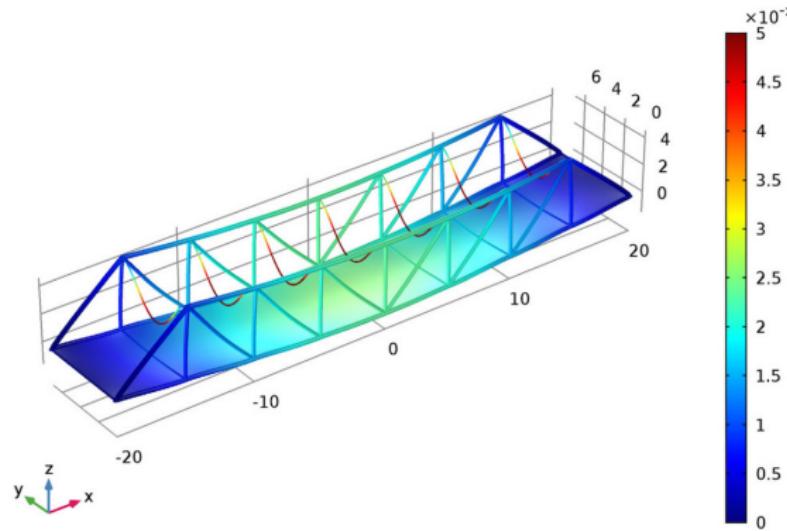
- Simple and easy to use

Cons

- Based on expensive experiments
- Deterministic loading chronology
- Not possible to consider initial damage



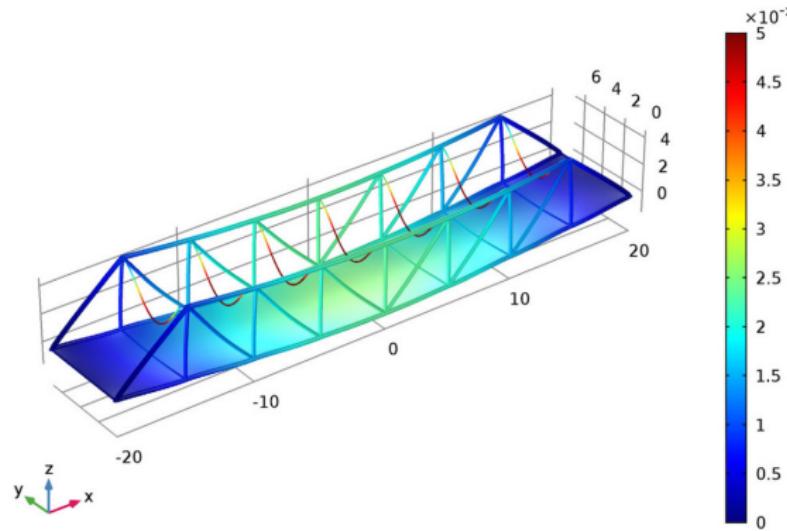
Numerical experiments



Displacement field under the bridge self weight [m]



Numerical experiments



Displacement field under the bridge self weight [m]

Computationally expensive

e.g. 1 cycle ~ 0.5 sec $\longleftrightarrow 10^9$ cycles ~ 16 years



Goals

Challenges

- Predict the damage for high number of cycles

- Limit the computational demands



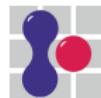
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Proposal

- Framework to incorporate model order reduction (MOR) techniques
- Develop MOR in time
- Different amplitudes, frequencies and random loadings
- Optimise in terms of accuracy vs. efficiency



What is model order reduction?

Singular value decomposition (SVD)

$$\underline{\underline{A}} \in \mathbb{R}^{n \times m}$$

$$= \underline{\underline{U}} \in \mathbb{R}^{n \times n}$$

$$\underline{\underline{\Sigma}} \in \mathbb{R}^{n \times m}$$

$$\underline{\underline{Z}}^T \in \mathbb{R}^{m \times m}$$



What is model order reduction?

