Large time increment approach for fatigue damage computations

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This contribution focuses on the use of an new method to reduce the computational demands of fatigue damage computations using continuum damage mechanics. The LArge Time INcrement (LATIN) method incorporates a model order reduction approach namely the proper generalised decomposition (PGD). LATIN has been extended to tackle damage problems.

1 Introduction

Non-linear mechanical behaviour such as (visco)plasticity or damage is generally tackled with time incremental methods where the time domain is subdivided into incremental steps. Then a numerical scheme is carried out consequently for each time step. In contrast, in the LArge Time INcrement (LATIN) method [1], an approximation of the total time history process is sought directly. LATIN has been well established to compute several types of problems including material non-linearities but not damage law which leads to a non-linear state equation. An extension of the method has been introduced by [2] to tackle fatigue damage and then to benefit from a promising numerical framework for fatigue damage computation.

2 LATIN-PGD

LATIN [1] involves an iterative sequence of two steps: tackling the linear equations such as the state equations and the mechanical equilibrium equation on one side (the global stage), and the nonlinear evolution equations on the other one (the local stage). The global stage, which is linear, can benefit from model reduction techniques such as the proper generalised decomposition (PGD), in order to obtain a substantial reduction of the computational cost.

3 Extension of LATIN to account for damage

When damage is incorporated, compared to the aforementioned LATIN method, a new difficulty due to the nonlinear elastic state law appears. In this case, the global stage is nonlinear and it is not straightforward to solve such a problem. This is why, it was suggested by [2] to incorporate the nonlinear elastic state law in the local step such that the global stage remains linear.

The quantities of interest are represented as s and the starting point $s_i \in \mathcal{A}$ is assumed to be known, typically from an elastic initialisation. The LATIN iterations after the initialisation are illustrated in Fig.1. In these iterations, plastic corrections are computed following two steps:

1. **Local step** (local in space)
   Solve the nonlinear constitutive equations in addition to the non-linear state equation at every space-time point, i.e. no integration of the nonlinear equations over the whole space domain. This step computes $\hat{s}_{i+\frac{1}{2}} \in \Gamma$ that satisfies the constitutive laws.

2. **Global step** (computationally expensive due to the integration over the space domain)
   The kinematically and statically admissible fields (KA and SA) are computed for the whole time-space domain with the help of a model reduction technique based on a separable form and a greedy algorithm which defines the space and time modes on-the-fly to obtain $s_{i+1} \in \mathcal{A}$.

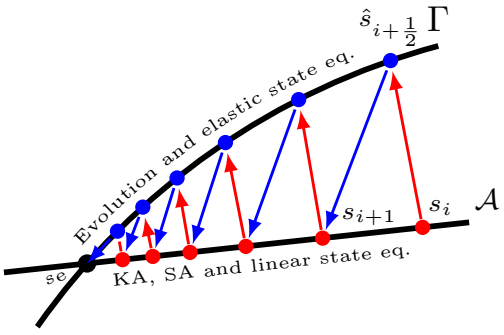

Fig. 1: Illustration of the LATIN iterations

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During the global step, an additive split of the stress and strain into two parts is introduced. One part depends only on the local step and consequently on the nonlinear state law while the other one is computed based on the plastic deformations. Therefore, Γ includes the solution of the evolution equations in addition to the elastic state law while in the global step, the linear hardening state law and the admissibility equation are solved.

These two steps form one LATIN iteration and they are repeated until convergence is reached at s_e. Note that at every local and global step, the quantities of interest for all the space-time points are approximated.

4 Damage prediction for a bar subjected to cyclic loading

The test case illustrated in Fig.2 is analysed within the small deformations framework. The material considered is a Cr-Mo steel at $580°\mathrm{C}$ [3]. The yield stress varies along the bars. An isotropic damage with microdefects closure effect is assumed and a standard viscoplastic model which is a variant of Chaboche and Marquis viscoplastic model [4] is considered.

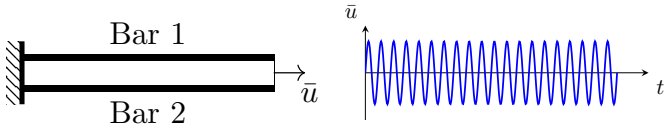

Fig. 2: Two bars under cyclic loading

The damage evolution is illustrated in Fig.3 where the material variability effect can be seen clearly. The convergence of the method is shown in Fig.4. With only three PGD pairs the relative error as defined in [2] can be minimised to approximately 10^{-9} within 25 LATIN iterations. The generated PGD time and space functions are plotted in Fig.5. The space functions, denoted by $\bar{\varepsilon}_i^{\mathrm{p}}$, reflect the variation of the yield stresses along each bar. The derivatives of the time functions are referred to as $\dot{\lambda}_i$. The first time function captures the dominant information of the system while the following ones carry less and less governing details.

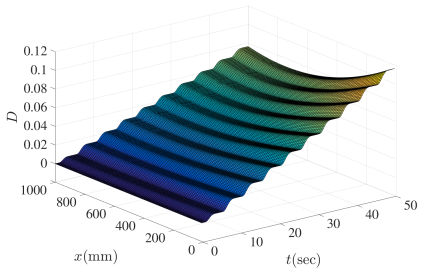

Fig. 3: Damage evolution in bar 1

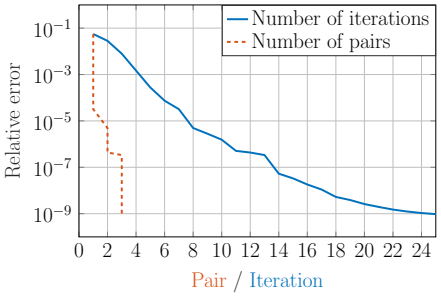

Fig. 4: Evolution of the relative error

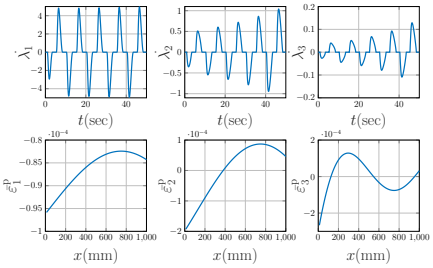

Fig. 5: Space-time modes approximating the plastic strain rate

5 Conclusion

The presented LATIN-PGD method has been extended in [2] to predict damage evolution of 2D structures undergoing non-proportional fatigue loading. This method is promising to conduct fatigue damage analysis. However, in the case of large number of cycles, more sophisticated time treatment is required to benefit from a substantial reduction in the computational cost and such development is in progress.

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