Model order reduction for fatigue analysis

S. Alameddin† , A. Fau†,
U. Nackenhorst†, D. Néron‡, P. Ladevèze‡
† IBNM, Leibniz Universität Hannover
‡ LMT, ENS Cachan, CNRS, Université Paris Saclay

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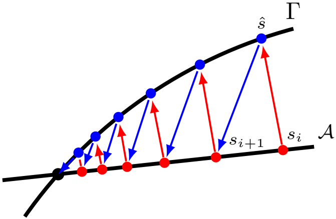

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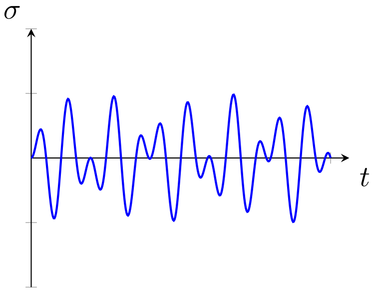
Fatigue damage

- Fluctuating loads



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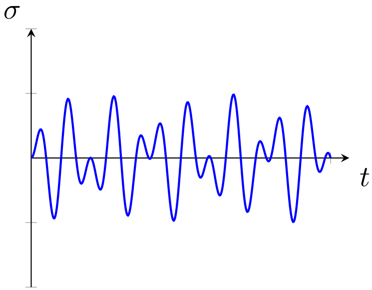

- Material degradation

Image by alegri / 4freephotos.com

Fatigue damage

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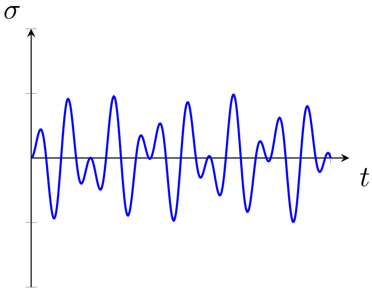

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Fatigue damage

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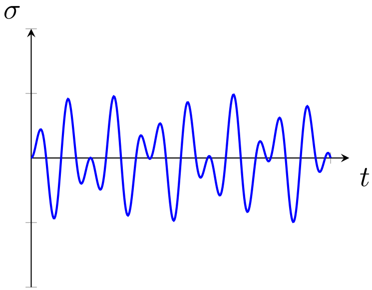
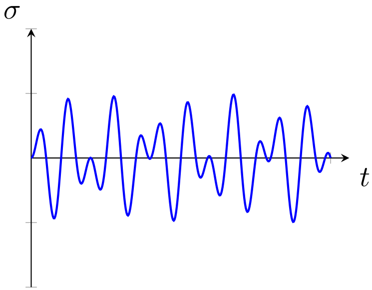


- Virtual experiments
- Continuum damage model
- Millions of cycles
- Macro crack initiation
- Computationally expensive

Fatigue damage

- Fluctuating loads



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Model order reduction (MOR) techniques

Outline

Outline

ROM for non-linear problems

- Short-term damage computations
  - Proper orthogonal decomposition (POD)
    [Kerfriden et al, 2011-2012, Ryckelynck, 2005-2011].
  - Proper generalised decomposition (PGD)
    [Bhattacharyya et al, 2017].
- Long-term fatigue computations
  - Temporal homogenisation
    [Fish and Yu, 2002; Devulder et al, 2010]
  - Space-time finite element method
    [Oden, 1969; Bhamare, 2014; Fritzen and Hassani, 2018]
  - Modified jump cycle approach
    [Bhattacharyya et al, 2018]

Previous works with LATIN

- An approach to include damage in a LATIN-PGD framework ☺
- Modified jump cycle approach to tackle large number of cycles ☺

- Limited to specific models and constant amplitude loads ☹
- No two-time scale, no time savings and many modes are generated ☹

- Generalised formulation for different nonlinear material models
- Efficient variable amplitude simulations

Outline

Mechanical problem

Admissibility equations

- Static admissibility

$$\nabla \cdot \boldsymbol{\sigma} + \boldsymbol{b} = \boldsymbol{0} \quad \text{in } \Omega$$

$$\boldsymbol{\sigma} \cdot \boldsymbol{n} = \bar{\boldsymbol{t}} \quad \text{on } \partial\Omega_{\mathrm{N}}$$