Singular value decomposition (SVD)

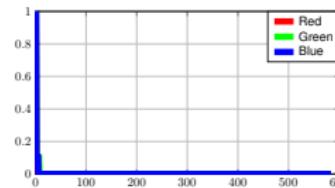
$$\underline{\underline{A}} \in \mathbb{R}^{n \times m} \approx \underline{\underline{\tilde{U}}} \in \mathbb{R}^{n \times N} \quad \underline{\underline{\Sigma}} \in \mathbb{R}^{N \times N}$$
$$\underline{\underline{\tilde{Z}}}^T \in \mathbb{R}^{N \times m}$$



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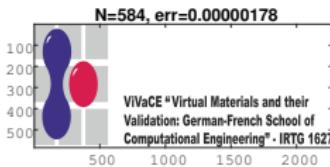
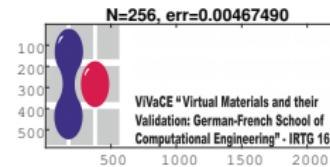
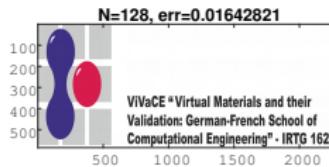
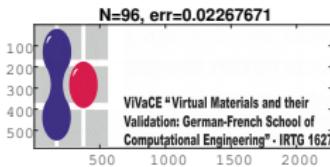
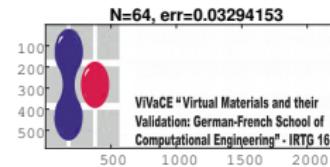
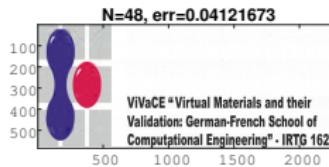
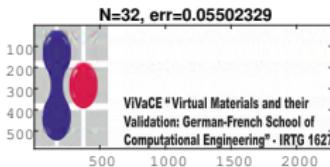
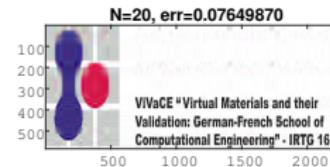
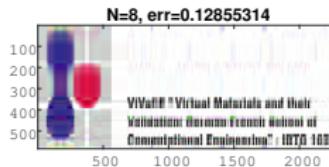
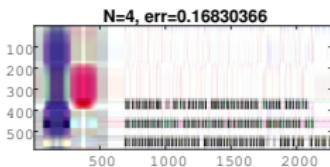
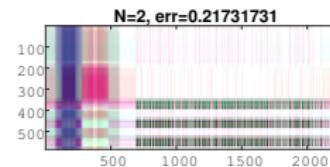
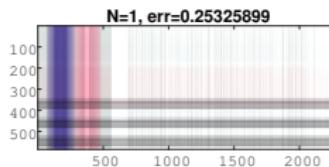
Singular values



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Image compression using SVD





Goals

Challenges

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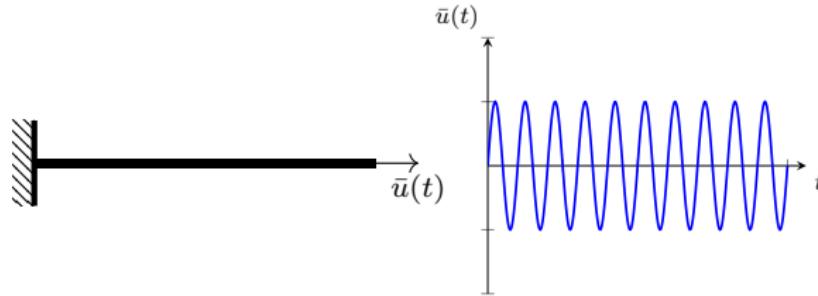
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- Framework to incorporate model order reduction (MOR) techniques
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Non-incremental framework

- Large time increment (LATIN) [Ladevèze, 1999]
 - At each iteration we obtain an approximation on the **whole time domain**.
 - At each iteration the balance equation is solved as a linear problem → convenient to apply MOR.

