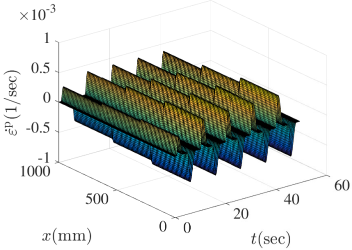
Model order reduction for fatigue analysis

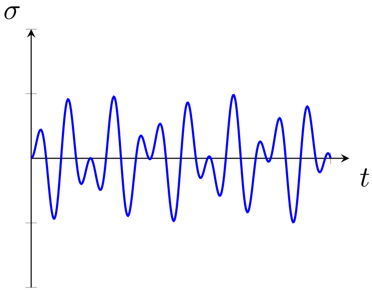
S. Alameddin†, M. Bhattacharyya†, A. Fau†,
U. Nackenhorst†, D. Néron‡, P. Ladevèze‡
† IBNM, Leibniz Universität Hannover
‡ LMT, ENS Cachan, CNRS, Université Paris Saclay
04. April 2018



Deutsche Forschungsgemeinschaft
IRTG-1627

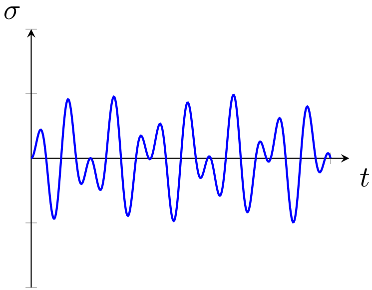
Fatigue damage

- Cyclic loading



Fatigue damage

- Cyclic loading

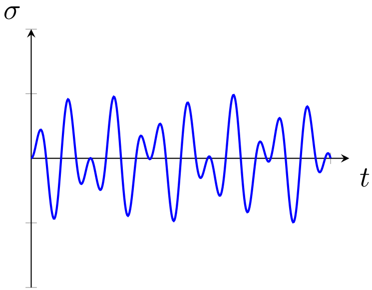


- Damage

Image by alegri / 4freephotos.com

Fatigue damage

- Cyclic loading

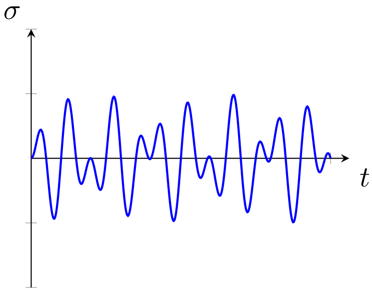


- Damage

Image by alegri / 4freephotos.com

Fatigue damage

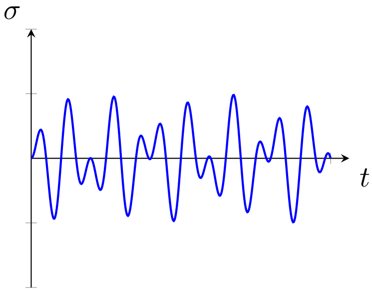
- Cyclic loading



- Virtual experiments
- Continuum damage model
- Millions of cycles
- Computationally expensive

Fatigue damage

- Cyclic loading



- Virtual experiments
- Continuum damage model
- Millions of cycles
- Computationally expensive

Model order reduction (MOR) techniques

LATIN overview

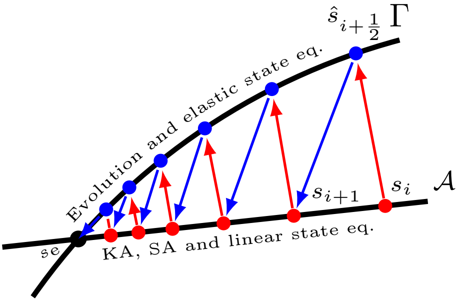
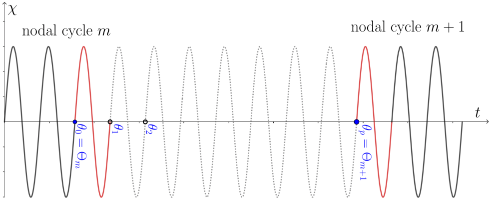