- Kinematic admissibility

$$\boldsymbol{\varepsilon} = \nabla^{\mathrm{s}} u \quad \text{in } \Omega$$

$$u = \bar{u} \qquad \text{on } \partial\Omega_{\mathrm{D}}$$

Nonlinear material model

- State equations

$$\boldsymbol{\sigma} = f(\psi, \boldsymbol{\varepsilon}^{\mathrm{e}})$$

$$\boldsymbol{\beta} = g(\psi, \boldsymbol{\alpha})$$

$$Y = q(\psi, D)$$

- Evolution equations

$$\dot{\boldsymbol{\varepsilon}}^{\mathrm{p}} = \hat{f}(\phi, \sigma)$$

$$\dot{\alpha} = \hat{g}(\phi, \beta)$$

$$\dot{D} = \hat{q}(\phi, D)$$

Solution algorithm

- Many LATIN algorithms for different non-linear problems
- A combination with some modifications
  - Local stage, given initial conditions

    Solve the state and evolution equations to get $\hat{s}$
  - Global stage

    Solve the admissibility equations to get s_{i+1}
  - Data flow between these stages

$$
\begin{aligned}
(\hat{\boldsymbol{\sigma}} - \boldsymbol{\sigma}_i) + \mathbb{H}^{+} \, (\hat{\boldsymbol{\varepsilon}} - \boldsymbol{\varepsilon}_i) &= \mathbf{0} \\
(\boldsymbol{\sigma}_{i+1} - \hat{\boldsymbol{\sigma}}) - \mathbb{H}^{-} (\boldsymbol{\varepsilon}_{i+1} - \hat{\boldsymbol{\varepsilon}}) &= \mathbf{0}
\end{aligned}
$$

  - Iterate until convergence with an energy error indicator

$$
\xi = \frac{\left\| s_{i+1} - \hat{s} \right\|}{\frac{1}{2} \left\| s_{i+1} + \hat{s} \right\|}, \qquad \left\| s \right\|^2 = \int_{[0,T]\times\Omega} \left(\boldsymbol{\sigma} : \mathbb{C}^{-1} \boldsymbol{\sigma} + \boldsymbol{\varepsilon} : \mathbb{C} \, \boldsymbol{\varepsilon} \right) \, \mathrm{d}\Omega \, \mathrm{d}t
$$

The global stage

- Weak form at iteration $i+1$

$$\int\limits_{[0,T]\times\Omega} \boldsymbol{\sigma}_{i+1} : \boldsymbol{\varepsilon}(u^*)\ \mathrm{d}\Omega\,\mathrm{d}t = \int\limits_{[0,T]\times\Omega} \boldsymbol{b}\cdot u^*\ \mathrm{d}\Omega\,\mathrm{d}t + \int\limits_{[0,T]\times\partial\Omega_N} \bar{\boldsymbol{t}}\cdot u^*\ \mathrm{d}S\,\mathrm{d}t, \quad \forall u^* \in \mathcal{U}_0$$

- Correction $\Delta\bullet_{i+1} = \bullet_{i+1} - \bullet_i$

$$\Delta\boldsymbol{\sigma}_{i+1} - \mathbb{H}^-\Delta\boldsymbol{\varepsilon}_{i+1} - \hat{f} = \mathbf{0}, \quad \hat{f} = \underbrace{(\hat{\boldsymbol{\sigma}} - \boldsymbol{\sigma}_i) - \mathbb{H}^-(\hat{\boldsymbol{\varepsilon}} - \boldsymbol{\varepsilon}_i)}_{known}$$

$$\int\limits_{[0,T]\times\Omega} \Delta\boldsymbol{\sigma}_{i+1} : \boldsymbol{\varepsilon}(\Delta u^*)\ \mathrm{d}\Omega\,\mathrm{d}t = 0$$

Proper Generalised Decomposition

- Low-rank approximation of the solution

$$u(\boldsymbol{x}, t) = \sum_{i=1}^{N} \lambda_i(t)\; v_i(\boldsymbol{x})$$

- Enriching with one mode

$$\Delta u = \lambda(t)\; v(\boldsymbol{x}) \qquad \Delta u^* = \lambda^*\; v + \lambda\; v^*$$

- Updating (μ) previously generated time modes

$$\Delta u = \sum_{i=1}^{\mu} \Delta\lambda_i(t)\; \underbrace{v_i(\boldsymbol{x})}_{known}$$