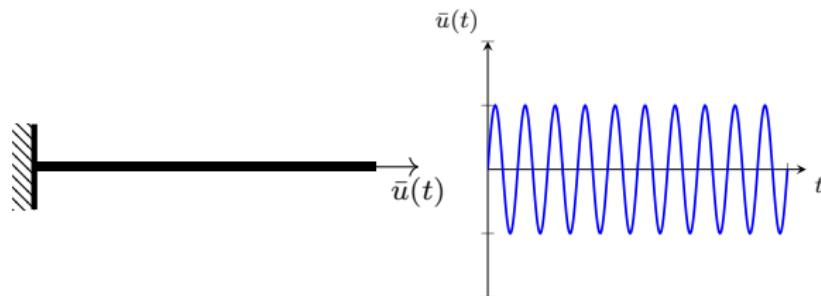


Uniaxial visco-plastic bar with cyclic loading without considering dynamic effects



Non-incremental framework

- Large time increment (LATIN) [Ladevèze, 1999]



Uniaxial visco-plastic bar with cyclic loading without considering dynamic effects

- $\psi(\varepsilon, \alpha) = \frac{1}{2} E (\varepsilon)^2 + \frac{1}{2} C (\alpha)^2$

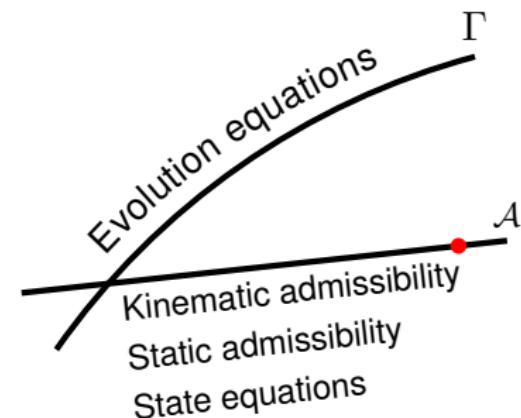
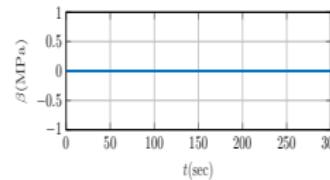
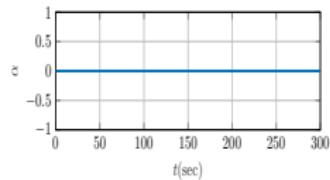
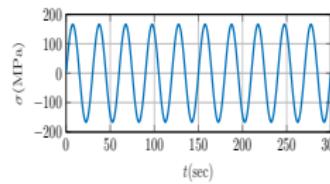
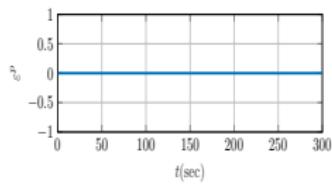
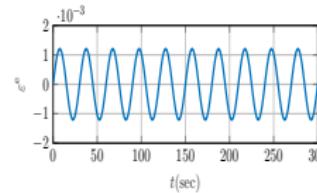
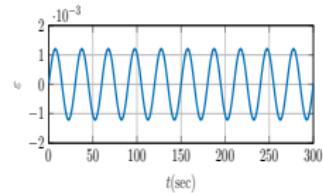
State equations: $\sigma = E \varepsilon^e, \quad \beta = C \alpha$

$$\phi^P(\sigma, \beta) = \frac{k}{n+1} \langle |\sigma - \beta| + \frac{a}{C} \beta^2 - \sigma_y \rangle_+^{n+1}$$