Illustration of the LATIN iterations

LATIN with two-time scale for ductile damage

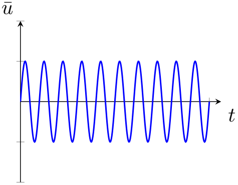
- The first and second nodal cycles are computed

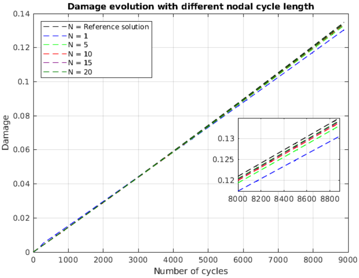


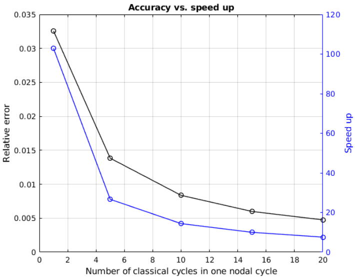
- The initial condition QoI are interpolated between the nodal cycles

$$\chi(t) = \frac{\Theta_{m+1} - \theta_k}{\Theta_{m+1} - \Theta_m}\chi(\tau_m) + \frac{\theta_k - \Theta_m}{\Theta_{m+1} - \Theta_m}\chi(\tau_{m+1})$$

Ductile damage example







LATIN for brittle damage

- Assumptions
  - Free energy function $\psi_{\mathrm{e}} = 1/2\,(1-D)\,E\,(\varepsilon^{\mathrm{e}})^2$
  - Damage dissipation potential

$$\varphi_{\mathrm{d}} = \frac{k_{\mathrm{d}}}{n_{\mathrm{d}}+1}\langle f^{\mathrm{d}}\rangle_{+}^{n_{\mathrm{d}}+1}, \qquad f^{\mathrm{d}} = Y - Y_0$$

Y_0 is the initial damage limit and k_{d} and n_{d} are the damage viscous parameters.

LATIN scheme for brittle damage

- The strain and displacement are initialised by the given B.C. only
- Global stage
  Purely elastic and the displacement is written in PGD form
  - The static admissibility

$$\int \sigma \, \varepsilon^* \;\, \mathrm{d}\Omega \, \mathrm{d}t = \int b \, u^* \;\, \mathrm{d}\Omega \, \mathrm{d}t + \int_{\partial\Omega_N} \bar{t} \, u^* \;\, \mathrm{d}s \, \mathrm{d}t.$$

  - In terms of corrections it becomes

$$\int \Delta\sigma \, \varepsilon^* \;\, \mathrm{d}\Omega \, \mathrm{d}t = 0.$$

LATIN scheme for brittle damage

- Global search direction

$$(\varepsilon^{\mathrm{e}}_{i+1} - \hat{\varepsilon}^{\mathrm{e}}_{i+1/2}) - E^{-1}(\sigma_{i+1} - \hat{\sigma}_{i+1/2}) = 0,$$
$$\Delta\varepsilon^{\mathrm{e}}_{i+1} - E^{-1}\Delta\sigma_{i+1} - \hat{f}^{\mathrm{e}} = 0,$$
$$\hat{f}^{\mathrm{e}} = -E^{-1}(\hat{\sigma}_{i+1/2} - \sigma_i) + (\hat{\varepsilon}^{\mathrm{e}}_{i+1/2} - \varepsilon^{\mathrm{e}}_i).$$

- Separate representation of the displacement

$$\Delta u = \lambda\, v, \qquad \Delta\varepsilon = \lambda\, \bar{\varepsilon} = \lambda\, \nabla v,$$

leads to the following variations

$$\Delta u^* = \lambda^*\, v + \lambda\, v^*, \qquad \Delta\varepsilon^* = \lambda^*\, \nabla v + \lambda\, \nabla v^*.$$