Static admissibility

- Static admissibility and global search direction

$$\int\limits_{[0,T]\times\Omega} \mathbb{H}^{-}\Delta\boldsymbol{\varepsilon}_{i+1} : \boldsymbol{\varepsilon}(\Delta u^{*})\ \mathrm{d}\Omega\,\mathrm{d}t = -\int\limits_{[0,T]\times\Omega} \hat{f} : \boldsymbol{\varepsilon}(\Delta u^{*})\ \mathrm{d}\Omega\,\mathrm{d}t$$

- Space problem: $\langle\bullet\rangle = \int\limits_{[0,T]} \bullet\ \mathrm{d}t$, given λ_j

$$\langle\lambda_j\lambda_j\rangle \int\limits_{\Omega} \nabla v^{*} : \mathbb{H}^{-}\nabla v_{j+1}\ \mathrm{d}\Omega = -\langle\lambda_j\rangle \int\limits_{\Omega} \nabla v^{*} : \hat{f}\,\mathrm{d}\Omega$$

- Time problem, given v_{j+1}:

$$\int\limits_{[0,T]} \lambda^{*} \left[\int\limits_{\Omega} \nabla v_{j+1} : \mathbb{H}^{-}\ \nabla v_{j+1}\,\mathrm{d}\Omega\right] \lambda_{j+1}\ \mathrm{d}t = -\langle\lambda^{*}\rangle \int\limits_{\Omega} \nabla v_{j+1} : \hat{f}\,\mathrm{d}\Omega$$

Algorithmic point of view

- Choosing the search directions as

$$\mathbb{H}^+ = 0, \qquad \mathbb{H}^- = \alpha\,\mathbb{C} \qquad \alpha \in]0,1]$$

- Enrichment

$$\gamma\,\underline{\underline{K}}\,\underline{v}_{i+1} = \underline{F} \qquad \gamma \in \mathbb{R} \qquad \underline{\underline{K}} \in \mathbb{R}^{n\times n} \qquad \underline{F} \in \mathbb{R}^{n}$$

$$a\,\underline{\lambda}_{i+1} = \underline{b} \qquad a \in \mathbb{R} \qquad b \in \mathbb{R}^{n_t}$$

- Update

$$\underline{\underline{A}}\,\{\Delta\underline{\lambda}_i\} = \underline{b} \qquad \underline{\underline{A}} \in \mathbb{R}^{N\times N} \qquad \underline{b} \in \mathbb{R}^{N\times n_t}$$

Multiple cycles with variable load amplitudes

- The stiffness
- The elastic solution
- An initial guess

Multiple cycles with variable load amplitudes

- The stiffness

  Computed only once

- The elastic solution

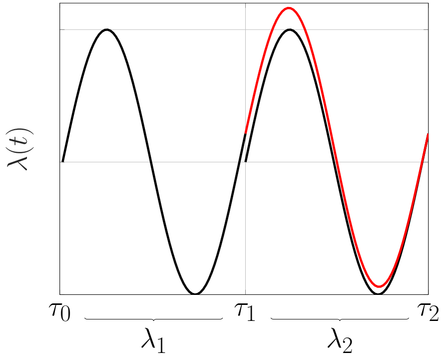
  Only once and parametrised over the load amplitude

- An initial guess

  From the previous cycle

The initial guess

- Time modes are shifted and scaled to ensure continuity



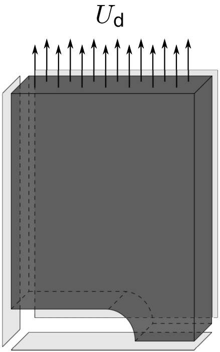
$$\lambda_2(t) = \lambda_1(t) + a\,t + b$$

$$\lambda_2(\tau_1) = \lambda_2(\tau_1) \qquad \lambda_2(\tau_2) = \lambda_2(\tau_1)$$