Evolution equations: $\dot{\varepsilon}^P = \frac{\partial \phi^P}{\partial \sigma}, \quad \dot{\alpha} = -\frac{\partial \phi^P}{\partial \beta}$

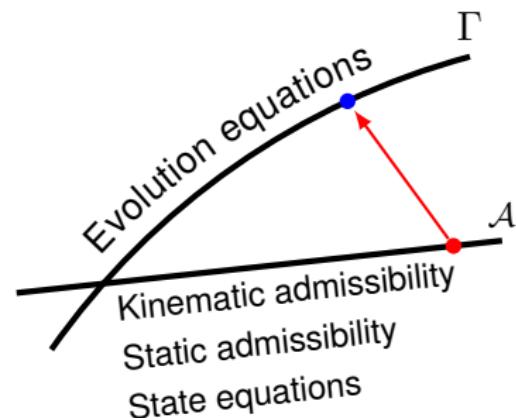
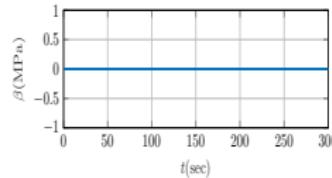
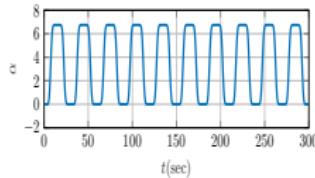
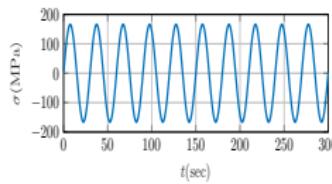
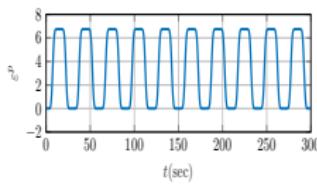
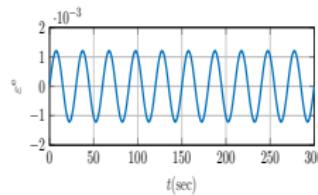
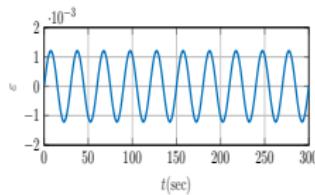


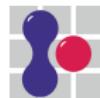
Time discretisation example



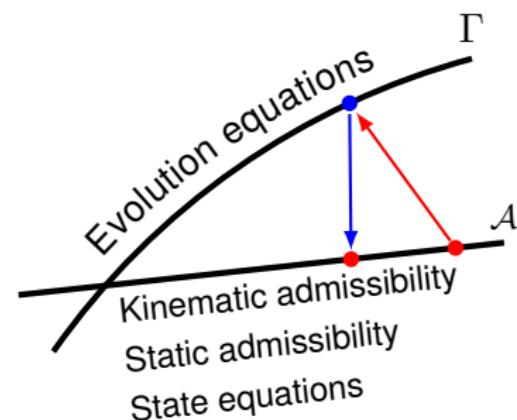
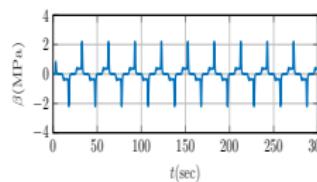
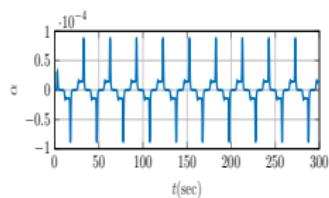
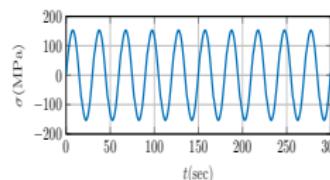
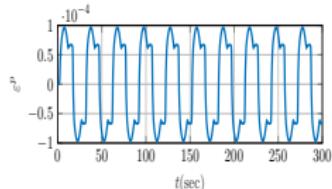
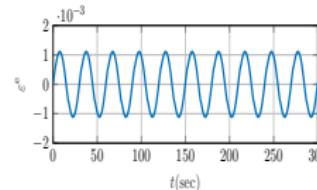
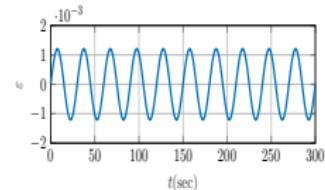


