MACM 203 Assignment 5
Spring 2025

This assignment is due <u>Tuesday March 4th at 10pm.</u> Upload your solutions to Crowdmark. Write your solutions as a single Matlab Live Script and export the script to PDF. Write the course number and assignment number as the title of the Matlab Live Script, followed by the table of contents, and then create a section for each part of the question.

Keep in mind that your assignment, including the source code, is a document that will be read in order to be marked. It has to be very clear and properly formatted.

Assignments should be written individually. You can discuss in groups, but you have to write your assignment yourself. In case of academic dishonesty SFU policies will be applied.

Preamble

This week's assignment focuses on dynamical systems.

Question 1 (20 marks)

Part (a)

At a company's headquarters there are three food outlets, labelled A,B,C, where employees eat lunch. Each employee eats on each weekday at exactly one outlet. The following observations were made:

- Among those employees who choose outlet A on a given day, 60% will choose outlet A on the following day, and the remaining employees will split their choices equally between outlet B and outlet C.

- Among those employees who choose outlet B on a given day, 50% will choose outlet B on the following day, and the remaining employees will split their choices equally between outlet A and outlet C.

- Among those employees who choose outlet C on a given day, 60% will choose outlet C on the following day, and all remaining employees will choose outlet A.

  Model this dynamical system as a Markov chain. Write down the transition matrix.

Part (b)

On Tuesday 10% of employees chose outlet A, 70% of employees chose outlet B, and 20% of employees chose outlet C. Determine the percentage of employees choosing each of the three outlets three days later (on Friday). Your calculation must be as simple as possible. Use words to present your answer (do not just write Matlab code).

Part (c)

Determine the percentage of employees choosing each of the three outlets after a long time. Use the best possible method. A calculation similar to part (b) is not acceptable. Use words to present your answer (do not just write Matlab code).

Part (d)

Download the file `a5.mat` from Canvas and load the file into your script. The matrix T contained in this file is the transition matrix for a dynamical system (Markov chain). Note this Markov chain is not regular.

Parts (e), (f) and (g) ask questions about this Markov chain. Let v be a steady state of this Markov chain, that is, v is a probability vector such that $Tv = v$. Denote the entries of v as

$$v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix}.$$

Part (e)

Determine a steady state with $v_1 = v_2 = 0.2$, or show that no such steady state exists. Explain your answer. Perform all calculations in Matlab.

Part (f)

Determine a steady state with $v_1 = v_3 = 0.2$, or show that no such steady state exists. Explain your answer. Perform all calculations in Matlab.

Part (g)

Determine a steady state with $v_1 = 0.2$, $v_2 = 0.1$, or show that no such steady state exists. Explain your answer. Perform all calculations in Matlab.