Outline

Numerical results

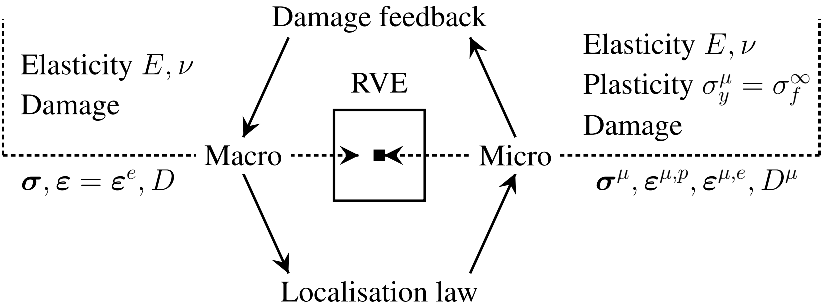
- A plate with a central groove subjected to cyclic loading (Cr-Mo steel at $20°\mathrm{C}$ and $580°\mathrm{C}$)



Quasi-brittle material model

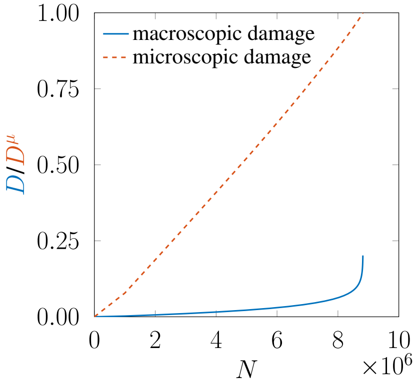
In collaboration with Bhattacharyya and Desmorat

- Two-scale damage model



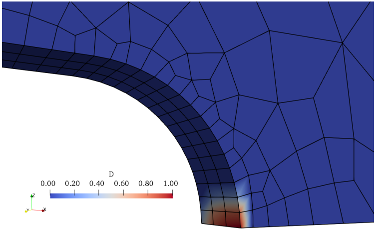
- With an adaptive jump cycle algorithm

Damage evolution

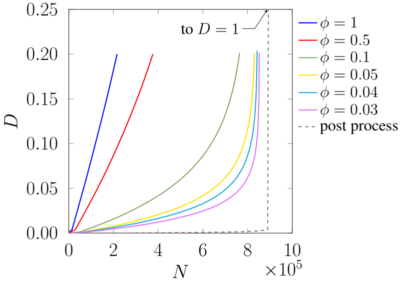


Damage evolution at the weakest Gauss point

Damage behaviour



Damage distribution
at the micro-scale



$$D = 1 - (1 - D^{\mu})^{\Phi}$$

Visco-plastic material model

- State equations

$$\boldsymbol{\sigma} = \mathbb{C}\ \boldsymbol{\varepsilon}^{\mathrm{e}}\ (1 - D)$$

$$\boldsymbol{\beta} = C\ \boldsymbol{\alpha}$$

$$Y = \frac{1}{2}\boldsymbol{\varepsilon}^{\mathrm{e}} : \mathbb{C}\ \boldsymbol{\varepsilon}^{\mathrm{e}}$$

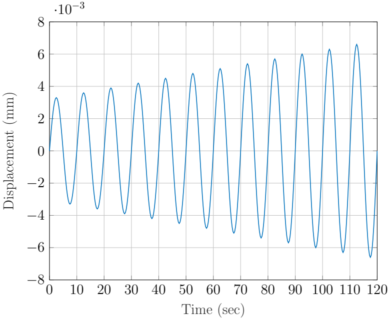
- Evolution equations

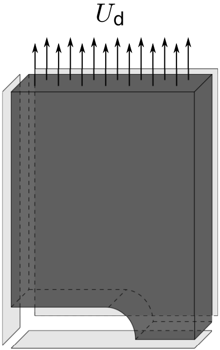
$$\dot{\boldsymbol{\varepsilon}}^{\mathrm{p}} = k\ \langle f^{\mathrm{p}} \rangle_{+}^{n}\ \left[\frac{3}{2}\frac{\boldsymbol{\tau}}{J_2(\boldsymbol{\tau})}\right]\frac{1}{1-D}$$

$$\dot{\alpha} = k\ \langle f^{\mathrm{p}} \rangle_{+}^{n}\ \left[\frac{3}{2}\frac{\boldsymbol{\tau}}{J_2(\boldsymbol{\tau})} - \frac{a}{C}\boldsymbol{\beta}\right]$$