LATIN scheme for brittle damage

- Static admissibility as a space and a time problems

$$\int \nabla v^* \left(\int \lambda \, \mathbf{C} \, \lambda \, \mathrm{d}t \right) \nabla v \, \mathrm{d}\Omega = \int \left(\int \hat{f}^{\mathrm{e}} \, \mathbf{C} \, \lambda \, \mathrm{d}t \right) \nabla v^* \, \mathrm{d}\Omega,$$

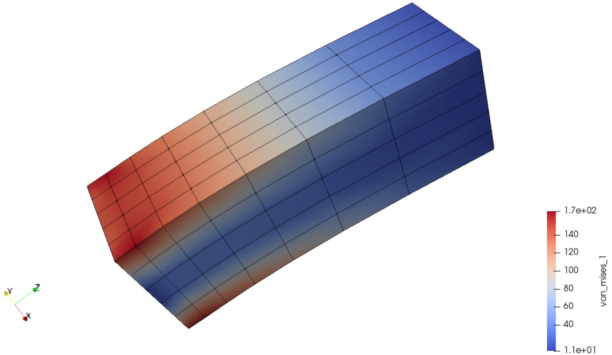
$$\int \lambda^* \left(\int \nabla v \, \mathbf{C} \, \nabla v \, \mathrm{d}\Omega \right) \lambda \, \mathrm{d}t = \int \lambda^* \left(\int \hat{f}^{\mathrm{e}} \, \mathbf{C} \, \nabla v \, \mathrm{d}\Omega \right) \mathrm{d}t.$$

- The strain space function can be computed through the kinematic admissibility relation as $\bar{\varepsilon} = \nabla v$.
- Local stage
  - The stress is taken from the last global stage
  - The damage evolution is computed

$$\dot{D} = \frac{\partial \varphi_{\mathrm{d}}}{\partial Y} = k_{\mathrm{d}} \langle f^{\mathrm{d}} \rangle_+^{n_{\mathrm{d}}}$$

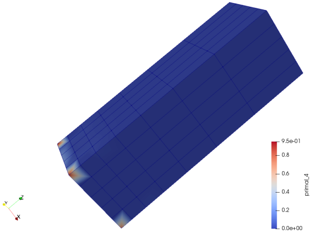
Brittle damage example

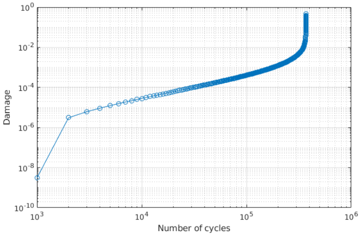
- Constant amplitude loading and adaptive time jumps



Brittle damage example

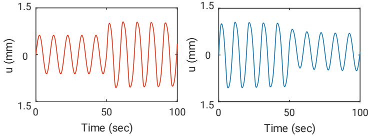
- Constant amplitude loading and adaptive time jumps