Time discretisation example





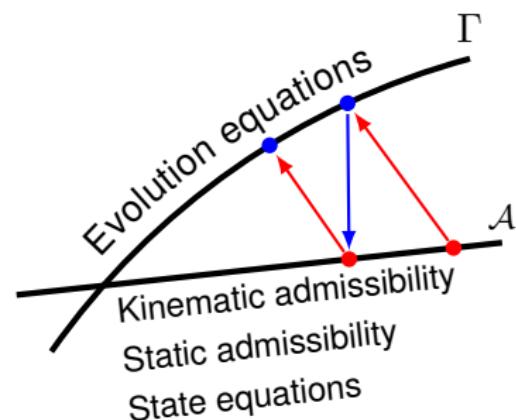
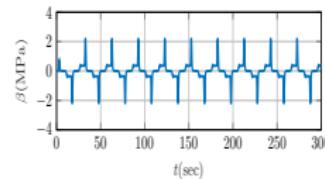
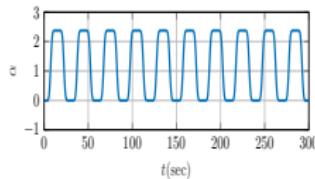
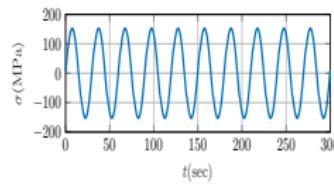
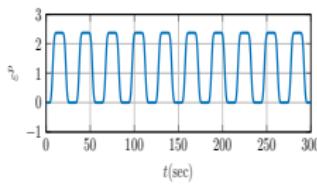
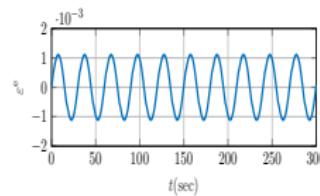
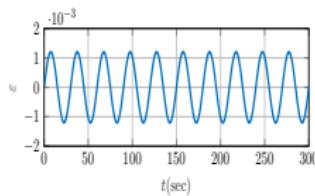
Time discretisation example



Linear step



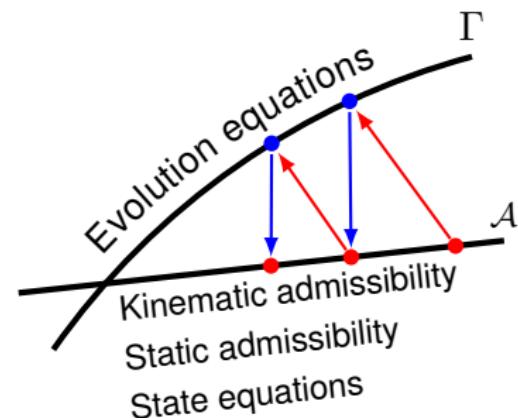
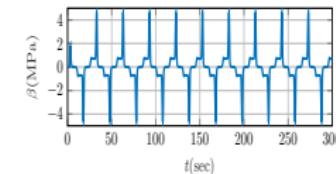
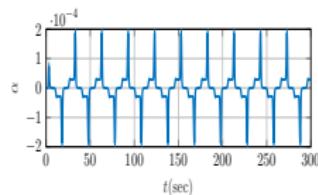
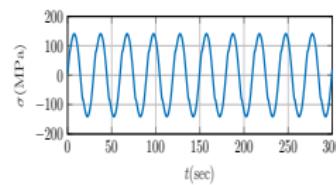
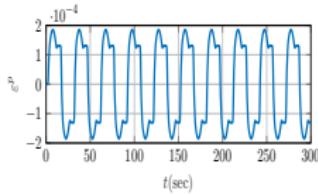
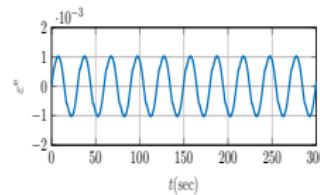
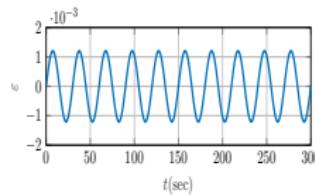
Time discretisation example