$$\dot{D} = k_{\mathrm{d}} \langle f^{\mathrm{d}} \rangle_{+}^{n_{\mathrm{d}}}$$

Variable amplitude





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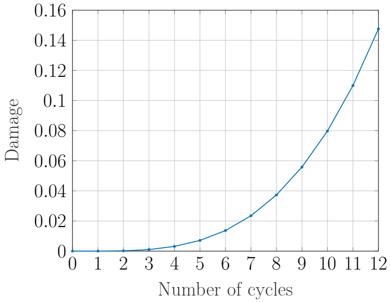
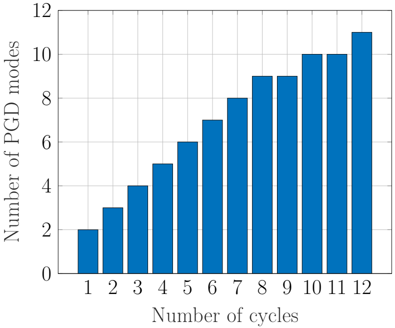
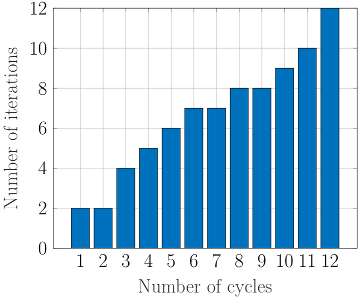
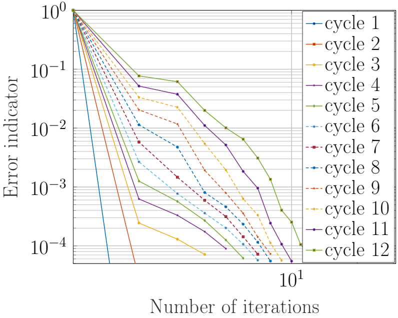




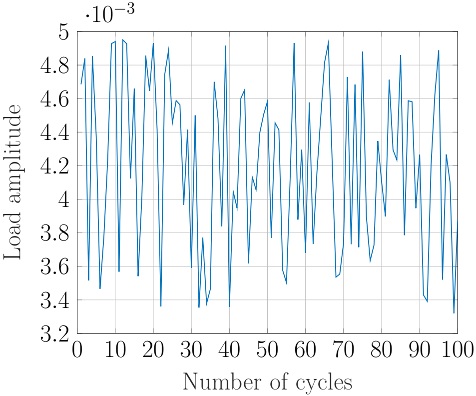
Variable amplitude





Random amplitudes

Cyclic load with the random amplitudes



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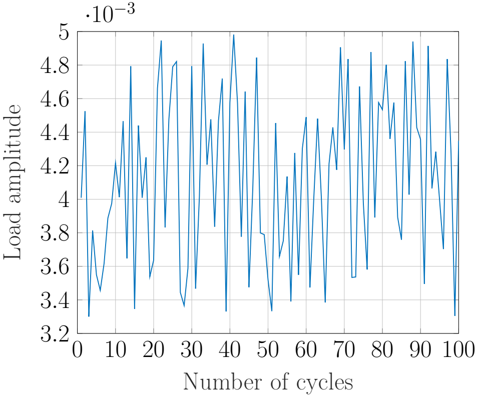

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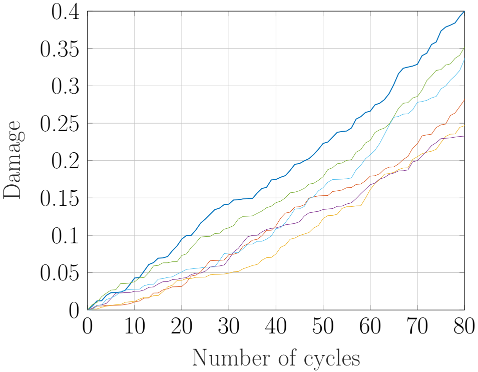

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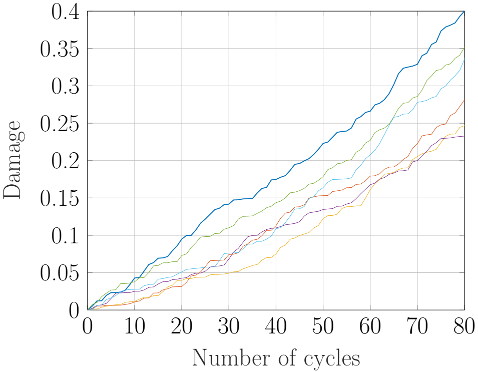

Number of modes is less than 20

Outline

Conclusion and future research

- Efficient cycle by cycle simulation for damage problems.
- Works for low and high cycle fatigue.