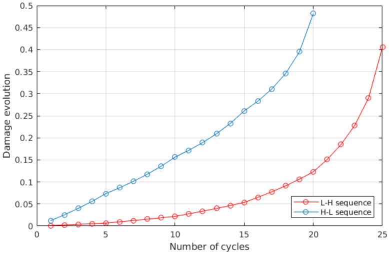




Variable loading

- L-H and H-L sequence

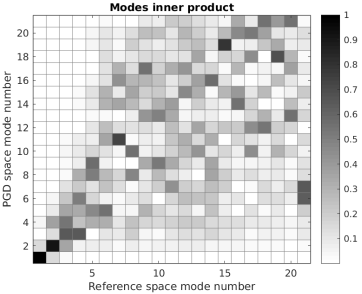


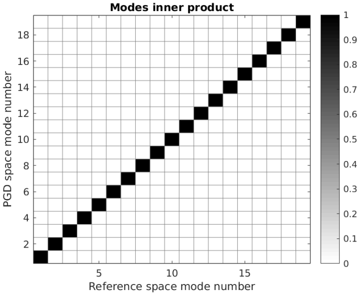


- Time modes are scaled to have a better initialisation

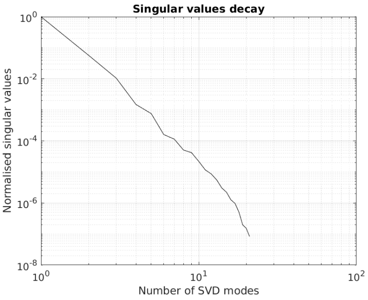
Modes optimisation

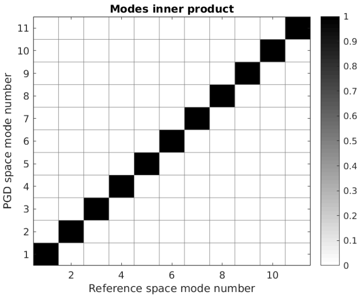
- Modified Gram Schmidt and SVD





Modes truncation





Conclusion and current research

- MOR approach for HCF
- It works for variable loading

Current objectives

- Virtual S-N curves
- Modes selection and optimisation for variable loading
- Reference point method or hyper-reduction
- Parametric loading
- Time homogenisation

Milestone plan

- First year

  Constitutive modelling, LATIN-PGD and Two-time scale
- Second year

  Brittle damage and different amplitudes and RPM
- Third year

  Parametric PGD and Computations and thesis writing

Milestone plan

- First year

  Constitutive modelling, LATIN-PGD and Two-time scale

- Second year

  Brittle damage and different amplitudes and RPM

- Third year

  Parametric PGD and Computations and thesis writing

Thank you for your attention!

- Time comparison

D. Néron et al, Time-space PGD for the rapid solution of 3D nonlinear parametrized problems in the many-query context, *IJNME*, 2015.

- LATIN convergence conditions

  Ladeveze 1999 [p84]

PGD existence and convergence

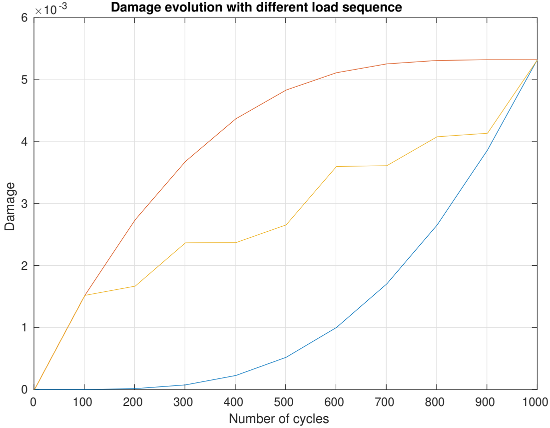
Ladeveze 1999 [p119]

PGD for solving PDE

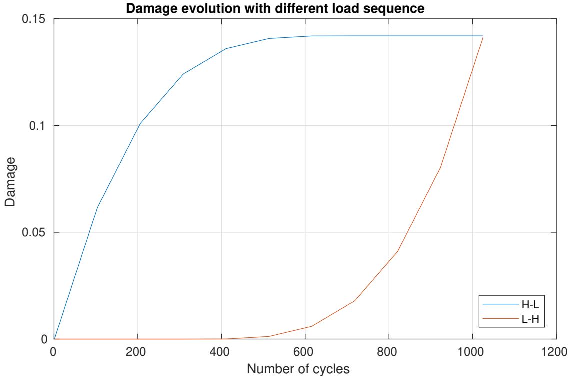
A. Nouy. A priori model reduction through proper generalized decomposition for solving time-dependent partial differential equations. Computer Methods In Applied Mechanics and Engineering, 199(23- 24):1603–1626, 2010.

Antonio Falco

Different loading sequence



Different loading sequence with elastic loading



The used mesh