Non-linear step



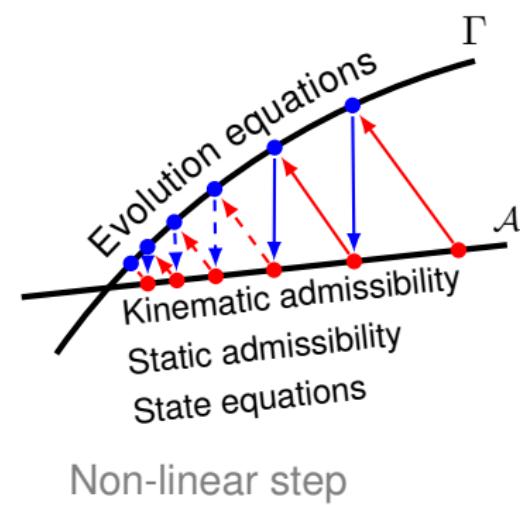
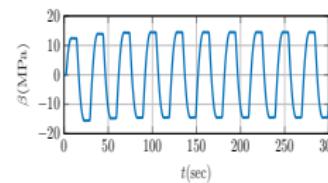
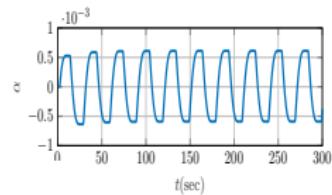
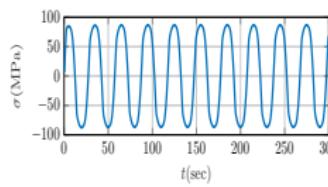
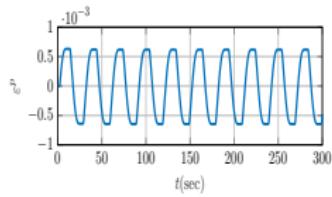
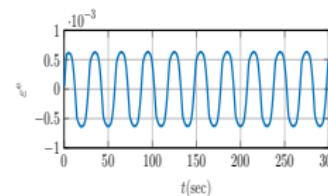
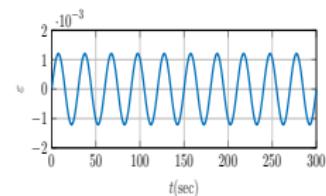
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Linear step