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Challenges

- The computation of the local stage.
- The integration of the error indicator and the time update.

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Future development

- Machine learning in the local stage.
- Reduced integration scheme.

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- The integration of the error indicator and the time update.

Future development

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Thank you for your attention

Github repository

https://github.com/dbeurle/neon

Quasi-brittle material model

Eshelby-Kröner localisation law

$$(\varepsilon^{\mu} - \varepsilon) = \gamma \left(\varepsilon^{\mu,p} - \varepsilon^{p}\right),$$

where γ is the Eshelby coefficient given by

$$\gamma = \frac{2}{15} \frac{4 - 5\nu}{1 - \nu}.$$

Quasi-brittle material model

- State equations

$$\tilde{\boldsymbol{\sigma}}^{\mu} = \mathbf{C}\boldsymbol{\varepsilon}^{\mu,e}$$

$$Y^{\mu} = R_v \frac{\left(\tilde{\sigma}^{\mu}_{eq}\right)^2}{2E}$$

$$\boldsymbol{\beta}^{\mu} = \mathbf{Q}\boldsymbol{\alpha}^{\mu}$$

- Evolution equations

$$\dot{\boldsymbol{\varepsilon}}^{\mu,p} = \frac{3}{2}\frac{\tilde{\boldsymbol{\sigma}}^{\mu} - \boldsymbol{\beta}^{\mu}}{\left(\tilde{\boldsymbol{\sigma}}^{\mu} - \boldsymbol{\beta}^{\mu}\right)_{eq}}\frac{\dot{\lambda}_p}{1 - D^{\mu}}$$

$$\dot{\boldsymbol{\alpha}}^{\mu} = \frac{3}{2}\frac{\tilde{\boldsymbol{\sigma}}^{\mu} - \boldsymbol{\beta}^{\mu}}{\left(\tilde{\boldsymbol{\sigma}}^{\mu} - \boldsymbol{\beta}^{\mu}\right)_{eq}}\dot{\lambda}_p$$

$$\dot{D}^{\mu} = \left(\frac{Y^{\mu}}{S}\dot{p}^{\mu}\right)^{s}$$

Crack closure effect

The effective stress

$$\tilde{\boldsymbol{\sigma}} = \frac{\boldsymbol{\sigma}_D}{1-D} + \left[\frac{\langle \sigma_H \rangle}{1-D} + \langle -\sigma_H \rangle\right] \mathbb{I}$$

The triaxiality function

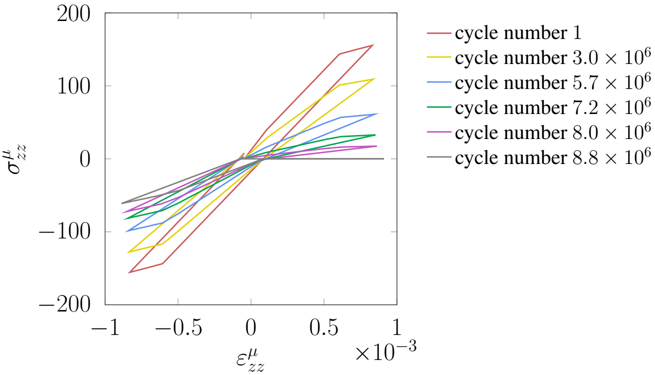
$$R_v = \frac{2}{3}(1+\nu) + 3(1-2\nu)\left\langle \frac{\sigma_H^\mu}{\sigma_{eq}^\mu} \right\rangle^2$$

$$\sigma_{eq}^\mu = \sqrt{\frac{3}{2}\sigma_{D_{ij}}^\mu \sigma_{D_{ij}}^\mu} \qquad \tilde{\sigma}_{eq}^\mu = \sqrt{\frac{3}{2}\tilde{\sigma}_{D_{ij}}^\mu \tilde{\sigma}_{D_{ij}}^\mu}$$

The micro-scale yield function f^μ is given as

$$f^\mu = \left(\tilde{\boldsymbol{\sigma}}_D^\mu - \boldsymbol{\beta}^\mu\right)_{eq} - \sigma_f^\infty,$$

Stress-strain response



Stress-strain diagram at certain cycles at the micro-scale

Visco-plastic material model

- Free energy function

$$\psi(\boldsymbol{\varepsilon}^{\mathrm{e}}, \alpha, D) = \frac{1}{2}\boldsymbol{\varepsilon}^{\mathrm{e}} : \mathbb{C}(1-D)\,\boldsymbol{\varepsilon}^{\mathrm{e}} + \frac{1}{2}\,C\,\boldsymbol{\alpha} : \boldsymbol{\alpha}$$

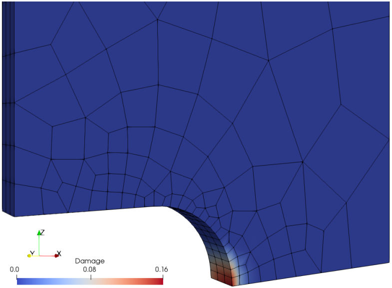
- Dissipation potential

$$\phi = \phi^{\mathrm{p}} + \phi^{\mathrm{d}}$$

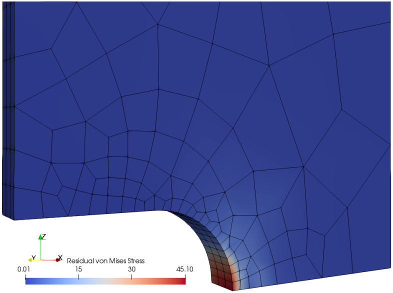
$$\phi^{\mathrm{p}} = \frac{k_{\mathrm{p}}}{n_{\mathrm{p}}+1}\langle f^{\mathrm{p}}\rangle_{+}^{n_{\mathrm{p}}+1}, \qquad f^{\mathrm{p}} = J_2(\boldsymbol{\tau}) + \frac{a}{2C}\,\boldsymbol{\beta} : \boldsymbol{\beta} - \sigma_{\mathrm{y}}$$

$$\phi^{\mathrm{d}} = \frac{k_{\mathrm{d}}}{n_{\mathrm{d}}+1}\langle f^{\mathrm{d}}\rangle_{+}^{n_{\mathrm{d}}+1}, \qquad f^{\mathrm{d}} = Y - Y_0$$

Variable amplitude

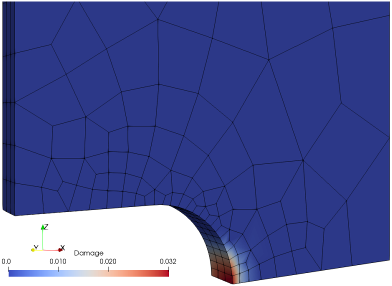


Damage distribution

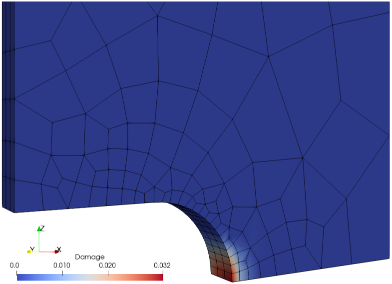


Von Mises stress distribution

Numerical results



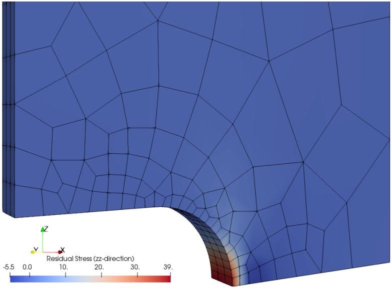
Newton-Raphson



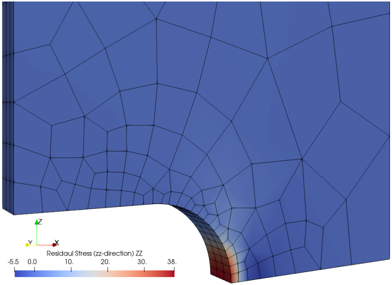
LATIN-PGD

Damage contour at the last time step

Numerical results



Newton-Raphson



LATIN-PGD

Residual stress distribution after removing the load

Differences

- State equations in the local stage
- PGD for the strain instead of the plastic one
- Cycle by cycle
- Modes rescaling

https://github.com/dbeurle/neon