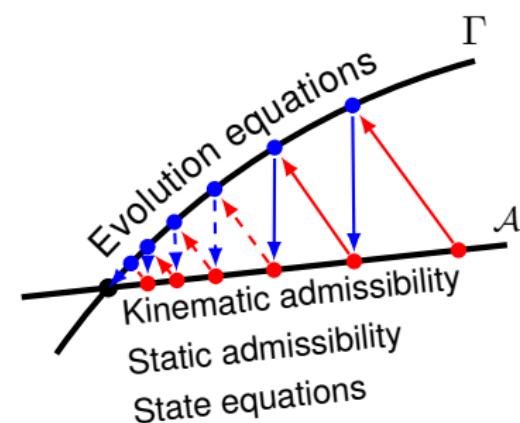
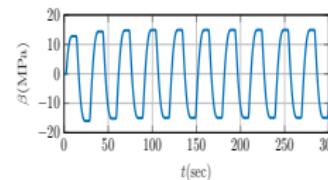
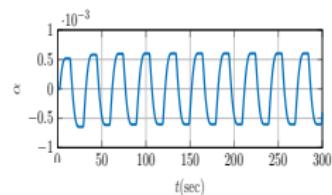
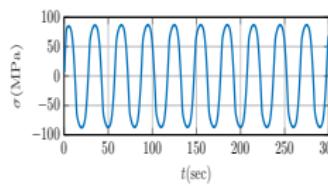
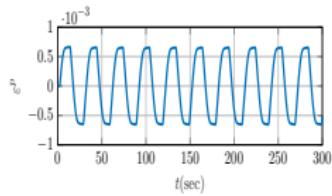
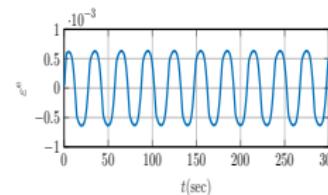
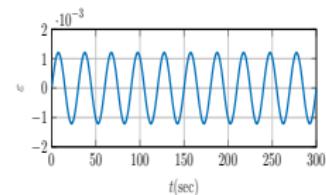
Time discretisation example



Non-linear step



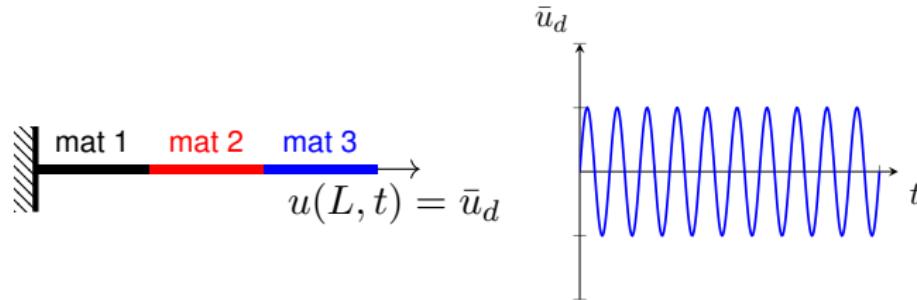
Time discretisation example



Linear step



Time-space discretisation



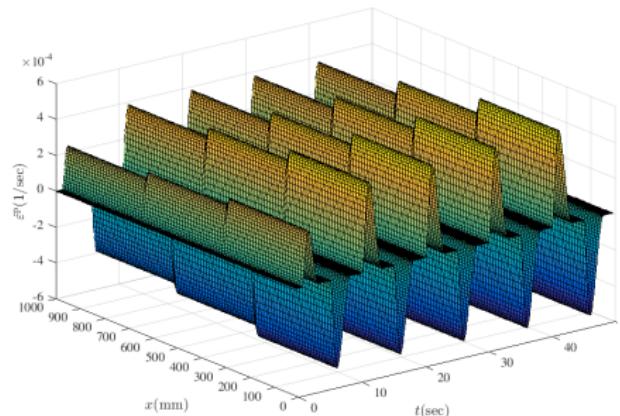
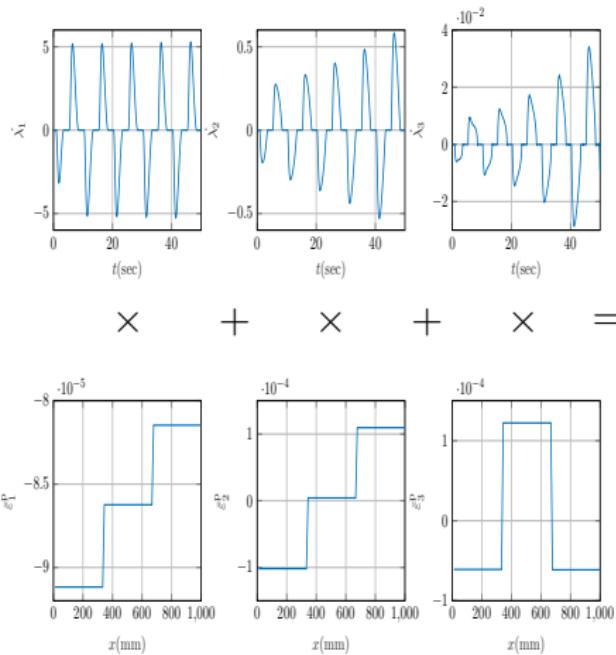
Separation of variables [Proper Generalised decomposition (PGD), Ladevèze 1999]

The quantities of interest are defined over the whole time-space domain

$$\varepsilon^P(x, t) \approx \sum_{i=1}^n \bar{\varepsilon}^P(x) \lambda_i(t)$$



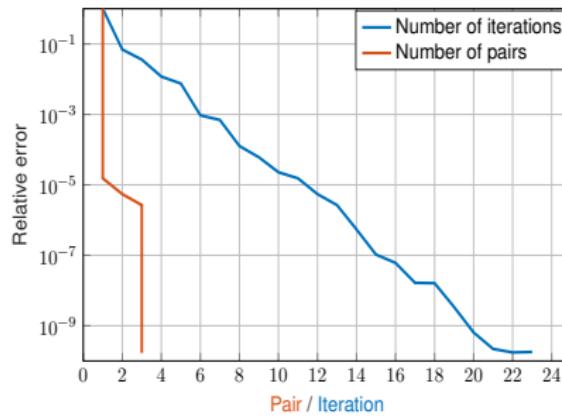
Time-space discretisation



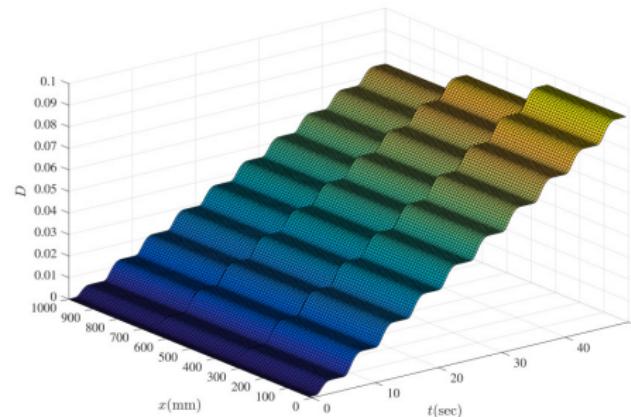
Plastic strain rate



Time-space discretisation



Error evolution



Damage evolution



Goals

Challenges

- Predict the damage for high number of cycles
- Limit the computational demands

Proposal

- Framework to incorporate model order reduction (MOR) techniques
- Develop MOR in time
- Different amplitudes, frequencies and random loadings
- Optimise in terms of accuracy vs. efficiency



Milestone plan

MATLAB			MATLAB			LMT						LMT		
LATIN	Constitutive modelling		LATIN + PGD			MOR in time			Different amplitudes					
O	N	D	J	F	M	A	M	J	J	J	A	S		
2016			2017											
LMT						LMT								
Different frequencies						Random loading								
O	N	D	J	F	M	A	M	J	J	J	A	S		
2017			2018											
LMT						LMT								
Computations and parametric study						Thesis writing								
O	N	D	J	F	M	A	M	J	J	J	A	S		
2018			2019											



Milestone plan

MATLAB			MATLAB			LMT				LMT
LATIN	Constitutive modelling		LATIN + PGD		MOR in time			Different amplitudes		
O 2016	N	D	J 2017	F	M	A	M	J	J	A S
LMT						LMT				
Different frequencies						Random loading				
O 2017	N	D	J 2018	F	M	A	M	J	J	A S
LMT						LMT				
Computations and parametric study						Thesis writing				
O 2018	N	D	J 2019	F	M	A	M	J	J	A S

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Thank you for your attention!